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A Robust Principal Component Analysis for Estimating Economic Growth in Nigeria in the Presence of Multicollinearity and Outlier

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Abstract: This study examined economic growth (RGDP) in relation to internal debt (INDT), external debt (EXDT), interest rate (RINR), exchange rate (REXR) and trade openness (OPEN) in the presence of multicollinearity and outlier. A quarterly data gathered from Central Bank of Nigeria from 1986 to 2021 were used. Exploratory data analysis and diagnostic carried out using variance inflation factor and Grubb's test revealed linear relation among the variables under investigation and ascertained the presence of multicollinearity and outlier in the data set. The principal component analysis revealed that INDT and EXDT accounts for 38.4% and 29.2% of the variance and as such their component PINDT and PEXDT were chosen to reduce the collinearity. Also, a robust M-estimation method results revealed that the impact of PINDT, PEXDT, RINR, REXR and OPEN on the RGDP were positive and significant for PEXDT and OPEN on the RGDP. Specifically, PINDT, PEXDT, RINR, REXR and OPEN increased the Nigeria's economic growth to the turn of 0.10%, 0.02%, 0.04%, 0.06% and 3.01% respectively during the period under consideration. Consequently, combining principal component with M-estimator of weighted bisquare with 4.685 turning and median centered as scale was revealed as the most efficient estimation technique that jointly addressed the two identified assumptions violation. This was based on predictive power of the fitted model that revealed M-estimator had minimum root mean square error (RMSE) and mean absolute error (MAE) when compared with the S-estimator and MM-estimator respectively. Therefore, it be concluded that economic challenges witnessed during the period under study greatly affected the identified determinants which in turn translated to the economic growth. Hence, a robust principal component regression technique remains the best and unbiased technique for modeling and estimating the parameters of a linear model when multicollinerity and outliers were jointly present in the data set.

Keywords: Economic Growth, Multicollinearity, Outlier, Principal Component and Robust Estimators.

1 Introduction

Nigerian economy poses several challenges to scholars and various researchers as a result of its structural, characteristics and outcomes and the contradictions inherent therein [1]. In the late 1960s, crude oil became the main driver of growth after a move from agricultural based economy. Since then, all diversification efforts made on the economy to provide better, stable and productive growth have not yeilded the desired result and success. Despite, the high-level of revenues generated from oil since its mainstay and the economic driver, it has failed to transform into the expected prosperity and development of the nation and its citizens. The ranking of the countries based on major development indicators still showed that the country is one of the poorest in the world. This is due to mismanagement, corruption and as such continuous poor performance put the growth of the economy under serious problem and threat. Poverty, avoidable disease and misery are the traceable attributes and characteristics to describe Nigerian citizens because those in the helm of affair of the nation has failed to demonstrate serious commitment, discipline and sacrifice in driving growth and progress.

Recently, Nigeria experienced a retarding growth in 2015 which ultimately deteriorate into harsh economic recession in 2016 and mid 2020 after a partial relief in 2018. [2] Identified shift in global monetary policy cycles as, a major reason for the country's economic wretchedness which has seriously impinge on Nigeria's financial market; particularly, the monetary policy rate for Nigeria which was situated at a level of 14% in the first quarter of 2017, as against the 12% mark in the first quarter of 2016 and 13% in 2015. Interest rate is one of the main drivers of gross domestic product, although indirectly through its consequence on investment. According to [3] and [4], it was asserted that the strength and the competitiveness of economy with other nations are determined by exchange rate as a vital economic measurement. [5] Stressed that both interest rate and exchange rate enhanced the economic competitiveness and the need for viable policy formulation and direction for better improvement. [6] and [7] stated the need to redesign the monetary policies so as to ensure a stable exchange rate and the potentials of enhancing industrial output growth.

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[8] Emphasized that the spending of government on capital project through foreign loans influenced the growth of Nigeria's economy and that foreign debt service cost hindered the growth of the economy. [9] Observed that economic growth is negatively affected by the external debt and as such the long-term growth was hindered even when it was economic growth-enhancing in the short-term. On the other hand, the effect of the domestic debt on the growth of the economy was in inverse order for both long and short term growth when compared with the external debt. Also, debt servicing constitutes a major hindrance to economic growth as it confirmed the effect of debt overhang. [7] Stated that the economic openness can be made to enhance economic growth via a sustainable diversification effort. [10] Asserted that in both the short and long run, trade openness had a serious constraint on the growth of the economy. Thus, it can be emphasized that the imports were greater than export in Nigeria and as such government need to engage in export led diversification to spur economic growth.

Empirically, economic growth various studied has been carried out using different macroeconomic variables as a determinant. Some of the work done includes: [9] investigated the effect of government debt on Nigeria's economic growth for the period (1980-2018). An Autoregressive Distributed Lag technique was empirically applied for analyzing the collected data. From the findings, it was discovered that the impact of external debt on economic growth was negative and as such the long term growth of the economy was hindered even when it was growth-enhancing in the short-term. On the other hand, the domestic debt impacted economic growth in inverse order for the long-term and short-term growth when compared with the debt incurred externally. Also, debt servicing hindered the economic growth as it confirmed the effect of debt overhang. [11] Examined the impact of external debt on economic growth in Nigeria (1980-2016). Secondary data gathered from the Statistical Bulletin of Central Bank Nigeria were analyzed by means of Generalized Method of Moments (GMM). Findings showed that external debt was positively and significantly related to Nigeria's economic growth.

[12] Investigated the effect of external debt on economic growth in Jordan from 2010 to 2017. A descriptive statistic adopted for the study during the period under examination revealed that external debt negatively and significantly affected economic growth. [8] Investigated the effect of external debt on the economic growth of Nigeria (1981-2017). The secondary data collected on the identified economic variables from CBN Statistical Bulletin and NBS were used for the study. The Granger Causality test and Error Correction Mechanism techniques revealed that debt stock from foreign nation and spending on capital projects by the government positively and significantly influenced the growth of the economy. It was further revealed from the study that the influence of the cost of servicing foreign debt on economic growth in Nigeria was insignificant. [13] Carried out a study to investigate the impact of external debt on economic growth in Nigeria (1999-2015). In the study, an econometric technique such as, Johansen Co-integration and Vector Error Correction Mechanism were employed to analyse the collected data. Findings showed that external debt and economic growth were inversely related in Nigeria.

