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Intuitionistic Fuzzy Generalized Conformable Fractional Derivative

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Abstract: The innovative idea of Atanassov's intuitionistic fuzzy sets (IFSs) is to get a more comprehensive and detailed description of the ambiguity and uncertainty by introducing a membership function and a nonmembership function. Each element in an IFS is represented by an ordered pair, which is known an intuitionistic fuzzy number (IFN). In this paper, We introduced a new definition of the generalized conformable fractional derivative of the intuitionistic fuzzy number-valued functions, Using this definition, we prove some results and with the help of α-cut set, the Hukuhara difference between intuitionistic fuzzy numbers are defined and proved. An intuitionistic fuzzy conformable nuclear decay equation with the initial condition given to show the new theorems and is solved under a new generalized conformable fractional derivative concept.

Keywords: Intuitionistic fuzzy number, intuitionistic fuzzy conformable fractional differentiability, fuzzy sets.

1 Introduction

We recall that the equation

$$\frac{dN(t)}{dt} = -\lambda .N(t),\ t \in I \qquad (1)$$
$$N(t_0) = N_0$$

which is known as the nuclear decay equation, where $N(t)$ the number of radionuclides present is in a given radioactive material, λ is the decay constant, and N_0 is the initial number of radionuclides. If we have uncertain information about the initial value N_0 of radionuclides present in the material, uncertainty is introduced in the model. Note that the phenomenon of nuclear disintegration is considered a stochastic process, uncertainty being introduced by the lack of information on the radioactive material under study. However, in some situations, there may be hesitation on the number of radionuclides present in the radioactive material. When the existence of nuclear disintegration has occurred, then the classical fuzzy nuclear decay equation is not capable to tackle the situation. Therefore, to analyze this situation, we incorporate an intuitionistic fuzzy environment in our proposed method, we consider N_0 being a triangular intuitionistic fuzzy number.
Fuzzy set theory was introduced by Zadeh in 1965 [1] and Atanassov developed the concept of fuzzy set theory to intuitionistic fuzzy set theory [2,3,4]. Fuzzy sets are only characterized by the degree of belongingness but an intuitionistic fuzzy set is characterized by two functions expressing the degree of belongingness and the degree of non-belongingness, respectively and so that the sum of both values is less than one[5,6,7,8]. Fuzzy sets are IFSs, but the opposite is not always true. Over the last few decades, IFS theory has been extensively investigated by many researchers and applied in a variety of fields including decision making and medical diagnosis and pattern recognition etc[9,10,11,12,13]. As far as we know, however, there are only a few investigations on the intuitionistic fuzzy differential equation.

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In [14] the Fuzzy generalized conformable fractional derivative depending just on the basic limit definition of the derivative, for $F: I \rightarrow \mathbb{R}_{\mathscr{F}}$ of order $q \in (0,1]$ of F at $t>0$

$$T_q(F)(t)=\lim_{\varepsilon \rightarrow 0^+} \frac{F\left(t+\varepsilon t^{1-q}\right) \ominus F(t)}{\varepsilon}=\lim_{\varepsilon \rightarrow 0^+} \frac{F(t) \ominus F\left(t-\varepsilon t^{1-q}\right)}{\varepsilon}.$$

This paper defines and studies some results for the generalized conformable fractional derivative of the intuitionistic fuzzy number-valued functions, and studies the solutions of intuitionistic fuzzy conformable nuclear decay equation.

2 Preliminaries

Let a set X be fixed. An intuitionistic fuzzy set $\tilde{A}^i$ in X is an object having the form $\tilde{A}^i=\{\langle x, \mu_{\tilde{A}^i}(x), v_{\tilde{A}^i}(x)\rangle\}$, where $\mu_{\tilde{A}^i}(x): X \rightarrow [0,1]$ and $v_{\tilde{A}^i}(x): X \rightarrow [0,1]$ define the degree of membership and degree of non-membership respectively, of the element $x \in X$ to the set $\tilde{A}^i$, which is subset of X, for every element of $x \in X$ $0 \leq \mu_{\tilde{A}^i}(x)+v_{\tilde{A}^i}(x) \leq 1$. Let $X=\mathbb{R}$

Definition 1. *Let $\mathbb{F}=\left\{\tilde{A}^i \mid \tilde{A}^i: \mathbb{R} \rightarrow [0,1]^2\right.$, satisfies $\left.(1)-(5)\right\}$: An intuitionistic fuzzy number $\tilde{A}^i$ is*

1.Normal i.e there is any $x_0, x_1 \in \mathbb{R}$ such that $\mu_{\tilde{A}^i}(x_0)=1$ and $v_{\tilde{A}^i}(x_1)=1$.
2.Convex for the membership function $\mu_{\tilde{A}^i}(x)$ i.e

$$\mu_{\tilde{A}^i}(\lambda x_1+(1-\lambda) x_2) \geq \min \left(\mu_{\tilde{A}^i}(x_1), \mu_{\tilde{A}^i}(x_2)\right) \forall x_1, x_2 \in \mathbb{R}, \lambda \in [0,1]$$

3.Concave for non-membership function $v_{\tilde{A}^i}(x)$ i.e

$$v_{\tilde{A}^{\bar{i}}}(\lambda x_1+(1-\lambda) x_2) \leq \max \left(v_{\tilde{A}^i}(x_1), v_{\tilde{A}^i}(x_2)\right) \forall x_1, x_2 \in \mathbb{R}, \lambda \in [0,1]$$

4.$\mu_{\tilde{A}^i}(x)$ is upper semi-continuous and $v_{\tilde{A}^i}(x)$ is lower semi-continuous and 5. $\operatorname{supp}(\mu_{\tilde{A}^i}, v_{\tilde{A}^i})=cl\{x \in \mathbb{R}: v_{\tilde{A}^i}(x)<1\}$ is bounded.
Then $\mathbb{IF}$ is called intuitionistic fuzzy space.

Remark. $\mathbb{IF}$ can be written as $\mathbb{IF}=\left[\mathbb{R}_{\mathscr{F}}^{+}, \mathbb{R}_{\mathscr{F}}^{-}\right]$ where $\mathbb{R}_{\mathscr{F}}^{+}$ and $\mathbb{R}_{\mathscr{F}}^{-}$ are two spaces of fuzzy numbers.

