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The Approach Repetition Rate Efficiency in Memorable Iterated Prisoner Dilemma Game

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Abstract: The Iterated Prisoner's Dilemma game has been widely matched with many applications in insurance, business, military, and biology to reach the optimum decision. So various aspects of this point have been discussed by researchers over the years. Besides, the use of the pre-previous unit to generate the new round has been also studied. However, the effect of repeating different approaches (regimes) in a single competition was not thoroughly investigated in previous studies. Therefore, the present study addresses the impact of repeating different approaches with different rates on the behaviour and payoff of strategies. Then, the payoff matrix was constructed for the sixteen strategies only generated by two-state automata and then analyzed using a Maple program. Taking into account the possibility of error in the player implementation of the strategy, this study demonstrated that defective strategies performed well over cooperative strategies

Keywords: Iterated games, Prisoner's dilemma, Finite automata, Game dynamics, Transition matrix, perturbed payoff.

1 Introduction

One of the main challenges faced by many mathematicians is the complex calculations in many mathematical models. Therefore, game theory is playing an important role in solving these complex mathematical models. Various theories in mathematical history illustrate the positive role of game theory [1,2,3,4,5].

Previous studies have proven the game theory's importance in policy, economy, biology, military, and many other sciences [6,7,8,9,10,11,12,13,14,15,16]. These studies have documented that the game is a competition among players who are looking for achieving goals [17,18,19,20]. The criteria of these studies are based on choosing between the game of chance, such as roulette, and the game of strategies, such as poker.

Iterated Prisoner Dilemma (IPD) is a game of strategies. The main idea of this game is the dilemma of the two players is to take one of two available decisions, namely, co-operation (deny) (C) or defection (confess) (D). Where the payoff of the two players will be (R) (Reward) if both players co-operate and will be (P) (Punish) if both defect. If one player plays (C) and the other plays (D), the co-operator payoff will be (S) (Sucker) and the defector will be (T) (Temptation) [21].

Research into repeated games has a long history. A lot of previous researches on IPD have focused on generating the new unit using the outcome of the immediately-previous one (memory one). However, delay or lack of knowledge of the immediately- previous unit is one of the most common problems reported in the single memory case [22]. Moreover, the impact of approach repetition rate in one competition on the average payoff was not thoroughly discussed in previous studies.

Therefore, the goals of the present research are the following: (1) to explore the generating of a new unit from the pre-previous one (memory two) [22], (2) to shed more light on calculating the payoff on the conflict between each strategy and the rest of the strategies used under noise effect, (3) to discuss the effect of approach repetition rate in every competition in the Iterated Prisoner Dilemma (IPD) player's payoff. The use of Maple makes it easy to include the approaches repetition rate in the payoff calculation process. The effect of noise on payoff calculations is also addressed. The remainder of

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the paper is organized as follows. The details of the methodology adopted in the study are explained in the next section. The results are presented and discussed next and the paper wraps up with the key findings and main conclusions.

2 States Generation Technique

In this study, the pre-previous unit has been used instead of the preceding unit immediately to increase the prisoner's dilemma memory. Thus, the first unit is used to generate the third unit, and the second unit is used to generate the fourth unit, and so on. The pre-previous unit is used because the previous unit is not known yet.

Unfortunately, sequential unit generation is associated with an increase in the number of strategies [23,24,25, 26]. So, finite two-state automata has a pivotal role in organizing the myriad of current strategies because it only outlines sixteen strategies to use [27].

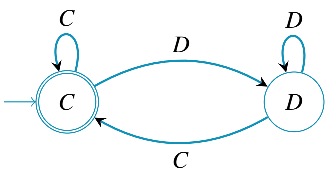

Fig. 1: Automata of TFT strategy

This research uses the two main traditional conditions $T > R > P > S$ and $2R > T + S$. Imhof (2007) pointed out that the player who chooses mutual cooperation receives a payoff higher than that in the case of switching between cooperation and defection [28]. PD is often described by this matrix.

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} R & S \\ T & P \end{pmatrix} \end{array} \tag{1}$$

The payoffs R, S, T, and P respectively, correspond to outcomes (C, C), (C, D), (D, C), and (D, D). The digital representation of strategies consists of zeroes and ones quadruples. Each digit represents the player's reaction when one of the four possible outcomes of each round appears (CC, CD, DC, and DD). The player's next decision will be D for 0 or C for 1. (1, 0, 0, 0) is called the Grim strategy which is the digital representation of S_8, and (1, 0, 1, 1) is called Tweedledee and represents S_{11}.

3 Contention Between S_8 Against S_{11}

After the imposition of the first two units, the generation of the new units will be as follows (All the payoffs that will be mentioned will be for the S_8-player).

In the first sequence, the two players are assumed to play (C) in the first two units, and then the new states are generated according to the states generation technique described previously, resulting in players playing (C) in each round. Since the payoff per unit is R, the repetition period of this sequence is one unit. This produces an average payoff with a value of (R). Therefore, this sequence and any other sequence having the same payoff value (R) will be called the approach X.

