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Some Properties of Odd Neighbor in D^c Dominating Sets

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Abstract: In this work, the study will continue of odd neighbor in D^c domination in graphs. The bounded of this number in general is determined. Also, the bound of this number in the graph which have at least three vertices of degree the order of the graph mines two is calculated. Moreover, compute this number for complement of tree is discussed. Finally, When is this number equal to one? in a graph and complement of a graph are been presented.

Keywords: odd neighbor in D^c dominating set, odd neighbor in D^c domination number, minimum dominating set, maximum degree, induced subgraph.

1 Introduction

The graph $G = (V, E)$ in this work is a finite, undirected, and simple. Most graph theory terminology that uses in this paper can be found in[2]. Specially, the size (m) and order (n) of G. The number of edges which are incidence with a vertex (v) is called the degree of that vertex and denoted by $deg(v)$. The minimum and maximum degree of a vertex is denoted by $\delta(G)$ and $\Delta(G)$. The degree of a vertex v, $deg(v)$, is the number of vertices adjacent to v. A vertex of degree one is called a pendent $(leaf)$. For a subset $S \subseteq V$, we define by S the subgraph induced by S. The open neighborhood and the closed neighborhood of v are denoted by $N(v)$ and $N[v] = N(v) \cup v$ respectively. The complement of a graph G is the graph with vertex set V and two vertices are adjacent in $\overline{G}$ if and only if they are not adjacent in G. For another graph theoretic terminology, we refer to Haynes et.al [2,3].
The domination in graphs has interest of researchers in recent times. It has taken a wide range of practical applications in most sciences. In mathematics in particular, it has become overlapped with most of its fields, as in general graph [4,5,6,7,8,9,10,11], topological graph[12,13,14], fuzzy graph[15,16,17], labeled graph[18,19],and topological indices [20,21,22, 23,24,25,26,27,28,29]. A set D of vertices in a graph G is a dominating set if every vertex in $V - D$ has a neighbor in D. The minimum cardinality of all dominating set is called the domination number and denoted by $\gamma(G)$.

Haynes, Hedetmiemi, and Slater[2] introduced the subject of domination and its variations. Omran and Aljanaby [30], introduced a new definition domination as the following: A dominating set D of graph G is called an odd neighbor in D^c dominating set of G if $|N(v) \cap (V - D)| = 0$ or odd $\forall v \in D$, then D is called odd neighbor in D^c dominating $(brieflyMOD^cS)$. The value of number of minimum cardinality of all DS is called the domination neighbor in D^c (briefly MOD^cS), and denoted by $\gamma_{odc}(G)$ [1]. In this paper, many new bounded of this number are been determined, especially when the graph G has at least three vertices have the degree $(n-2)$. Moreover, many properties of this number are been calculated. Finally, the number of the complement of a graph, especially a tree graph is been discussed.

Proposition 1*[30] Consider P_n be a path graph with n vertices, so $\gamma_{odc}(P_n) = \lceil \frac{n}{2} \rceil$.*

Proposition 2 *1.If C_n is cycle graph, then,*
$$\gamma_{odc}(C_n) = \begin{cases} \frac{n+1}{2}, & if\, if n \equiv 1,2,3 \mod 4; \\ \frac{n}{2}, & if n \equiv 0 \mod 4\,. \end{cases}$$

Proposition 3 *1.If K_n is cycle graph, then,*
$$\gamma_{odc}(K_n) = \begin{cases} 1, & if\, if\ n \text{ is even;} \\ 2, & if\ n \text{ is odd}\,. \end{cases}$$

Proposition 4 *1.If C_n is cycle graph, then,*
$$\gamma_{odc}(\overline{C_n}) = \begin{cases} 3, & if\, if\ n = 3,5; \\ 2, & otherwise\,. \end{cases}$$

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2 Main results

Theorem 5. *Let G be a graph with n vertices and maximum degree Δ:*

1.If Δ is odd, then $\lceil \frac{n}{\Delta+1} \rceil \leq \gamma_{odc} \leq n-\Delta$
2.Δ is even, then $\lceil \frac{n}{\Delta} \rceil \leq \gamma_{odc} \leq n-\Delta+1$