Definition 2. *If $\tilde{A}^i$ is an intuitionistic fuzzy number α-cut is given by*

$$\left[\tilde{A}^i\right]^\alpha=\left\{\left[A^{+}\right]^\alpha,\left[A^{-}\right]^\alpha ; \alpha \in [0,1]\right\}$$

$$\text{where } \left[A^{-}\right]^\alpha=\{x \in \mathbb{R}: v_{\tilde{A}^i}(x) \leq 1-\alpha\}, \quad \left[A^{+}\right]^\alpha=\{x \in \mathbb{R}: \mu_{\tilde{A}^i}(x) \geq \alpha\}.$$

It is expressed as $\left[\tilde{A}^i\right]^\alpha=\left\{\left[A_1^{+\alpha}, A_2^{+\alpha}\right],\left[A_1^{-\alpha}, A_2^{-\alpha}\right] ; \alpha \in [0,1]\right\}$

(i)$A_1^{+\alpha}$ and $A_2^{-\alpha}$ will be continuous, monotonic increasing function of α
(ii)$A_2^{+\alpha}$ and $A_1^{-\alpha}$ will be continuous, monotonic decreasing function of α
(iii)$A_1^{+1}=A_2^{+1}; A_1^{-0}=A_2^{-0}$.

Definition 3. *A Triangular Intuitionistic Fuzzy Number (TIFN) $\tilde{A}^i$ is an intuitionistic fuzzy in $\mathbb{R}$ with following membership function $(\mu_{\tilde{A}^i}(x))$ and non-membership $(v_{\tilde{A}^i}(x))$*

$$\mu_{\tilde{A}^i}(x)=\begin{cases}\frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2 \\ \frac{a_3-x}{a_3-a_2}, & a_2 \leq x \leq a_3 \\ 0, & \text{otherwise.}\end{cases} \quad \text{and} \quad v_{\tilde{A}^i}(x)=\begin{cases}\frac{a_2-x}{a_2-a_1^{\prime}}, & a_1^{\prime} \leq x \leq a_2 \\ \frac{x-a_2}{a_3^{\prime}-a_2}, & a_2 \leq x \leq a_3^{\prime} \\ 1, & \text{otherwise.}\end{cases}$$

Where $a_1^{\prime}<a_1<a_2^{\prime}<a_2<a_3^{\prime}$ and $\mu_{\tilde{A}^i}(x), v_{\tilde{A}^i}(x) \leq 0.5$ for $\mu_{\tilde{A}^i}(x)=v_{\tilde{A}^i}(x)$ $\forall x \in \mathbb{R}$. This TIFN is denoted by $\tilde{A}^i=(a_1, a_2, a_3; a_1^{\prime}, a_2^{\prime}, a_3^{\prime})$ We will write :

1.$\tilde{A}^i>0$ if $a_1^{\prime}>0$,
2.$\tilde{A}^i \geq 0$ if $a_1^{\prime} \geq 0$,
3.$\tilde{A}^i<0$ if $a_3^{\prime}<0$
4.$\tilde{A}^i \leq 0$ if $a_3^{\prime} \leq 0$. and

$$\left[A^{+}\right]^{\alpha}=\left[a_{1}+\alpha\left(a_{2}-a_{1}\right), a_{3}-\alpha\left(a_{3}-a_{2}\right)\right] \text{ and } \left[A^{-}\right]^{\alpha}=\left[a_{1}^{\prime}+\alpha\left(a_{2}-a_{1}^{\prime}\right), a_{3}^{\prime}-\alpha\left(a_{3}^{\prime}-a_{2}\right)\right]$$

For $\tilde{A}^{i}, \tilde{B}^{i} \in \mathbb{IF}$ and $\lambda \in \mathbb{R}$, the addition and scaler-multiplication are defined as follows

$$\left[\left(\tilde{A}^{i}+\tilde{B}^{i}\right)\right]^{\alpha}=\left(\left[A^{+}\right]^{\alpha}+\left[B^{+}\right]^{\alpha},\left[A^{-}\right]^{\alpha}+\left[B^{-}\right]^{\alpha}\right)$$

$$\left[\lambda \tilde{A}^{i}\right]^{\alpha}=\begin{cases}\left(\left[\lambda A_{1}^{+\alpha}, \lambda A_{2}^{+\alpha}\right],\left[\lambda A_{1}^{-\alpha}, \lambda A_{2}^{-\alpha}\right]\right) & \lambda \geq 0 \\ \left(\left[\lambda A_{2}^{+\alpha}, \lambda A_{1}^{+\alpha}\right],\left[\lambda A_{2}^{-\alpha}, \lambda A_{1}^{-\alpha}\right]\right), & \lambda<0\end{cases}$$

Define $d: \mathbb{R}_{\mathscr{F}}^{+} \times \mathbb{R}_{\mathscr{F}}^{+} \rightarrow \mathbb{R}_{+} \cup\{0\}$ and $d: \mathbb{R}_{\mathscr{F}}^{-} \times \mathbb{R}_{\mathscr{F}}^{-} \rightarrow \mathbb{R}_{+} \cup\{0\}$ by the equation

$$d(u, v)=\sup_{\alpha \in[0,1]} d_{H}\left(\left[u^{+}\right]^{\alpha},\left[v^{+}\right]^{\alpha}\right), \quad \text{for all } u^{+}, v^{+} \in \mathbb{R}_{\mathscr{F}}+$$
$$d(u, v)=\sup_{\alpha \in[0,1]} d_{H}\left(\left[u^{-}\right]^{\alpha},\left[v^{-}\right]^{\alpha}\right), \quad \text{for all } u^{-}, v^{-} \in \mathbb{R}_{\mathscr{F}}^{-}$$

where d_H is the Hausdorff metric.

$$d_{H}\left(\left[u^{+}\right]^{\alpha},\left[v^{+}\right]^{\alpha}\right)=\max \left\{\left|u_{1}^{+\alpha}-v_{1}^{+\alpha}\right|,\left|u_{2}^{+\alpha}-v_{2}^{+\alpha}\right|\right\}$$
$$d_{H}\left(\left[u^{-}\right]^{\alpha},\left[v^{-}\right]^{\alpha}\right)=\max \left\{\left|u_{1}^{-\alpha}-v_{1}^{-\alpha}\right|,\left|u_{2}^{-\alpha}-v_{2}^{-\alpha}\right|\right\}$$

It is well known that $\left(\mathbb{R}_{\mathscr{F}}^{+}, d\right)$ and $\left(\mathbb{R}_{\mathscr{F}}^{-}, d\right)$ are complete metric spaces [15] We adopt the general definition of a intuitionistic fuzzy number given in [16,17,18,19,20,21] Let $I=(0, a) \subset \mathbb{R}$ be an interval.