S_8 :	C	C	C	C	C	C	C	Average
S_{11}:	C	C	C	C	C	C	C	Payoff
Payoff	R	R	R	R	R	R	R	R

In the second sequence, the S_8-player is assumed to play (C) in the first two units and the S_{11}-player is assumed to play (C) in the first unit and (D) in the second. The new units are produced by the same previously explained technique. The repetition period of this sequence is four units with payoffs (R, T, R, P). The value of the average payoff for this sequence will be (2R+P+T)/4. Thus, any sequence that has a payoff (2R+P+T)/4 will be called the approach Y.

S_8:	C	C	C	D	C	D	C	Average
S_{11}:	C	D	C	C	C	D	C	Payoff
	R	S	R	T	R	P	R	$\frac{2R+P+T}{4}$

In the third sequence, approach Y will be obtained (as in the second sequence).

S_8:	C	D	C	D	C	D	C	Average
S_{11}:	C	C	C	D	C	C	C	Payoff
	R	T	R	P	R	T	R	$\frac{2R+P+T}{4}$

In the fourth sequence, approach Y will be obtained.

S_8:	C	D	C	D	C	D	C	Average
S_{11}:	C	D	C	C	C	D	C	Payoff
	R	P	R	T	R	P	R	$\frac{2R+P+T}{4}$

In the fifth sequence, approach Y will be obtained.

S_8:	C	C	D	C	D	C	D	Average
S_{11}:	D	C	C	C	D	C	C	Payoff
	S	R	T	R	P	R	T	$\frac{2R+P+T}{4}$

A new approach emerged at the sixth sequence with a four-unit repetition period (T, T, P, P) and an average payoff ((P+T)/2). Approach Z is the third and last approach. No new approaches will appear in the next sequences.

S_8:	$\underline{C}$	$\underline{C}$	D	D	D	D	D	Average
S_{11}:	$\underline{D}$	$\underline{D}$	C	C	D	D	C	Payoff
	S	S	T	T	P	P	T	$\frac{P+T}{2}$

The three approaches (X, Y, Z) will be repeated in the next remaining sequences.

In the seventh sequence, approach Z will be obtained.

S_8:	$\underline{C}$	$\underline{D}$	D	D	D	D	D	Average
S_{11}:	$\underline{D}$	$\underline{C}$	C	D	D	C	C	Payoff
	S	T	T	P	P	T	T	$\frac{P+T}{2}$

In the eighth sequence, approach Z will be obtained.

S_8:	$\underline{C}$	$\underline{D}$	D	D	D	D	D	Average
S_{11}:	$\underline{D}$	$\underline{D}$	C	C	D	D	C	Payoff
	S	P	T	T	P	P	T	$\frac{P+T}{2}$

In the ninth sequence, approach Y will be obtained.

S_8:	$\underline{D}$	$\underline{C}$	D	C	D	C	D	Average
S_{11}:	$\underline{C}$	$\underline{C}$	D	C	C	C	D	Payoff
	T	R	P	R	T	R	P	$\frac{2R+P+T}{4}$

In the tenth sequence, approach Z will be obtained.

S_8:	$\underline{D}$	$\underline{C}$	D	D	D	D	D	Average
S_{11}:	$\underline{C}$	$\underline{D}$	D	C	C	D	D	Payoff
	T	S	P	T	T	P	P	$\frac{P+T}{2}$

In the eleventh sequence, approach Z will be obtained.

S_8:	$\underline{D}$	$\underline{D}$	D	D	D	D	D	Average
S_{11}:	$\underline{C}$	$\underline{C}$	D	D	C	C	D	Payoff
	T	T	P	P	T	T	P	$\frac{P+T}{2}$

In the twelfth sequence, approach Z will be obtained

S_8:	$\underline{D}$	$\underline{D}$	D	D	D	D	D	Average
S_{11}:	$\underline{C}$	$\underline{D}$	D	C	C	D	D	Payoff
	T	P	P	T	T	P	P	$\frac{P+T}{2}$

In the thirteenth sequence, approach Y will be obtained.

S_8:	$\underline{D}$	$\underline{C}$	D	C	D	C	D	Average
S_{11}:	$\underline{D}$	$\underline{C}$	C	C	D	C	C	Payoff
	P	R	T	R	P	R	T	$\frac{2R+P+T}{4}$

In the fourteenth sequence, approach Z will be obtained.

S_8:	$\underline{D}$	$\underline{C}$	D	D	D	D	D	Average
S_{11}:	$\underline{D}$	$\underline{D}$	C	C	D	D	C	Payoff
	P	S	T	T	P	P	T	$\frac{P+T}{2}$

In the fifteenth sequence, approach Z will be obtained.

S_8:	$\underline{D}$	$\underline{D}$	D	D	D	D	D	Average
S_{11}:	$\underline{D}$	$\underline{C}$	C	D	D	C	C	Payoff
	P	T	T	P	P	T	T	$\frac{P+T}{2}$

In the sixteenth sequence, approach Z will be obtained.

