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Profit Analysis Study of Two-Dissimilar-Unit Warm Standby System under Different Weather Conditions

A. Kamal[1], *M. A. W. Mahmoud*[2], *R. M. EL-Sagheer*[2,3,*] *and M. A. Moaz*[4]

[1]Mathematical and Computer Science Department, Faculty of Science Port-Said University, Port-Said 42522, Egypt
[2]Mathematics Department, Faculty of Science, Al-Azhar University, Nasr City 11884, Cairo, Egypt
[3]High Institute of Computer and Management Information System, First Statement, New Cairo 11865, Cairo, Egypt
[4]Department of Physics and Mathematical Science, Faculty of Engineering Port-Said University, Port-Said 42522, Egypt

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Abstract: In this paper, we study a warm standby repairable system that consists of two dissimilar units. One of these units is a good quality unit while the other one is of substandard quality that might need some repairs or replacement by another substandard unit upon failure. The system works under two different weather conditions, normal and abnormal. The unit operates under normal weather conditions, but incase of abnormal weather conditions, the system stops and the unit fails. In this paper, we analyze the steady state transition probabilities, mean sojourn time, mean time to failure, steady state availability of the system. We also performed busy period analysis of repairman and cost benefit analysis of the system. All of the previously mentioned analyses were done by using regenerative point technique.

Keywords: Warm Standby; MTSF; Busy period; Cost benefit estimated

1 Introduction

Nowadays, system reliability has an important role in both industry and manufacturing, with redundancy being one of the most common techniques used to enhance system reliability. The research on the cold standby system, the warm standby system, and the hot standby system has made great progress over the past decades. In case of the cold standby system, the system assumes that the spare parts will not fail during the storage period, while the hot standby system assumes that the spare parts have the same failure rate as the working part. In this regard, the warm standby system that assumes the spare parts may fail in the storage period with a smaller failure rate than the working part is more suitable for modeling the degradation process of the spare parts.

Extensive researches on the reliability analysis for the standby redundant systems have already been carried out in the literature. For the cold standby redundancy systems, [1] dealt with a two-unit cold standby system considering hardware failure, human error failure and preventive maintenance (PM) in which all time distributions are assumed to be arbitrary. [2] studied reliability measures of a cold standby system with preventive maintenance and repair. [3] discussed Reliability analysis of a two-unit cold standby system with arbitrary distributions and change in units. With regard to the hot standby model, Rizwan, [4] provided a reliability analysis of a programable logic controllers system which was studied as a two-unit hot standby system, and the real data had been used from an industrial system for the purpose. [5] discussed the reliability analysis of a two-unit hot standby redundant system with repairable failures and non-repairable failures by using probability analysis, definite integral and the supplementary variable technique.

The warm standby repairable system has also been explored, for example, [6] conducted the reliability indexes and the steady-state availability with different parameters of a warm standby repairable system with repairman vacation under Poisson shocks. [7] obtained the performance measures of a three-unit warm standby system with dependent structure, whereas the lifetimes of online unit, standby units, and the repair time of failed units are governed by quadrivariate exponential law. [8] analyzed cost benefit of two similar warm standby systems subject to failure due to melting of glaciers and severe storms caused by global warming and failure rate as Gamma distribution. Kumar, Pawar and [9] analyzed

* Corresponding author e-mail: rashadmath27@gmail.com

economically a warm standby system with single server. the purpose of the present study is to obtain reliability measures of a system of non identical units with warm standby approach. [10] studied a warm standby redundant repairable systems with two different units and assumed that unit 1 follows a geometric process after repair, and some reliability indexes were obtained by using the supplementary variables approach. In addition, the researches for warm standby repairable systems have continuously developed, including the papers [11,12,13] and others.
For weather conditions [14,15] used the idea of two weather conditions (normal and abnormal) in a single-unit system. [16] discussed reliability and economic analysis of a system operating under different weather conditions. [17] analyzed steady state of an operating system with repair at different levels of damages subject to inspection and weather conditions. In this paper we study a two-dissimilar-unit warm standby repairable system with priority in use. We present a study for a warm standby system consisting of two units where one unit is of good quality and another unit is of substandard quality, which may need some repair or replacement by another sub-standard unit upon failure. The system is affected by different weather conditions.

2 Assumptions

- Two dissimilar units in the system. In the beginning, one unit is in operative mode and the another is in standby mode (Warm standby).
- When operative unit get failure, the Warm standby unit may be turn to operative mode and failure unit move for repair.
- The first unit has priority for operating, first unit is of good quality and another unit is of substandard quality, which may need some repair or replacement by another substandard unit after getting failed.
- If the weather is in normal case, the unit operates, and if the weather is in abnormal case, the system stops, and the operate unit fails.
- The connected switch is perfect.
- All times are independent and exponentially distributed.

3 Notations

E	Set of regenerative states.
$q_{ij}(t), Q_{ij}(t)$	PDF and CDF of time for the system transits from stateS_i to S_j.
P_{ij}	Transition probability from S_i to S_j.
λ_1, λ_2	parameter of the failure rate of unit 1,unit 2 respectively.
μ_1, μ_2	The parameter of repair rate of unit 1,unit 2 respectively.
α	parameter of normal weather rate.
β	parameter of abnormal weather rate.
z	parameter of second unit replacement rate.
τ	Probability that the Standby unit is ready.
$(1-\tau)$	Probability that the Standby unit is not ready.
θ	Probability that the second unit repair.
$(1-\theta)$	Probability that the second unit replacement.
η_{ij}	contribution to mean sojourn time in state S_i, when system transits direct to S_j.
$M_i(t)$	p $\{$system is up initially in state S_i is up at t without passing through any other regenerative state$\}$.
$M_i(s)$	$\int P\{$system sojourns in state S_i for at least time t$\}dt$.
$\prod_i(t)$	CDF of time to system failure starting from state S_i.
$AV_i(t)$	p $\{$ The system is up at time t starting at state $S_i\}$.
$C(t)$	The net revenue of the system in $(o,t]$.
$G(t)$	Cdf of the repair time at state $S_i$with first unit.
$G'(t)$	Cdf of the repair time at state S_i with second unit.
$\circledast$	Convolution.
$*$	Laplace transforms.

3.1 Symbols for the states of the system

- s_{II} unit 2 is in warm standby state.