[14] In their study examined the relation between external borrowing by the government and economic growth between 1990 and 2015. A continuous increase in Oman's external debt in the financing budget on yearly basis served as the motivation for the study. In the study, an Autoregressive Distributed Lag cointegration method was adopted in analyzing the data. Thus, it was discovered from the result that external borrowing by the government negatively and significantly affected the economic growth in Oman. It was further discovered that gross fixed capital positively and significantly impacted the Oman's economic growth during the period examined. [15] Analyzed the impact of external debt on economic growth in Nigeria between 1985 and 2015. The data gathered for the study were analyzed using ordinary least square regression method, unit root test, Johansen cointegration and error correction test. From the results, a negative and insignificant was discovered as the impact of debt servicing on the growth of the economy in Nigeria. Also, it was discovered that the stock of external debt positively and significantly affects the index of economic growth. The effects of other control variables used in the study were positive and significant on economic growth. A long-run relationship and unidirectional causality were established between external debt and economic growth during the period under consideration.

[16] Carried out study to assess the impact of trade openness on economic growth among ECOWAS countries (1975-2017). A non-stationary heterogeneous dynamic panel models through the application of Pooled Mean Group (PMG) and Mean Group (MG) estimators were employed to explore the secondary data gathered for the study. In the findings, it was revealed that trade openness had positive effects on growth in ECOWAS countries in the long-run but mixed effects in the short-run. Thus, it was emphasized that the cooperation among ECOWAS member countries need to be improved as this would help economic actors in the region to access international markets and to be more strategic in term of trade and competitiveness through export consortia. [10] Investigated the dynamic impact of trade openness on the economic growth in Nigerian economy (1980-2016). The secondary data that were sourced from the CBN Statistical Bulletin were used. The techniques and diagnostic test carried out in the studied were: unit root, cointegration and error correction model. As a result, it was discovered trade openness had a negative impact on the economic growth, both in the short run and the long run. Thus, it can be emphasized that the imports were greater than export in Nigeria and as such, they need the government to engage in export led diversification to spur economic growth.

[17] Conducted research on the impacts of international trade on Nigeria's economic growth for the period 1985-2015. The variables considered in the study were: interest rate, the balance of trade, exports and trade openness. The analytic techniques adopted were unit root, cointegration and vector error correction model and it was found that the relationship between the variables under investigation were insignificant in the long run. Also, it was found that economic growth and the trade openness were unidirectionally related. [18] Carried out study on trade openness makes sense, using Nigeria trade policy as yardstick. An Autoregressive Conditional Heteroscedasticity (ARCH), Generalized Autoregressive Conditional Heteroscedasticity (GARCH) and Pairwise Granger causality were used in the study. The findings showed that trade openness and economic growth were significantly related. Also, it was found that interest rate and exchange rate were significant in determining economic growth in Nigeria. Thus, it was emphasized that trade policymakers should put into consideration the policy environment before the formulation of trade policies that may not be implemented.

[19] Investigated the causal relationship that exists between trade openness and economic growth in Nigeria. The study was done to examine the pre and the post Structural Adjustment Programme (SAP) which were the period 1970Q1-1985Q4 and 1986- 2011. The data gathered on the following variables: Trade Openness, Government Expenditure and Investment were examined using an Augmented Dickey Fuller and the Pillips Perron for the unit root and Granger Causality and Cointegration method. Thus, it was discovered that openness caused economic growth more in the SAP period than in the pre SAP period. [20] examined the nexus between the economic openness and productivity growth of Nigeria (1970-2010). Secondary data collected on real gross domestic product, openness, real interest rate, exchange rate and unemployment were analysed using Ordinary Least Square (OLS) Method. The result revealed a positive and statistically significant relationship between trade openness and economic growth. Thus, emphasized the need for proper utilization of revenue from export, diversification of the economy as well as the encouragement of export promotion policies.

[21] Examined the determinants of the external trade in Nigeria with the aimed of identifying main factors that influence external trade growth so as to formulate economic policy. The secondary data were gathered from CBN, IFS and WB on total trade (dependent variable) and GDP, inflation rate, capacity utilization, exchange rate, government expenditure, interest rate, import and export (independent variables). The least square regression method employed for the study revealed that GDP, inflation rate, capacity utilization, exchange rate and export were positively and significantly influenced external trade. Also, it was further revealed that government expenditure, interest rate and import had a negative influenced on external trade. [22] Examined whether there are long run relationships between the trade openness and the economic growth in Nigeria for the period 1970–2010. In the research, an OLS was adopted to examine existing relationship between the variables. The results obtained showed that trade openness had a great influence on economic growth in Nigeria. Thus, it can be emphasized that a conducive atmosphere such as reformed institutional structures should be put in place to enhance better economic growth.

[7] Examined the impact of the exchange rate, as an important determinant of economic growth in Nigeria between 1980 and 2019. Secondary data used were sourced from the CBN Statistical Bulletin and an econometric technique such as: Unit Root, Cointegration, and Error Correction Model were employed. In the result, it was indicated that exchange had a positive and significant impact on economic growth. The result further indicated that economic openness had a negative impact on economic growth. Thus, government should make more effort to redesign the monetary policies so as to ensure stable exchange rate. Also, economic openness can be made to enhance economic growth a sustainable diversification effort. [5] Empirically, studied the contribution of interest rate and exchange rate on economic competitiveness in Nigeria. The data collected on interest rate, exchange rate and gross domestic product the proxy for economic competitiveness spanning the period 1981-2016 were examined. The result obtained from the OLS technique employed revealed that both interest rate and exchange rate had a significant impact on economic competitiveness. However, it was found that the contribution of the exchange rate to economic competitiveness was greater than that of interest rate and as such, the need for government to improve exchange rate system through viable policy formulation.