S_8:	$\underline{D}$	$\underline{D}$	D	D	D	D	D	Average
S_{11}:	$\underline{D}$	$\underline{D}$	C	C	D	D	C	Payoff
	P	P	T	T	P	P	T	$\frac{P+T}{2}$

Due to the imposition of the first two units, sixteen sequences will be produced. The S_8 average payoff for each sequence will be one of the following three discussed payoffs. Table 1 clarifies the average payoff of each approach in the first row, its corresponding sequences in the second row, the number of repetitions for each approach (the number of sequences in which each approach appeared) in the third row, and the repetition ratio (the number of repetitions of any approach divided by the total number of sequences (16)) in the fourth row.

Table 1: Approaches and their corresponding payoffs.

Approach	X	Y	Z
Payoff	ρ_1=R	$\rho_2=\frac{2R+P+T}{4}$	$\rho_3=\frac{P+T}{2}$
Sequence(s)	1	2, 3, 4, 5, 9, 13	6, 7, 8, 10, 11, 12, 14, 15, 16
Repetitions	1	6	9
Repetitions Ratio	$N_1=\frac{1}{16}$	$N_2=\frac{6}{16}$	$N_3=\frac{9}{16}$

The average payoff may be affected by several types of errors. Only the implementation error will be discussed.

4 Noise Effect

Assuming the probability to play C after the appearance of one of the four traditional outcomes (CC, CD, DC, DD) is P = (p_1, p_2, p_3, p_4) for the first player and Q = (q_1, q_2, q_3, q_4) for the second player. Then, the perturbed (error-affected) payoff can be calculated using the following transition matrix.

$$
\begin{array}{c} \\ R \\ S \\ T \\ P \end{array}
\begin{array}{c}
\begin{array}{cccc} R & S & T & P \end{array} \\
\begin{pmatrix}
p_1 q_1 & p_1(1-q_1) & (1-p_1)q_1 & (1-p_1)(1-q_1) \\
p_2 q_3 & p_2(1-q_3) & (1-p_2)q_3 & (1-p_2)(1-q_3) \\
p_3 q_2 & p_3(1-q_2) & (1-p_3)q_2 & (1-p_3)(1-q_2) \\
p_4 q_4 & p_4(1-q_4) & (1-p_4)q_4 & (1-p_4)(1-q_4)
\end{pmatrix}
\end{array}
\quad (2)
$$

Assuming $\pi = (\pi_1, \pi_2, \pi_3, \pi_4)$ is the left eigenvector of the matrix (2) for eigenvalue 1. Furthermore, this vector is the unique stationary distribution for infinitely repeated games. The following equation calculates the payoff of player P against Q.

$$E(P,Q) = R\pi_1 + S\pi_2 + T\pi_3 + P\pi_4. \tag{3}$$

Sometimes the stochastic matrix (2) may contain many zeroes because p_i and q_i are zeroes and ones. This makes the vector π isn't uniquely defined. This pushed us to directly calculate this vector for every contention by mutations. We assume that each player makes a wrong decision (plays C when the transition rule specifies D or plays D when the transition rule specifies C). When any player makes a wrong decision, the approach may be changed which in turn changes the average payoff. These changes will be studied separately for each round in the repeated period of each approach in each contention. The contention of S_8 against S_{11} will be as follows.

Approach	Mutation
a) **Approach X**:	
It has two mutations because its repetition period is one unit.	
• If S_8 plays D instead of C	$X \rightarrow Y$
• If S_{11} plays D instead of C	$X \rightarrow Y$
b) **Approach Y**:	
It has six mutations because its repetition period is four units with one repeating unit.	
• If S_8 plays D instead of C	$Y \rightarrow Z$
• If S_8 plays C instead of D when S_{11} C	$Y \rightarrow X$
• If S_8 plays C instead of D when S_{11} D	$Y \rightarrow Y$
• If S_{11} plays D instead of C when S_8 C	$Y \rightarrow Z$
• If S_{11} plays D instead of C when S_8 D	$Y \rightarrow Y$
• If S_{11} plays C instead of D	$Y \rightarrow Y$
b) **Approach Z**:	
It has four mutations because its repetition period is four units with two repeating units.	
• If S_8 plays C instead of D when S_{11} D	$Z \rightarrow Z$
• If S_8 plays C instead of D when S_{11} C	$Z \rightarrow Y$
• If S_{11} plays C instead of D	$Z \rightarrow Z$
• If S_{11} plays D instead of C	$Z \rightarrow Z$

5 Perturbed Payoff

Using this direct method (expected mutations), the resulting transition matrix for the competition between S_8 against S_{11} will be as follows.

$$\begin{array}{c|ccc} & X & Y & Z \\ \hline X & 0 & 1 & 0 \\ Y & 1/6 & 3/6 & 2/6 \\ Z & 0 & 1/4 & 3/4 \end{array} \tag{4}$$

Every element in the previous matrix represents the probability that each approach (row approaches) may be changed to another approach (column approaches) or remains the same when a wrong decision occurs. The first row in this matrix represents the probabilities of converting from approach X to any approach. By studying the possible mutations in approach X, it is evident that when a wrong decision occurs in approach X, approach X will be changed to (approach Y) in all possible mutations with a probability of 100% and will not convert to approach Z and will not remain on approach X, so the value of the element in the intersection between the first row (approach X) and second column (approach Y) in the previous matrix is one and the rest of the elements in the same row are zeros. Approach Y (at the second row) has a possible six mutations, only one mutation will convert approach Y to approach X (with probability (1/6) at the first column), two mutations to approach Z (with probability (2/6) at the third column), and three mutations will remain the same (with probability (3/6) at the second column). Approach Z has four possible mutations in its period of repetition, no mutations will lead to approach X (with probability zero at the first column), one mutation to approach Y (with probability (1/4) at the second column), and three mutations will remain the same (with probability (3/4) at the third column).

