Progress in Fractional Differentiation and Applications
An International Journal

http://dx.doi.org/10.18576/pfda/080407

Effect of Slip on Generalized Viscous Nanofluid with Thermal Flux with Effective Fractional Derivative

Usman Majeed[1], *Muhammad Imran Asjad*[2], *Muhammad Irfan Qadir*[1] *and Ali Akgül*[3,4,*]

[1]Department of Mathematics, University of Engineering and Technology, Lahore, Pakistan
[2]Department of Mathematics, University of Management and Technology, Lahore, Pakistan
[3]Department of Mathematics, Art and Science Faculty, Siirt University, 56100 Siirt, Turkey
[4]Department of Mathematics, Mathematics Research Center, Near East University, Near East Boulevard, PC: 99138, Nicosia /Mersin 10, Turkey

Received: 21 May 2022, Revised: 22 Jun. 2022, Accepted: 24 Aug. 2022.
Published online: 1 Oct. 2022.

Abstract: In this article, the laminar boundary layer flow of a non-compressible viscous fluid with carbon nanotubes over an infinite vertical plate is taken into account. Our goal is to study the effects of slip boundary condition on the generalized viscous nanofluid over an infinite vertically positioned plate. The unsteady fractional Prabhakar derivative is used to introduce the correlated fractional system of the governing equations. We find the analytical expression of velocity of the fluid using the Laplace transform method. The results obtained in case of no slip effect are compared with the classical results. The impacts of fractional and physical factors are depicted graphically.

Keywords: Nanofluid, slip effect, Laplace transform, carbon nanotubes, vertical plate**.**

1 Introduction

Heat transfer has many industrial and technological applications in addition to several scientific concerns. Some examples of heat transfer are fiber coating and wire, filtration, a polymer sheet, space ship construction, cooled or heated storage room, artificial fiber, chemical processing equipment, nuclear reactors, geothermal system, and heat exchangers [1]. Fluids are utilized in industry for heat transfer between two mediums. A nanofluid is a substance that contains nanoparticles. Choi [2] of the Argonne USA in 1995 was firstly used the term nanofluid. Nanofluids are considered to be the upcoming generation heat transfer substances because they have large possibilities to increase heat transfer capability in comparison with the liquids [3]. These fluids have large heat conductivity and increase the heat transfer rate for a long time. The formation and the characterization of nanofluids have been investigated by many researchers in recent years [4,5,6,7,8,9,10,11].

The study of nanofluids is interesting because of their numerous industrial and biology applications. They also have applications in many engineering works like welding equipment, polymer extrusion, automobile engine, glass fiber production, microwave tubes, enhancement in the critical heat flux and electronic cooling system.

The researchers are fascinated by the improvement of heat transfer conventional fluids. A generic mechanism for the enhancement of heat exchange is the use of solid nanoparticles scattered into the base fluid. Different materials can be used in the formation of nanofluid such as silver, brass, ethylene glycol, etc. When the nanosized solid particles are suspended into the conventional fluids, they increase their thermal conductivity. The thermal conductivity of solid particle is larger than fluids. For instance, the thermal conductivity of carbon nanotubes (CNTs) is 3000 and for a diamond is 3300. The metallic materials have different values of thermal conductivities such as copper, aluminum, magnesium, and zinc have the values 397, 226, 151, and 112, respectively, whereas thermal conductivity of base fluid such as engine oil, polydimethylsiloxane and water is 0.25, 0.15 and 0.61, respectively. So, the fluids that contain the suspended solid materials can increase the thermal conductivity in comparison with the liquids.

Eastman et al. [12] observed that by having water be the base fluid with a 5 percent volume of copper oxide (CuO) nanoparticles, thermal conductivity rises by around 60 percent for the nanofluid. Wang [13] investigated the thermal conductivity of CuO and aluminum oxide (AL_2O_3) by putting them into engine oil, distilled water, and ethylene glycol. They found that different base fluids have different enhancements in their thermal conductivities. The enhancement of heat

*Corresponding author e-mail: aliakgul@siirt.edu.tr

conductivity of copper distilled water nanofluid was discussed by Xuan et al. [14] and showed a 56 percent increase of thermal conductivity for the 5 percent volume fraction of the nanofluid.

Das et al. [15] showed through experiments that when nanofluid like CuO and (AL_2O_3) nanoparticles are placed in water, the heat conductivity increased at high temperatures. Murshed et al. [16] recorded the maximum increase percentage of 33% on heat conductivity for 5 percent volumetric filling of titanium dioxide (TiO2) nanoparticles in water.

Many researchers are interested in CNTs because of their physical characteristics, such as their high thermal and electrical conductivity [17]. Choi et al [18] studied that a little number of CNTs suspended in base fluid improved its heat conductivity. The effective heat conductivity of suspended CNTs in synthetic poly oil was measured, and a 160 percent rise in heat conductivity of oil was reported for 1 percent volume fraction of CNTs. According to the estimation in [19], the heat conductivity of copper (Cu) in the base fluid (ethylene glycol) nanofluid is increased by 40 percent having 0.3 percent volume of Cu nanoparticles. A model of heat transfer in aqueous suspensions of carbon nanotube was investigated in [20] and found that for 0.5 percent to 1.0 percent volume of carbon nanotubes in water, thermal conductivity rises from 35 percent to 79 percent.

Hwang et al. [21] used multiwall carbon nanotubes (MCNTs) i.e fullerene (F), CuO, and silicon dioxide (SiO_2) for the increment of thermal conductivity of conventional fluids like distilled water (DI), ethylene glycol, and oil. After this research, they concluded that the multiwall CNTs silicon dioxide have low thermal conductivity while water fluids have higher thermal conductivity.

Saeed et al. [22] investigated mass transference and thermal conduction of free convection with wall slip boundary conditions for the flow on a vertical plate. The idea concept of "coefficient of slip" for slip arising adjacent to the wall was developed by Helmholtz [23]. The damping of a vibrating disk in gas was studied by Kundt et al. [24] and found that pressure was inversely proportioned to the "coefficient of slip". The wall slip conditions have many essential applications such as in medical sciences, micro, and nanofluids lubrication, extrusion, frictional study, biological fluids, and in flows through porous media [25-26].

Generally, in fluid problems, we use two boundary conditions. The "no-slip" boundary condition is one of the foundations on which the mechanics of the linearly viscous liquid is developed. This condition is widely used for non-Newtonian fluids and occurs when the fluid has zero velocity relative to the boundary. The slip boundary is applied to the wall. The velocity function is supposed to be discontinued in this condition *i.e.*, a relative movement between the boundary and the fluid [27]. Navier [28] proposed the slip boundary condition in which slip velocity was dependent on the shear stress.