[6] Examined the effect of exchange rate fluctuation on industrial output in Nigeria (1986-2015). The study adopted gross domestic product as a proxy for industrial output growth as the response variable, while exchange rate, inflation, interest rate and net exports were used as explanatory variables. Data sourced from the National Bureau of Statistics and CBN Statistical Bulletin were analysed using ADF and PP unit root, Johansen co-integration, Pairwise Granger causality and VECM. The results showed a unidirectional causality from exchange rate to industrial output growth, that is, exchange rate positively and significantly influenced industrial output and the influence was more seen than the other variables examined in this study. Thus, its establishing exchange rate potentials of enhancing industrial output growth in Nigeria.

[23] Investigated the effect of exchange rate fluctuation on economic growth in Nigeria using time series data spanning (1970-2012). The study adopted exchange rate, inflation, money supply and oil revenue as the explanatory variables, while gross domestic product was used to proxy economic growth which was the response variable. Secondary data for the study were obtained from CBN Statistical Bulletin. A multiple linear regression technique was used for the analysis of the collected data. A mixed finding was obtained which indicated that a floating exchange rate was better than fixed exchange rate in determining a sustainable economic growth.

[24] Examined the estimation of regression coefficients in the presence of multicollinearity. The study focused on the contribution of agriculture, industry and services to the overall GDP in Nigeria over a period of 1960-2011. Data extracted from the CBN website were analysed using multiple regression method and variance inflation factor (VIF) was used as a tool for detecting the presence of multicollinearity. After various transformations were carried out, the result of the differenced log-transformation of the components showed that the VIF reduced. It was further revealed that agriculture, industry and services were positively related to GDP during the period under study in Nigeria. The result also showed that industry had the highest contribution which was an indication that industry was key contributor to the Nigeria economy. The result also revealed that agriculture, industry and services played significant roles in the growth and development of Nigeria economy.

[1] Investigated an efficient estimation technique for economic growth and its determinants for Nigeria in the presence of multicollinearity. The macroeconomic variables considered in the study were economic growth (RGDP), internal debt, external debt, interest rate, exchange rate and trade openness. An exploratory data analysis and the variance inflation factor carried out revealed the presence of multicollinearity. Thus, a ridge regression method was adopted and it was found that a ridge regression technique with appropriate ridge constant was a robust method that was efficient to estimate economic growth in Nigeria.

However, based on various literature reviews, the presence of outlier and multicollinearity problem as common assumptions violation in classical linear model have not been examined together or jointly put into consideration in various analyses been carried out. As such, the detection of outlier and multicollinearity among macroeconomic variables becomes an important problem in modelling, analysis and inference about the fitted regression model. Specifically, we were unable to find a study that investigate the existing relationship among the economic growth (RGDP), internal debt (INDT), external debt (EXDT), interest rate (RINR), exchange rate (REXR) and trade openness (OPEN) for Nigeria that jointly put outlier and multicollinearity problems into consideration in their study thus, a worthy gap in literature to be filled. We also aim to find an estimate of the parameters for RGDP considering INDT, EXDT, RINR, REXR and OPEN as the drivers in the presence of multicollinearity and outliers using principal component regression with robust estimation technique that was lacking in various work and study previously carried out to enhance the efficiency of the estimated parameters.

2 Materials and Methods

In this study, the macroeconomic data gathered from the CBN statistical bulletin that were used to examine the existing relationship among economic growth (RGDP) as endogeneous variable, internal debt (INDT), external debt (EXDT), interest rate (RINR), exchange rate (REXR), and trade openness (OPEN) between 1986Q1-2021Q1. The variables assumed a linear model stated in both functional and econometrical form as (1) and (2)

$$\text{RGDP} = \text{F(INDT, EXDT, RINR, REXR, OPEN)} \quad (1)$$

$$\text{RGDP} = \propto_0 + \propto_1 \text{ INDT} + \propto_2 \text{ EXDT} + \propto_3 \text{ RINR} + \propto_4 \text{ REXR} + \propto_5 \text{ OPEN} + \epsilon_i \quad (2)$$

Where, $\propto_i$ and ϵ_i are the parameters to be estimated and error term respectively.

However, the multiple linear regression model stated in (1) and (2) were transformed and expressed in general form as given in (3) where Y represent the dependent variable RGDP and $\boldsymbol{X}$ represent the explanatory variables INDT, EXDT, RINR, REXR and OPEN.

$$\mathbf{Y} = \mathbf{X}'\boldsymbol{\beta} + \boldsymbol{\varepsilon} \quad (3)$$

The ordinary least square estimator of $\boldsymbol{\beta}$ is given in (4)

$$\widehat{\boldsymbol{\beta}} = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\mathbf{Y} \quad (4)$$

The covariance matrix of $\boldsymbol{\beta}$ can be obtained as given in (5)

$$\text{Cov}(\widehat{\boldsymbol{\beta}}) = \sigma^2(\boldsymbol{X}'\boldsymbol{X})^{-1} \quad (5)$$

where $\boldsymbol{Y}$ is an observational vector of dimension $n \times 1$, $\boldsymbol{X}$ is an $n \times p$ data matrix of regressors, $\boldsymbol{\beta}$ is a $p \times 1$ vector of regression coefficient and ε is an $n \times 1$ disturbance vector.

Outlier and multicollinearity problem as common assumptions violation in classical linear model and these shall be simultaneously addressed in this study using a robust estimation technique. According to [25], it was noted that an outlier is an observation that appears to be inconsistent with other observations in a given dataset which can influences or causes a substantial change of some important aspects of the regression analysis such as estimated parameters and the variance or standard error. Also, the presence of outliers can lead to biased estimation of the parameters, misspecification of the model and inappropriate predictions. Thus, the need for diagnostics in order to detect the presence of outlier and multicollinearity. Testing for outliers can be done using any of the following test: Dixon's test, Grubbs' test, Cochran's C test, and Bartlett test while, correlation analysis, variance inflation factor, tolerance level, eigenvalue and condition number, can be used to detect the presence of multicollinearity. In this study, Grubbs' test and variance inflation factor were used to establish the violation of these assumptions.