The perturbed payoff of every approach can be calculated using the previous matrix. This can be done for any approach by using row values for that approach. Every value in the specified row will be multiplied by the corresponding column approach payoff value and these values will be added together to get the perturbed payoff of that row. Approach Y perturbed payoff can be calculated as follows.

$$R_2 = \frac{1}{6} \times \rho_1 + \frac{3}{6} \times \rho_2 + \frac{2}{6} \times \rho_3 \tag{5}$$

$$R_2 = \frac{1}{6} \times R + \frac{3}{6} \times \frac{2R+P+T}{4} + \frac{2}{6} \times \frac{2R+7P+7T}{16} \tag{6}$$

$$R_2 = \frac{10R+7P+7T}{24} \tag{7}$$

6 Approach repetition rate effect

Table 2: Approaches and their corresponding perturbed payoffs.

Approach	X	Y	Z
Payoff	ρ_1=R	$\rho_2=\frac{2R+P+T}{4}$	$\rho_3=\frac{P+T}{2}$
Perturbed payoff	$R_1=\frac{P+2R+T}{4}$	$R_2=\frac{7P+7T+10R}{24}$	$R_3=\frac{7P+2R+7T}{16}$
Sequence(s)	1	2, 3, 4, 5, 9, 13	6, 7, 8, 10, 11, 12, 14, 15, 16
Repetitions	1	6	9
Repetitions Ratio	$N_1=\frac{1}{16}$	$N_2=\frac{6}{16}$	$N_3=\frac{9}{16}$

Table 3: 16x16 payoff matrix.

	S_0	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}
S_0	0,0,1,0	1,0,1,0	0,0,1,0	1,0,1,0	5,0,11,0	1,0,0,0	1,0,1,0	1,0,0,0	0,0,1,0	1,0,1,0	0,0,1,0	1,0,1,0	1,0,1,0	1,0,0,0	11,0,5,0	1,0,0,0
S_1	0,0,1,1	0,1,1,0	1,0,1,1	0,1,1,0	66,0,95,95	46,41,41,0	1,0,0,0	14,5,5,0	0,0,1,1	1,1,1,0	1,0,1,1	1,1,1,0	26,0,11,11	1,0,0,0	1,0,0,0	1,0,0,0
S_2	0,0,1,0	1,0,1,1	5,0,14,5	1,0,0,1	0,0,1,0	1,1,1,0	0,0,1,0	1,1,1,0	0,0,1,0	1,0,1,1	41,1,46,41	1,0,0,1	11,11,26,0	1,1,0,0	95,95,66,0	1,1,0,0
S_3	0,0,1,1	0,1,1,0	1,0,0,1	1,1,1,1	0,0,1,1	0,1,1,0	1,1,1,1	0,1,1,0	0,0,1,1	1,1,1,1	1,0,0,1	1,0,0,1	1,1,1,1	1,1,0,0	1,1,0,0	1,1,0,0
S_4	0,0,11,5	95,0,95,66	0,0,1,0	1,0,1,0	1,0,2,1	7,0,0,1	5,0,11,0	1,0,0,0	0,0,11,5	95,0,95,66	0,0,1,0	1,0,1,0	8,0,5,3	7,0,0,1	11,0,5,0	1,0,0,0
S_5	0,0,0,1	0,41,41,46	0,1,1,1	0,1,1,0	1,0,0,7	7,5,5,7	15,11,11,11	46,41,41,0	1,0,0,0	11,11,11,15	1,1,1,1	1,1,1,0	1,0,0,1	7,0,0,1	1,0,0,0	1,0,0,0
S_6	0,0,1,1	0,0,0,1	0,0,1,0	1,1,1,1	0,0,11,5	11,11,11,15	0,0,1,0	1,1,1,0	0,0,5,11	0,0,0,1	11,11,15,11	1,1,0,1	11,11,13,13	95,95,0,66	95,95,66,0	1,1,0,0
S_7	0,0,0,1	0,5,5,14	0,1,1,1	0,1,1,0	0,0,0,1	0,41,41,46	0,1,1,1	0,1,1,0	0,0,0,1	0,0,0,1	1,1,0,1	1,1,0,1	11,11,0,26	95,95,0,66	1,1,0,0	1,1,0,0
S_8	0,0,1,0	1,0,1,0	0,0,1,0	1,0,1,0	5,0,11,0	0,0,0,1	11,0,5,0	1,0,0,0	0,1,7,0	95,66,95,0	0,1,7,0	95,66,95,0	5,3,8,0	11,5,0,0	2,1,1,0	11,5,0,0
S_9	0,0,1,1	0,1,1,1	1,0,1,1	1,1,1,1	66,0,95,95	15,11,11,11	1,0,0,0	1,0,0,0	0,66,95,95	0,1,0,0	11,15,11,11	0,1,0,0	13,13,11,11	5,11,0,0	11,5,0,0	1,1,0,0
S_{10}	0,0,1,0	1,0,1,1	41,0,46,41	1,0,0,1	0,0,1,0	1,1,1,1	11,11,15,11	1,1,0,1	0,1,7,0	11,15,11,11	5,7,7,5	41,46,0,41	0,1,1,0	0,1,0,0	0,7,1,0	0,1,0,0
S_{11}	0,0,1,1	0,1,1,1	1,0,0,1	1,0,0,1	0,0,1,1	0,1,1,1	1,1,0,1	1,1,0,1	0,66,95,95	0,1,0,0	41,46,0,41	5,14,0,5	0,26,11,11	0,1,0,0	0,1,0,0	0,1,0,0
S_{12}	0,0,1,1	11,0,11,26	0,11,26,11	1,1,1,1	3,0,5,8	1,0,0,1	13,11,13,11	26,11,0,11	0,3,8,5	11,13,11,13	0,1,1,0	11,26,11,0	1,1,1,1	8,5,0,3	5,8,3,0	1,1,0,0
S_{13}	0,0,0,1	0,0,0,1	0,1,0,1	0,1,0,1	1,0,0,7	1,0,0,7	66,95,0,95	66,95,0,95	0,5,0,11	0,11,0,5	0,1,0,0	0,1,0,0	3,5,0,8	1,2,0,1	5,11,0,0	5,11,0,0
S_{14}	0,0,5,11	0,0,0,1	0,95,66,95	0,1,0,1	0,0,5,11	0,0,0,1	0,95,66,95	0,1,0,1	0,1,1,2	0,5,0,11	0,7,1,0	0,1,0,0	0,8,3,5	0,11,0,5	0,7,1,0	0,1,0,0
S_{15}	0,0,0,1	0,0,0,1	0,1,0,1	0,1,0,1	0,0,0,1	0,0,0,1	0,1,0,1	0,1,0,1	0,5,0,11	0,1,0,1	0,1,0,0	0,1,0,0	0,1,0,1	0,11,0,5	0,1,0,0	0,1,0,0