- o_I, o_{II} unit 1, unit 2 are in operating state respectively.
- o_{Id}, o_{IId} the first operating unit, the second operating unit stopped because the weather is abnormal.
- w_g Normal weather.
- w_d Abnormal weather.
- r_I, r_{II} unit 1, unit 2 are under repair respectively.
- rep_{II} unit 2 is under replacement.
- wr_I, wr_{II} unit 1, unit 2 are waiting for repair respectively.

The system can be in any one of the following states
$S_0 = (o_I, s_{II}, w_g)$, $S_1 = (r_I, o_{II}, w_g)$, $S_2 = (r_I, wr_{II}, w_g)$,
$S_3 = (r_I, o_{IId}, w_d)$, $S_4 = (r_I, wr_{II}, w_d)$, $S_5 = (o_I, r_{II}, w_g)$,
$S_6 = (o_I, rep_{II}, w_g)$, $S_7 = (o_{Id}, s_{II}, w_d)$, $S_8 = (o_{Id}, r_{II}, w_d)$,
$S_9 = (o_{Id}, rep_{II}, w_d)$, $S_{10} = (wr_I, r_{II}, w_g)$, $S_{11} = (wr_I, r_{II}, w_d)$,
$S_{12} = (wr_I, rep_{II}, w_d)$, $S_{13} = (wr_I, rep_{II}, w_g)$.

Up states:S_0, S_1, S_5, S_6. **Down states:**$S_2, S_3, S_4, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{13}$.

3.2 Transition probabilities and mean sojourn time

We use regenerative technique to obtain the transition probabilities. All points are regenerative points. Let $T_1(\equiv 0), T_2, T_0, ...$ denote the epochs at which the system enters any state $S_i \in E$ let X_n denote the state visited at epoch T_n+, *i.e.* just after transition at T_n. $\{X_n, T_n\}$ is a Markov renewel process with state space E and $Q_{ij}(t) = P[X_{n+1} = j, T_{n+1} - T_n \preceq t \setminus X_n = i]$, is the semi Markov kernel over E. Since the transition probabilities $P_{ij} = Q_{ij}(\infty)$ then the non-zeros P_{ij}'s are

$$P_{01} = \frac{\tau\lambda_1}{\beta+\lambda_1}, \quad P_{02} = \frac{(1-\tau)\lambda_1}{\beta+\lambda_1}, \quad P_{03} = \frac{\tau\beta}{\beta+\lambda_1}, \quad P_{04} = \frac{(1-\tau)\beta}{\beta+\lambda_1},$$
$$P_{01} + P_{02} + P_{03} + P_{04} = 1,$$

$$P_{10} = \frac{\mu_1}{\beta+\lambda_2+\mu_1}, \quad P_{12} = \frac{\lambda_2}{\beta+\lambda_2+\mu_1}, \quad P_{14} = \frac{\beta}{\beta+\lambda_2+\mu_1},$$
$$P_{10} + P_{12} + P_{14} = 1,$$

$$P_{24} = \frac{\beta}{\beta+\mu_1}, \quad P_{25} = \frac{\theta\mu_1}{\beta+\mu_1}, \quad P_{26} = \frac{(1-\theta)\mu_1}{\beta+\mu_1},$$
$$P_{24} + P_{25} + P_{26} = 1,$$

$$P_{31} = \frac{\alpha}{\alpha+\mu_1}, \quad P_{37} = \frac{\mu_1}{\alpha+\mu_1},$$
$$P_{31} + P_{37} = 1,$$

$$P_{42} = \frac{\alpha}{\alpha+\mu_1}, \quad P_{48} = \frac{\theta\mu_1}{\alpha+\mu_1}, \quad p_{49} = \frac{(1-\theta)\mu_1}{\alpha+\mu_1},$$
$$P_{42} + P_{48} + P_{49} = 1,$$

$$P_{50} = \frac{\mu_2}{\beta+\mu_2+\lambda_1}, \quad P_{5(10)} = \frac{\lambda_1}{\beta+\mu_2+\lambda_1}, \quad P_{5(11)} = \frac{\beta}{\beta+\mu_2+\lambda_1},$$
$$P_{51} + P_{5(10)} + P_{5(11)} = 1,$$

$$P_{60} = \frac{z}{\beta+\lambda_1+z}, \quad P_{6(12)} = \frac{\beta}{\beta+\lambda_1+z}, \quad P_{6(13)} = \frac{\lambda_1}{\beta+\lambda_1+z},$$

$$P_{60}+P_{6(12)}+P_{6(13)}=1,$$

$$P_{70}=1,$$

$$P_{85}=\frac{\alpha}{\alpha+\mu_2},\quad P_{87}=\frac{r_2}{\alpha+\mu_2},$$
$$P_{85}+P_{87}=1,$$

$$P_{96}=\frac{\alpha}{\alpha+z},\quad P_{97}=\frac{z}{\alpha+z},$$
$$P_{96}+P_{97}=1,$$

$$P_{(10)1}=\frac{\mu_2}{\beta+\mu_2},\quad P_{(10)(11)}=\frac{\beta}{\beta+\mu_2},$$
$$P_{(10)1}+P_{(10)(11)}=1,$$

$$P_{(11)(10)}=\frac{\alpha}{\alpha+\mu_2},\quad P_{(11)3}=\frac{\mu_2}{\alpha+\mu_2},$$
$$P_{(11)(10)}+P_{(11)3}=1,$$

$$P_{(12)(13)}=\frac{\alpha}{\alpha+z},\quad P_{(12)3}=\frac{z}{\alpha+z},$$
$$P_{(12)(13)}+P_{(12)3}=1,$$

$$P_{(13)(12)}=\frac{\beta}{z+\beta},\quad P_{(13)1}=\frac{z}{z+\beta},$$
$$P_{(13)(12)}+P_{(13)1}=1.$$