Grubbs' test

The Grubbs' test for outliers was recommended by International Statistical Organization (ISO) and as such it will be used in this study. This test compares the deviation of the suspect value from the sample mean with the standard deviation of the sample. The suspect value is the value that is furthest away from the mean. In order to use Grubbs' test for an outlier, the null hypothesis (H_0) is there no outliers in the dataset under investigation. The statistic G_m is calculated as expressed in (6):

$$\widehat{\mathbf{G}}_{\mathbf{m}} = \frac{|\mathbf{X}_{\mathbf{s}}^{*} - \overline{\mathbf{X}}|}{\mathbf{S}} \qquad (6)$$

$$\mathbf{G}_{\mathbf{m}} = \frac{|\textbf{Suspect value} - \overline{\mathbf{X}}|}{\mathbf{S}}$$

Under the assumption multicollinearity, correlation coefficients of independent variables are computed even though strong correlation coefficients do not necessarily imply the presence of multicollinearity, it can be a suspect and as such it can be ascertained by checking the variance inflation factor (VIF) and condition number (CN).

Variance Inflation Factor

The VIF is given by (7)

$$\text{VIF} = \frac{1}{1 - R_i^2}, i = 1, 2, \ldots, r \qquad (7)$$

and R_i^2 represents the squared multiple correlation coefficients when X_1 is regressed on the remaining X_{1+i} independent variables. According to [26], it was opined that VIF increased the variability of the estimated coefficients. In other words, it inflated the variance of the coefficient in comparison to what can be obtained when the variables were uncorrelated with any other variable in the model. Thus, VIF greater than 10 indicates a statistically significant multicollinearity. In such situation, the ordinary least square estimator does not possess the optimum statistical property hence, the need for alternative estimator that can address the situation.

Robust Estimator: M-Estimator

The most common general method of robust regression is M-estimation, introduced by [27]. It is nearly as efficient as OLS. Rather than minimizing the sum of squared errors as the objective, the M-estimate minimizes a function ρ of the errors. The M-estimate objective function is given in (8) as:

$$min \sum_{i=1}^{n} p \frac{e_i}{S} = \min \sum_{i=1}^{n} \rho \left(\frac{Y_i - X'\hat{\beta}_i}{S} \right) \qquad (8)$$

Where S is an estimate of scale often formed from linear combination of the residuals. The function ρ gives the contribution of each residual to the objective function. A reasonable ρ should have the following properties:

$\rho(e) \geq 0,\ \rho(0) = 0, \rho(e) = \rho(-e))$, and $(\rho(e_i) \geq \rho(e_i^{;})\ for\ |e_i| \geq |e_i^{;}|$

The system of normal equations to solve this minimization problem is found by taking partial derivatives with respect to β and setting them equal to 0, yielding (9),

$$\sum_{i=1}^{n} \varphi \left(\frac{Y_i - X'\hat{\beta}_i}{S} \right) X_i = 0 \qquad (9)$$

Where φ is a derivative of ρ. The choice of the φ function is based on the preference of how much weight to assign outliers. Newton-Raphson and Iteratively Reweighted Least Squares (IRLS) are the two methods to solve the M-estimates in nonlinear normal equations. IRLS expresses the normal equations as:

$$X'\varphi X\hat{\beta}_i = X'\varphi Y \qquad (10)$$

Robust Estimator: S-Estimator

According to [28], S estimator which is derived from a scale statistics in an implicit way, corresponding to $s(\theta)$ where $s(\theta)$ is a certain type of robust M-estimate of the scale of the residuals $e_1(\theta),\ e_2(\theta), \dots ,\ e_n(\theta)$. They are defined by minimization of the dispersion of the residuals:

minimize $S(e_1(\theta),\ e_2(\theta), \dots ,\ e_n(\hat{\theta}))$ with final scale estimate stated in (11)

$$\hat{\sigma} = S(e_1(\theta),\ e_2(\theta), \dots ,\ e_n(\hat{\theta})). \quad (11)$$

The dispersion $e_1(\theta),\ e_2(\theta), \dots ,\ e_n(\hat{\theta})$ is defined as the solution in (12)

$$\frac{1}{n}\sum_{i=1}^{n} P\left(\frac{e_i}{S}\right) = K \quad (12)$$

Where K is a constant and $P\left(\frac{e_i}{s}\right)$ is the residual function. Based on [29], Tukey's biweight function was suggested and is defined as expressed in (13)

$$p(x) = \begin{cases} \frac{x^2}{2} - \frac{x^4}{2c^2} + \frac{x^6}{6c^4}\ for\ |x| \le c \\ \frac{c^2}{6}\ for\ |x| > c \end{cases} \quad (13)$$

Setting c = 1.5476 and K = 0.1995 gives 50% breakdown point [29].

Robust Estimator: MM-Estimator

MM-estimation is special type of M-estimation [30]. MM-estimators combine the high asymptotic relative efficiency of M-estimators with the high breakdown of class of estimators called S-estimators. It was among the first robust estimators to have these two properties simultaneously. The MM refers to the fact that multiple M-estimation procedures are carried out in the computation of the estimator. MM-estimator was described in three stages as follows:

Stage 1. A high breakdown estimator is used to find an initial estimate, which we denote $\tilde{\beta}$. The estimator needs to be efficient. Using this estimate, the residuals are computed as given in (14)

$$r_i(\beta) = y_i - x_i^T\tilde{\beta} \quad (14)$$

Stage 2. Using these residuals from the robust fit and where K is a constant and the objective as given in (15)

$$\frac{1}{n}\sum_{i=1}^{n} \rho\left(\frac{r_i}{s}\right) = K \quad (15)$$

function ρ, an M-estimate of scale with 50% BDP is computed. This $s(r_1(\tilde{\beta}), r_2(\tilde{\beta}), \dots r_n(\tilde{\beta})$

Is denoted s_n The objective function used in this stage is labeled ρ_0.