M. Tahir [29] worked on the non-integer derivative and slip wall effects on the heat transfer flow by using Caputo-Fabrizio derivatives. Gossaye [30] find the slip effect in the flow and heat transfer on the nanofluid over a Stretching sheet by using the optimal homotopy asymptotic method. A. Malvandi et al. [31] find the slip effect of a nanofluid over a stretching sheet on time-dependent stagnation point flow.

Fractional calculus has been recently used in several research areas because of its capability of portraying the memory effects of various physical phenomena. Fractional calculus is now being utilized successfully in a variety of scientific areas, such as electrochemistry, signals processing, mechatronics, viscoelasticity, mathematical biology, mechanics, and dynamics.

We are considering the viscous incompressible fluid with CNTs above an infinite vertically positioned plate. Our goal is to study the generalized viscous nanofluid with slip effect along with the Prabhakar-like heat transfer over an infinite vertically positioned plate. Slip boundary condition affects only the velocity and temperature of the fluid. The unsteady Prabhakar derivative (PD) is employed to introduce the correlated fractional system of the governing equations. The utilization of Prabhakar operators with specified coefficients might be a useful strategy for constructing a mathematical model that matches practical and theoretical results closely. Three governing equations are used namely momentum equation, energy balance equation, and Fourier's law of heat flux. To obtain the dimensionless governing equations, we introduce seven dimensionless quantities and substitute them into the governing equations. Using the Laplace transformation method, we find the analytical solutions for the fluid's velocity and compare with no slip effect. Thermal properties [32] of carbon nanotubes and water are defined in.

Table 1: Physical properties**.**

Physical properties	Water	SWCNTs
Density	997.1	2600
Heat capacity	4179	425.0
Thermal conductivity	0.613	6600

Thermal coefficient	21.00	1.500

2 Mathematical Formations

Consider a viscous incompressible fluid containing CNTs that flows across an infinite vertical plate with $f(t)$ being a piecewise continuous function. The temperature of the wall is $T_\infty + (T_w - T_\infty) f(t)$, with $f(0) = 0$ and T_∞ is the ambient temperature.

In this study, we have shown the Cartesian coordinate system in Fig. 1.The temperature and velocity fields are considered be the functions of t and y. Therefore $\hat{i}$ denotes the unit vector along the horizontal axis and fluid velocity will be $\vec{v}(y,t) = u(y,t)\hat{i}$.

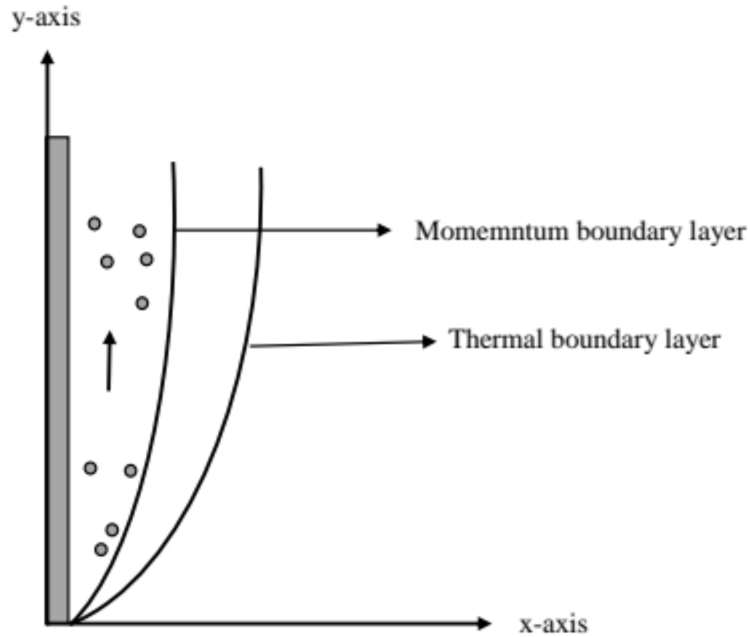

Fig.1: Vertical heated plate.

In the prior assumption, the governing equation for the Boussinesq's approximation without pressure gradient can be given as follows [33]

The momentum equation

$$\rho_{nf} \frac{\partial u}{\partial t} = \mu_{nf} \frac{\partial^2 u}{\partial y^2} + g(\rho\beta)_{nf} \left[T - T_\infty \right], \quad (1)$$

the energy equation

$$(\rho C_p)_{nf} \frac{\partial T}{\partial t} = -\frac{\partial q}{\partial y}, \quad (2)$$

and the Fourier's heat flux law

$$q = -k_{nf} \frac{\partial T}{\partial y}, \quad (3)$$

where u, ρ_{nf}, q and k_{nf} are the velocity, density, heat flux, and thermal conductivity, respectively, of the nanofluid, μ_{nf}, $(C_p)_{nf}$, T and β_{nf} is the viscosity, specific heat, temperature, and thermal coefficient of the nanofluid.

From equation (1),(2) and (3), we assume the following boundary and initial conditions for velocity and temperature,

when $t = 0, u = 0$, $T = T_\infty$, $y \geq 0$, (4) for $y=0,\ u-\eta\dfrac{\partial u}{\partial y}=v_0H(t)e^{at},\ T=T_\infty+(T_w-T_\infty)f(t),\ t\ge 0,$ (5)

for $y\to\infty, u\to\infty\ ,\ T\to T_\infty,\ t\ge 0,$ (6)

where $v_0>0$ is the characteristic velocity and $H(t)$ be the Heaviside unit step function.
The equations describing the thermophysical parameters of nanofluid are given as follows [34]

$$\mu_{nf}(1-\phi)^{2.5}=\mu_f,\quad \rho_{nf}-(1-\phi)\rho_f=\phi\rho_{CNT},$$

$$(\rho C_p)_f(1-\phi)=(\rho C_p)_{nf}-\phi(\rho C_p)_{CNT},\left(\rho\beta\right)_f(1-\phi)=\left(\rho\beta\right)_{nf}-\phi\left(\rho\beta\right)_{CNT},$$

$$\frac{k_f}{k_{nf}}1-\phi+2\phi\left(\frac{k}{k_{CNT}-k_f}\right)\ln\left(\frac{k_{CNT}+k_f}{2k_f}\right)=1-\phi+2\phi\left(\frac{k_f}{k_{CNT}-k_f}\right)\ln\left(\frac{k_{CNT}+k_f}{2k_f}\right),$$

where ϕ is the volume fraction of nanofluid.