Table 4: 16x16 payoff matrix for Axelrod's values.

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	S_0	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}
S_0	1	3	1	3	2.25	5	3	5	1	3	1	3	3	5	3.75	5
S_1	0.5	2	2	2	1.6602	3.0781	5	3.75	0.5	3	2	3	2.9375	5	5	5
S_2	1	2	1.625	2.5	1	3	1	3	1	2	1.9609	2.5	2.375	4	3.2266	4
S_3	0.5	2	2.5	2.25	0.5	2	2.25	2	0.5	2.25	2.5	2.5	2.25	4	4	4
S_4	0.6875	2.2266	1	3	1.75	4.375	2.25	5	0.6875	2.2266	1	3	2.8125	4.375	3.75	5
S_5	0	1.2813	1.3333	2	0.625	2.2917	2.4792	3.0781	5	2.0625	2.25	3	2.5	4.375	5	5
S_6	0.5	0	1	2.25	0.6875	2.0625	1	3	0.3125	0	2.1458	2.6667	2.1042	2.9688	3.2266	4
S_7	0	0.8333	1.3333	2	0	3.4167	1.3333	2	0	0	2.6667	2.6667	1.8333	2.9688	4	4
S_8	1	3	1	3	2.25	0	3.75	5	1.25	3	1.25	3	2.625	4.375	3.5	4.375
S_9	0.5	1.3333	2	2.25	1.6602	2.4792	5	5	1.1445	3	2.3125	3	2.3958	3.625	4.375	4
S_{10}	1	2	1.9606	2.5	1	2.25	2.1458	2.6667	1.25	2.3125	2.2083	2.6797	2	3	2.75	3
S_{11}	0.5	1.3333	2.5	2.5	0.5	1.3333	2.6667	2.6667	1.1445	3	2.6797	2.7917	1.8542	3	3	3
S_{12}	0.5	1.375	1.2292	2.25	1.25	2.5	2.3125	3.3958	1.0625	2.1875	2	3	2.25	3.4375	3.25	4
S_{13}	0	0	1.5	1.5	0.625	0.625	2.4023	2.4023	0.9375	2.0625	3	3	1.875	2.75	3.625	3.625
S_{14}	0.3125	0	1.3711	1.5	0.3125	0	1.3711	1.5	1	0.9375	2.75	3	1.6875	2.0625	2.75	3
S_{15}	0	0	1.5	1.5	0	0	1.5	1.5	0.9375	1.5	3	3	1.5	2.0625	3	3

Considering the number of repetitions of approaches in contention between S_8xS_{11} sequences founded in Table 2 fifth row. The perturbed payoff may be influenced by this repetition ratio founded in Table 2 sixth row and recalculated as follows.

$$E(S_i, S_j) = N_1 \times R_1 + N_2 \times R_2 + N_3 \times R_3 \tag{8}$$

$$E(S_8, S_{11}) = \frac{1}{16} \times \frac{2R+P+T}{4} + \frac{6}{16} \times \frac{10R+7P+7T}{24} + \frac{9}{16} \times \frac{2R+7P+7T}{16} \tag{9}$$

$$E(S_8, S_{11}) = \frac{66R+95P+95T}{256} \tag{10}$$

This procedure will be repeated for every contention between every two strategies and then put into Table 3 which represents the conflict payoff between any two strategies used in this paper under the effect of approach repetition rate.