Stage 3. The MM-estimator can now defined as an M-estimator of β using a re-descending score function expressed in (16)

$$\emptyset_1(u) = \frac{\partial \rho_1(u)}{\partial u} \quad (16)$$

And the scale estimate s_n obtained from stage 2. So an MM-estimator $\hat{\beta}$ is defined as a solution

To expression in (17):

$$\sum_{i=1}^{n} x_{ij}\ \emptyset_1\left(\frac{y_i - x_i^T\tilde{\beta}}{s_n}\right) = 0\ \ j = 1, 2 \dots , p \quad (17)$$

Principal Component Regression Technique

This technique provides a unified way to handle multicollinearity which requires some calculations that are not usually included in standard regression analysis. The principle component analysis follows from the fact that every linear regression model can be restated in terms of a set of orthogonal explanatory variables. These new variables are obtained as linear combinations of the original explanatory variables. They are referred to as the principal components. Consider the following model in (3): Where Y is an $nx1$ matrix of response variable, X is an nxp matrix of the independent variables, β is a $px1$ vector of unknown constants, and ε is an $nx1$ vector of random errors. There exists a matrix V, satisfying (18)

$$V'(X'X)V = \Lambda \; and \; V'V = VV' = 1 \quad (18)$$

Where Λ is a diagonal matrix with ordered characteristics roots of $X'X$ on the diagonal. The characteristic roots are denoted by $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_p$ V may be used to calculate a new set of explanatory variables, namely as expressed in (19) and (20)

$$Z = \left(Z_{(1)}, Z_{(2)}, \dots, Z_{(p)}\right) \quad (19)$$

$$XV = \left(X_{(1)}, X_{(2)}, \dots, X_{(p)}\right) \quad (20)$$

That are linear functions of the original explanatory variables. The $Z's$ are referred to as principal components. Thus the regression model can be restated in terms of the principal components as stated in 21 and 22:

$$Y = Z\alpha + \varepsilon, where \; Z = XV, \alpha = V\beta \quad (21)$$

$$Z'Z = V'X'XV = V'V\Lambda V'V \quad (22)$$

The least square estimator of α and the variance covariance matrix of $\hat{\alpha}$ are given in (23) and (24):

$$\hat{\alpha} = (Z'Z)^{-1} Z'Y = \Lambda^{-1} Z'Y \quad (23)$$

$$Var(\hat{\alpha}) = \sigma^2 (Z'Z)^{-1} = \sigma^2 \Lambda^{-1} \quad (24)$$

Thus a small eigenvalue of $X'X$ implies that the variance of the corresponding regression coefficient will be large. From expression in in (17) and (22) which can be put in (25) as:

$$Z'Z = V'X'XV = V'V\Lambda V'V = \Lambda \quad (25)$$

This is often refer to the eigenvalues λ_j as the variance of the jth principal component. If all λ_j equal to unity, the original regressors are orthogonal, while if a λ_j is exactly equal to zero, then it implies a perfect linear relationship between the original regressors. One or more near to zero implies that multicollinearity is present. The principal component regression approach combats multicollinearity by using less than the full set of principal components in the model. To obtain the principal components estimators, assume that the regressors are arranged in order of decreasing eigenvalues $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_p > 0$. In principal components regression the principal components corresponding to near zero eigenvalues are removed from the analysis and least squares applied to the remaining components as in (21) that can be expressed in (26) as:

$$Y = XVV'\beta + \varepsilon = Y = Z\alpha + \varepsilon \quad (26)$$

Where $Z = XV, \alpha = V'\beta$ and $V = \left(v_1, v_2, v_3, \dots, v_p\right) = (V_r, V_{p-r})$ is a pxp orthogonal matrix with expression in (27)

$$\left(V_r, V_{p-r}\right)' X'X\left(V_r, V_{p-r}\right) = A = \begin{pmatrix} A_r & 0 \\ 0 & A_{p-r} \end{pmatrix} \quad (27)$$

Where$0 < r \leq p$, $A = diag\left(v_1, v_2, v_3, \dots, v_p\right)$, $A_r = diag(v_1, v_2, v_3, \dots, v_r)$, $A_{p-r} = diag\left(v_{r+1}, v_{r+2}, v_{r+3}, \dots, v_p\right)$ and $v_1 \geq v_2 \geq v_3 \geq \dots \geq v_p > 0$ are the ordered eigenvalues of $X'X$. By definition expression obtained from (26) and put in (28) as:

$$Z = XV = \left(Z_r, Z_{p-r}\right) \quad (28)$$

is the nxp matrix of the principal components (PCs), where $Z_i = XV_i$ is the ith PC. Let Z_{p-r} contains PCs corresponding to near zero eigenvalues, which implies the partitioning of Z into Z_r and Z_{p-r}, such that Z_{p-r} will be eliminated. Thus, expression in (26) can further be written and expressed in (29) as:

$$Y = Z_r\alpha_r + Z_{p-r}\alpha_{p-r}\alpha + \varepsilon \quad (29)$$

The least square estimator of α is given in (30) as:

$$\hat{\alpha} = (A)^{-1}Z'Y \quad (30)$$

From expression in (10) and (30), the M-estimator of α can be given in (31) as:

$$\hat{\alpha}_M = \left(A_\varphi\right)^{-1} Z'\hat{\varphi}Y \quad (31)$$

Where $A_\varphi = Z'\hat{\varphi}Y, \hat{\varphi}$ is the derivative of p

Based on (29) and (30), the principal component estimator of α can be given in (32) as:

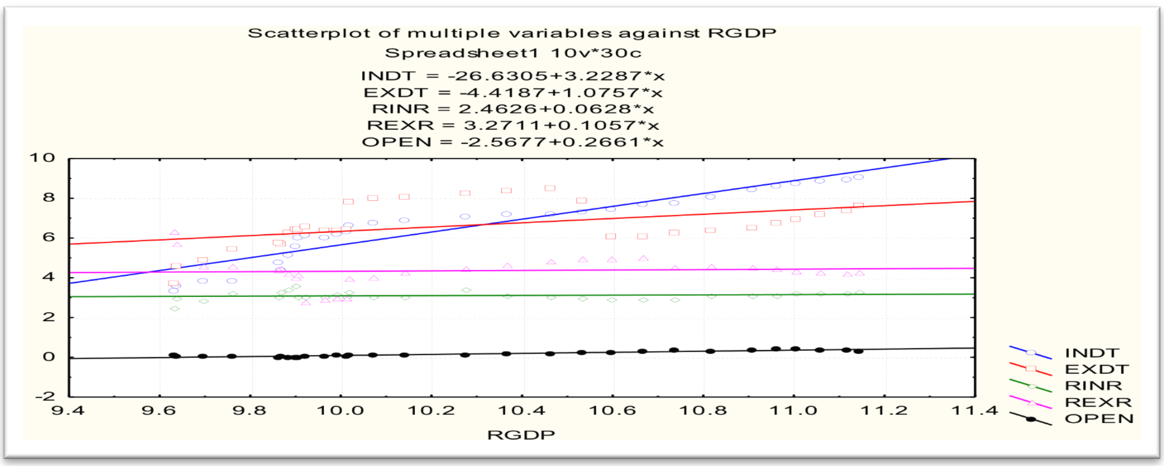
$$\hat{\alpha}_{PC} = (A_r)^{-1} Z_r{}' Y \quad (32)$$

Thus, in the next session, result and discussion were done under the followings sub-heading: data exploration, descriptive analysis, test for outliers and multicollinearity as well as fitting robust principal component regression model and its associated diagnostics carried out would be presented.

3 Result and Discussion

3.1 Data Exploration

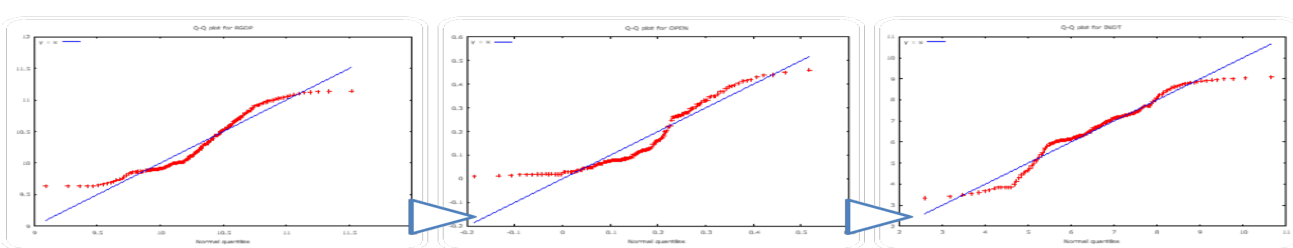
The characteristics of the economic growth (RGDP) and drivers such as internal debt (INDT), external debt (EXDT), interest rate (RINR), exchange rate (REXR) and trade openness (OPEN) were described in this section using trend so as to explore the existing relationship among the variables mentioned. Thus, it was revealed that INDT, EXDT, RINR, REXR, and OPEN had either inverse or direct relationship with the RGDP in Nigeria and as such the need to examine the descriptive and diagnostic properties of the model such as outliers and multicollinearity among the variables. In view of this, a linear model is fitted and the presence of outliers and multicollinearity as assumptions violation among macroeconomic variables is examined so as to ensure efficient estimate of the parameters. Thus, in Figure 1, the plot that showed the relation between the macroeconomic variables under consideration was displayed. This was done to show the direction of relationship between the economic growth (RGDP) in Nigeria and the drivers mentioned in this study.

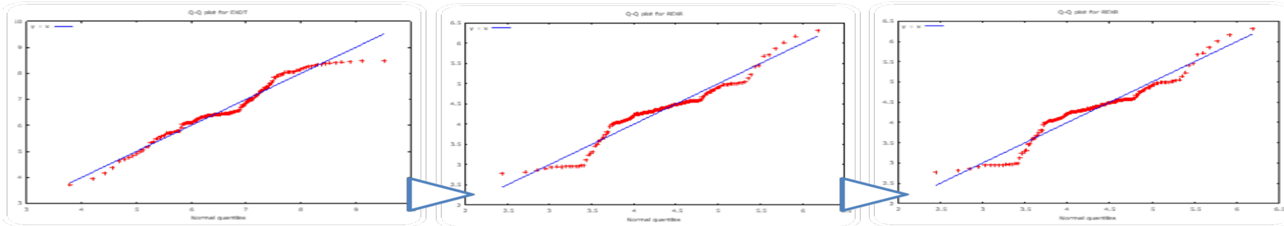


Source: Researcher's Computation, 2022

Figure 1: The graph showing the trend of independent variables in relation to RGDP

In Figures 1, the scattered plot that showed various trends in mean and variance of the independent variables such as INDT, EXDT, RINR, REXR and OPEN and how they were related to the RGDP during the period under study. Thus, in this study, a linear relationship was found between the identified drivers and economic growth in Nigeria. To further establish the linear relationship of the economic growth and the aforementioned drivers, we carried out Grubb's test for the presence of outliers among the macroeconomic variables which is presented in Table 3. Also, in Table 1 and Figure 2, the test and graph to examine the nature of distribution of the macroeconomic variables were considered.





