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A Generalized Class of Circular Designs Strongly Balanced for Neighbor Effects

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Abstract: Minimal strongly balanced neighbor designs are useful (i) to minimize the bias due to neighbor effects economically, and (ii) to estimate the direct effect and neighbor effects independently. Such designs can easily be obtained for v odd and are available in literature. In this article, A generalized class of minimal circular designs strongly balanced for neighbor effects in blocks of equal and two different sizes have been constructed in which only $v/2$ unordered pairs of treatments do not appear as neighbors, where v is the number of treatments.

Keywords: Bias due to neighbor effects; Neighbor designs; Minimal designs; Neighbor balanced designs.
Mathematics Subject Classification (2010): 05B05; 62K10; 62K05.

1 Introduction

Minimal strongly balanced neighbor designs (SBNDs) are well known to balance the neighbor effects economically as well as to estimate the direct and neighbor effects independently. These minimal designs can be constructed only for odd number of treatments (v). If each treatment has all other treatments as its neighbors exactly once, (i) excluding itself, design is neighbor balanced, (ii) including itself, design is strongly balanced. A circular design is called minimal generalized strongly balanced neighbor designs-I (MCGSBNDs-I) in which only v/2 unordered pairs of treatments do not appear as neighbors while remaining ones appears once. [1] suggested neighbor balanced designs (NBDs) in non-circular blocks. [2] introduced neighbor designs in research of virus. [3] and [4] showed that NBDs minimize the bias due to neighbor effects. [5] gave the brief review on NBDs since 1967. [6] derived the designs which are totally balanced to estimate direct and neighbor effects. [7] constructed the one sided right neighbors designs. [8] developed some methods to construct circular NBDs. [9] suggested that partially balanced neighbor designs (PBNDs) should be used if minimal NBDs cannot be generated. [10] presented minimal circular PBNDs (MCPBNDs) for some specific cases. [11] developed a series of CPBNDs for v = n. [12] presented two new series of non-binary CPBNDs. In this article, MCGSBNDs-I are constructed in blocks of equal sizes and two different sizes.

2 Method of cyclic shifts

This method was developed by [13]. Logic behind of its Rule I is described here to construct MCGSBNDs. Consider in this article, $m = (v-2)/2$ and '$v-a$'as the complement of an element 'a'.

- $A = [1, 2, ..., m]$ will produce MCGSBND-I for $v = 2ik$ if sum of A is divisible by v otherwise, to make it replace one or more values with their complements.

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Divide the resultant elements of A in i classes of size k such that the sum of each class should be divisible of v. Then delete any one value from each class, to get i sets of shifts which generate MCGSBNDs-I in equal blocks sizes.

Example 2.1. $[4,5,7]$ and $[1,2,3]$ produce MCSPBND-I for $v = 16$ and $k = 4$.

Proof: Since $m = 7$ for $v = 16$, therefore, consider $A = [0,1,2,..,7]$ to get MCGSBND-I. Sum of elements of A is 28. To make the sum divisible by 16, replace '6'with its complement '10'. Resultant $A = [0,1,2,3,4,5,10,7]$ is divided into two groups $[0,4,5,7]$ and $[1,2,3,10]$ of size 4. Deleting one element of each group, $[4,5,7]$ and $[1,2,3]$ will produce MCGSBND-I.

To complete the array from a set $[4,5,7]$, take 16 blocks. Assign $0,1,2,...,15$ in first cell of each block. For second cell elements, add 4 (mod 16) to each of first cell element. For third cell elements, add 5 (mod 16) to each of second cell element. Then add 7.

Table 1. Arrays obtained from $[4,5,7]$

B1	B2	B3	B4	B5	B6	B7	B8	B9	B10	B11	B12	B13	B14	B15	B16
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
4	5	6	7	8	9	10	11	12	13	14	15	0	1	2	3
9	10	11	12	13	14	15	0	1	2	3	4	5	6	7	8
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Take 16 more blocks and complete the arrays from [1,2,3] in the similar way.

Table 2. Arrays obtained from $[1,2,3]$

B17	B18	B19	B20	B21	B22	B23	B24	B25	B26	B27	B28	B29	B30	B31	B32
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	0
3	4	5	6	7	8	9	10	11	12	13	14	15	0	1	2
6	7	8	9	10	11	12	13	14	15	0	1	2	3	4	5

Table 1 and Table 2 jointly present the MCGSBND-I for $v = 16$ and k = 4. In this design, unordered pairs (0,8), (1,9), (2,10), (3,11), (4,12), (5,13), (6,14), (7,15) will not appear as neighbors.