Strategies behaviour can be studied using domination. To study the behaviour of any two strategies (S_i x S_j) with each other, the four jointed entries between the two strategies in Table 1 must be extracted and reused in the next matrix.

$$\begin{array}{cc} & \begin{array}{cc} S_i & S_j \end{array} \\ \begin{array}{c} S_i \\ S_j \end{array} & \begin{pmatrix} a_{ii} & a_{ij} \\ a_{ji} & a_{jj} \end{pmatrix} \end{array} \tag{11}$$

S_i and S_j are equivalent if $a_{ii} = a_{ji}$ and $a_{ij} = a_{jj}$. But S_i dominates S_j when one of these two inequalities $a_{ii} \geq a_{ji}$ and $a_{ij} \geq a_{jj}$ is attained.

Table 4 is created by substituting with Axelrod's values in Table 3.Table 5 shows the behavior of the strategies when Axelrod's values are substituted. The next section will go over these findings.

Table 5 Dominating strategies for Axelrod's values

Strategy	Dominating strategies
S_0	S_2, S_8, S_{10}
S_1	$S_0, S_3, S_4, S_8, S_{10}$
S_2	S_1, S_9, S_{10}, S_{11}
S_3	$S_0, S_4, S_8, S_9, S_{11}$
S_4	S_0, S_8
S_5	S_0, S_1, S_2, S_4, S_9
S_6	$S_0, S_1, S_2, S_3, S_4, S_5, S_8, S_9, S_{10}, S_{11}, S_{12}$
S_7	$S_0, S_1, S_2, S_3, S_4, S_8, S_9, S_{11}, S_{12}$
S_8	S_5, S_{10}
S_9	S_0, S_1, S_8
S_{10}	$S_5, S_9, S_{11}, S_{14}, S_{15}$
S_{11}	$S_0, S_1, S_4, S_5, S_8, S_9, S_{12}, S_{15}$
S_{12}	$S_0, S_1, S_2, S_4, S_8, S_9$
S_{13}	$S_0, S_1, S_2, S_3, S_4, S_5, S_8, S_9, S_{12}$
S_{14}	$S_0, S_1, S_2, S_3, S_4, S_5, S_7, S_8, S_9, S_{12}, S_{13}, S_{15}$
S_{15}	$S_0, S_1, S_2, S_3, S_4, S_5, S_7, S_8, S_9, S_{12}, S_{13}$

7 Results and discussion

The effect of approach repetition rate on strategy payoff is analyzed in this research. This analysis is performed by using the results computed by a maple program. Axelrod's values are used to enhance the results of this study.

One interesting finding in Table 5 is that each strategy is outperformed by at least two strategies. Nevertheless, some strategies seem to have strong properties, and strategy (S_8) is one of the strong strategies. This strategy attacks the largest number of strategies (exactly twelve) including several strong strategies like the All D strategy (S_0) and Pavlov (win-stay, lose-shift (WSLS)) strategy (S_9). Unfortunately, the (S_8) strategy was invaded by two strategies like the Tit-for-Tat (TFT) strategy (S_{10}) and strategy (S_5). This means that there is no absolute dominant strategy that cannot be conquered. This directs any player to use more than one strategy to achieve better results. The (S_0), (S_4), and (S_9) strategies also show good performance.

On the other hand, there are weak strategies that allow a large number of strategies to invade it. In general, the (S_6) strategy is considered the weakest of the strategies. Not only (S_6) strategy cannot overcome any strategy, but also eleven strategies can crush it. In addition, the (S_{14}) and (S_{15}) strategies are poor strategies defeated by at least eleven strategies. This means that co-operative strategies in this algorithm cannot withstand in front of defective ones.

8 Conclusion

This study has been developed to conclude the behavior of strategies under the influence of the approach repetition rate. Only the finite two-state automata sixteen strategies are used. Instead of the usage of the immediately previous unit, this study uses the pre-previous state to create the new state. The effect of movement error is also taken into account.

This study has demonstrated that the (S_8) (1, 0, 0, 0) strategy has great behaviour. The (S_8) strategy is defective and is called the grim strategy and only co-operates with its opponent if the two players co-operate in the unit used to generate the new state. Turning now to strategy (S_6) which is considered an incompetent strategy. Foolish strategy (S_6) (0, 1, 1, 0) does not co-operate unless the opponent's strategy opposes its decision in the pre-previous state.