Introducing the following dimensionless quantities

$$y^*=\frac{yv_0}{\upsilon_f},\ \upsilon_f=\frac{\mu_f}{\rho_f},\ t^*=\frac{v_0^2t}{\upsilon_f},\ u^*=\frac{u}{v_0},\ T^*=\frac{T-T_\infty}{T_w-T_\infty},\ q^*=\frac{q}{q_0},\ q_0=\frac{k_f(T_w-T_\infty)v_0}{\upsilon_f},\ \Pr=\frac{(\mu C_p)_f}{k_f},$$

$$Gr=\frac{g(\upsilon\beta)_f(T_w-T_\infty)}{v_0^3} \tag{7}$$

We substitute these dimensionless quantities into Equations $(1)-(6)$, then using thermophysical parameters and ignoring the star notation, the following dimensionless governing equations are obtained:

$$a_0\frac{\partial u(y,t)}{\partial t}=a_1\frac{\partial^2u(y,t)}{\partial y^2}+a_2GrT(y,t), \tag{8}$$

$$b_0\Pr\frac{\partial T(y,t)}{\partial t}=\frac{\partial q(y,t)}{\partial y}, \tag{9}$$

$$q(y,t)=-c_0\frac{\partial T(y,t)}{\partial y}, \tag{10}$$

In this article, we have taken into account a mathematical model that describes the generalized thermal memory effects. For seeking this goal, we introduce the generalized Fourier's law based on Prabhakar's fractional derivative [35]

$$q(y,t)=-{}^CD^{\gamma}_{\alpha,\beta,a}\frac{\partial T(y,t)}{\partial y}, \tag{11}$$

The dimensionless form of generalized Fourier law depends on fractional Prabhakar's derivative i.e,

$$q(y,t)=-c_0\,{}^CD^{\gamma}_{\alpha,\beta,a}\frac{\partial T(y,t)}{\partial y}$$

The temperature field for the above system can be found in and is given below[33]

$$\overline{T}(y,s)=F(s)e^{-\sqrt{\frac{b_0\Pr s}{s^{\beta}(1-as^{-\alpha})^{\gamma}}}y} \tag{12}$$

where

$$a_0=(1-\phi)+\phi\frac{\rho_{CNT}}{\rho_f},a_1=\frac{1}{(1-\phi)^{2.5}},a_2=(1-\phi)+\phi\frac{\left(\rho\beta\right)_{CNT}}{\left(\rho\beta\right)_f},$$

$$b_0=(1-\phi)+\phi\frac{(\rho C_p)_{CNT}}{(\rho C_p)_f},c_0=\frac{1-\phi+2\phi\left(\frac{k}{k_{CNT}-k_f}\right)\ln\left(\frac{k_{CNT}+k_f}{2k_f}\right)}{1-\phi+2\phi\left(\frac{k_f}{k_{CNT}-k_f}\right)\ln\left(\frac{k_{CNT}+k_f}{2k_f}\right)}.$$

The dimensionless form of initial and boundary conditions is:

For $t=0, u=0, T=0, y\geq 0,$ (13)

for $y=0,\ u-\gamma\frac{\partial u}{\partial y}=H(t)e^{bt},\ T=f(t), t\geq 0,$ (14)

for $y\to\infty, u\to\infty, T\to 0, t\geq 0.$ (15)

where the regularized Prabhakar derivative is defined as fallows [36]

$$\begin{aligned}{}^{C}D^{\gamma}_{\alpha,\beta,a}f(p)&=E^{-\gamma}_{\alpha,m-\beta,a}f^{(m)}(p)\\&=e^{-\gamma}_{\alpha,m-\beta}(a;p)*f^{(m)}(p)\\&=\int_0^p(p-n)^{m-\beta-1}E^{-\gamma}_{\alpha,m-\beta}(a(p-n)^{\alpha})f^{(m)}(n)dn,\end{aligned}\tag{16}$$

where $f^{(r)}$ represents the rth derivative of $f(p)$, and $r=[\beta]$ is the floor function.

In Equation (16), $E^{\gamma}_{\alpha,\beta,a}f(p)=\int_0^p(p-n)^{\beta-1}E^{\gamma}_{\alpha,\beta}(a(p-n)^{\alpha})f(n)dn$ denotes the Prabhakar integral.

Here $E^{\gamma}_{\alpha,\beta}(h)=\sum_{m=0}^{\infty}\frac{\Gamma(\gamma+m)h^n}{m!\Gamma(\gamma)\Gamma(\alpha m+\beta)}$, $\alpha,\beta,\gamma,\ h\in C$, $\mathrm{Re}(\alpha)\geq 0$ is the Mittag-Leffler function with three-parameters [37]. The function $e^{\gamma}_{\alpha,\beta}(a;t)=t^{\beta-1}E^{\gamma}_{\alpha,\beta}(at^{\alpha})$, $t\in R$, $\alpha,\beta,\gamma,a\in C$, $\mathrm{Re}(\alpha)\geq 0$ is known as the Prabhakar kernel [38]. The Laplace transform of the Regularized Prabhakar derivative [39]

$$\begin{aligned}L\left\{{}^{C}D^{\gamma}_{\alpha,\beta,a}f(p)\right\}&=L\left\{e^{-\gamma}_{\alpha,m-\beta}(a;p)*f^{(m)}(p)\right\}\\&=L\left\{e^{-\gamma}_{\alpha,m-\beta}(a;p)\right\}L\left\{f^{(m)}(p)\right\}\\&=s^{\beta-m}(1-as^{-\alpha})^{\gamma}L\left\{f^{(m)}(p)\right\}.\end{aligned}\tag{17}$$

2.1 Solution of the problem

Taking Laplace transform of Equations (14), (8), (15), (13), we obtain

$$a_0s\bar{u}(y,s)=a_1\frac{\partial^2\bar{u}(y,s)}{\partial y^2}+a_2Gr\bar{T}(y,s),\tag{18}$$

$$\bar{u}(0,s)-\gamma\frac{\partial\bar{u}(0,s)}{\partial y}=\frac{1}{s-b},\ \lim_{y\to\infty}\bar{u}(y,s)=0.\tag{19}$$

By substitution Equation (12) into Equation (18), we obtain

$$\frac{\partial^2\bar{u}(y,s)}{\partial y^2}-d_0s\bar{u}(y,s)=-d_1GrF(s)e^{-\sqrt{\frac{b_0\Pr s^{1-\beta}}{(1-as^{-\alpha})^{\gamma}}}y},\tag{20}$$

where $$d_0=\frac{a_0}{a_1},d_1=\frac{a_2}{a_1}.$$

$$\bar{u}(y,s)=Ae^{\sqrt{d_0 s}y}+Be^{-\sqrt{d_0 s}y}+\frac{d_1}{d_0}\frac{GrF(s)e^{-\sqrt{\frac{b_0 \Pr s^{1-\beta}}{(1-as^{-\alpha})^{\gamma}}}y}}{\left(s-\frac{b_0 \Pr s^{\beta}}{d_0(1-as^{-\alpha})^{\gamma}}\right)}. \tag{21}$$