3 Constructors to obtain MCGSBNDs-I

In this section, Constructors are developed, using Rule I to obtain MCGSBNDs-I for $v = 2ik$. In this study $m = (v-2)/2$.

Constructor 3.1: If $m(\text{mod } 4) \equiv 3$, then MCGSBNDs-I can be obtained from $A = [0,1,2,...,(3m-1)/4,(3m+7)/4,(3m+11)/4,...,m,5(m+1)/4]$.

Proof: Let S be sum of elements in A.
$S = 0+1+2+...+[(3m-1)/4]+[(3m+7)/4]+[(3m+11)/4]+...+m+[5(m+1)/4]$
$= 0+1+2+...+[(3m-1)/4]+[(3m+3)/4]+[(3m+7)/4]+[(3m+11)/4]+...+m+[5(m+1)/4]-[(3m+3)/4]$
$= [0+1+2,...+m]+[(2m+2)/4]$
$= [m(m+1)/2]+2(m+1)/4$
$= 2(m+1)(m+1)/4$ Since $v = 2(m+1)$
$= v(m+1)/4.$ As m (mod 4) $\equiv$ 3 then $(m+1)/4$ will be integer.

Constructor 3.2: If $m(\text{mod } 4) \equiv 0$, then MCGSBNDs-I can be obtained from $B = [0,1,2,...,m]$.

Proof: Let S be sum of elements in B.
$S = 0+1+2+,...+m$
$= (m+1)m/2$
$= 2(m+1)m/4$ Since $v = 2(m+1)$
$= v(m/4)$ As m (mod 4) $\equiv$ 0 then $m/4$ will be integer.

Hence proved that S is divisible by v.

4 Construction of MCGSBNDs-I in equal block sizes

4.1 MCGSBNDs-I in equal block sizes for m (mod 4) $\equiv 3$

Here, MCGSBNDs-I are constructed from i sets of shifts for $v = 2ik$ and m (mod 4) $\equiv 3$. These sets will be generated from $A = [0,1,2,...,(3m-1)/4,(3m+7)/4,(3m+11)/4,...,m,5(m+1)/4]$ in the following manner.

- Divide values of A in i classes of size k such that sum of values in each class is divisible by v.
- Deleting any one value from each class, resulting are i sets of shifts which produce the MCGSBND-I.

Construction 4.1.1. MCGSBNDs-I can be produced from i sets for $v = 2ik$, $k = 4l$, i and l integer.

Example 4.1.1. For $v = 8, k = 4, m = 3, l = 1, i = 1$ then $B = [0,1,2,5]$. Hence MCGSBNDs-I can be obtained from [1, 2, 5] for $v = 8$ and $k = 4$.

Designs constructed through this method for $v \leq 100$ and k = 4, 8, 12 and 16 are presented as Table 1 in Appendix.

Construction 4.1.2. MCGSBNDs-I may be produced from i sets for $v = 2ik, k = 4l+2, i$ even and l integer.

Example 4.1.2. $S_1 = [1,11,3,4,5]$ and $S_2 = [6,7,8,10,15]$ produce MCGSBNDs-I for $v = 24$ and k = 6.

Designs constructed through this method for $v \leq 100$ and k = 6, 10, 14 and 18 are presented as Table 2 in Appendix.

Construction 4.1.3. MCGSBNDs-I may be produced from i sets for $v = 2ik, i = 4w+3$, k (odd) > 3.

Example 4.1.3. $S_1 = [1,4,17,18]$, $S_2 = [6,7,8,9]$, $S_3 = [12,13,14,16]$ and $S_4 = [2,3,5,11]$ produce MCGSBNDs-I for $v = 40$ and $k = 5$.

Designs constructed through this method for $v \leq 100$ and $5 \leq k$ (odd) ≤ 11 are presented as Table 3 in Appendix.

4.2 MCGSBNDs-I in equal block sizes for m (mod 4) $\equiv 0$

In this Section, MCGSBNDs-I are constructed from i sets of shifts for $v = 2ik$ and m (mod 4) $\equiv 0$. These sets will be generated from $B = [0,1,2,..,m]$ in the following manner.

- Divide values of B in i classes of k size such that sum of values in each group is divisible by v.
- Deleting one value (any) from each group, resulting are i sets of shifts which produce MCGSBND-I.

Construction 4.2.1. MCGSBNDs-I may be produced from i sets for $v = 2ik$, $i = 4w+1$, $k = 4u+1$.

Example 4.2.1. $S_1 = [4,9,12,22]$, $S_2 = [6,7,8,24]$, $S_3 = [10,11,13,15]$, $S_4 = [19,20,21,23]$ and $S_5 = [2,14,16,18]$ produce MCGSBNDs-I for $v = 50$ and k = 5.