9 Appendix: Maple program

> restart;
> With (linalg):
> $k1 := 12; k2 := 13; h(C) := D; h(D) := C; f((C,C)) := R; f((C,D)) := S; f((D,C)) := T; f((D,D)) := p;$

$Q := (L1,L2,N)->B[L1,N][1,1] <> B[L2,N][1,1] or B[L1,N][2,1] <> B[L2,N][2,1] : G(1) := 0;$
$G(2) := 1; G(3) := 1; NN := 0; w := x->$ spline([2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17], $[6,6,7,7,8,8,9,9,10,10,11,11,12,12,13,13]$, x, linear):
$> g(0,C,C) := D; g(0,C,D) := D; g(0,D,C) := D; g(0,D,D) := D; g(1,C,C) := D; g(1,C,D) := D; g(1,D,C) := D; g(1,D,D) := C; g(2,C,C) := D; g(2,C,D) := D; g(2,D,C) := C; g(2,D,D) := D; g(3,C,C) := D; g(3,C,D) := D; g(3,D,C) := C; g(3,D,D) := C; g(4,C,C) := D; g(4,C,D) := C; g(4,D,C) := D; g(4,D,D) := D; g(5,C,C) := D; g(5,C,D) := C; g(5,D,C) := D; g(5,D,D) := C; g(6,C,C) := D; g(6,C,D) := C; g(6,D,C) := C; g(6,D,D) := D; g(7,C,C) := D; g(7,C,D) := C; g(7,D,C) := C; g(7,D,D) := C; g(8,C,C) := C; g(8,C,D) := D; g(8,D,C) := D; g(8,D,D) := D; g(9,C,C) := C; g(9,C,D) := D; g(9,D,C) := D; g(9,D,D) := C; g(10,C,C) := C; g(10,C,D) := D; g(10,D,C) := C; g(10,D,D) := D; g(11,C,C) := C; g(11,C,D) := D; g(11,D,C) := C; g(11,D,D) := C; g(12,C,C) := C; g(12,C,D) := C; g(12,D,C) := D; g(12,D,D) := D; g(13,C,C) := C; g(13,C,D) := C; g(13,D,C) := D; g(13,D,D) := C; g(14,C,C) := C; g(14,C,D) := C; g(14,D,C) := C; g(14,D,D) := D; g(15,C,C) := C; g(15,C,D) := C; g(15,D,C) := C; g(15,D,D) := C; B[0,0] :=$
$matrix(2,1,[C,C]); B[1,0] :=$
$matrix(2,1,[C,C]); B[0,1] :=$
$matrix(2,1,[C,C]); B[1,1] :=$
$matrix(2,1,[C,D]); B[0,2] :=$
$matrix(2,1,[C,D]); B[1,2] :=$
$matrix(2,1,[C,C]); B[0,3] :=$
$matrix(2,1,[C,C]); B[1,3] :=$
$matrix(2,1,[D,C]); B[0,4] :=$
$matrix(2,1,[D,C]); B[1,4] :=$
$matrix(2,1,[C,C]); B[0,5] :=$
$matrix(2,1,[C,D]); B[1,5] :=$
$matrix(2,1,[C,D]); B[0,6] :=$
$matrix(2,1,[C,C]); B[1,6] :=$
$matrix(2,1,[D,D]); B[0,7] :=$
$matrix(2,1,[D,C]); B[1,7] :=$
$matrix(2,1,[C,D]); B[0,8] :=$
$matrix(2,1,[C,D]); B[1,8] :=$
$matrix(2,1,[D,C]); B[0,9] :=$
$matrix(2,1,[D,D]); B[1,9] :=$
$matrix(2,1,[C,C]); B[0,10] :=$
$matrix(2,1,[D,C]); B[1,10] :=$
$matrix(2,1,[D,C]); B[0,11] :=$
$matrix(2,1,[C,D]); B[1,11] :=$
$matrix(2,1,[D,D]); B[0,12] :=$
$matrix(2,1,[D,D]); B[1,12] :=$
$matrix(2,1,[C,D]); B[0,13] :=$
$matrix(2,1,[D,C]); B[1,13] :=$
$matrix(2,1,[D,D]); B[0,14] :=$
$matrix(2,1,[D,D]); B[1,14] :=$
$matrix(2,1,[D,C]); B[0,15] :=$
$matrix(2,1,[D,D]); B[1,15] := matrix(2,1,[D,D]);$