Source: Researcher's Computation, 2022

Figure 2: The Q-Q Plot showing the Normality of each Variable

In Figure 2, the Q-Q plot showed that economic growth (RGDP) and the drivers such as internal debt (INDT), external debt (EXDT), interest rate (RINR), exchange rate (REXR) and trade openness (OPEN) were from normal population that contained outliers. This is based on the evidence that the red lines representing the observations for each identified variables were not perfectly related to the expected values showing with the blue line in Figure 2. That is, if all but a few points fall on a line there is presence outlier. In Table 1, the probability value of Jarque-Bera which are 0.0043 < 0.05, 0.0004 < 0.05 and 0.0005 < 0.05 respectively revealed that RGDP, RINR and OPEN were normally distributed. However, Shapiro-Wilk test statistics which is the most efficient test for normality showed that 0.9290, 0.9564, 0.9652, .09673, 0.9431 and 0.8899 for RGDP, INDT, EXDT, RINR, REXR OPEN respectively were not closer to 1 and the probability value of the Shapiro-Wilk statistics with p-value < 0.05 revealed that all the macroeconomic variables under investigation were from normally distributed population that contained outliers as indicated in Table 1. Also, in Table 2, the strong correlation between INDT and OPEN with correlation coefficient of 0.8201 is an indication for the presence of multicollinearity among the explanatory variables. Thus, the need carried out further diagnostics to detect outlier and multicollinearity as evidently presented in Table 3

3.2 Descriptive Analysis

In Table 1 the descriptive statistic such as mean, minimum, maximum, standard deviation, variance, skewness, kurtosis and others were presented for the macroeconomic variables under consideration in this study.

Table 1: Descriptive Analysis Result

	RGDP	INDT	EXDT	RINR	REXR	OPEN
Mean	10.3046	6.6288	6.6573	3.1051	4.3175	0.1665
Median	10.2054	6.7393	6.4629	3.1139	4.3803	0.1200
Maximum	11.1422	9.0867	8.4950	3.5860	6.3197	0.4600
Minimum	9.6316	3.3478	3.7246	2.4849	2.7763	0.0100
Std. Dev.	0.4503	1.4988	1.0655	0.1929	0.6951	0.1302
Skewness	0.3710	-0.4095	-0.1634	-0.5734	-0.1748	0.6710
Kurtosis	1.8572	2.5576	2.7076	4.1714	3.6271	2.1246
Jarque-Bera	10.9083	5.0906	1.1299	15.7879	3.0287	15.0811
Probability	0.0043	0.0785	0.5684	0.0004	0.2120	0.0005
Shapiro-Wilk	0.9290	0.9564	0.9652	0.9673	0.9431	0.8899
Probability	0.0000	0.0002	0.0012	0.0018	0.0000	0.0000
Sum	1452.950	934.6548	938.6716	437.8212	608.7708	23.4700
Sum Sq. Dev.	28.3891	314.4906	158.9331	5.2110	67.6489	2.3744
Observations	141	141	141	141	141	141

Source: Researcher's Computation, 2022

Table 1 showed the descriptive analysis of results of the economic variables such as RGDP, INDT, EXDT, RINR, REXR, and OPEN under investigation in this study. The average values of RGDP during the period under study stood at 10.3046 and it ranged from 9.6315 to 11.1422. The mean value of INDT and EXDT were 6.6288 and 6.6572 which were ranged between 3.3478 to 9.0867 and 3.7245 to 8.4950 respectively. While, the average values of RINR, REXH and OPEN were 3.1051, 4.3175 and 0.16645 respectively. It was observed that RINR, REXH and OPEN were ranged from 2.4849 to 3.5860, 2.7763 to 6.3197 and 0.01 to 0.46 in that order in the study period. The values 0.4503, 1.4988, 1.0655, 0.1929, 0.6951 and 0.1302 revealed the rate at which RGDP, INDT, EXDT, RINR, REXR and OPEN deviated from their respective mean values.

The skewness and kurtosis as shown in the result gave explanation about the distribution and shape of the economic variables under investigation. The skewness result showed that the RGDP (0.3710) and OPEN (0.6709) were positively skewed that is, skewed to the right of the mean and it was also discovered that INDT (-0.4095), EXDT (-0.1634), RINR (-0.5734) and REXR (-0.1748) were negatively skewed that is skewed to the left of the mean. The kurtosis results revealed that all the macroeconomic variables under consideration were platykurtic with the kurtosis coefficient index less than 3 except RINR and REXR which were mesokurtic thus, emphasized the flattering beyond the level of normal distribution.

Table 2: Correlation Matrix

	INDT	EXDT	RINR	REXR	OPEN
INDT	1.000000	0.607348	0.127553	-0.092361	0.820054
EXDT	0.607348	1.000000	0.387585	-0.289828	0.253438
RINR	0.127553	0.387585	1.000000	-0.459411	-0.008967
REXR	-0.092361	-0.289828	-0.459411	1.000000	0.241640
OPEN	0.820054	0.253438	-0.008967	0.241640	1.000000

Source: Researchers' Computation, 2022

The correlation coefficients presented in Table 2 showed the extent of relationship that exist among the explanatory variables under consideration such as INDT, EXDT, RINR, REXR and OPEN. From the Table 2, it was discovered that INDT was positively correlated with EXDT, RINR and OPEN with correlation coefficient of 0.61, 0.13 and 0.82 respectively. The study also revealed a positive correlation between the EXDT and RINR, EXDT and OPEN, REXR and OPEN with correlation coefficient of 0.39, 0.25 and 0.24 respectively. Thus, the high or strong correlation between the INDT and OPEN revealed the need to test for the presence of multicollinearity problem.

Therefore, in order to check for the outliers and multicollinearity among the macroeconomic variables INDT, EXDT, RINR, REXR, and OPEN used as the drivers of economic growth (RGDP), we carried out Grubb's test which is an International Statistical Organization (ISO) recommended test for large data in detecting outliers and variance inflation factor (VIF) to detect multicollinearity among the identified variables used in this study and in Table 3, the results were presented.