Designs constructed through this method for $v \leq 100$, k = 5, 9 and 13 are presented as Table 4 in Appendix.

Construction 4.2.2. MCGSBNDs-I may be constructed from i sets for $v = 2ik$, $i = 4w+3$, $k = 4u+3$.

Example 4.2.2. $S_1 = [1,2,3,4,5,7]$, $S_2 = [9,10,11,12,13,14]$ and $S_3 = [6,8,16,17,18,19]$ produce MCGSBNDs-I for $v = 42$ and k = 7.

Designs constructed through this method for $v \leq 100$, k = 7, 11 and 15 are presented as Table 5 in Appendix

5 Construction of MCGSBNDs-I in two different block sizes

5.1 MCGSBNDs-I in block sizes k_1 and k_2 for m (mod 4) $\equiv 3$

In this Section, MCGSBNDs-I can be produced from (i+1) sets of shifts for $v = 2ik_1 + 2k_2$, and m (mod 4) $\equiv 3$. These sets will be generated from $A = [0,1,2,...,(3m-1)/4,(3m+7)/4,(3m+11)/4,...,m,5(m+1)/4]$ in the following manner.
(i) Divide values of A in i classes of k_1 size and one of k_2 size such that sum of the values in each class is divisible by v.
(ii) Deleting any one value from each class, resulting are $i+1$ sets which produce the required design.

- $k_2 = k_1 - 1$

Construction 5.1.1. MCGSBNDs-I may be produced from $(i+1)$ sets for $v = 2ik_1 + 2k_2$, $k_1 = 4u+1$, $k_2 = k_1 - 1$, $i = 4w$.

Example 5.1.1. $[2,9,16,20],[5,10,11,22],[7,8,13,14],[19,21,23,30]$ and $[12,15,17]$ produce MCGSBNDs-I for $v = 48$, $k_1 = 5$ and $k_2 = 4$.

Construction 5.1.2. MCGSBNDs-I may be produced from $(i+1)$ sets for $v = 2ik_1 + 2k_2$, $k_1 = 4u+3$, $k_2 = k_1 - 1$, $i = 4w+2$.

Example 5.1.2. $[4,5,9,16,19,25],[7,11,13,14,17,18]$ and $[3,6,8,10,12]$ produce MCGSBNDs-I for $v = 40$, $k_1 = 7$ and $k_2 = 6$.

- $k_2 = k_1 - 2$

Construction 5.1.3. MCGSBNDs-I may be produced from $(i+1)$ sets for $v = 2ik_1 + 2k_2$, $k_1 = 4l+2$, $k_2 = k_1 - 2$, i even.

Example 5.1.3. $[3,4,5,8,10],[7,9,13,14,15]$ and $[1,11,20]$ produce MCGSBNDs-I for $v = 32$, $k_1 = 6$ and $k_2 = 4$.

Construction 5.1.4. MCGSBNDs-I may be produced from $(i+1)$ sets for $v = 2ik_1 + 2k_2$, k_1 (odd) > 3, $k_2 = k_1 - 2, i = 4w+1$.

Example 5.1.4. $[3,6,9,10,11,12]$ and $[4,5,7,8]$ produce MCGSBNDs-I for $v = 24$, $k_1 = 7$ and $k_2 = 5$.

5.2 MCGSBNDs-I in block sizes k_1 and k_2 for m (mod 4) $\equiv 0$

In this Section, MCGSBNDs-I can be produced from $(i+1)$ sets of shifts for $v = 2ik_1 + 2k_2$, and m (mod 4) $\equiv 0$. These sets will be generated from $B = [0,1,2,...,m]$ in the following manner.
(i) Divide values of B in i classes of k_1 size and one set of k_2 size such that sum of the values in each class is divisible by v.
(ii) Deleting any one value from each class, resulting are $i+1$ sets of shifts which produce the required design.

- $k_2 = k_1 - 1$

Construction 5.2.1. MCGSBNDs-I may be produced from $(i+1)$ sets for $v = 2ik_1 + 2k_2$, $k_1 = 2+4l$, $k_2 = k_1 - 1$, i even and l integer.

Example 5.2.1. $[3,5,6,7,11],[4,8,9,12]$ and $[13,14,15,16]$ produce MCGSBNDs-I for $v = 34$, $k_1 = 6$ and $k_2 = 5$.

Construction 5.2.2. MCGSBNDs-I may be produced from $(i+1)$ sets for $v = 2ik_1 + 2k_2$, k_1 (odd) > 3, $k_2 = k_1 - 1, i = 4w+1$.