3 The Intuitionistic Fuzzy Conformable Fractional Differentiability

Definition 4. *Let $\tilde{u}^{i}, \tilde{v}^{i} \in \mathbb{IF}$. If there exists $\tilde{w}^{i} \in \mathbb{IF}$ such that $\tilde{u}^{i}=\tilde{v}^{i}+\tilde{w}^{i}$ then $\tilde{w}^{i}$ is called the iH-difference of $\tilde{u}^{i}$ and $\tilde{v}^{i}$ and it is denoted by $\tilde{u}^{i} \ominus_{i} \tilde{v}^{i}$*

Theorem 1. *If $\tilde{u}^{i}, \tilde{v}^{i} \in \mathbb{IF}$, then the α-cut set of the iH-difference $\tilde{u}^{i}$ and $\tilde{v}^{i}$ is H-difference of membership function and non-membership function of $\tilde{u}^{i}, \tilde{v}^{i}$*

Proof. Suppose that the iH-difference $\tilde{u}^{i}$ and $\tilde{v}^{i}$ is $\tilde{w}^{i}$, then

$$\tilde{u}^{i} \ominus_{i} \tilde{v}^{i}=\tilde{w}^{i} \Longleftrightarrow \tilde{u}^{i}=\tilde{v}^{i}+\tilde{w}^{i}$$

and by α-cut set we have $\left[\tilde{u}^{i}\right]^{\alpha}=\left[\tilde{v}^{i}\right]^{\alpha}+\left[\tilde{w}^{i}\right]^{\alpha}$ i.e

$$\left[u^{+}\right]^{\alpha}=\left[v^{+}\right]^{\alpha}+\left[w^{+}\right]^{\alpha},\left[u^{-}\right]^{\alpha}=\left[v^{-}\right]^{\alpha}+\left[w^{-}\right]^{\alpha}$$

then $u^{+} \ominus v^{+}=w^{+}, u^{-} \ominus v^{-}=w^{-}$

Definition 5. *Let $\tilde{F}^{i}: I \rightarrow \mathbb{IF}$ be intuitionistic fuzzy function. q^{th} order " intuitionistic fuzzy conformable fractional derivative " of $\tilde{F}^{i}$ is defined by*

$$T_{q}\left(\tilde{F}^{i}\right)(t)=\lim_{\varepsilon \rightarrow 0^{+}} \frac{\tilde{F}^{i}\left(t+\varepsilon t^{1-q}\right) \ominus_{i} \tilde{F}^{i}(t)}{\varepsilon}=\lim_{\varepsilon \rightarrow 0^{+}} \frac{\tilde{F}^{i}(t) \ominus_{i} \tilde{F}^{i}\left(t-\varepsilon t^{1-q}\right)}{\varepsilon}.$$

for all $t>0, q \in(0,1)$. Let $\left(\tilde{F}^{i}\right)^{(q)}(t)$ stands for $T_{q}\left(\tilde{F}^{i}\right)(t)$. Hence

$$\left(\tilde{F}^{i}\right)^{(q)}(t)=\lim_{\varepsilon \rightarrow 0^{+}} \frac{\tilde{F}^{i}\left(t+\varepsilon t^{1-q}\right) \ominus_{i} \tilde{F}^{i}(t)}{\varepsilon}=\lim_{\varepsilon \rightarrow 0^{+}} \frac{\tilde{F}^{i}(t) \ominus_{i} \tilde{F}^{i}\left(t-\varepsilon t^{1-q}\right)}{\varepsilon}.$$

If $\tilde{F}^{i}$ is q-differentiable in some I, and $\lim_{t \rightarrow 0^{+}} F^{(q)}(t)$ exists, then

$$\left(\tilde{F}^{i}\right)^{(q)}(0)=\lim_{t \rightarrow 0^{+}}\left(\tilde{F}^{i}\right)^{(q)}(t)$$

and the limits (in the metric d)

Definition 6. *Let $\tilde{F}^i : I \to \mathbb{IF}$ and $t \in I.q^{th}$ order we say that $\tilde{F}^i$ is q-differentiable at t, if there exist elements $T_q(F^+)(t) \in \mathbb{R}^+_{\mathscr{F}}, T_q(F^-)(t) \in \mathbb{R}^-_{\mathscr{F}}$ such that For all $\varepsilon > 0$ sufficiently small, $\exists F^+\left(t+\varepsilon t^{1-q}\right) \ominus F^+(t), F^+(t) \ominus_i F^+\left(t-\varepsilon t^{1-q}\right)$ and $\exists F^-(t+\ \varepsilon t^{1-q}) \ominus F^-(t), F^-(t) \ominus_i F^-\left(t-\varepsilon t^{1-q}\right)$*

$$T_qF^+(t) = \lim_{\varepsilon\to 0^+} \frac{F^+\left(t+\varepsilon t^{1-q}\right) \ominus F^+(t)}{\varepsilon} = \lim_{\varepsilon\to 0^+} \frac{F^+(t) \ominus_i F^+\left(t-\varepsilon t^{1-q}\right)}{\varepsilon}$$
$$T_qF^-(t) = \lim_{\varepsilon\to 0^+} \frac{F^-\left(t+\varepsilon t^{1-q}\right) \ominus F^-(t)}{\varepsilon} = \lim_{\varepsilon\to 0^+} \frac{F^-(t) \ominus_i F^-\left(t-\varepsilon t^{1-q}\right)}{\varepsilon}$$

for all $t > 0, q \in (0,1)$. Let $\left(\tilde{F}^i\right)^{(q)}(t)$ stands for $T_q\left(\tilde{F}^i\right)(t)$. Hence

$$\left(F^+\right)^{(q)}(t) = \lim_{\varepsilon\to 0^+} \frac{F^+\left(t+\varepsilon t^{1-q}\right) \ominus F^+(t)}{\varepsilon} = \lim_{\varepsilon\to 0^+} \frac{F^+(t) \ominus F^+\left(t-\varepsilon t^{1-q}\right)}{\varepsilon}$$
$$\left(F^-\right)^{(q)}(t) = \lim_{\varepsilon\to 0^+} \frac{F^-\left(t+\varepsilon t^{1-q}\right) \ominus F^-(t)}{\varepsilon} = \lim_{\varepsilon\to 0^+} \frac{F^-(t) \ominus F^-\left(t-\varepsilon t^{1-q}\right)}{\varepsilon}$$

If $\tilde{F}^i$ is q-differentiable in some I, and $\lim_{t\to 0^+} F^{(q)}(t)$ exists, then

$$\left(F^+\right)^{(q)}(0) = \lim_{t\to 0^+} \left(F^+\right)^{(q)}(t)$$
$$\left(F^-\right)^{(q)}(0) = \lim_{t\to 0^+} \left(F^-\right)^{(q)}(t)$$

and the limits (in the metric d)

Remark. From the definition, it directly follows that if $\tilde{F}^i$ is q-differentiable then the multi-valued mapping F^+_α and F^-_α are q-differentiable for all $\alpha \in [0,1]$ and

$$T_qF^+_\alpha = \left[\left(F^+\right)^{(q)}(t)\right]^\alpha \quad \text{and} \quad T_qF^-_\alpha = \left[\left(F^-\right)^{(q)}(t)\right]^\alpha \tag{2}$$

Here $T_qF^+_\alpha$ and $T_qF^-_\alpha$ are denoted the fuzzy conformable fractional derivative [14] of F^+_α and F^-_α of order q.