3.3 Mean sojourn times

The unconditional mean time taken by the system to transit from any regenerative state S_i when time is counted from epoch of entrance into state S_j is given by

$$\eta_{01}=\frac{\tau\lambda_1}{(\beta+\lambda_1)^2},\quad \eta_{02}=\frac{(1-\tau)\lambda_1}{(\beta+\lambda_1)^2},\quad \eta_{03}=\frac{\tau\beta}{(\beta+\lambda_1)^2},\quad \eta_{04}=\frac{(1-\tau)\beta}{(\beta+\lambda_1)^2},$$

$$\eta_{24}=\frac{\beta}{(\beta+\mu_1)^2},\quad \eta_{25}=\frac{\theta\mu_1}{(\beta+\mu_1)^2},\quad \eta_{26}=\frac{(1-\theta)\mu_1}{(\beta+\mu_1)^2},$$

$$\eta_{31}=\frac{\alpha}{(\alpha+\mu_1)^2},\quad \eta_{37}=\frac{\mu_1}{(\alpha+\mu_1)^2},$$

$$\eta_{42}=\frac{\alpha}{(\alpha+\mu_1)^2},\quad \eta_{48}=\frac{\theta\mu_1}{(\alpha+\mu_1)^2},\quad \eta_{49}=\frac{(1-\theta)\mu_1}{(\alpha+\mu_1)^2},$$

$$\eta_{50}=\frac{\mu_2}{(\beta+\mu_2+\lambda_1)^2},\quad \eta_{5(10)}=\frac{\lambda_1}{(\beta+\mu_2+\lambda_1)^2},\quad \eta_{5(11)}=\frac{\beta}{(\beta+\mu_2+\lambda_1)^2},$$

$$\eta_{60}=\frac{z}{(\beta+\lambda_1+z)^2},\quad \eta_{6(12)}=\frac{\beta}{(\beta+\lambda_1+z)^2},\quad \eta_{6(13)}=\frac{\lambda_1}{(\beta+\lambda_1+z)^2},$$

$$\eta_{70} = \frac{1}{\alpha},$$

$$\eta_{85} = \frac{\alpha}{(\alpha+\mu_2)^2}, \quad m_{87} = \frac{\mu_2}{(\alpha+\mu_2)^2},$$

$$\eta_{96} = \frac{\alpha}{(\alpha+z)^2}, \quad \eta_{97} = \frac{z}{(\alpha+z)^2},$$

$$\eta_{(10)1} = \frac{\mu_2}{(\beta+\mu_2)^2}, \quad \eta_{(10)(11)} = \frac{\beta}{(\beta+\mu_2)^2},$$

$$\eta_{(11)(10)} = \frac{\alpha}{(\alpha+\mu_2)^2}, \quad \eta_{(11)3} = \frac{\mu_2}{(\alpha+\mu_2)^2},$$

$$\eta_{(12)(13)} = \frac{\alpha}{(\alpha+z)^2}, \quad \eta_{(12)3} = \frac{z}{(\alpha+z)^2},$$

$$\eta_{(13)(12)} = \frac{\beta}{(z+\beta)^2}, \quad \eta_{(13)1} = \frac{z}{(z+\beta)^2}.$$

Mean sojourn time in state S_i which is given by $M_i(s) = \sum_j \eta_{ij}$

$$M_0(s) = \frac{1}{\beta+\lambda_1}, \quad M_1(s) = \frac{1}{\beta+\lambda_2+\mu_1}, \quad M_2(s) = \frac{1}{\beta+\mu_1},$$

$$M_3(s) = \frac{1}{\alpha+\mu_1}, \quad M_4(s) = \frac{1}{\alpha+\mu_1}, \quad M_5(s) = \frac{1}{\beta+\mu_2+\lambda_1},$$

$$M_6(s) = \frac{1}{\beta+\lambda_1+z}, \quad M_7(s) = \frac{1}{\alpha}, \quad M_8(s) = \frac{1}{\alpha+\mu_2},$$

$$M_9(s) = \frac{1}{\alpha+z}, \quad M_{10}(s) = \frac{1}{\beta+\mu_2}, \quad M_{11}(s) = \frac{1}{\alpha+\mu_2},$$

$$M_{12}(s) = \frac{1}{\alpha+z}, \quad M_{13}(s) = \frac{1}{z+\beta}.$$

3.4 Mean time to system failure MTSF

According to the arguments of theory of regenerative processes, we obtain the following relation for $\overline{\prod}_0(t)$

$$\overline{\prod}_0(t) = e^{-(\beta+\lambda_1)} + q_{01}(t) \circledast \overline{\prod}_1(t), \tag{1}$$

$$\overline{\prod}_1(t) = e^{-(\beta+\lambda_2+\mu_1)} + q_{10}\overline{\prod}_0(t), \tag{2}$$

$$\overline{\prod}_5(t) = e^{-(\beta+\mu_2+\lambda_1)} + q_{50}\overline{\prod}_0(t), \tag{3}$$

$$\overline{\prod}_6(t) = e^{-(\beta+\lambda_1+z)} + q_{60}\overline{\prod}_0(t). \tag{4}$$

Taking Laplace transform (LT) for equations (1), (2) and (3) and solving for $\overline{\prod}_0^*(s)$ considering $S = 0$, We have the mean time to system failure MTSF as follows

$$MTSF = \frac{N_0}{D_0}, \tag{5}$$

where

$$D_0 = 1 - (P_{10}P_{01}),$$

and

$$N_0 = M_0(s) + M_1(s)P_{01}.$$

4 Availability Analysis

From the arguments used in the theory of regenrative processes, the point wise availabilities $AV_i(t)$ where $i = 0, 1, 2, 5, 6, 11, 12$.we obtain the following recursive relations.

$$\begin{aligned} AV_0(t) = {} & M_0(t) + (q_{03}(t) \circledast q_{37}(t) \circledast q_{70}(t) + q_{04}(t) \circledast q_{48}(t) \circledast q_{87}(t) \circledast q_{70}(t) \\ & + q_{04}(t) \circledast q_{49}(t) \circledast q_{97}(t) \circledast q_{70}(t)) \circledast AV_0(t) + (q_{01}(t) + q_{03}(t) \circledast q_{31}(t)) \circledast AV_1(t) \\ & + (q_{02}(t) + q_{04}(t) \circledast q_{42}(t)) \circledast AV_2(t) + (q_{04}(t) \circledast q_{48}(t) \circledast q_{85}) \circledast AV_5(t) \\ & + (q_{04}(t) \circledast q_{49}(t) \circledast q_{96}(t)) \circledast AV_6(t), \end{aligned} \tag{6}$$

$$\begin{aligned} AV_1(t) = {} & M_1(t) + (q_{10}(t) + q_{14}(t) \circledast q_{48}(t) \circledast q_{87}(t) \circledast q_{70}(t) \\ & + q_{14}(t) \circledast q_{49}(t) \circledast q_{97}(t) \circledast q_{70}(t))AV_0 + (q_{12}(t) + q_{14}(t) \circledast q_{42}(t)) \circledast AV_2(t) \\ & + (q_{14}(t) \circledast q_{48}(t) \circledast q_{85}(t)) \circledast AV_5(t) + (q_{14}(t) \circledast q_{49}(t) \circledast q_{96}(t))AV_6(t), \end{aligned} \tag{7}$$