Example 5.2.2. $[3,4,5,6]$ and $[2,7,8]$ produce MCGSBNDs-I for $v = 18$, $k_1 = 5$ and $k_2 = 4$.

- $k_2 = k_1 - 2$

Construction 5.2.3. MCGSBNDs-I may be produced from $(i+1)$ sets for $v = 2ik_1 + 2k_2$, $k_1 = 4u+1$, $k_2 = k_1 - 2$, $i = 4w+2$.

Example 5.2.3. $[3,5,7,11], [2,6,8,9]$ and $[10,12]$ produce MCGSBND-I for $v = 26$, $k_1 = 5$ and $k_2 = 3$.

Construction 5.2.4. MCGSBNDs-I may be produced from $(i+1)$ sets for $v = 2ik_1 + 2k - 2$, $k_1 = 4u+3$, $k_2 = k_1 - 2$, $i = 4w$.

Example 5.2.4. $[8,16,24,25,26,29], [11,15,17,27,30,31], [3,5,6,10,19,21], [13,14,22,23,28,32]$ and $[9,12,18,20]$ produce MCGSBNDs-I for $v = 66$, $k_1 = 7$ and $k_2 = 5$.

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Appendix

Table 1: MCGSBNDs-I for $v \leq 100$ and $k = 4, 8, 12$ and 16

v	k	Set(s) of Shifts
8	4	[1,2,5]
16	4	[3,4,7]+[1,5,10]
24	4	[2,10,11]+[5,7,8]+[3,6,15]
32	4	[2,9,20]+[6,7,15]+[8,10,11]+[5,13,14]
40	4	[3,17,18]+[6,8,25]+[9,13,14]+[10,11,12]+[5,16,19]
48	4	[6,20,21]+[7,9,30]+[10,11,19]+[12,14,17]+[13,15,16]+[3,22,23]
56	4	[3,16,35]+[7,19,24]+[10,11,27]+[12,14,25]+[15,18,22]+[9,17,26]+[13,20,23]
64	4	[3,19,40]+[9,22,25]+[11,15,28]+[13,14,30]+[16,18,29]+[12,17,31]+[6,26,27]+[20,21,23]
72	4	[3,22,45]+[7,25,34]+[9,20,35]+[14,15,30]+[17,18,21]+[19,23,29]+[10,24,33]+ [11,26,31]+[12,28,32]
80	4	[3,25,50]+[7,29,38]+[11,28,32]+[14,15,39]+[13,26,37]+[20,21,23]+[22,24,33]+ [8,31,36]+[17,18,35]+[19,27,34]
88	4	[3,28,55]+[12,34,35]+[14,22,39]+[15,24,43]+[16,26,38]+[21,23,25]+[9,36,42]+ [18,29,30]+[10,32,41]+[20,27,37]+[17,31,40]
96	4	[3,31,60]+[7,40,44]+[19,28,39]+[14,27,42]+[17,26,45]+[22,23,30]+[24,25,32]+ [20,29,46]+[11,34,47]+[16,33,38]+[12,37,41]+[18,35,43]
16	8	[1,2,3,4,5,7,10]
32	8	[9,10,11,13,14,15,20]+[1,2,3,5,6,7,8]
48	8	[30,16,20,21,22,23,9]+[10,11,12,13,14,15,17]+[1,2,5,6,7,8,19]
64	8	[2,3,5,6,7,9,31]+[10,11,12,14,15,18,40]+[17,20,25,27,28,29,30]+[4,13,19,21,22,23,26]
80	8	[2,3,4,5,7,8,50]+[12,13,14,15,20,36,39]+[16,19,21,23,24,25,26]+[22,32,33,34,35,37,38]+ [10,17,18,27,28,29,31]
96	8	[5,6,7,32,38,41,60]+[10,11,12,13,14,15,20]+[18,19,21,22,23,25,47]+[28,29,31,40,42,45,46]+ [4,8,33,34,35,37,39]+[9,16,24,26,30,43,44]
24	12	[1,2,3,4,5,6 ,7,8,10,11,15]
48	12	[6,9,14,16,19 ,20,17,22,23,30]+[2,3,4,7,8 ,10,11,12,13,21]
72	12	[24,26,28,30,32 ,31,33,34,35,45]+[14,15,16,18,19 ,20,21,22,23,25]+ [1,2,3,5,7 ,8,9,10,11,12]
96	12	[24,38,39,41,43 ,44,45,46,47,60]+[6,14,15,17,18 ,19,20,21,22,23]+ [27,28,29,31,32 ,34,33,35,37,42]+[2,3,4,7,8 ,9,10,11,12,25]
32	16	[1,2,3,4,5,6,7,8,9,10,11,13,14,15,20]
64	16	[17,18,19,20,21,22,23,25,26,27,29,28,30,31,40]+[1,2,3,4,5,6,7,9,10,11,12,13,14,15,16]
96	16	[18,17,34,35,37,38,39,40,42,43,44,45,46,60,47]+ [33,19,20,21,22,23,24,25,27,28,29,30,31,32,18]+[3,4,5,6,7,9,10,11,12,13,14,15,16,26,41]