However, for the converse result we have the following:

Theorem 2. *Let $\tilde{F}^i : I \to \mathbb{IF}$ be q-differentiable. Denote $F^+_\alpha(t) = \left[\left(f^+_1\right)^\alpha(t), \left(f^+_2\right)^\alpha(t)\right]$ and $F^-_\alpha(t) = \left[\left(f^-_1\right)^\alpha(t), \left(f^-_2\right)^\alpha(t)\right], \alpha \in [0,1]$. Then $\left(f^+_1\right)^\alpha(t), \left(f^+_2\right)^\alpha(t), \left(f^-_1\right)^\alpha(t)$ and $\left(f^-_2\right)^\alpha(t)$ are q-differentiable and*

$$\left[\left(\tilde{F}^i\right)^{(q)}(t)\right]^\alpha = \left\{\left[\left(F^+\right)^{(q)}(t)\right]^\alpha, \left[\left(F^-\right)^{(q)}(t)\right]^\alpha\right], \alpha \in [0,1]\right\}$$

where
$$\left[\left(F^+\right)^{(q)}(t)\right]^\alpha = \left[\left(f^+_1\right)^{\alpha(q)}(t), \left(f^+_2\right)^{\alpha(q)}(t)\right], \left[\left(F^-\right)^{(q)}(t)\right]^\alpha = \left[\left(f^-_1\right)^{\alpha(q)}(t), \left(f^-_2\right)^{\alpha(q)}(t)\right]$$

Proof. If $\varepsilon > 0, q \in (0,1]$ and $\alpha \in [0,1]$, let us consider $\tilde{F}^i$ to be iH-differentiable function, then using Theorem 1 we have :

$$\begin{aligned}\left[\tilde{F}^i\left(t+\varepsilon t^{1-q}\right) \ominus_i \tilde{F}^i(t)\right]^\alpha &= \left\{\left[F^+\left(t+\varepsilon t^{1-q}\right) \ominus F^+(t)\right]^\alpha; \left[F^-\left(t+\varepsilon t^{1-q}\right) \ominus F^-(t)\right]^\alpha, \alpha \in [0,1]\right\} \\ &= \left\{\left[\left(f^+_1\right)^\alpha\left(t+\varepsilon t^{1-q}\right) - \left(f^+_1\right)^\alpha(t), \left(f^+_2\right)^\alpha\left(t+\varepsilon t^{1-q}\right) - \left(f^+_2\right)^\alpha(t)\right]\right. \\ &\quad \left. ;\ \left[\left(f^-_1\right)^\alpha\left(t+\varepsilon t^{1-q}\right) - \left(f^-_1\right)^\alpha(t), \left(f^-_2\right)^\alpha\left(t+\varepsilon t^{1-q}\right) - \left(f^-_2\right)^\alpha(t)\right]\right\}\end{aligned}$$

Dividing by ε, we have :

$$\frac{\left[\tilde{F}^i\left(t+\varepsilon t^{1-q}\right)\ominus_i \tilde{F}^i(t)\right]^\alpha}{\varepsilon}=\left\{\frac{\left[F^+\left(t+\varepsilon t^{1-q}\right)\ominus F^+(t)\right]^\alpha}{\varepsilon};\frac{\left[F^-\left(t+\varepsilon t^{1-q}\right)\ominus F^-(t)\right]^\alpha}{\varepsilon},\alpha\in[0,1]\right\}$$
$$=\left\{\left[\frac{\left(f_1^+\right)^\alpha\left(t+\varepsilon t^{1-q}\right)-\left(f_1^+\right)^\alpha(t)}{\varepsilon},\frac{\left(f_2^+\right)^\alpha\left(t+\varepsilon t^{1-q}\right)-\left(f_2^+\right)^\alpha(t)}{\varepsilon}\right]\right.$$
$$\left.;\left[\frac{\left(f_1^-\right)^\alpha\left(t+\varepsilon t^{1-q}\right)-\left(f_1^-\right)^\alpha(t)}{\varepsilon},\frac{\left(f_2^-\right)^\alpha\left(t+\varepsilon t^{1-q}\right)-\left(f_2^-\right)^\alpha(t)}{\varepsilon}\right]\right\}$$

Similarly, we obtain :

$$\frac{\left[\tilde{F}^i(t)\ominus_i \tilde{F}^i\left(t-\varepsilon t^{1-q}\right)\right]^\alpha}{\varepsilon}=\left\{\frac{\left[F^+(t)\ominus F^+\left(t-\varepsilon t^{1-q}\right)\right]^\alpha}{\varepsilon};\frac{\left[F^-(t)\ominus F^-\left(t-\varepsilon t^{1-q}\right)\right]^\alpha}{\varepsilon},\alpha\in[0,1]\right\}$$
$$=\left\{\left[\frac{\left(f_1^+\right)^\alpha(t)-\left(f_1^+\right)^\alpha\left(t-\varepsilon t^{1-q}\right)}{\varepsilon},\frac{\left(f_2^+\right)^\alpha(t)-\left(f_2^+\right)^\alpha\left(t-\varepsilon t^{1-q}\right)}{\varepsilon}\right]\right.$$
$$\left.;\left[\frac{\left(f_1^-\right)^\alpha(t)-\left(f_1^-\right)^\alpha\left(t-\varepsilon t^{1-q}\right)}{\varepsilon},\frac{\left(f_2^-\right)^\alpha(t)-\left(f_2^-\right)^\alpha\left(t-\varepsilon t^{1-q}\right)}{\varepsilon}\right]\right\}$$

and passing to the limit gives the theorem.

4 The Generalized Intuitionistic Fuzzy Conformable Fractional Differentiability

Let $c\in\mathbb{IF}$ and $g:I\to\mathbb{R}_+$ be q-differentiable for some $q\in(0,1]$. Define $F:I\to\mathbb{IF}$ by $F(t)=c.\,g(t)$ where $F=(f^+,f^-)$ and $f^+\in\mathbb{R}^+_{\mathscr{F}},f^-\in\mathbb{R}^-_{\mathscr{F}}$, for all $t\in I$. Firstly, let us suppose that $g^{(q)}>0$. Then by $g^{(q)}(t)=\lim_{\varepsilon\to 0}\frac{g\left(t+\varepsilon t^{1-q}\right)-g(t)}{\varepsilon}$ it follows that for $\varepsilon>0$ sufficiently small we have $g\left(t+\varepsilon t^{1-q}\right)-g(t)=w\left(t,\varepsilon t^{1-q}\right)>0$. Multiplying by c, it follows $c.\,g\left(t+\varepsilon t^{1-q}\right)=c.\,g(t)+c.\,w\left(t,\varepsilon t^{1-q}\right)$ i.e there exists the H-difference $f^+\left(t+\varepsilon t^{1-q}\right)\ominus f^+(t)$ and $f^-\left(t+\varepsilon t^{1-q}\right)\ominus f^-(t)$ then there exists the iHdifference $F\left(t+\varepsilon t^{1-q}\right)\ominus_i F(t)$. Similary, by $g^{(q)}(t)=\lim_{\varepsilon\to 0}\frac{g(t)-g\left(t-\varepsilon t^{1-q}\right)}{\varepsilon}$, reasoning as above we get there exists the iH-difference $F(t)\ominus_i F\left(t-\varepsilon t^{1-q}\right)$ too. Also, simple reasoning shows in this case that $F^{(q)}(t)=c.\,g^{(q)}(t)$ for some $q\in(0,1]$. Now, if we suppose $g^{(q)}<0$, we easily see that we cannot use the above kind of reasoning to prove that the iH-differences $F\left(t+\varepsilon t^{1-q}\right)\ominus_i F(t),F(t)\ominus_i F\left(t-\varepsilon t^{1-q}\right)$ and the conformable fractional derivative $F^{(q)}(t)$ exist. Consequently, by Definition 6 we cannot say that exists $F^{(q)}(t)$. This shortcoming can be solved by introducing some generalized concepts of the conformable fractional derivative as follows.
We consider the following definition.