$$\begin{aligned} AV_2(t) = {} & (q_{24}(t) \circledast q_{48}(t) \circledast q_{87}(t) \circledast q_{70}(t) \\ & + q_{24}(t) \circledast q_{49}(t) \circledast q_{97}(t) \circledast q_{70}(t))AV_0(t) + (q_{24}(t) \circledast q_{42}(t)) \circledast AV_2(t) \\ & + (q_{25}(t) + q_{24}(t) \circledast q_{48}(t) \circledast q_{85}(t)) \circledast AV_5(t) + (q_{26}(t) \\ & + q_{24}(t) \circledast q_{49}(t) \circledast q_{96}(t)) \circledast AV_6(t), \end{aligned} \tag{8}$$

$$\begin{aligned} AV_5(t) = {} & M_5(t) + (q_{50}(t)) \circledast AV_0(t) + (q_{5(10)}(t) \circledast q_{(10)1}(t))) \circledast AV_1(t) \\ & + (q_{5(10)}(t) \circledast q_{(10)(11)}(t) + q_{5(11)}(t)) \circledast AV_{11}(t), \end{aligned} \tag{9}$$

$$\begin{aligned} AV_6(t) = {} & M_6(t) + (q_{60}(t)) \circledast AV_0(t) + (q_{6(13)}(t) \circledast q_{(13)1}(t)) \circledast AV_1(t) \\ & + (q_{6(12)}(t) + q_{6(13)}(t) \circledast q_{(13)(12)}(t)) \circledast AV_{12}(t), \end{aligned} \tag{10}$$

$$\begin{aligned} AV_{11}(t) = {} & (q_{(11)3}(t) \circledast q_{37}(t) \circledast q_{70}(t)) \circledast AV_0(t) \\ & + (q_{(11)(10)}(t) \circledast q_{(10)1}(t) + q_{(11)3}(t) \circledast q_{31}(t)) \circledast AV_1(t) \\ & + (q_{(11)(10)}(t) \circledast q_{(10)(11)}(t)) \circledast AV_{11}(t), \end{aligned} \tag{11}$$

$$\begin{aligned} AV_{12}(t) = {} & (q_{(12)(3)}(t) \circledast q_{37}(t) \circledast q_{70}(t)) \circledast AV_0(t) + (q_{(12)(13)}(t) \circledast q_{(13)1}(t) \\ & + q_{(12)3} \circledast q_{31}) \circledast AV_1(t) + (q_{(12)(13)}(t) \circledast q_{(13)(12)}(t)) \circledast AV_{12}(t). \end{aligned} \tag{12}$$

Taking LT for equation (6), (7), (8), (9), (10), (11) and (12) and solve for AV_0^*, then we get the steady state availability of the system AV_0 in the form,

$$AV_0 = AV_0(\infty) = \lim_{x \to 0} SAV_0^*(S) = \frac{N_1}{D_1}. \tag{13}$$

$$\begin{aligned} h_1 = {} & (\eta_{03}P_{37}P_{70} + P_{03}\eta_{37}P_{70} + P_{03}P_{37}\eta_{70} + \eta_{04}P_{48}P_{87}P_{70} + P_{04}\eta_{48}P_{87}P_{70} \\ & + P_{04}P_{48}\eta_{87}P_{70} + P_{04}P_{48}P_{87}\eta_{70} + \eta_{04}P_{49}P_{97}P_{70} + P_{04}\eta_{49}P_{97}P_{70} + P_{04}P_{49}\eta_{97}P_{70} \\ & + P_{04}P_{49}P_{97}\eta_{70} + \eta_{01} + \eta_{03}P_{31} + P_{03}\eta_{31} + \eta_{02} + \eta_{04}P_{42} + P_{04}\eta_{42} + \eta_{04}P_{48}P_{85} \\ & + P_{04}\eta_{48}P_{85} + P_{04}P_{48}\eta_{85} + \eta_{04}P_{49}P_{96} + P_{04}\eta_{49}P_{96} + P_{04}P_{49}\eta_{96}), \end{aligned}$$

$$\begin{aligned} h_2 = {} & (\eta_{10} + \eta_{14}P_{48}P_{87}P_{70} + P_{14}\eta_{48}P_{87}P_{70} + P_{14}P_{48}\eta_{87}P_{70} + P_{14}P_{48}P_{87}\eta_{70} \\ & + \eta_{14}P_{49}P_{97}P_{70} + P_{14}\eta_{49}P_{97}P_{70} + P_{14}P_{49}\eta_{97}P_{70} + P_{14}P_{49}P_{97}\eta_{70} + \eta_{12} \\ & + \eta_{14}P_{42} + P_{14}\eta_{42} + \eta_{14}P_{48}P_{85} + P_{14}\eta_{48}P_{85} + P_{14}P_{48}\eta_{85} + \eta_{14}P_{49}P_{96} \\ & + P_{14}\eta_{49}P_{96} + P_{14}P_{49}\eta_{96}), \end{aligned}$$

$$\begin{aligned} h_3 = {} & (\eta_{24}P_{48}P_{87}P_{70} + P_{24}\eta_{48}P_{87}P_{70} + P_{24}P_{48}\eta_{87}P_{70} + P_{24}P_{48}P_{87}\eta_{70} \\ & + \eta_{24}P_{49}P_{97}P_{70} + P_{24}\eta_{49}P_{97}P_{70} + P_{24}P_{49}\eta_{97}P_{70} + P_{24}P_{49}P_{97}\eta_{70} \\ & + \eta_{24}P_{42} + P_{24}\eta_{42} + \eta_{25} + \eta_{24}P_{48}P_{85} + P_{24}\eta_{48}P_{85} + P_{24}P_{48}\eta_{85} \\ & + \eta_{26} + \eta_{24}P_{49}P_{96} + P_{24}\eta_{49}P_{96} + P_{24}P_{49}\eta_{96}), \end{aligned}$$

$$h_4 = (\eta_{50} + \eta_{5(10)}P_{(10)1} + P_{5(10)}\eta_{(10)1} + \eta_{5(10)}P_{(10)(11)} + P_{5(10)}\eta_{(10)(11)} + \eta_{5(11)}),$$

$$h_5 = (\eta_{60} + \eta_{6(13)}P_{(13)1} + P_{6(13)}\eta_{(13)1} + \eta_{6(12)} + \eta_{6(13)}P_{(13)(12)} + P_{6(13)}\eta_{(13)(12)}),$$