Table 2: MCGSBNDs-I for $v \leq 100$ and $k = 6, 10, 14$ and 18

v	k	Set(s) of Shifts
24	6	[6,7,8,10,15]+[1,3,4,5,11]
48	6	[6,16,19,23,30]+[7,8,9,11,12]+[13,15,17,20,21]+[3,4,5,14,22]
72	6	[2,3,5,30,31]+[8,9,10,11,28]+[16,17,32,33,34]+[20,22,23,25,35]+ [13,24,26,29,45]+[4,14,15,18,21]
96	6	[3,4,5,22,60]+[9,10,11,12,46]+[14,15,16,17,21]+[7,19,20,23,26]+ [28,29,30,42,45]+[31,32,35,33,37]+[25,27,43,44,47]+[34,38,39,40,41]
40	10	[3,4,5,6,7,8,9,17,19]+[1,10,11,12,13,14,16,18,25]
80	10	[2,3,4,5,6,7,8,9,35]+[12,13,14,15,16,17,19,18,25]+[20,22,23,24,26,27,28,29,31]+ [21,32,33,34,36,37,38,39,50]
56	14	[14,15,16,17,18,19,20,23,24,25,26,27,35]+[2,3,4,5,6,7,8,9,10,11,12,13,22]
72	18	[3,4,5,6,7,8,9,10,11,13,12,14,15,16,17,29,35]+ [1,18,19,20,21,22,23,24,25,26,28,30,31,32,33,34,45]

Table 3: MCGSBNDs-I for $v \leq 100$ and $5 \leq k$ (odd) ≤ 11

v	k	Set(s) of Shifts
40	5	[1,17,18,4]+[6,7,8,9]+[16,12,13,14]+[11,2,3,19]
80	5	[36,2,3,4]+[32,6,8,9]+[21,12,13,14]+[16,17,18,19]+[11,22,23,24]+[26,27,28,29]+ [7,1,33,34]+[31,37,38,39]
56	7	[2,3,4,5,6,35]+[9,10,15,17,26,27]+[13,14,16,18,19,20]+[7,11,22,23,24,25]
72	9	[3,4,5,6,7,9]+[11,12,13,14,15,16]+[20,23,24,26,25,31]+[8,21,22,28,29,30]
88	11	[2,3,4,5,6,7,8,10,9,33]+[25,33,34,37,38,39,40,41,42,43]+ [22,26,27,28,30,29,31,32,35,36]+[13,14,15,16,17,18,19,20,21,23]

Table 4: MCGSBNDs-I for $v \leq 100$, $k = 5, 9$ and 13

v	k	Set(s) of Shifts
10	5	[1,2,3,4]
50	5	[4,9,12,22]+[6,7,8,24]+[10,11,13,15]+[19,20,21,23]+[2,14,16,18]
90	5	[3,6,39,40]+[9,14,31,35]+[11,13,16,42]+[15,18,22,25]+[17,19,23,24]+[36,37,38,41]+ [12,20,26,27]+[30,34,43,44]+[4,21,32,33]
18	9	[1,2,3,4,5,6]
90	9	[3,4,5,6,7,9]+[12,13,14,15,16,17]+[23,24,25,26,31,36]+[19,20,29,34,35,37]+ [1,8,21,27,28,30]
26	13	[1,2,3,4,5,6,7,8,9,10,11,12]

Table 5: MCGSBNDs-I for $v \leq 100$, $k = 7, 11$ and 15

v	k	Set(s) of Shifts
42	7	[2,3,4,5]+[15,11,12,13]+[6,8,16,17]
98	7	[3,4,5,7]+[9,10,11,12]+[18,19,20,29]+[23,25,26,27]+[24,31,32,33]+ [15,17,21,42]+[6,36,37,38]
66	11	[2,3,4,5,6,7,8,9]+[2,3,4,5,6,7,8,9]+[13,14,15,16,17,19,18,22]
90	15	[44,4,5,6,7,8,9,10,11,12,13,32,14,3]+[43,18,19,20,21,22,23,25,24,27,26,29,30,17]+ [1,15,28,31,33,34,35,36,37,38,39,40,41,42]