The solution of Equation (21) subject to condition in Equation (19), we obtain

$$\bar{u}(y,s)=\frac{e^{-y\sqrt{d_0 s}}}{(s-b)(1+\gamma\sqrt{d_0 s})}-d_3\frac{GrF(s)e^{-y\sqrt{d_0 s}}}{(1+\gamma\sqrt{d_0 s})(s-d_2 w(s))}\left(1+\gamma\sqrt{b_0 w(s)}\right)+d_3\frac{GrF(s)e^{-y\sqrt{b_0 w(s)}}}{s-d_2 w(s)} \tag{22}$$

where $w(s)=\frac{\Pr s^{1-\beta}}{(1-as^{-\alpha})^{\gamma}}, d_2=\frac{b_0}{d_0}, d_3=\frac{d_1}{d_0}$.

$$\bar{u}(y,s)=\overline{M_1}(s)+d_3\overline{M_2}(s)+d_3\overline{M_3}(s). \tag{23}$$

Taking inverse Laplace transform on both sides, we get

$$u(y,t)=M_1(t)+d_3 M_2(t)+d_3 M_3(t), \tag{24}$$

where $$M_1(t)=\frac{-y\sqrt{d_0}e^{\frac{-y^2 d_0}{4t}}}{2t\sqrt{\pi t}}*H(t)e^{\frac{a}{\eta}t}*\frac{e^{\frac{t}{(\gamma d_0)^2}}erfc\left(\frac{\sqrt{t}}{\gamma d_0}\right)}{\sqrt{\pi t}},$$

$$M_2(t)=erfc\left(\frac{y\sqrt{d_0}}{2\sqrt{t}}\right)*\sum_{t=0}^{\infty}\sum_{l=0}^{\infty}(\gamma d_0)^l(d_2 \Pr)^t e^{\gamma t}_{\alpha,\beta t-\frac{l}{2}}(a;t)\left[H(t)-\gamma(b_0 \Pr)^{1/2}e^{\gamma/2}_{\alpha,\frac{\beta+1}{2}}(a;t)\right],$$

$$M_3(t)=d_3 Grf(t)*\left[\sum_{k=0}^{\infty}\frac{(d_2)^k(-y)^k(b_0 \Pr)^{\frac{k}{2}}}{k!}e^{\frac{\gamma k}{2}}_{\alpha,1-\frac{(1-\beta)k}{2}}(a;t)-erfc\left(\frac{y\sqrt{b_0}}{2\sqrt{t}}\right)\right].$$

In particular for no slip condition, we substitute $\gamma=0$ in Equation (22) to obtain

$$\bar{u}(y,s)=\frac{e^{-y\sqrt{d_0}\sqrt{s}}}{(s-b)}+d_3\frac{GrF(s)}{\left(1-d_2 \Pr s^{-\beta}\right)}\left[\frac{e^{-y\sqrt{b_0 \Pr}\sqrt{s^{1-\beta}}}}{s}-\frac{e^{-y\sqrt{d_0}\sqrt{s}}}{s}\right]. \tag{25}$$

Taking inverse Laplace transform on both side, we get

$$u(y,t)=e^{bt}*\frac{-y\sqrt{d_0}e^{\frac{-y^2 d_0}{4t}}}{2t\sqrt{\pi t}}+d_3\sum_{k=0}^{\infty}\frac{Gr(d_2 \Pr)^k t^{\beta k}}{\beta k!}*\left[\sum_{k=0}^{\infty}\frac{(-y)^k(b_0 \Pr)^{\frac{k}{2}}t^{\left(\frac{\beta-1}{2}\right)k}}{k!\left(\frac{\beta-1}{2}\right)k!}-erfc\left(\frac{y\sqrt{d_0 Pr}}{2\sqrt{t}}\right)\right]. \tag{26}$$

3 Results and Discussion

The impact of slip of a generalized viscous unsteady nanofluid with thermal flux was studied in natural convection flows over an infinite vertically heated plate. The Prabhakar fractional derivative has been utilized in the governing equations to account for extended memory effects. Figure 2-9 represent the analytical solution of the velocity of the nanofluid. We considered the thermophysical parameter values for nanoparticles (CNTs) and water (base fluid) as $C_{pf}=4179$, $C_{ps}=425$, $\rho_s=2600$, $\rho_f=997.1$, $k_s=6600$ and $k_f=0.613$.

The influence of the fractional parameters on velocity with the slip effect profiles has been cleared in Figure 2 to Figure 6.

In Figure 2-3, the influence of fractional parameters α and β on the velocity field is shown at two distinct values of time and it is pointed out that velocity increased by increasing these parameter. For both times, the velocity increases by increasing the values of these parameters. Figure 4 shows the velocity effect due to change in the fractional parameter γ and the velocity decreases fastly by increasing the value of γ. Figure 5 is shown to describe the effect of volume fraction ϕ on the velocity field. The velocity of the nanofluid decreases with the increase in the value of the ϕ. This decrease in velocity is due to the slip effect on the nanofluid. The comparison between fractional and classical models is illustrated in Figure 6. For small time, the values in the classical model are smaller than those in the fractional model. However this trend reverses in the case of large values of time.

The slip effect depends on the fractional parameter γ. If the value of fractional parameter gamma is nonzero it is considered slip boundary condition otherwise it will be no slip boundary condition. The effects of fractional parameter γ with slip and no slip conditions are considered. The graph of slip and no slip conditions will overlap in the case of a small value of gamma as can be seen in Figure 6. In Figure 7 and Figure 8 the behavior of velocity of the nanofluids for the large value of parameter gamma is shown. It can be seen that the velocity of the fluid decreases as the slip effect becomes significant. Figure 9 explains that for increasing the slip parameter γ, the velocity of the fluid decreases.

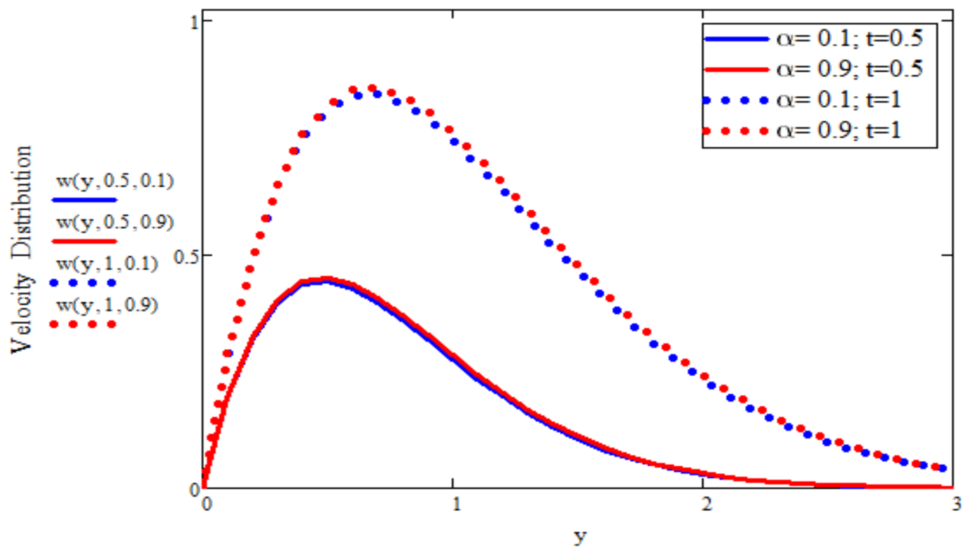

Fig.2: Velocity profiles for different combinations of parameter values. Here $a = 0.4$, $\beta = 0.1$, $\gamma = 0.1$, $\Pr = 3$ and $\phi = 0.001$.