Definition 7. *Let $\tilde{F}^i:I\to\mathbb{IF}$ be an intuitionistic fuzzy function and $q\in(0,1]$. One says, $\tilde{F}^i$ is $q_{(1)}$-differentiable at point $t>0$ if there exists an element $\left(F^+\right)^{(q)}(t)\in\mathbb{R}^+_{\mathscr{F}}$ and $\left(F^-\right)^{(q)}(t)\in\mathbb{R}^-_{\mathscr{F}}$ such that for all $\varepsilon>0$ sufficiently near to 0, there exist $\tilde{F}^i\left(t+\varepsilon t^{1-q}\right)\ominus_i\tilde{F}^i(t),\tilde{F}^i(t)\ominus_i\tilde{F}^i\left(t-\varepsilon t^{1-q}\right)$ and the limits (in the metric d)*

$$\lim_{\varepsilon\to 0^+}\frac{\tilde{F}^i\left(t+\varepsilon t^{1-q}\right)\ominus\tilde{F}^i(t)}{\varepsilon}=\lim_{\varepsilon\to 0^+}\frac{\tilde{F}^i(t)\ominus\tilde{F}^i\left(t-\varepsilon t^{1-q}\right)}{\varepsilon}=\left(\tilde{F}^i\right)^{(q)}(t)\tag{3}$$

$\tilde{F}^i$ is $q_{(2)}$-differentiable at $t>0$ if for all $\varepsilon<0$ sufficiently near to 0, there exist $\tilde{F}^i\left(t+\varepsilon t^{1-q}\right)\ominus\tilde{F}^i(t),\tilde{F}^i(t)\ominus\tilde{F}^i\left(t-\varepsilon t^{1-q}\right)$

$$\lim_{\varepsilon\to 0^-}\frac{\tilde{F}^i\left(t+\varepsilon t^{1-q}\right)\ominus_i\tilde{F}^i(t)}{\varepsilon}=\lim_{\varepsilon\to 0^-}\frac{\tilde{F}^i(t)\ominus_i\tilde{F}^i\left(t-\varepsilon t^{1-q}\right)}{\varepsilon}=\left(\tilde{F}^i\right)^{(q)}(t)\tag{4}$$

If $\tilde{F}^i$ is $q_{(n)}$-differentiable at $t>0$, we denote its q-derivatives ($q\in(0,1]$) by $\left(\tilde{F}^i\right)^{(q)}_n(t)$, for $n=1,2$.

Theorem 3. *If $g:I\to\mathbb{R}$ is conformable fractional derivative on I such that $g^{(q)}$ has at most finite number of roots in I and $c\in\mathbb{IF}$, the $F(t)=c.\,g(t)$ is generalized intuitionistic fuzzy conformable fractional derivative on I and $F^{(q)}(t)=c.\,g^{(q)}(t),\ \forall t\in I,\ q\in(0,1]$*

Proof. For $t \in I$ and $q \in (0,1]$ we have the possibilities:

1. case (i) $g(t) > 0,\ g^{(q)}(t) > 0$, For $\varepsilon > 0$ Let

$$g^{(q)}(t) = \lim_{\varepsilon \to 0^+} \frac{g\left(t+\varepsilon t^{1-q}\right)-g(t)}{\varepsilon} = \lim_{\varepsilon \to 0^+} \frac{g(t)-g\left(t-\varepsilon t^{1-q}\right)}{\varepsilon}$$

sufficiently small, $g\left(t+\varepsilon t^{1-q}\right) > 0, g\left(t-\varepsilon t^{1-q}\right) > 0$,

$$g\left(t+\varepsilon t^{1-q}\right)-g(t) = \delta_1\left(t,\varepsilon t^{1-q}\right) > 0, \quad g(t)-g\left(t-\varepsilon t^{1-q}\right) = \delta_2\left(t,\varepsilon t^{1-q}\right) > 0$$

i.e $g\left(t+\varepsilon t^{1-q}\right) = g(t)+\delta_1\left(t,\varepsilon t^{1-q}\right) > 0, g(t) = g\left(t-\varepsilon t^{1-q}\right)+\delta_2\left(t,\varepsilon t^{1-q}\right) > 0$.
Multiplying by $c \in \mathbb{IF}$, we get that there exist $F\left(t+\varepsilon t^{1-q}\right) \ominus_i F(t), F(t) \ominus_i F\left(t-\varepsilon t^{1-q}\right)$ and that

$$F^{(q)}(t) = \lim_{\varepsilon \to 0^+} \frac{F\left(t+\varepsilon t^{1-q}\right) \ominus_i F(t)}{\varepsilon} = \lim_{\varepsilon \to 0^+} \frac{F(t) \ominus_i F\left(t-\varepsilon t^{1-q}\right)}{\varepsilon} = c.\ g^{(q)}(t)$$

i.e F is generalized conformable fractional derivative by definition 7 (i).

2. case (ii) $g(t) > 0,\ g^{(q)}(t) < 0$, For $\varepsilon < 0$ Let

$$g^{(q)}(t) = \lim_{\varepsilon \to 0^-} \frac{g\left(t+\varepsilon t^{1-q}\right)-g(t)}{\varepsilon} = \lim_{\varepsilon \to 0^-} \frac{g(t)-g\left(t-\varepsilon t^{1-q}\right)}{\varepsilon}$$

sufficiently small, $g\left(t+\varepsilon t^{1-q}\right) > 0, g\left(t-\varepsilon t^{1-q}\right) > 0$,

$$g\left(t+\varepsilon t^{1-q}\right)-g(t) = \delta_1\left(t,\varepsilon t^{1-q}\right) > 0, \quad g(t)-g\left(t-\varepsilon t^{1-q}\right) = \delta_2\left(t,\varepsilon t^{1-q}\right) > 0$$

i.e $g\left(t+\varepsilon t^{1-q}\right) = g(t)+\delta_1\left(t,\varepsilon t^{1-q}\right) > 0, g(t) = g\left(t-\varepsilon t^{1-q}\right)+\delta_2\left(t,\varepsilon t^{1-q}\right) > 0$. Multiplying by $c \in \mathbb{IF}$, we get that there exist $F\left(t+\varepsilon t^{1-q}\right) \ominus_i F(t), F(t) \ominus_i F\left(t-\varepsilon t^{1-q}\right)$ and that

$$F^{(q)}(t) = \lim_{\varepsilon \to 0^-} \frac{F\left(t+\varepsilon t^{1-q}\right) \ominus_i F(t)}{\varepsilon} = \lim_{\varepsilon \to 0^-} \frac{F(t) \ominus_i F\left(t-\varepsilon t^{1-q}\right)}{\varepsilon} = c.\ g^{(q)}(t)$$

i.e F is generalized conformable fractional derivative by definition 7 (ii).