$$h_6 = (\eta_{(11)3}P_{37}P_{70} + P_{(11)3}\eta_{37}P_{70} + P_{(11)3}P_{37}\eta_{70} + \eta_{(11)(10)}P_{(10)1} + P_{(11)(10)}\eta_{(10)1} + \eta_{(11)3}P_{31} + P_{(11)3}\eta_{31} + \eta_{(11)(10)}P_{(10)(11)} + P_{(11)(10)}\eta_{(10)(11)}),$$

$$h_7 = (\eta_{(12)3}P_{37}P_{70} + P_{(12)3}\eta_{37}P_{70} + P_{(12)3}P_{37}\eta_{70} + \eta_{(12)(13)}P_{(13)1} + P_{(12)(13)}\eta_{(13)1} + \eta_{(12)3}P_{31} + P_{(12)3}\eta_{31} + \eta_{(12)(13)}P_{(13)(12)} + P_{(12)(13)}\eta_{(13)(12)}).$$

$$b1 = \begin{vmatrix} 1 & -(L_7) & -(L_{10}) & -(L_{13}) & 0 & 0 \\ 0 & 1-(L_8) & -(L_{11}) & -(L_{14}) & 0 & 0 \\ -(L_2) & 0 & 1 & 0 & -(L_{15}) & 0 \\ -(L_3) & 0 & 0 & 1 & 0 & -(L_{17}) \\ -(L_4) & 0 & 0 & 0 & 1-(L_{16}) & 0 \\ -(L_5) & 0 & 0 & 0 & 0 & 1-(L_{18}) \end{vmatrix}$$

$$b_2 = \begin{vmatrix} -(L_1) & -(L_6) & -(L_9) & -(L_{12}) & 0 & 0 \\ 0 & 1-(L_8) & -(L_{11}) & -(L_{14}) & 0 & 0 \\ -(L_2) & 0 & 1 & 0 & -(L_{15}) & 0 \\ -(L_3) & 0 & 0 & 1 & 0 & -(L_{17}) \\ -(L_4) & 0 & 0 & 0 & 1-(L_{16}) & 0 \\ -(L_5) & 0 & 0 & 0 & 0 & 1-(L_{18}) \end{vmatrix}$$

$$b3 = \begin{vmatrix} -(L_1) & -(L_6) & -(L_9) & -(L_{12}) & 0 & 0 \\ 1 & -(L_7) & -(L_{10}) & -(L_{13}) & 0 & 0 \\ -(L_2) & 0 & 1 & 0 & -(L_{15}) & 0 \\ -(L_3) & 0 & 0 & 1 & 0 & -(L_{17}) \\ -(L_4) & 0 & 0 & 0 & 1-(L_{16}) & 0 \\ -(L_5) & 0 & 0 & 0 & 0 & 1-(L_{18}) \end{vmatrix}$$

$$b4 = \begin{vmatrix} -(L_1) & -(L_6) & -(L_9) & -(L_{12}) & 0 & 0 \\ 1 & -(L_7) & -(L_{10}) & -(L_{13}) & 0 & 0 \\ 0 & 1-(L_8) & -(L_{11}) & -(L_{14}) & 0 & 0 \\ -(L_3) & 0 & 0 & 1 & 0 & -(L_{17}) \\ -(L_4) & 0 & 0 & 0 & 1-(L_{16}) & 0 \\ -(L_5) & 0 & 0 & 0 & 0 & 1-(L_{18}) \end{vmatrix}$$

$$b5 = \begin{vmatrix} -(L_1) & -(L_6) & -(L_9) & -(L_{12}) & 0 & 0 \\ 1 & -(L_7) & -(L_{10}) & -(L_{13}) & 0 & 0 \\ 0 & 1-(L_8) & -(L_{11}) & -(L_{14}) & 0 & 0 \\ -(L_2) & 0 & 1 & 0 & -(L_{15}) & 0 \\ -(L_4) & 0 & 0 & 0 & 1-(L_{16}) & 0 \\ -(L_5) & 0 & 0 & 0 & 0 & 1-(L_{18}) \end{vmatrix}$$

$$b6 = \begin{vmatrix} -(L_1) & -(L_6) & -(L_9) & -(L_{12}) & 0 & 0 \\ 1 & -(L_7) & -(L_{10}) & -(L_{13}) & 0 & 0 \\ 0 & 1-(L_8) & -(L_{11}) & -(L_{14}) & 0 & 0 \\ -(L_2) & 0 & 1 & 0 & -(L_{15}) & 0 \\ -(L_3) & 0 & 0 & 1 & 0 & -(L_{17}) \\ -(L_5) & 0 & 0 & 0 & 0 & 1-(L_{18}) \end{vmatrix}$$

$$b7 = \begin{vmatrix} -(L_1) & -(L_6) & -(L_9) & -(L_{12}) & 0 & 0 \\ 1 & -(L_7) & -(L_{10}) & -(L_{13}) & 0 & 0 \\ 0 & 1-(L_8) & -(L_{11}) & -(L_{14}) & 0 & 0 \\ -(L_2) & 0 & 1 & 0 & -(L_{15}) & 0 \\ -(L_3) & 0 & 0 & 1 & 0 & -(L_{17}) \\ -(L_4) & 0 & 0 & 0 & 1-(L_{16}) & 0 \end{vmatrix}$$

Where

$$L_1 = P_{01} + P_{03}P_{31}, \quad L_2 = P_{5(10)}P_{(10)1}, \quad L_3 = P_{6(13)}P_{(13)1},$$
$$L_4 = P_{(11)(10)}P_{(10)1} + P_{(11)3}P_{31}, \quad L_5 = P_{(12)(13)}P_{(13)1} + P_{(12)3}P_{31},$$
$$L_6 = P_{02} + P_{04}P_{42}, \quad L_7 = P_{12} + P_{14}P_{42}, \quad L_8 = P_{24}P_{42},$$
$$L_9 = P_{04}P_{48}P_{85}, \quad L_{10} = P_{14}P_{48}P_{85}, \quad L_{11} = P_{25} + P_{24}P_{48}P_{85},$$
$$L_{12} = P_{04}P_{49}P_{96}, \quad L_{13} = P_{14}P_{49}P_{96}, \quad L_{14} = P_{26} + P_{24}P_{49}P_{96},$$
$$L_{15} = P_{5(10)}P_{(10)(11)} + P_{5(11)}, \quad L_{16} = P_{(11)(10)}P_{(10)(11)},$$
$$L_{17} = P_{6(12)} + P_{6(13)}P_{(13)(12)}, \quad L_{18} = P_{(12)(13)}P_{(13)(12)}.$$

$$N_1 = (M_0(s)b_1 - M_1(s)b_2 - M_5(s)b_4 + M_6(s)b_5),$$
$$D_1 = h_1b_1 - h_2b_2 + h_3b_3 - h_4b_4 + h_5b_5 - h_6b_6 + h_7b_7.$$

5 Busy Period Analysis

Let $G_i(t)$ be the probability that the repairman is busy due to repair of the failed failed unit at instant t, given that the system entered the regenerative state S_i at t = 0.