Proof. Let D be a γ_{odc}-set.
Case1: If Δ is odd, then to proof the lower bound since every vertex can dominate at most itself and $\Delta(G)$ other vertices such that each vertex in D is adjacent to odd number of vertices in $V-D$. Hence, $\gamma_{odc} \geq \lceil \frac{n}{\Delta+1} \rceil$.
Now, to proof the upper bound. Let u be vertex that has maximum degree $\Delta(G)$.Then the vertex u dominates $N[u]$ vertices. Suppose the other vertices in $V-N[u]$ are dominate themselves only such that achieve the definition of OD^cS. Then the set $V-N(u)$ is odd neighbor in D^c dominating set since Δ is odd. Since $|V|=n$ and $|N(u)|=\Delta$, then $|D|=|V-N(u)|=n-\Delta$. So $\gamma_{odc} \leq n-\Delta$.
Case2. If Δ is even by the same way in case1 every vertex can dominate at most itself and $\Delta(G)-1$, then $\gamma_{odc} \geq \lceil \frac{n}{\Delta} \rceil$
To proof upper bound by the same hypothesis in case1u dominates $N[u]-w$ vertices for any vertex $w \in N[u]$. So $|D| = |V-(N(u)-w| = n-\Delta+1$. Hence $\gamma_{odc} \leq n-\Delta+1$.

Theorem 6. *Consider G be a graph of order n and $\forall v \in V(G)$, $deg(v) = even \geq 2$, then $\gamma_{odc}(G) \geq \frac{n}{\Delta}$*

Proof. Proof: Let D is γ_{odc}-set of G and $\forall v \in V(G)$, $deg(v) = even$ Then by definition every vertex of D must adjacent to at least some odd number $(say\ m)$ such that $m \leq$. Since all vertices in V is even, then every vertex in D must adjacent to $2k+1$ vertices in $V-D$ such that $k=1,2,\ldots,\frac{n}{2-1}$. That is, $N(D)=V(G)$. Since every vertex $v \in D$ has at most Δ neighbors. Then $\Delta\gamma_{odc} \geq |V| = n$. So, by dividing this inequality by Δ we get $\gamma_{odc}(G) \geq \frac{n}{\Delta}$.

Corollary 7 *Let G be a graph of order n and $\forall v \in V(G), deg(v) = even \geq 2$*
If $\Delta \leq \frac{n}{k}$ for some positive integer k, then $\gamma_{odc}(G) \geq k$.

Proof. By Theorem, $\gamma_{odc}(G) \geq \frac{n}{\Delta}$.
If $\Delta \leq \frac{n}{k}$, then substitution yields $\gamma_{odc}(G) \geq k$.

Theorem 8. *For any connected graph of order n has at least three vertices of degree $n-2$, then $\gamma_{odd}(G) \leq 4$.*