3. case (iii) $g(t) < 0,\ g^{(q)}(t) > 0$ and case (iv) $g(t) < 0,\ g^{(q)}(t) < 0$ are similar to the proofs of the above cases $(i),(ii)$.

Remark. In the previous definition, $q_{(1)}$-differentiable corresponds to definition 7 so this differentiability concept is a generalization of definition 6 and obviously more general. For instance, for $F(t) = c.\ g(t)$ with $g^{(q)}(t_0) < 0$, we have

$$F^{(q)}(t_0) = c.\ g^{(q)}(t_0)$$

Theorem 4. *Let $\tilde{F}^i : I \to \mathbb{IF}$ be intuitionistic fuzzy function, $\tilde{F}^i(t) = [F_\alpha^+(t), F_\alpha^-(t)]$*

$$\text{where } F_\alpha^+(t) = \left[\left(f_1^+\right)^\alpha(t), \left(f_2^+\right)^\alpha(t)\right] \text{ and } F_\alpha^-(t) = \left[\left(f_1^-\right)^\alpha(t), \left(f_2^-\right)^\alpha(t)\right], \alpha \in [0,1]$$

(i)If $\tilde{F}^i$ is $q_{(1)}$-differentiable, then $\left(f_1^+\right)^\alpha(t), \left(f_2^+\right)^\alpha(t), \left(f_1^-\right)^\alpha(t)$ and $\left(f_2^-\right)^\alpha(t)$ are q-differentiable and

$$\left[\left(\tilde{F}^i\right)^{(q_{(1)})}(t)\right]^\alpha = \left\{\left[\left(f_1^+\right)^{\alpha(q)}(t), \left(f_2^+\right)^{\alpha(q)}(t)\right], \left[\left(f_1^-\right)^{\alpha(q)}(t), \left(f_2^-\right)^{\alpha(q)}(t)\right]\right\}$$

(ii)If $\tilde{F}^i$ is $q_{(2)}$-differentiable, then $\left(f_1^+\right)^\alpha(t), \left(f_2^+\right)^\alpha(t), \left(f_1^-\right)^\alpha(t)$ and $\left(f_2^-\right)^\alpha(t)$ are q-differentiable and

$$\left[\left(\tilde{F}^i\right)^{(q_{(2)})}(t)\right]^\alpha = \left\{\left[\left(f_2^+\right)^{\alpha(q)}(t), \left(f_1^+\right)^{\alpha(q)}(t)\right], \left[\left(f_2^-\right)^{\alpha(q)}(t), \left(f_1^-\right)^{\alpha(q)}(t)\right]\right\}$$

Proof. (i)See demonstration of Theorem 2.
(ii)If $\varepsilon < 0$, $q \in (0,1]$ and $\alpha \in [0,1]$, then we have

$$\left[\tilde{F}^i\left(t+\varepsilon t^{1-q}\right) \ominus_i \tilde{F}^i(t)\right]^\alpha = \left\{\left[F^+\left(t+\varepsilon t^{1-q}\right) \ominus F^+(t)\right]^\alpha ; \left[F^-\left(t+\varepsilon t^{1-q}\right) \ominus F^-(t)\right]^\alpha, \alpha \in [0,1]\right\}$$
$$= \left\{\left[\left(f_1^+\right)^\alpha\left(t+\varepsilon t^{1-q}\right) - \left(f_1^+\right)^\alpha(t), \left(f_2^+\right)^\alpha\left(t+\varepsilon t^{1-q}\right) - \left(f_2^+\right)^\alpha(t)\right]\right.$$
$$\left. ; \left[\left(f_1^-\right)^\alpha\left(t+\varepsilon t^{1-q}\right) - \left(f_1^-\right)^\alpha(t), \left(f_2^-\right)^\alpha\left(t+\varepsilon t^{1-q}\right) - \left(f_2^-\right)^\alpha(t)\right]\right\}$$

and, multiplying by $\frac{1}{\varepsilon}$ and see proof theorem 6 in [14] we have:

$$\frac{\left[\tilde{F}^i(t) \ominus_i \tilde{F}^i\left(t-\varepsilon t^{1-q}\right)\right]^\alpha}{\varepsilon} = \left\{\frac{\left[F^+(t) \ominus F^+\left(t-\varepsilon t^{1-q}\right)\right]^\alpha}{\varepsilon} ; \frac{\left[F^-(t) \ominus F^-\left(t-\varepsilon t^{1-q}\right)\right]^\alpha}{\varepsilon},\ \alpha \in [0,1]\right\}$$
$$= \left\{\left[\frac{\left(f_2^+\right)^\alpha(t) - \left(f_2^+\right)^\alpha\left(t-\varepsilon t^{1-q}\right)}{\varepsilon}, \frac{\left(f_1^+\right)^\alpha(t) - \left(f_1^+\right)^\alpha\left(t-\varepsilon t^{1-q}\right)}{\varepsilon}\right]\right.$$
$$\left. ; \left[\frac{\left(f_2^-\right)^\alpha(t) - \left(f_2^-\right)^\alpha\left(t-\varepsilon t^{1-q}\right)}{\varepsilon}, \frac{\left(f_1^-\right)^\alpha(t) - \left(f_1^-\right)^\alpha\left(t-\varepsilon t^{1-q}\right)}{\varepsilon}\right]\right\}$$