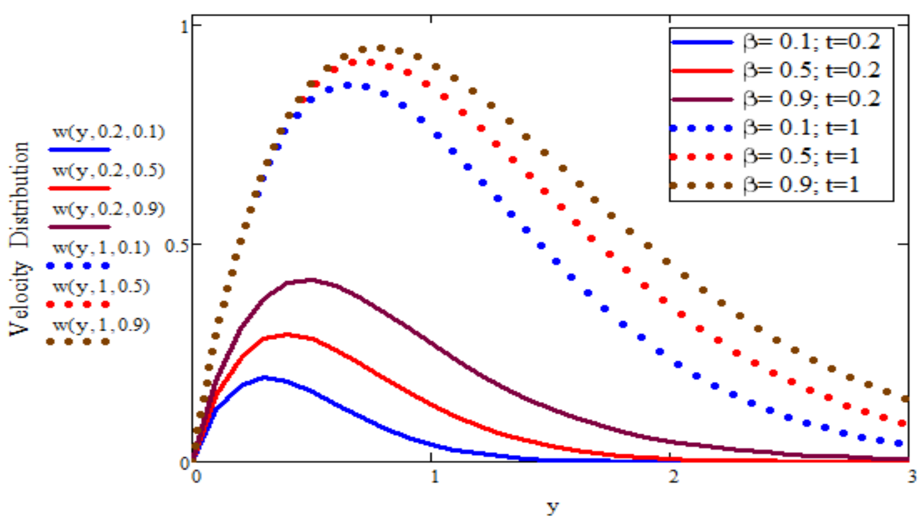

Fig. 3: Velocity profiles for different combinations of parameter values. Here $a = 0.4$, $\alpha = 0.1$, $\gamma = 0.1$, $\Pr = 3$ and $\phi = 0.001$.

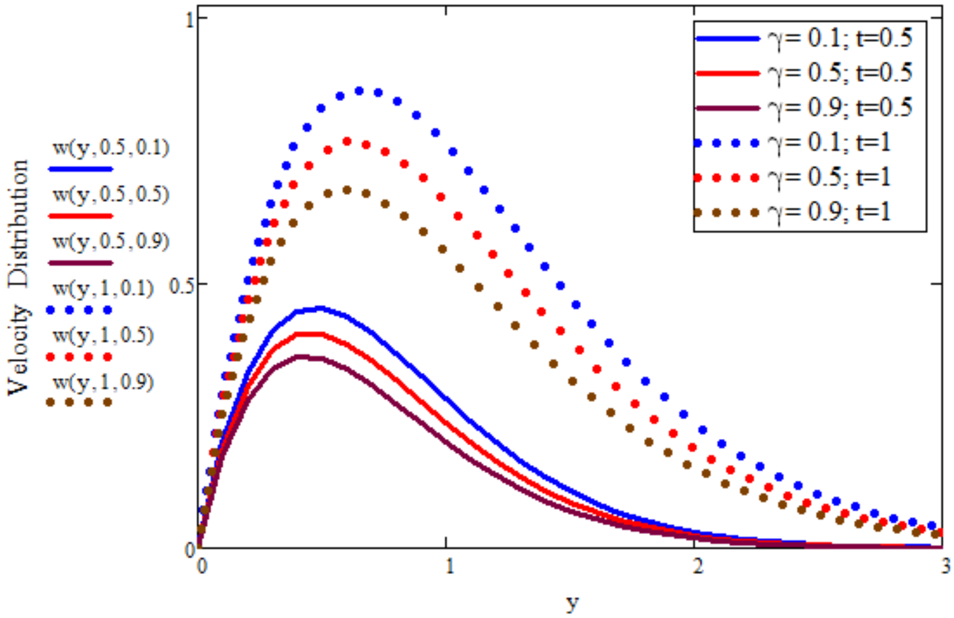

Fig.4: Velocity profiles for different combinations of parameter values. Here $a = 0.4$, $\alpha = 0.1$, $\beta = 0.1$, $\Pr = 3$ and $\phi = 0.001$.

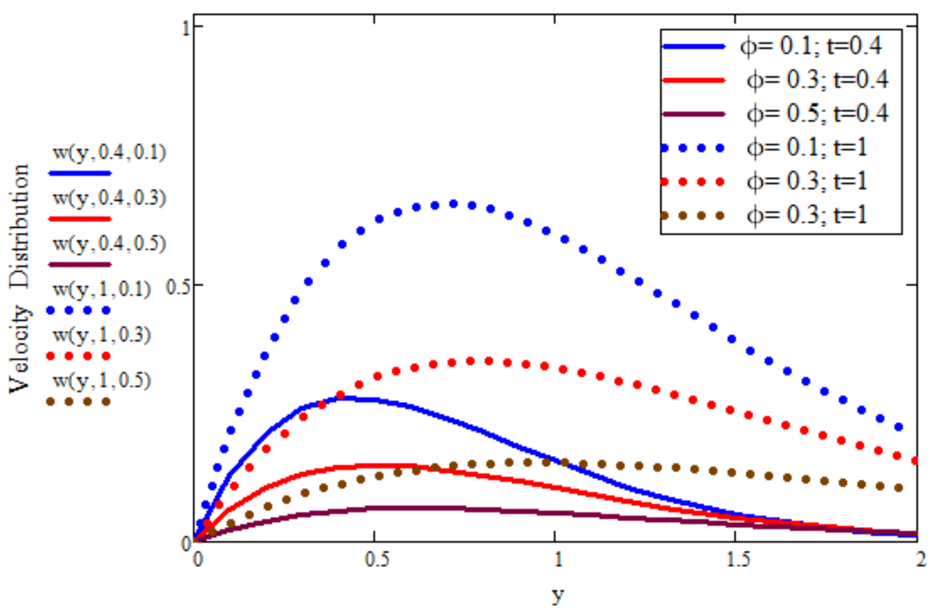

Fig.5: Velocity profiles for different combinations of parameter values. Here $a = 0.4$, $\alpha = 0.1$, $\beta = 0.1$, $\gamma = 0.1$ and $\Pr = 3$.