Proof. Suppose that G has three vertices say $V(H) = v_i, i = 1,2,3$ have degree $n-2$ such that H is subgraph of G. If one of them vertices independent to others, then this vertex has $n-3$ degree but that contradiction, so every vertex in H is adjacent to at least one vertex of them. So, an induced subgraph of H is connected subgraph and it is path of order three. In case the vertex that not adjacent to $v_i \in V(H) \forall i lie in V(G)-V(H)$ then an induced subgraph of H is complete of order three, so $H \equiv K_3$ or P_3. Then there are two cases depending on whether the order is even or odd.
Case If n is odd, then there are two subcases.
Subcase If $v_1, v_2, v_3 \equiv K_3$, then we distinguish three cases as follows.
1) If these three vertices are not adjacent to the same vertex $(say\ u)$ then two cases are distinguished as follows.
I) If u has odd or zero degree, then $D = u, v_1$ is MOD^c S, so $\gamma_{odc}(G) = 2$.
II) If u has even degree, then two cases are distinguished as follows.
A. If $N(u)$ has a vertex of odd degree say u_1, then $D = u_1, v_1, v_2$ is MOD^cS, so $\gamma_{odc}(G) = 3$.
B. If all vertices belong to the set $N(u)$ have even degree, then $D = u, u_2, v_1, v_2$, where $u_2 \in N(u)$. One can be concluded that the set D is MOD^cS, so $\gamma_{odc}(G) = 4$.
2) If two vertices say v_1,and v_2 are not adjacent to the same vertex say u_1 and the vertex v_3 is not adjacent to the vertex u_2, where u_1 and u_2 are different, then four cases are distinguished as follows.
A) If the vertex u_1 has odd degree and the vertex u_2 has even degree, then the set $D = u_1, v_2$ is minimum OD^cS, so $\gamma_{odc}(G) = 2$.
B) If the vertex u_1 has even degree and the vertex u_2 has odd degree, then the set $D = u_2, v_3$ is MOD^cS, so $\gamma_odc(G) = 2$.
C) If two vertices u_1 and u_2 have odd degree, then and set D in case A or B is obtained the result.
D) If two vertices u_1 and u_2 have even degree, $D = v_1, v_2, v_3$ is minimal OD^c S. So, $\gamma_{odc}(G) = 3$.
3) If The vertex v_1 is not adjacent to the vertex say u_1 and the vertex v_2 is not adjacent to the vertex say u_2 and the vertex v_3 is not adjacent to the vertex say u_3 where the vertices u_1, u_2,and u_3 are different, then there are cases as follows.
A. If at least one vertex from the set $S = u_1, u_2, u_3$ has odd degree sayu_1, then the set $D = u_1, v_1$ is MOD^cS, so $\gamma_{odc}(G) = 2$.
B. If all vertices in the set S have even degree, then two cases are distinguished as follows.
B_1. If there is two vertices of the set S are adjacent say u_1, u_2, the if $N(u_1) \cap N(u_2) = V(G)$, then the set $D = u_1, u_2$ is MOD^cS, so $\gamma_{odc}(G) = 2$.
B_2. If there is no two vertices of the set S are adjacent, then the set $D = v_1, v_2, v_3$ is MOD^cS, so $\gamma_{odc}(G) = 3$.
Subcase2. If $v_1, v_2, v_3 \equiv P_3$, then the vertices v_1 and $v_3$are not adjacent. Then the set $D = v_1, v_3$ is MOD^c S. So, $\gamma_odc(G) = 2$.
Case 2. If n is even, then if G has at least one vertex v of degree $n-1$, then $D = v$ and $\gamma_{odd}(G) = 1$. Otherwise, there are two subcases:
subcase1. If $v_1, v_2, v_3 \equiv K_3$, then we distinguish three cases as follows.
1) If these three vertices are not adjacent to the same vertex $(say\ u)$ then there are two cases as follows.
I) If u has odd, then there are two cases as follows.

A) If $\equiv v \in N(u)$ such that v has even degree, then $D = v, v_1$ is minimal OD^cS.
B) If all vertices in $N(u)$ has odd degree, then $D = u, v_1, v_2$ is the MOD^cS, so $\gamma_{odc}(G) = 3$.
II) If u has even degree, then there are three cases as follows.
A. If $N(u)$ has a vertex of even degree say u_1, then $D = u_1, v_1$ is MOD^cS, so $\gamma_{odc}(G) = 2$.
B. If all vertices belong to the set $N(u)$ have odd degree, then let $D = u, u_2, v_1$, where $u_2 \in N(u)$. One can be concluded that the set D is MOD^cS, so $\gamma_{odc}(G) = 3$.
2) If two vertices say v_1,and v_2 are not adjacent to the same vertex say u_1 and the vertex v_3 is not adjacent to the vertex u_2, where u_1 and u_2 are different, then there are two cases as follows.
A. If the vertex u_1 has even degree, then the set $D = u_1, v_3$ is a MOD^cS, so $\gamma_{odc}(G) = 2$.
B. If the vertex u_2 has even degree, then the set $D = u_2, v_1$ is a MOD^cS, so $\gamma_odc(G) = 2$.
C. If the vertex u_1 and u_2 have odd degree, then the set $D = u_1, v_1, v_2$ is a MOD^cS, so $\gamma_odc(G) = 3$.
3) If The vertex v_1 is not adjacent to the vertex say u_1 and the vertex v_2 is not adjacent to the vertex say u_2 and the vertex $v_3$is not adjacent to the vertex say u_3 where the vertices u_1, u_2,and u_3 are different, then the set $D = v_1, v_2$ is MOD^cS, so $\gamma_{odc}(G) = 2$.
Subcase2. If $v_1, v_2, v_3 \equiv P_3$, by hypothesis v_1 are v_3 pendent vertices of P_3. Then $D = v_1, v_2$ is MOD^cS. So, $\gamma_{odc}(G) = 2$. From all cases above, the result is obtained.