Similarly, we obtain

$$\frac{\left[\tilde{F}^i(t) \ominus_i \tilde{F}^i\left(t-\varepsilon t^{1-q}\right)\right]^\alpha}{\varepsilon} = \left\{\frac{\left[F^+(t) \ominus F^+\left(t-\varepsilon t^{1-q}\right)\right]^\alpha}{\varepsilon} ; \frac{\left[F^-(t) \ominus F^-\left(t-\varepsilon t^{1-q}\right)\right]^\alpha}{\varepsilon}, \alpha \in [0,1]\right\}$$
$$= \left\{\left[\frac{\left(f_2^+\right)^\alpha(t) - \left(f_2^+\right)^\alpha\left(t-\varepsilon t^{1-q}\right)}{\varepsilon}, \frac{\left(f_1^+\right)^\alpha(t) - \left(f_1^+\right)^\alpha\left(t-\varepsilon t^{1-q}\right)}{\varepsilon}\right]\right.$$
$$\left. ; \left[\frac{\left(f_2^-\right)^\alpha(t) - \left(f_2^-\right)^\alpha\left(t-\varepsilon t^{1-q}\right)}{\varepsilon}, \frac{\left(f_1^-\right)^\alpha(t) - \left(f_1^-\right)^\alpha\left(t-\varepsilon t^{1-q}\right)}{\varepsilon}\right]\right\}$$

and passing to the limit we have

$$\left[\left(\tilde{F}^i\right)^{(q(2)}\right)(t)\right]^\alpha = \left\{\left[\left(f_2^+\right)^{\alpha(9)}(t), \left(f_1^+\right)^{(q)}(t)\right], \left[\left(f_2^-\right)^{\alpha(q)}(t), \left(f_1^-\right)^{\alpha(q)}(t)\right]\right\}.$$

Theorem 5. *Let* $q \in (0,1]$
(i)*If* $\tilde{F}^i$ *is (1)-differentiable and* $\tilde{F}^i$ *is* $q_{(1)}$*-differentiable then*

$$T_{q_{(1)}}\tilde{F}^i(t) = t^{1-q}D_1^1\tilde{F}^i(t)$$

(ii)*If* $\tilde{F}^i$ *is (2)-differentiable and* $\tilde{F}^i$ *is* $q_{(2)}$*-differentiable then*

$$T_{q_{(2)}}\tilde{F}^i(t) = t^{1-q}D_2^1\tilde{F}^i(t)$$

Note that the definition of (n)-differentiable or $\left(D_n^1\right)$ for $n \in 1,2$ see [17,18,19,22]

Proof. We present the details only for the case (i), since the other case is analogous. Let $h = \varepsilon t^{1-q}$ in Definition 7 , and then $\varepsilon = t^{q-1}h$. Therefore, If $\varepsilon > 0$ and $\alpha \in [0,1]$, we have

$$\left[\tilde{F}^i\left(t+\varepsilon t^{1-q}\right) \ominus_i \tilde{F}^i(t)\right]^\alpha = \left\{\left[F^+\left(t+\varepsilon t^{1-q}\right) \ominus F^+(t)\right]^\alpha ; \left[F^-\left(t+\varepsilon t^{1-q}\right) \ominus F^-(t)\right]^\alpha, \alpha \in [0,1]\right\}$$
$$= \left\{\left[\left(f_1^+\right)^\alpha\left(t+\varepsilon t^{1-q}\right) - \left(f_1^+\right)^\alpha(t), \left(f_2^+\right)^\alpha\left(t+\varepsilon t^{1-q}\right) - \left(f_2^+\right)^\alpha(t)\right]\right.$$
$$\left. ; \left[\left(f_1^-\right)^\alpha\left(t+\varepsilon t^{1-q}\right) - \left(f_1^-\right)^\alpha(t), \left(f_2^-\right)^\alpha\left(t+\varepsilon t^{1-q}\right) - \left(f_2^-\right)^\alpha(t)\right]\right\}$$

Dividing by ε, we have

$$\frac{\left[\tilde{F}^i\left(t+\varepsilon t^{1-q}\right)\ominus_i \tilde{F}^i(t)\right]^\alpha}{\varepsilon} = \left\{\frac{\left[F^+\left(t+\varepsilon t^{1-q}\right)\ominus F^+(t)\right]^\alpha}{\varepsilon};\frac{\left[F^-\left(t+\varepsilon t^{1-q}\right)\ominus F^{-(t)]^\alpha}\right.}{\varepsilon},\alpha\in[0,1]\right\}$$

$$= \left\{\left[\frac{\left(f_1^+\right)^\alpha\left(t+\varepsilon t^{1-q}\right)-\left(f_1^+\right)^\alpha(t)}{\varepsilon},\frac{\left(f_2^+\right)^\alpha\left(t+\varepsilon t^{1-q}\right)-\left(f_2^+\right)^\alpha(t)}{\varepsilon}\right]\right.$$

$$\left.;\left[\frac{\left(f_1^-\right)^\alpha\left(t+\varepsilon t^{1-q}\right)-\left(f_1^-\right)^\alpha(t)}{\varepsilon},\frac{\left(f_2^-\right)^\alpha\left(t+\varepsilon t^{1-q}\right)-\left(f_2^-\right)^\alpha(t)}{\varepsilon}\right]\right\}$$

and passing to the limit

$$\lim_{\varepsilon\to0^+}\frac{\left[\tilde{F}^i\left(t+\varepsilon t^{1-q}\right)\ominus_i \tilde{F}^i(t)\right]^\alpha}{\varepsilon} = \lim_{\varepsilon\to0^+}\left\{\left[\frac{\left(f_1^+\right)^\alpha\left(t+\varepsilon t^{1-q}\right)-\left(f_1^+\right)^\alpha(t)}{\varepsilon},\frac{\left(f_2^+\right)^\alpha\left(t+\varepsilon t^{1-q}\right)-\left(f_2^+\right)^\alpha(t)}{\varepsilon}\right]\right.$$

$$\left.;\left[\frac{\left(f_1^-\right)^\alpha\left(t+\varepsilon t^{1-q}\right)-\left(f_1^-\right)^\alpha(t)}{\varepsilon},\frac{\left(f_2^-\right)^\alpha\left(t+\varepsilon t^{1-q}\right)-\left(f_2^{-\alpha}\right)^\alpha(t)}{\varepsilon}\right]\right\}$$

$$= \lim_{h\to0^+}\left\{\left[\frac{\left(f_1^+\right)^\alpha(t+h)-\left(f_1^+\right)^\alpha(t)}{t^{q-1}h},\frac{\left(f_2^+\right)^\alpha(t+h)-\left(f_2^+\right)^\alpha(t)}{t^{q-1}h}\right]\right.$$

$$\left.;\left[\frac{\left(f_1^-\right)^\alpha(t+h)-\left(f_1^-\right)^\alpha(t)}{t^{q-1}h},\frac{\left(f_2^-\right)^\alpha(t+h)-\left(f_2^-\right)^\alpha(t)}{t^{q-1}h}\right]\right\}$$

$$= t^{1-q}\lim_{h\to0^+}\left\{\left[\frac{\left(f_1^+\right)^\alpha(t+h)-\left(f_1^+\right)^\alpha(t)}{h},\frac{\left(f_2^+\right)^\alpha(t+h)-\left(f_2^+\right)^\alpha(t)}{h}\right]\right.$$

$$\left.;\left[\frac{\left(f_1^-\right)^\alpha(t+h)-\left(f_1^-\right)^\alpha(t)}{h},\frac{\left(f_2^-\right)^\alpha(t+h)-\left(f_2^-\right)^\alpha(t)}{h}\right]\right\}$$

$$= t^{1-q}\left\{\left[\left(f_1^+\right)^{\alpha'}(t),\left(f_2^+\right)^{\alpha'}(t)\right],\left[\left(f_1^-\right)^{\alpha'}(t),\left(f_2^-\right)^{\alpha'}(t)\right]\right\}$$