5.1 Expected busy period with first unit

By using probabilistic arguments, we obtain

$$\begin{aligned} G_0(t) = {} & (q_{03}(t) + q_{04}(t)) \circledast \overline{R_1} + (q_{03}(t) \circledast q_{37}(t) \circledast q_{70}(t) \\ & + q_{04}(t) \circledast q_{48}(t) \circledast q_{87}(t) \circledast q_{70}(t) + q_{04}(t) \circledast q_{49}(t) \circledast q_{97}(t) \circledast q_{70}(t)) \circledast G_0(t) \\ & + (q_{01}(t) + q_{03}(t) \circledast q_{31}(t))G_1(t) + (q_{02}(t) + q_{04}(t) \circledast q_{42}(t)) \circledast G_2(t) \\ & + (q_{04}(t) \circledast q_{48}(t) \circledast q_{85}(t)) \circledast G_5(t) + (q_{04}(t) \circledast q_{49}(t) \circledast q_{96}(t)) \circledast G_6(t), \end{aligned} \tag{14}$$

$$\begin{aligned} G_1(t) = {} & (1 + q_{14}(t)) \circledast \overline{R_1} + (q_{10}(t) + q_{14}(t) \circledast q_{48}(t) \circledast q_{87}(t) \circledast q_{70}(t) \\ & + q_{14}(t) \circledast q_{49}(t) \circledast q_{97} \circledast q_{70})G_0(t) + (q_{12}(t) + q_{14}(t) \circledast q_{42}) \circledast G_2(t) \\ & + (q_{14}(t) \circledast q_{48}(t) \circledast q_{85}) \circledast G_5(t) + (q_{14}(t) \circledast q_{49}(t) \circledast q_{96}(t)) \circledast G_6(t), \end{aligned} \tag{15}$$

$$\begin{aligned} G_2(t) = {} & (1 + q_{24}(t)) \circledast \overline{R_1} + (q_{24}(t) \circledast q_{48}(t) \circledast q_{87}(t) \circledast q_{70}(t) \\ & + q_{24}(t) \circledast q_{49}(t) \circledast q_{97}(t) \circledast q_{70}(t))G_0(t) + (q_{24}(t) \circledast q_{42}(t)) \circledast G_2(t) \\ & + (q_{25}(t) + q_{24}(t) \circledast q_{48}(t) \circledast q_{85}(t)) \circledast G_5(t) + (q_{26}(t) \\ & + q_{24}(t) \circledast q_{49}(t) \circledast q_{96}(t)) \circledast G_6(t), \end{aligned} \tag{16}$$

$$\begin{aligned} G_5(t) = {} & (q_{50}(t)) \circledast G_0(t) + (q_{5(10)}(t) \circledast q_{(10)1}(t)) \circledast G_1(t) \\ & + (q_{5(10)}(t) \circledast q_{(10)(11)}(t) + q_{5(11)}(t)) \circledast G_{11}(t) \end{aligned} \tag{17}$$

$$G_6(t) = (q_{60}(t)) \circledast G_0(t) + (q_{6(13)}(t) \circledast q_{(13)1}(t)) \circledast G_1(t) \\ +(q_{6(12)}(t) + q_{6(13)}(t) \circledast q_{(13)(12)}(t)) \circledast G_{12}(t), \tag{18}$$

$$G_{11}(t) = (q_{(11)3}(t)) \circledast \overline{R_1} + (q_{(11)3}(t)) \circledast q_{37}(t) \circledast q_{70}(t)) \circledast G_0(t) \\ +(q_{(11)(10)}(t) \circledast q_{(10)1}(t) + q_{(11)3}(t) \circledast q_{31}(t)) \circledast G_1(t) \\ +(q_{(11)(10)}(t) \circledast q_{(10)(11)}(t)) \circledast G_{11}(t), \tag{19}$$

$$G_{12}(t) = (q_{(12)3}(t)) \circledast \overline{R_1} + (q_{(12)3}(t)) \circledast q_{37}(t) \circledast q_{70}(t)) \circledast G_0(t) \\ +(q_{(12)(13)}(t) \circledast q_{(13)1}(t) + q_{(12)3}(t) \circledast q_{31}(t)) \circledast G_1(t) \\ +(q_{(12)(13)}(t) \circledast q_{(13)(12)}(t) \circledast G_{12}(t). \tag{20}$$

Using LT to solve equations (14), (15), (16), (17), (18), (19) and (20) for $G_0^*(s)$, We have the expected busy period with repair in steady state as follows

$$G_0 = G_0(\infty) = \frac{N_2}{D_1}, \tag{21}$$

where

$$N_2 = \overline{R}^*(0)\{(P_{03} + P_{04})b_1 - (1 + P_{14})b_2 + (1 + P_{24})b_3 - (P_{(11)3})b_6 + (P_{(12)3})b_7\}, \tag{22}$$

and

$$\overline{R}^*(0) = \frac{1}{\mu_1}.$$

5.2 Expected busy period with second unit

By using probabilistic arguments, we obtain

$$G'_0(t) = (q_{04}(t) \circledast q_{48}(t)) \circledast \overline{R'_2} + (q_{03}(t) \circledast q_{37}(t) \circledast q_{70}(t) \\ +q_{04}(t) \circledast q_{48}(t) \circledast q_{87}(t) \circledast q_{70}(t) + q_{04}(t) \circledast q_{49}(t) \circledast q_{97}(t) \circledast q_{70}(t)) \circledast G'_0(t) \\ +(q_{01}(t) + q_{03}(t) \circledast q_{31}(t))G'_1(t) + (q_{02}(t) + q_{04}(t) \circledast q_{42}(t)) \circledast G'_2(t) \\ +(q_{04}(t) \circledast q_{48}(t) \circledast q_{85}(t)) \circledast G'_5(t) + (q_{04}(t) \circledast q_{49}(t) \circledast q_{96}(t)) \circledast G'_6(t), \tag{23}$$