$>$ for N from 0 by 1 to 15 do for k_0 from 1 by 1 to 17 do if($k_0 > 3$) then $G(k_0)$:=0: end if; for L from 2 by 1 to 50 do $a_1 := g(k_1, B[L-2,N][1,1], B[L-2,N][2,1])$; $a_2 := g(k_2, B[L-2,N][2,1], B[L-2,N][1,1])$; $B[L,N] := matrix(2,1,[a_1,a_2])$; if ($k_0 = 1$ and $L = 50$) then $NN := 0$; end if; if ($k_0 = 2$ and $L = 6$) then $B[6,N][1,1] := h(B[6,N][1,1])$; $NN := 1$; end if; if ($k_0 = 3$ and $L = 6$) then $B[6,N][2,1] := h(B[6,N][2,1])$; $NN := 2$; end if; if ($k_0 = 4$ and $L = 7$ and $Q(7,6,N)$) then $NN := NN + 1$; $G(4) := 1$; $B[7,N][1,1] := h(B[7,N][1,1])$; end if; if ($k_0 = 5$ and $L = 7$ and $Q(7,6,N)$) then $G(5) := 1; B[7,N][2,1] := h(B[7,N][2,1])$; $NN := NN+1$; end if; if ($k_0 = 6$ and $L = 8$ and $Q(8,7,N)$ and $Q(8,6,N)$) then $G(6) := 1$; $B[8,N][1,1] := h(B[8,N][1,1])$; $NN := NN+1$; end if; if ($k_0 = 7$ and $L = 8$ and $Q(8,7,N)$ and $Q(8,6,N)$) then $G(7) := 1$; $B[8,N][2,1] := h(B[8,N][2,1])$; $NN := NN+1$; end if; if ($k_0 = 8$ and $L = 9$ and $Q(9,8,N)$ and $Q(9,7,N)$ and $Q(9,6,N)$) then $G(8) := 1$; $B[9,N][1,1] := h(B[9,N][1,1])$; $NN := NN+1$; end if; if ($k_0 = 9$ and $L = 9$ and $Q(9,8,N)$ and $Q(9,7,N)$ and $Q(9,6,N)$) then $G(9) := 1$; $B[9,N][2,1] := h(B[9,N][2,1])$; $NN := NN+1$; end if; if ($k_0 = 10$ and $L = 10$ and $Q(10,9,N)$ and $Q(10,8,N)$ and $Q(10,7,N)$ and $Q(10,6,N)$) then $G(10) := 1$; $B[10,N][1,1] := h(B[10,N][1,1])$; $NN := NN+1$; end if; if ($k_0 = 11$ and $L = 10$ and $Q(10,9,N)$ and $Q(10,8,N)$ and $Q(10,7,N)$ and $Q(10,6,N)$) then $G(11) := 1$; $B[10,N][2,1] := h(B[10,N][2,1])$; $NN := NN+1$; end if; if ($k_0 = 12$ and $L = 11$ and $Q(11,10,N)$ and $Q(11,9,N)$ and $Q(11,8,N)$ and $Q(11,7,N)$ and $Q(11,6,N)$) then $G(12) := 1$; $B[11,N][1,1] := h(B[11,N][1,1])$; $NN := NN+1$; end if; if ($k_0 = 13$ and $L = 11$ and $Q(11,10,N)$ and $Q(11,9,N)$ and $Q(11,8,N)$ and $Q(11,7,N)$ and $Q(11,6,N)$) then $G(13) := 1$; $B[11,N][2,1] := h(B[11,N][2,1])$; $NN := NN+1$; end if; if ($k_0 = 14$ and $L = 12$ and $Q(12,11,N)$ and $Q(12,10,N)$ and $Q(12,9,N)$ and $Q(12,8,N)$ and $Q(12,7,N)$ and $Q(12,6,N)$) then $G(14) := 1$; $B[12,N][1,1] := h(B[12,N][1,1])$; $NN := NN+1$; end if; if ($k_0 = 15$ and $L = 12$ and $Q(12,11,N)$ and $Q(12,10,N)$ and $Q(12,9,N)$ and $Q(12,8,N)$ and $Q(12,7,N)$ and $Q(12,6,N)$) then $G(15) := 1$; $B[12,N][2,1] := h(B[12,N][2,1])$; $NN := NN+1$; end if; if ($k_0 = 16$ and $L = 13$ and $Q(13,12,N)$ and $Q(13,11,N)$ and $Q(13,10,N)$ and $Q(13,9,N)$ and $Q(13,8,N)$ and $Q(13,7,N)$ and $Q(13,6,N)$) then $G(16) := 1$; $B[13,N][1,1] := h(B[13,N][1,1])$; $NN := NN+1$; end if; if ($k_0 = 17$ and $L = 13$ and $Q(13,12,N)$ and $Q(13,11,N)$ and $Q(13,10,N)$ and $Q(13,9,N)$ and $Q(13,8,N)$ and $Q(13,7,N)$ and $Q(13,6,N)$) then $G(17) := 1; B[13,N][2,1] := h(B[13,N][2,1])$; $NN := NN + 1$; end if; $p_1(L) := f(B[L,N][1,1], B[L,N][2,1])$; if ($L = 50$) then $av(k_0,N,k_1,k_2) := sum(p_1(i)/24,$

$i = w(k_0) + 8..w(k_0) + 31)$; end if; if $(L = 50)$ then print $("k_0", k_0, "L", L, "NN", NN, "G(k_0)", G(k_0),$ $"N", N, "B0", B[0,N], B[1,N],$ $"k_1", k_1, "k_2", k_2,$ $"av", av(k_0, N, k_1, k_2))$; end if;if $(L = 50$ and $k_0 = 17)$ then $avp(N, k_1, k_2) := sum(G(i_1) * av(i_1, N, k_1, k_2)/NN,$ $i_1 = 2..17)$; print $("avp", avp(N, k_1, k_2))$; end if;if $(N = 15$ and $L = 50$ and $k_0 = 17)$ then $avg(k_1, k_2) := sum(avp(J, k1, k2)/16, J = 0..15)$; print $("avg", avg(k_1, k_2))$; end if; end do ; end do ; end do ;

Conflict of Interest The authors declare that they have no conflict of interest.

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