Similarly, we obtain

$$\frac{\left[\tilde{F}^i(t)\ominus_i \tilde{F}^i\left(t-\varepsilon t^{1-q}\right)\right]^\alpha}{\varepsilon} = \left\{\frac{\left[F^+(t)\ominus F^+\left(t-\varepsilon t^{1-q}\right)\right]^\alpha}{\varepsilon};\frac{\left[F^-(t)\ominus F^-\left(t-\varepsilon t^{1-q}\right)\right]^\alpha}{\varepsilon},\alpha\in[0,1]\right\}$$

$$= \left\{\left[\frac{\left(f_1^+\right)^\alpha(t)-\left(f_1^+\right)^\alpha\left(t-\varepsilon t^{1-q}\right)}{\varepsilon},\frac{\left(f_2^+\right)^\alpha(t)-\left(f_2^+\right)^\alpha\left(t-\varepsilon t^{1-q}\right)}{\varepsilon}\right]\right.$$

$$\left.;\left[\frac{\left(f_1^-\right)^\alpha(t)-\left(f_1^-\right)^\alpha\left(t-\varepsilon t^{1-q}\right)}{\varepsilon},\frac{\left(f_2^-\right)^\alpha(t)-\left(f_2^-\right)^\alpha\left(t-\varepsilon t^{1-q}\right)}{\varepsilon}\right]\right\}$$

and passing to the limit and $\varepsilon = t^{t-1}h$ gives

$$T_{q_{(1)}}F(t) = t^{1-q}\left\{\left[\left(f_1^+\right)'(t),\left(f_2^+\right)^{\alpha'}(t)\right],\left[\left(f_1^-\right)^{\alpha'}(t),\left(f_2^-\right)^{\alpha'}(t)\right]\right\}.$$

Remark. Let $q\in(0,1]$ and $F^+\in\mathbb{R}^+_{\mathscr{F}}, F^-\in\mathbb{R}^-_{\mathscr{F}}$

(i)If $\tilde{F}^i$ is (1)-differentiable and $\tilde{F}^i$ is $q_{(1)}$-differentiable then

$$T_{q_{(1)}}\tilde{F}^i(t) = \left\{t^{1-q}D_1^1F^+(t), t^{1-q}D_1^1F^-(t)\right\}$$

(ii)If $\tilde{F}^i$ is (2)-differentiable and $\tilde{F}^i$ is $q_{(2)}$-differentiable then

$$T_{q_{(2)}}\tilde{F}^i(t) = \left\{t^{1-q}D_2^1F^+(t), t^{1-q}D_2^1F^-(t)\right\}$$

Theorem 6. *Let $q \in (0,1]$. If $\tilde{F}^i, \tilde{G}^i : I \to \mathbb{IF}$ are q-differentiable at point $t \in I$ and $\lambda \in \mathbb{R}$ then*

$$\left(\tilde{F}^i + \tilde{G}^i\right)^{(q)} = \left(\tilde{F}^i\right)^{(q)} + \left(\tilde{G}^i\right)^{(q)}$$
$$\text{and } \left(\lambda \tilde{F}^i\right)^{(q)} = \lambda \left(\tilde{F}^i\right)^{(q)}$$

*Proof.*By Definition 6 and Definition 7 the statement of the theorem follows easily.

5 Applications to Intuitionistic Fuzzy Definition of Fractional Derivative

Let us consider the equation :

$$\frac{d^q N(t)}{dt^q} = -\lambda \cdot N(t),\ q \in (0,1] \qquad (5)$$
$$N(t_0) = N_0, \quad t \in I, \quad N_0 \in \mathbb{IF}$$

which is known as nuclear decay equation, where $\frac{d^q N(t)}{dt}$ means conformable derivative of function N(t). With the help of Theorem 5 , we can write Eq(5) as follows.

$$t^{1-q}\frac{dN(t)}{dt} = -\lambda \cdot N(t),\ q \in (0,1]$$
$$N(t_0) = N_0, \quad t \in I, \quad N_0 \in \mathbb{IF}$$

Let $I = [0,1]$ and $N_0 = \left(5,7,9;3,7,11\right)$, the α-cut of

$$N_0 = \{[5+2\alpha, 9-2\alpha], [3+4\alpha, 11-4\alpha]; \quad \alpha \in [0,1]\}.$$

The exact solution of equation (5) under $q_{(1)}$-differentiability is given by

$$\left(N_1^+\right)^\alpha(t) = (2\alpha-2)e^{\frac{t^q}{q}} + 7e^{\frac{-t^q}{q}}$$
$$\left(N_2^+\right)^\alpha(t) = -(2\alpha-2)e^{\frac{t^q}{q}} + 7e^{\frac{-t^q}{q}}$$
$$\left(N_1^-\right)^\alpha(t) = (4\alpha-4)e^{\frac{t^q}{q}} + 7e^{-t^q}q$$
$$\left(N_2^-\right)^\alpha(t) = -(4\alpha-4)e^{\frac{t^q}{q}} + 7e^{\frac{-t^q}{q}}$$

The exact solution of equation (5) under $q_{(2)}$-differentiability is given by

$$\left(N_1^+\right)^\alpha(t) = (5+2\alpha)e^{\frac{-t^q}{q}}$$
$$\left(N_2^+\right)^\alpha(t) = (9-2\alpha)e^{\frac{-t^q}{q}}$$
$$\left(N_1^-\right)^\alpha(t) = (3+4\alpha)e^{\frac{-t^q}{q}}$$
$$\left(N_2^-\right)^\alpha(t) = (11-4\alpha)e^{\frac{-t^q}{q}}.$$

6 Conclusion

In this study, we demonstrated that the generalized difference $\ominus_i$ represents a particular case of the interactive difference, namely the iH-difference one that is based on H-difference, Using this definition for developing and proving some results for intuitionistic fuzzy conformable differentiability. We introduced and proved the generalized conformable fractional derivative of the intuitionistic fuzzy number-valued functions, we provided under some weak conditions, existence solutions to intuitionistic fuzzy fractional Nuclear decay equation, which is interpreted by using the generalized conformable intuitionistic derivatives concept.
We suggest studying intuitionistic fuzzy fractional differential equations with the use of the generalized conformable differentiability concept for further research. In addition, we propose to extend the results of the present paper and to combine them with the results in [13,23,24,25] for intuitionistic fuzzy fractional differential equations.

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