Corollary 9*For any graph of order n has at least three vertices of degree $n-k-2$ and k isolated vertices, then $\gamma_{odd}(G) \leq 4+k$.*

*Proof.*Suppose that G has at least three vertices of order $n-k-2$ and k isolated vertices, then G has $k+1$ component such that k isolated vertices and subgraph H of order $m = n-k$. Then $\gamma_{odd}(G-H) = k$.
It is clear H has at least three vertices of order $m-2$ and by theorem9 $\gamma_{odd}(H) \leq 4$. So, $\gamma_{odd}(G) \leq 4+k$.

Corollary 10 *1.For any tree graph T of order n and $l \geq 3$ such that l is the number of pendent and s is the number of support vertices in T,*

$$\gamma_{odc}(\overline{T}) = \begin{cases} 3, & \text{if if nis even ands} = 1\text{; or see below } F \\ 2, & \text{if n is even and s¿1 or n is odd ands=1 or see } H\text{ .} \end{cases}$$

$F =$ or n is odd and $s > 1$ and all support vertices in T have even degree in $\overline{T}$
$H =$ or n is odd and $s > 1$ and there is a support vertex in T has odd degree in $\overline{T}$

*Proof.*Suppose that $v_1, v_2, \ldots, v_l$ the set of pendent vertices and $u_1, u_2, \ldots, u_s$ is set of support vertices. It is clear that $deg(vi) = n-2$.
If $l \leq 3$, then T is path and proof this case by proposition 1
If $l \geq 3$, there are two cases as follows
Case1. If n is even, since $deg(v_i)$ is even for$i = 1, \ldots$,l , then there are two cases:
I) If all pendant adjacent to one support vertex say u that mean G is isomorphic to star graph, then $D = v_1, v_2, u$ is minimum odd neighbor in D^c dominating set. Thus, $\gamma_{odc}(\overline{T}) = 3$.
II) If the pendant vertices are adjacent to more than one support vertex, then the set $D = v_1, v_2$ where the vertices v_1, v_2 are adjacent to different two support vertices is minimum odd neighbor in D^c dominating set. Thus, $\gamma_{odc}(\overline{T}) = 2$.
Case2. If n is odd, then there are two cases:
I) If all pendant adjacent to one support vertex say u that mean G is isomorphic to star graph, then $D = v_1, u$ is minimum odd neighbor in D^c dominating set. Thus, $\gamma_{odc}(\overset{T}{}) = 2$.
II) If the pendant vertices are adjacent to more than one support vertex, then there are two cases:
A) If there is a support vertex say w_1 has odd degree, then $D = v_i, w_1$, wherev_i is a pendant vertex that adjacent to w_1 in the graph T is minimum odd neighbor inD^c dominating set in the graph $\overline{T}$. Thus, $\gamma_{odc}(\overline{T}) = 2$.
B) If all support vertices have even degree, then the set $D = v_1, v_2, v_3$ is minimum odd neighbor inD^c dominating set in the graph $\overline{T}$. Thus, $\gamma_{odc}(\overline{T}) = 3$.
Then, we get the result.