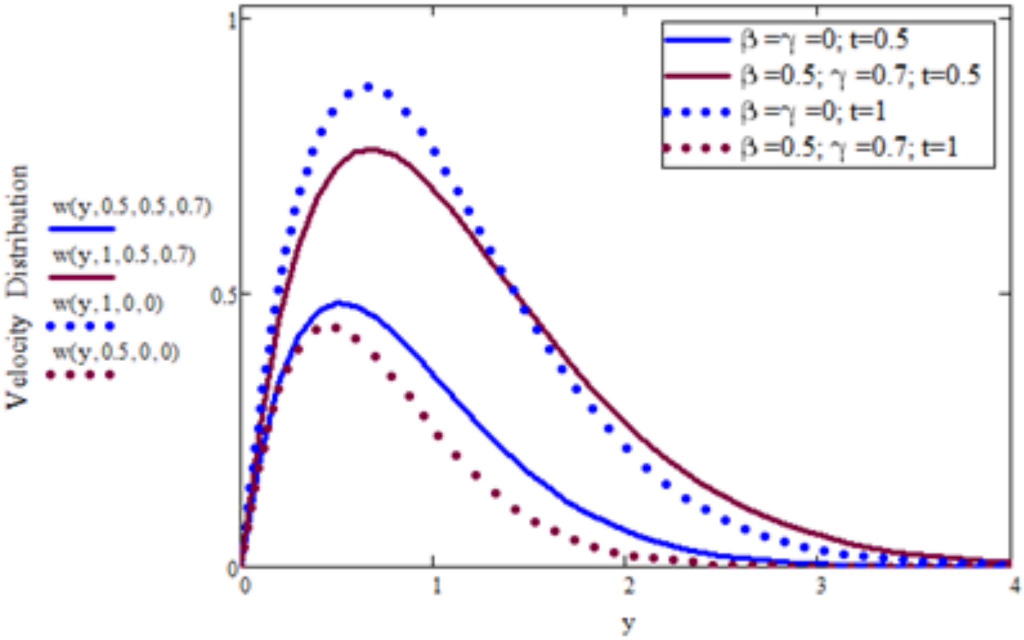

Fig. 6: Velocity profiles for different combinations of parameter values. Here $a = 0.4$, $\alpha = 0.1$, $\Pr = 3$ and $\phi = 0.001$

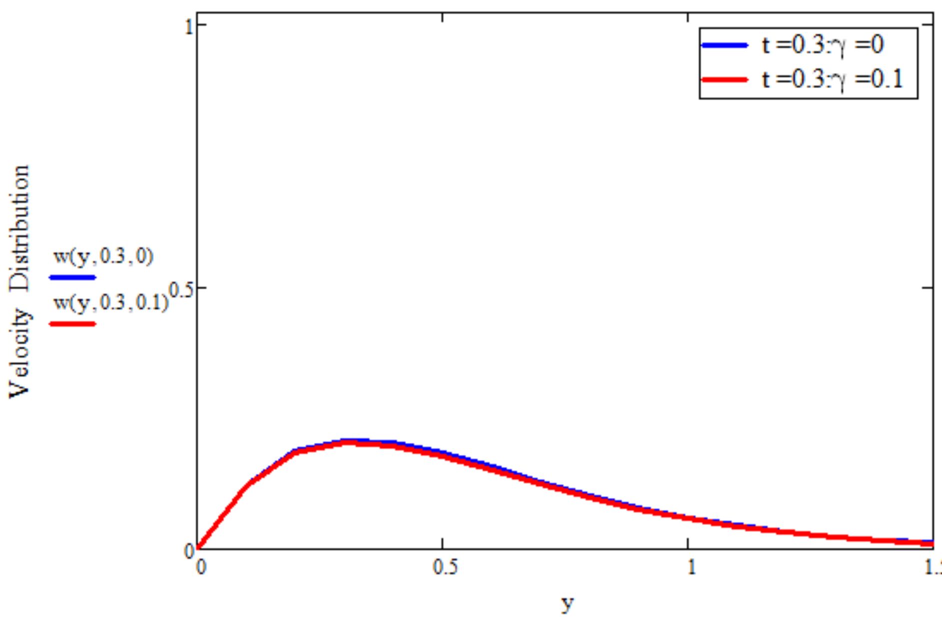

Fig. 7: Velocity profiles for different combinations of parameter values. Here $a = 0.4$, $\alpha = 0.1$, $\beta = 0.1$, $\Pr = 3$ and $\phi = 0.001$.

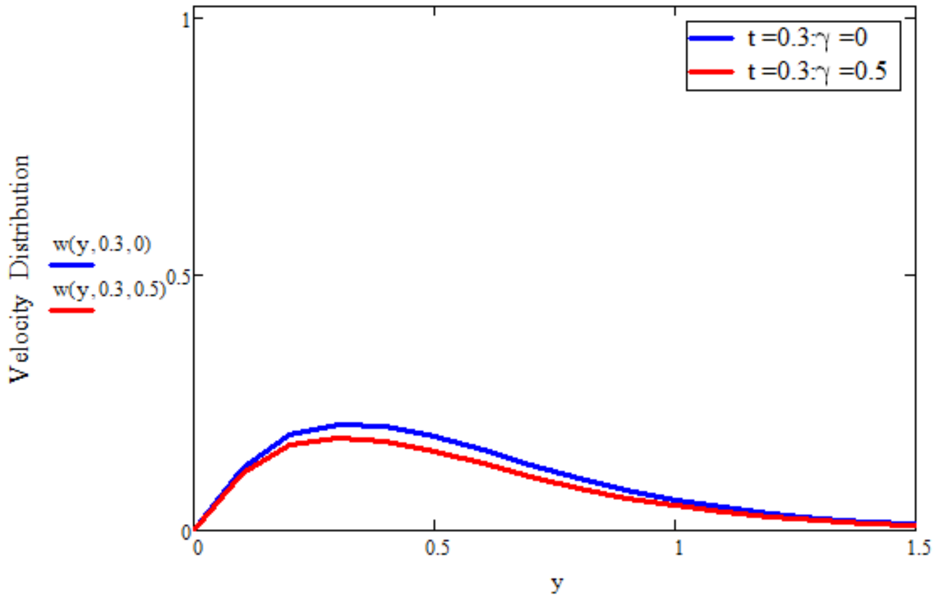

Fig.8: Velocity profiles for different combinations of parameter values. Here $a = 0.4$, $\alpha = 0.1$, $\beta = 0.1$, $\Pr = 3$ and $\phi = 0.001$.