$$G'_1(t) = (q_{14}(t) \circledast q_{48}(t)) \circledast \overline{R'_2} + (q_{10}(t) + q_{14}(t) \circledast q_{48}(t) \circledast q_{87}(t) \circledast q_{70}(t) \\ +q_{14}(t) \circledast q_{49}(t) \circledast q_{97}(t) \circledast q_{70}(t))G'_0(t) + (q_{12}(t) + q_{14}(t) \circledast q_{42}(t)) \circledast G'_2(t) \\ +(q_{14}(t) \circledast q_{48}(t) \circledast q_{85}(t)) \circledast G'_5(t) + (q_{14}(t) \circledast q_{49}(t) \circledast q_{96}(t)) \circledast G'_6(t), \tag{24}$$

$$G'_2(t) = (q_{24}(t) \circledast q_{48}(t)) \circledast \overline{R'_2} + (q_{24}(t) \circledast q_{48}(t) \circledast q_{87}(t) \circledast q_{70}(t) \\ +q_{24}(t) \circledast q_{49}(t) \circledast q_{97}(t) \circledast q_{70}(t))G'_0(t) + (q_{24}(t) \circledast q_{42}(t)) \circledast G'_2(t) \\ +(q_{25}(t) + q_{24}(t) \circledast q_{48}(t) \circledast q_{85}(t)) \circledast G'_5(t) \\ +(q_{26}(t) + q_{24}(t) \circledast q_{49}(t) \circledast q_{96}(t)) \circledast G'_6(t), \tag{25}$$

$$G'_5(t) = (1 + q_{5(10)}(t)) \circledast \overline{R'_2} + (q_{50}(t)) \circledast G'_0(t) \\ +(q_{5(10)}(t) \circledast q_{(10)1}(t)) \circledast G'_1(t) + (q_{5(10)}(t) \circledast q_{(10)(11)}(t) \\ +q_{5(11)}(t)) \circledast G'_{11}(t), \tag{26}$$

$$G'_6(t) = (q_{60}(t)) \circledast G'_0(t) + (q_{6(13)}(t) \circledast q_{(13)1}(t)) \circledast G'_1(t) \\ +(q_{6(12)}(t) + q_{6(13)}(t) \circledast q_{(13)(12)}(t)) \circledast G'_{12}(t), \tag{27}$$

$$G'_{11}(t) = (1 + q_{(11)(10)}(t)) \circledast \overline{R'_2} + (q_{(11)3}(t)) \circledast q_{37}(t) \circledast q_{70}(t)) \circledast G'_0(t) \\ +(q_{(11)(10)}(t) \circledast q_{(10)1}(t) + q_{(11)3}(t) \circledast q_{31}(t)) \circledast G'_1(t) \\ +(q_{(11)(10)}(t) \circledast q_{(10)(11)}(t)) \circledast G'_{11}(t), \tag{28}$$

$$G'_{12}(t) = (q_{(12)3}(t)) \circledast q_{37}(t) \circledast q_{70}(t)) \circledast G'_0(t) + (q_{(12)(13)}(t) \circledast q_{(13)1}(t)$$
$$+q_{(12)3}(t) \circledast q_{31}(t)) \circledast G'_1(t) + (q_{(12)(13)}(t) \circledast q_{(13)(12)}(t) \circledast G'_{12}(t). \tag{29}$$

Using LT to solve equations (23), (24), (25), (26), (27), (28) and (29) for $G^*_0(s)$, We have the expected busy period with repair in steady state as follows

$$G'_0 = G'_0(\infty) = \frac{N_3}{D_1}, \tag{30}$$

where

$$N_3 = \overline{R}'^*(0)\{(P_{04}P_{48})b_1 - (P_{14}P_{48})b_2 + (P_{24}P_{48})b_3 - (1+P_{5(10)})b_4 - (1+P_{(11)(10)})b_6\}, \tag{31}$$

and

$$\overline{R}'^*(0) = \frac{1}{\mu_2}.$$

5.3 Cost benefit analysis

This section, we calculate the expected profit to the system in the period $(0,t]$ by calculating the deference between total revenue and total cost of repair

$$C(t) = K_1\omega_{up}(t) - K_2\omega_r(t) - K_3\omega_{r'}(t), \tag{32}$$

Where, K_1 is the revenue per unit of up time, K_2 is cost per unit of repair of the first unit, and K_3 is cost per unit of repair of the second unit .

$$\omega_{up}(t) = \int_0^t AV_0(t)dt, \tag{33}$$

$$\omega_r(t) = \int_0^t G_0(t)dt, \tag{34}$$

$$\omega_{r'}(t) = \int_0^t G'_0(t)dt. \tag{35}$$

using (5.33), (5.34) and (5.35) we obtain

$$C^*(s) = K_1\omega^*_{up}(s) - K_2\omega^*_r(s) - K_3\omega^*_{r'}(s).$$

Therefore the expected revenue per unit time in steady state is given by

$$C = \lim_{t\to\infty}\frac{C(t)}{t} = \lim_{s\to 0} s^2 C^*(s) = \frac{K_1N_1 - K_2N_2 - K_3N_3}{D_1}. \tag{36}$$

6 Numerical Example

By setting $K_1 = 500, K_2 = 5, K_3 = 2$, figures display the variation of MTSF, Availability, Busy period1, Busy period2 and Cost benefit, for different values of $\theta, \tau, \beta, \alpha, \mu_1, \mu_2, z, \lambda_1$ *and* λ_2.

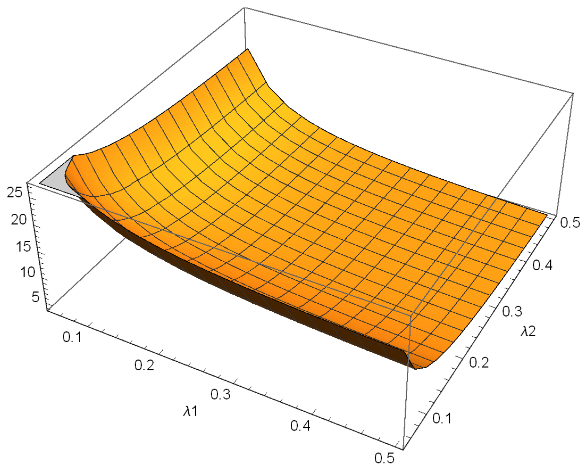

Fig. 1: MTSF with $\theta = 0.8$, $\tau = .99$, $\beta = .005$, $\alpha = .001$, $\mu_1 = .01$, $\mu_2 = .002$, $z = .03$, $\lambda_1 = 0.05$ *to* 0.5 and $\lambda_2 = 0.05$ *to* 0.5.