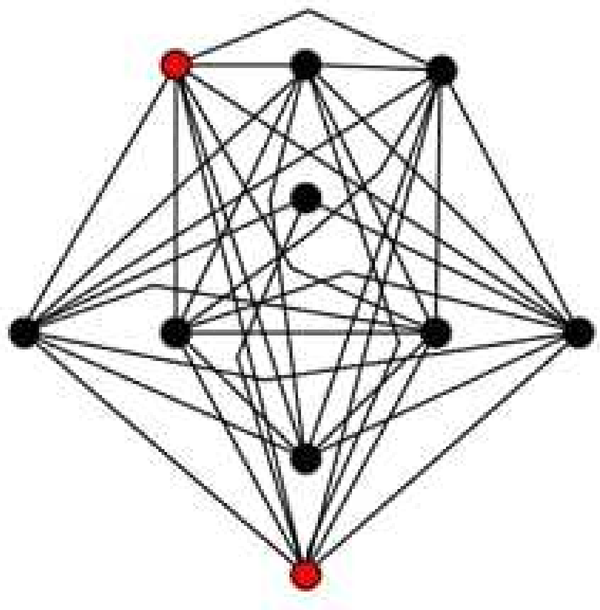

Fig. 1: OD^cS of Tree graph has six pendent vertices and his complement.(a) .

Proposition 11*Let G be any graph of order n, then $\gamma_{odc}(G) = 1$ if and only if n is even and $\Delta = n-1$.*

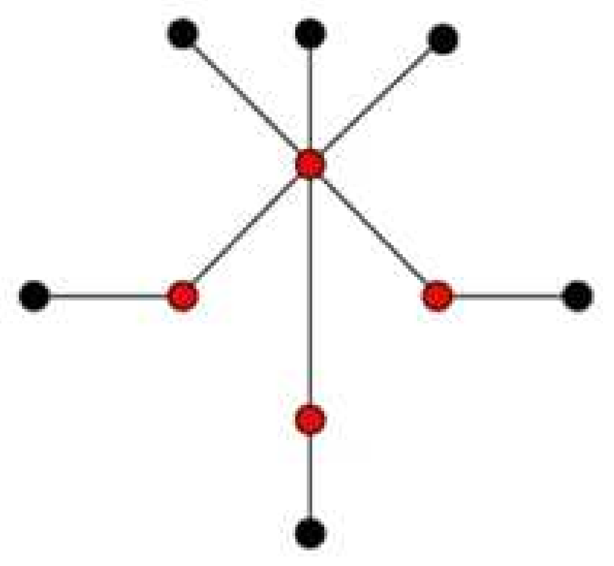

Fig. 2: OD^cS of Tree graph has six pendent vertices and his complement.(b)

*Proof.*If $\gamma_{odc}(G) = 1$, then G has an odd neighbor in D^c dominating set say D contains one vertex say v such that it dominates all other vertices in $V(G)$, then degree of this vertex is$n-1$, and since $\Delta(G) \leq n-1$, so $\Delta = n-1$. Since D is odd neighbor in D^c dominating set, then $deg(v) = n-1$ is odd, then n is even
. Conversely, let n is even and $\Delta = n-1$, then the vertex has degree Δ dominates all other vertices in $V(G)$ and since has degree odd since n is even. So $\gamma_{odd}(G) = 1$.

Proposition 12. *Let G be any graph of order n, then* $\gamma_{odc}(\overline{G}) = 1$ *if and only if n is even and G has at least one isolated vertex.*

*Proof.*Let $\gamma_{odc}(\overline{G}) = 1$ and G has one isolated vertex, then there is a vertex has degree $n-1$ in $\overline{G}$ and this vertex is an isolated in G.
If n is odd, then $n-1$ is even and the dominating set has one vertex but this set is not OD^cS. So, n must be even.
Conversely, if n is even and G has at least one isolated vertex, then this isolated vertex has odd degree in $\overline{G}$ and it represents dominating set. Hence, $\gamma_{odc}(\overline{G}) = 1$.

3 Conclusion

Throughout this paper, many new bounded of the domination graph mentioned above are been calculated. Moreover, the number of the graph G has at least three vertices have the degree $(n-2)$ is been proved. Also, the bounds of this number of the complement of a graph is discussed.

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Conflict of Interest

The authors declare that they have no conflict of interest.

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