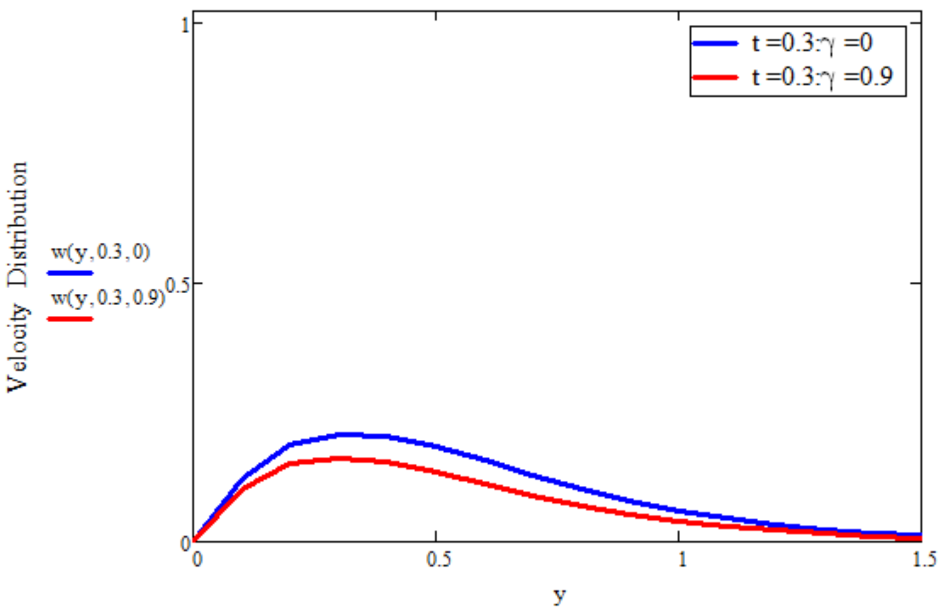

Fig.9: Velocity profiles for different combinations of parameter values. Here $a = 0.4$, $\alpha = 0.1$, $\beta = 0.1$, $\Pr = 3$ and $\phi = 0.001$.

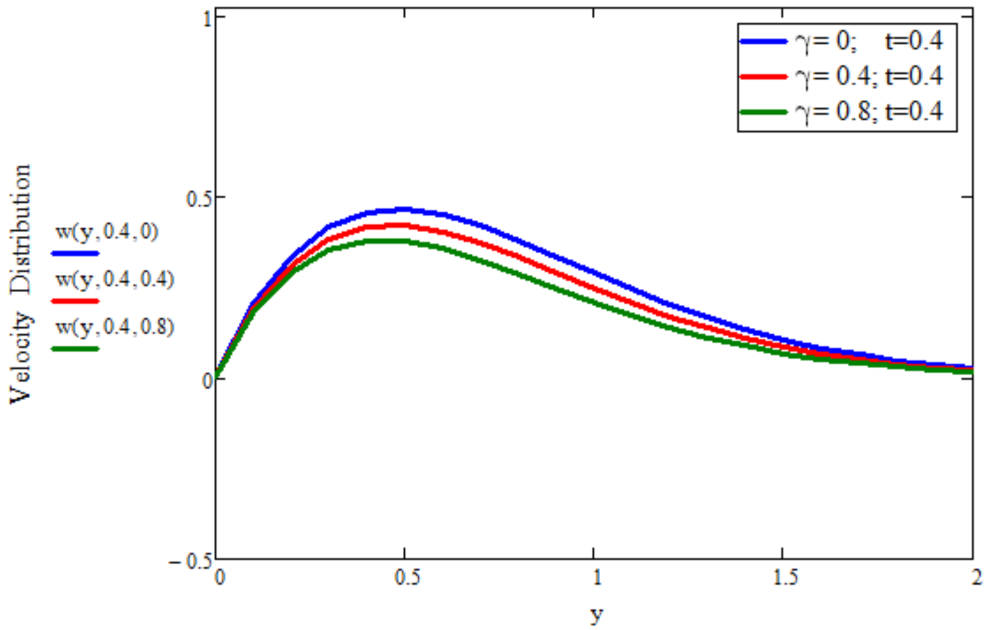

Fig.10: Velocity profiles for different combinations of parameter values. Here $a = 0.4$, $\alpha = 0.1$, $\beta = 0.1$, $\Pr = 3$ and $\phi = 0.001$.

4 Conclusions

The purpose of this paper is to investigate the effects of slip on generalized viscous nanofluid Prabhakar heat transport near an infinite vertically heated plate. The unsteady fractional Prabhakar derivative is used to introduce the correlated fractional system of the governing equations. We find the exact solution of the velocity of the fluid by using the Laplace transform method and compare with no slip condition. We analysed the results obtained and the finding are presented as follows:

- Velocity profile increases by increasing the values of α and β but decreases by increasing the value of γ.
- Velocity profile decreases by increasing the value of volume fraction ϕ.
- For the small-time result of a classical model is less than the fractional model, whereas for large time influence is opposite.
- For the small value of fractional parameter γ the graphs of velocity with or without slip show the overlapping behavior.
- The velocity decreases by increasing the values of γ and the graph of velocity with no slip become comparable with the graph of velocity with slip effect.
- For increasing the slip affect the velocity of fluid decreases.

This means that the fluid flow can be reduced by introducing the slip effect. i.e. the fractional fluid has less velocity than the ordinary fluid. Moreover, the fluid velocity is a decreasing function of gamma and time.

References

[1] Y. Jaluria, *Natural convection heat and mass transfer,* Pergamon Press, Oxford, UK, 1980.

[2] S. U. Choi and J. A. Eastman, *Enhancing thermal conductivity of fluids with nanoparticles*, Argonne National Lab., United States, 1995.

[3] X.-Q. Wang and A. S. Mujumdar, A review on nanofluids-part I: theoretical and numerical investigations, *Braz. J. Chem. Eng*. **25**, 613-630 (2008).

[4] M. Sheikholeslami, M. Gorji-Bandpy and K. Vajravelu, Lattice Boltzmann simulation of magnetohydrodynamic natural convection heat transfer of Al2O3–water nanofluid in a horizontal cylindrical enclosure with an inner triangular cylinder, *Int. J. Heat Mass Transf.* **80**, 16- 25 (2015).