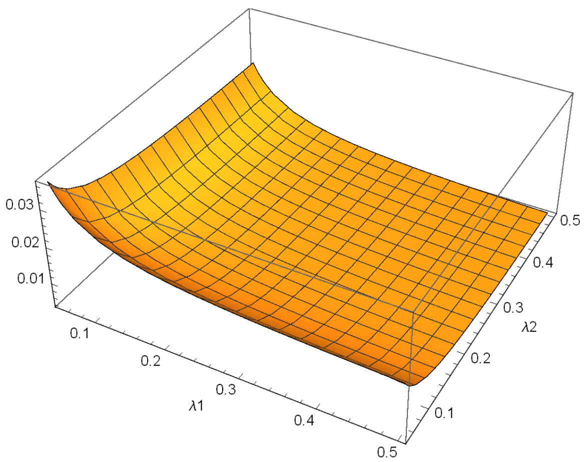

Fig. 2: Availability with $\theta = 0.8$, $\tau = .99$, $\beta = .005$, $\alpha = .001$, $\mu_1 = .01$, $\mu_2 = .002$, $z = .03$, $\lambda_1 = 0.05$ *to* 0.5 and $\lambda_2 = 0.05$ *to* 0.5.

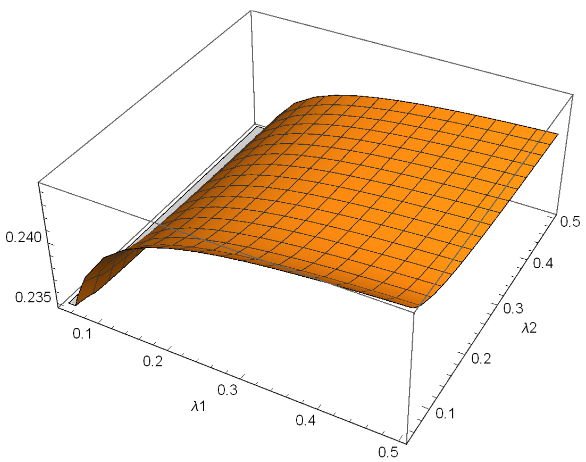

Fig. 3: Busy period1 with $\theta = 0.8$, $\tau = .99$, $\beta = .005$, $\alpha = .001$, $\mu_1 = .01$, $\mu_2 = .002$, $z = .03$, $\lambda_1 = 0.05$ *to* 0.5 and $\lambda_2 = 0.05$ *to* 0.5.

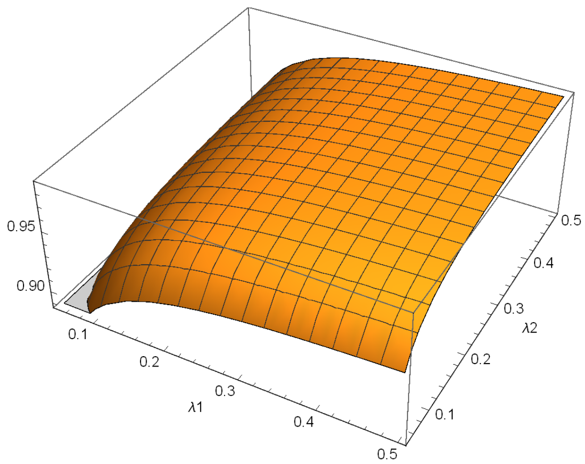

Fig. 4: Busy period2 with $\theta = 0.8$, $\tau = .99$, $\beta = .005$, $\alpha = .001$, $\mu_1 = .01$, $\mu_2 = .002$, $z = .03$, $\lambda_1 = 0.05\ to\ 0.5$ and $\lambda_2 = 0.05\ to\ 0.5$.

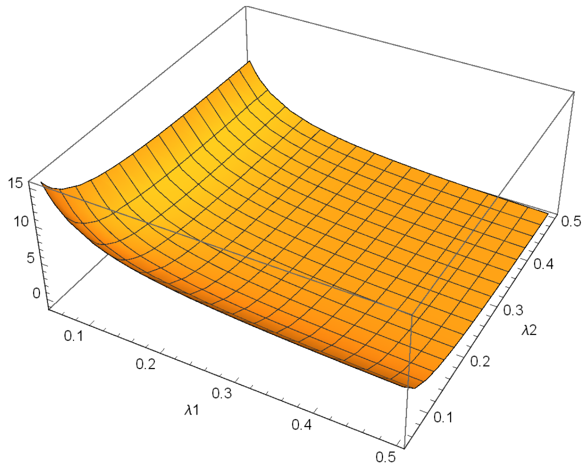

Fig. 5: Cost benefit with $\theta = 0.8$, $\tau = .99$, $\beta = .005$, $\alpha = .001$, $\mu_1 = .01$, $\mu_2 = .002$, $z = .03$, $K_1 = 500, K_2 = 5, K_3 = 2$, $\lambda_1 = 0.05\ to\ 0.5$ and $\lambda_2 = 0.05\ to\ 0.5$.

7 Conclusion

This paper provides the reliability analysis for a warm standby repairable system that consists of two dissimilar units. One of these units is a good quality unit while the other one is of substandard quality that might need some repairs or replacement by another substandard unit upon failure. The system works under two different weather conditions, normal and abnormal. The unit operates under normal weather conditions, but incase of abnormal weather conditions, the system stops and the unit fails. we successfully obtained some reliability measures of the system such as, the MTSF, the availability analysis, the expected busy period and the expected profit of the system.

- The mean time to system failure increases with decreasing the failure rate of unit $1(\lambda_1)$ and the failure rate of unit 2 (λ_2).
- The Availability increases with decreasing the failure rate of unit $1(\lambda_1)$ and the failure rate of unit 2 (λ_2).
- The busy period with first and second unit increase with increasing the failure rate of unit $1(\lambda_1)$ and the failure rate of unit 2 (λ_2).
- The cost benefit increases with decreasing the failure rate of unit $1(\lambda_1)$ and the failure rate of unit 2 (λ_2).

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