[5] S. Das and R. Jana, Natural convective magneto-nanofluid flow and radiative heat transfer past a moving vertical plate, *Alex. Eng. J.* **54,** 55-64 (2015).
[6] R. Ellahi, M. Hassan and A. Zeeshan, Aggregation effects on water base Al2O3—nanofluid over a permeable wedge in mixed convection, *Asia-Pac. J. Chem. Eng.* 11(2), 179- 186 (2016).
[7] C. Kleinstreuer and Z. Xu, Mathematical modeling and computer simulations of nanofluid flow with applications to cooling and lubrication, *Fluids.* **1**(2), 16 (2016).
[8] S. K. Das and U. Stephen, A review of heat transfer in nanofluids, *Adv. Heat Transf.* **41**, 81-197 (2009).
[9] S. K. Das, S. U. Choi, W. Yu and T. Pradeep, *Nanofluids: Science and Technology*, John Wiley & Sons, Hoboken and New Jersey (2007).
[10] M. Sheikholeslami and R. Ellahi, Electrohydrodynamic nanofluid hydrothermal treatment in an enclosure with sinusoidal upper wall, *Appl. Sci.* **5**(3), 294-306 (2015).
[11] M. R. Safaei, G. Ahmadi, M. S. Goodarzi, A. Kamyar and S. Kazi, Boundary layer flow and heat transfer of FMWCNT/water nanofluids over a flat plate, *Fluids.* **1**(4), 31 (2016).
[12] J. A. Eastman, U. Choi, S. Li, L. Thompson and S. Lee, Enhanced thermal conductivity through the development of nanofluids, *MRS (OPL),* **457** (1996).
[13] X. Wang, X. Xu and S. U. Choi, Thermal conductivity of nanoparticle-fluid mixture, *J. Thermophys. Heat Trans***.** **13**(4), 474-480 (1999).
[14] Y. Xuan and Q. Li, Heat transfer enhancement of nanofluids, *Int. J. Heat Fluid Flow.* **21**(1), 58-64 (2000).
[15] S. K. Das, N. Putra, P. Thiesen and W. Roetzel, Temperature dependence of thermal conductivity enhancement for nanofluids, *J. Heat Transfer.* **125**(4), 567-574 (2003).
[16] S. Murshed, K. Leong and C. Yang, Enhanced thermal conductivity of TiO2—water-based nanofluids, *Int. J. Therm. Sci.* **44**(4), 367-373 (2005).
[17] P. Kim, L. Shi, A. Majumdar and P. L. McEuen, Thermal transport measurements of individual multiwalled nanotubes, *PRL*. **87(**21), 215502 (2001).
[18] S. Choi, Z. G. Zhang, W. Yu, F. Lockwood, and E. Grulke, Anomalous thermal conductivity enhancement in nanotube suspensions, *Appl. Phys. Lett*. **79**(14), 2252-2254 (2001).
[19] J. A. Eastman, S. Choi, S. Li, W. Yu, and L. Thompson, Anomalously increased effective thermal conductivities of ethylene glycol-based nanofluids containing copper nanoparticles, *Appl. Phys. Lett.* **78**(6), 718-720 (2001).
[20] Y. Ding, H. Alias, D. Wen and R. A. Williams, Heat transfer of aqueous suspensions of carbon nanotubes (CNT nanofluids), *Int. J. Heat Mass Transf.* **49**(1-2), 240-250 (2006).
[21] Y. J. Hwang, J. Lee, C. Lee, Y. Jung, S. Cheong, C. Lee, et al., Stability and thermal conductivity characteristics of nanofluids, *Thermochim. Acta*. **455**(1-2), 70-74 (2007).
[22] S. U. Jan, S. U. Haq, S. I. A. Shah and I. Khan, Heat and mass transfer of free convection flow over a vertical plate with chemical reaction under a wall–slip effect**,** *Arab. J. Sci. Eng.* **44**(12), 9869-9887 (2019).
[23] H. Helmholtz and G.V. *Piotrowski, Liquids dripping through friction,* Hof & Stts. print, **d. XL**, 607, (1860).
[24] A. Kundt and E. Warburg, on the friction and heat conduction of rarefied gases, *Ann. Phys.* 231(10), 337-365 (1875).
[25] R. Pit, H. Hervet and L. Leger, Friction and slip of a simple liquid at a solid surface, *Tribol. Lett.* **7**(2), 147-152, (1999).
[26] T. D. Blake, Slip between a liquid and a solid: DM Tolstoi's (1952) theory reconsidered, *Colloids Surf.* **47**, 135-145 (1990).
[27] M. A. Day, The no-slip condition of fluid dynamics, *Erkenntnis.* **33**(3), 285-296 (1990).
[28] C. Navier, On the laws of fluid motion, *Mem. Acad. Roy. Science. Inst. France.* **6**, 389-440 (1827).
[29] M. Tahir, M. Imran, N. Raza, M. Abdullah and M. Aleem, Wall slip and non-integer order derivative effects on the heat transfer flow of Maxwell fluid over an oscillating vertical plate with a new definition of fractional Caputo-Fabrizio derivatives, *Results Phys.***7**, 1887-1898 (2017).
[30] G. A. Adem and N. Kishan, Slip effects in a flow and heat transfer of a nanofluid over a nonlinearly stretching sheet using optimal homotopy asymptotic method, *Int. J. Eng. Manuf. Sci*. **8**(1), 25-46 (2018).
[31] A. Malvandi, F. Hedayati and D. Ganji, Slip effects on unsteady stagnation point flow of a nanofluid over a stretching sheet, *Powder Technol*. **253**, 377-384 (2014).
[32] S. Ghadikolaei, K. Hosseinzadeh, M. Hatami, D. Ganji and M. Armin, Investigation for squeezing flow of ethylene glycol (C2H6O2) carbon nanotubes (CNTs) in rotating stretching channel with nonlinear thermal radiation, *J. Mol. Liq.* **263**, 10-21 (2018).
[33] T. Elnaqeeb, N. A. Shah and I. A. Mirza, Natural convection flows of carbon nanotubes nanofluids with Prabhakar-like thermal transport, *Math. Methods Appl. Sci*. (2020).
[34] R. U. Haq, S. N. Kazmi and T. Mekkaoui, Thermal management of water based SWCNTs enclosed in a partially heated trapezoidal cavity via FEM, *Int. J. Heat Mass Transf.* **112**(1), 972-982 (2017).

[35] M. I. Asjad, M. Zahid, Y. M. Chu and D. Baleanu, Prabhakar fractional derivative and its applications in the transport phenomena containing nanoparticles, *Therm. Sci.* **25**(2), 411-416 (2021).
[36] A. Fernandez and D. Baleanu, Classes of operators in fractional calculus: a case study, *Math. Methods Appl. Sci*. **44**(11), 9143-9162 (2021).
[37] A. Fernandez, D. Baleanu and H. Srivastava, Series representations for fractional-calculus operators involving generalised Mittag-Leffler functions, *Commun. Nonlinear Sci. Numer. Simul*. **67**(1), 517-527 (2019).
[38] A. Bokhari, R. Belgacem, S. Kumar, D. Baleanu and S. Djilali, Projectile motion using three parameter Mittag-Leffler function calculus, *Math. Comput. Simul.* **195**(1), 22-30 (2022).
[39] A. Fernandez, T. Abdeljawad and D. Baleanu, Relations between fractional models with three-parameter Mittag-Leffler kernels, Adv. Differ. Equ. **20**(1), 1-13 (2020).