Table 3: Test for Outliers and Multicollinearity using Grubb's Test and VIF

Variable	Grubb's Value	G (Critical value	VIF
INDT	2.1890	3.4970	14.26569
EXDT	2.7520	3.4970	3.528954
RINR	3.2150	3.4970	1.4369
REXR	2.8800	3.4970	1.757099
OPEN	2.2540	3.4970	9.564367

Source: Researcher's Computation, 2022

The result presented in Table 3 showed the Grubb's test carried out with the null hypothesis (H_0) stated as "there is outliers in the data set under consideration". The rule of thumb for this test is that if the calculated Grubb's value is less than the critical value, we do not reject H_0. Therefore, from the result presented in Table 3, it was discovered that the Grubb's value of 1.860, 2.189, 2.752, 3.215, 2.880 and 2.254 were less than the Grubb's critical value of 3.497 for RGDP, INDT, EXDT, RINR, REXR, OPEN respectively. Thus, it can be affirmed from the result that an outliers were in the macroeconomic data set under investigation. Also, in Table 3, the result of variance inflation factor (VIF) for examining the presence of multicollinearity was presented and the VIF for the variables revealed that INDT, EXDT, RINR, REXR and OPEN were 14.2657, 3.5290, 1.4369, 1.7571, and 9.5644 respectively. Thus, according to [26], the result of VIF 14.2657 > 10.00 indicated the presence and statistical significance of multicollinerity caused by INDT. Based on this evidence that affirmed the presence of outlier and multicollinearity as an assumption violation for linear model, a robust principal component analysis was adopted to address the problems in order to obtain an estimation of the parameters of linear model that can efficiently predict the economic growth. In Table 4, we present the principal component result

Table 4: Principal Components Analysis

Principal Components Analysis					
Included observations: 141					
Computed using: Ordinary correlations					
Extracting 5 of 5 possible components					
Eigenvalues: (Sum = 5, Average = 1)					
Number	Value	Difference	Proportion	Cumulative Value	Cumulative Proportion
1	1.9193	0.4610	0.3839	1.9193	0.3839
2	1.4584	0.7565	0.2917	3.3777	0.6755
3	0.7018	0.1927	0.1404	4.0795	0.8159
4	0.5091	0.0977	0.1018	4.5886	0.9177
5	0.4114	---	0.0823	5.0000	1.0000
Eigenvectors (loadings):					
Variable	PINDT	PEXDT	PRINR	PREXR	POPEN
INDT	0.3809	0.4720	-0.6488	0.3821	0.2552
EXDT	0.5914	0.0284	0.4141	-0.3279	0.6086
RINR	0.5113	-0.3499	0.2953	0.6464	-0.3333
REXR	-0.4158	0.4655	0.5121	0.5046	0.3059
OPEN	0.2660	0.6613	0.2411	-0.2719	-0.5999
Ordinary correlations:					
	INDT	EXDT	RINR	REXR	OPEN
INDT	1.0000				
EXDT	0.2635	1.0000			
RINR	0.0893	0.4605	1.0000		
REXR	-0.0865	-0.3116	-0.4153	1.0000	
OPEN	0.4240	0.2946	-0.0336	0.1780	1.0000

Source: Researcher's Computation, 2022

In Table 4, the principal components technique was used to reduce multicollinearity in the data set to be used for the estimation. The reduction is accomplished by using less than the full set of principal components to explain the variation in the response variable. Thus, from Table 4, the principal components of the explanatory variables such as INDT, EXDT, RINR, REXR and OPEN were given in (33), (34), (35), (36), and (37) as follows:

$$PINDT = 0.3809INDT + 0.5914EXDT + 0.5113RINR - 0.4158REXR + 0.2660OPEN \quad (33)$$

PEXDT = 0.4720INDT + 0.0284EXDT - 0.3499RINR +0.4655REXR + 0.6613OPEN (34)

PRINR = - 0.6488INDT + 0.4141EXDT + 0.2953RINR + 0.5121REXR + 0.2411OPEN (35)

PREXR = 0.3821INDT - 0.3279EXDT + 0.6464RINR + 0.5046REXR - 0.2719OPEN (36)

POPEN = 0.2552INDT + 0.6086EXDT - 0.3333 RINR + 0.3059REXR - 0.5999OPEN (37)

Thus, the economic growth the model in relation to its identified drivers in this study can be written in principal components form as expressed (38):

$$RGDP = \propto_0 + \propto_1 PINDT + \propto_2 PEXDT + \propto_3 PRINR + \propto_4 PREXR + \propto_5 POPEN + \epsilon_i \quad (38)$$

Also, from Table 4 it was discovered that the first and second component accounts for 38.4 and 29.2 percent of the variance and as such the remaining components were insignificant. Hence, the first and second components were chosen. Then the linear regression model to be fitted for RGDP and its identified drivers can be given by (39):

$$RGDP = \propto_1 PINDT + \propto_2 PEXDT + \propto_3 RINR + \propto_4 REXR + \propto_5 OPEN + \epsilon_i \quad (39)$$

The estimated value of $\propto_i$ can be obtaining by the equation stated and the results were given in Table 5

Table 5: Robust Principal Component Regression Analysis Results

Dependent Variable: RGDP								
Method: Robust Least Squares								
Sample: 1986Q 2021Q								
Included observations: 141								
	PCR-estimator		M-estimator		S-estimator		MM-estimator	
Variable	Coefficient	Prob.	Coefficient	Prob.	Coefficient	Prob.	Coefficient	Prob.
C	9.4749	0.0000	9.3942	0.0000	8.9974	0.0000	9.7604	0.0000
PINDT	0.1090	0.0278	0.1011	0.0312	0.0626	0.0059	0.1001	0.0258
PEXDT	0.0095	0.7466	0.0187	0.5239	-0.1527	0.0000	-0.0703	0.0121
RINR	0.0090	0.9680	0.0429	0.8476	0.1392	0.1981	-0.0844	0.6924
REXR	0.0685	0.2216	0.0582	0.2911	0.0913	0.0006	0.0705	0.1811
OPEN	2.8630	0.0000	3.0085	0.0000	2.5145	0.0000	2.6842	0.0000
Robust Statistics								
R-squared	0.9420		0.8163		0.8497		0.7156	
Adjusted R-squared	0.9299		0.7780		0.8184		0.6564	
Rw-squared			0.9510				0.9608	
Adjust Rw-squared			0.9510				0.9608	
Akaike info criterion	-1.0602		29.0875				48.4633	
Schwarz criterion	-0.7800		41.9449				57.9980	
Deviance			0.3606		0.0084		0.3163	
Scale			0.1294		0.0917		0.0917	
Durbin-Watson stat	1.5232							
Log likelihood	21.9032							
F-statistic	77.9224							

Prob(F-statistic)	0.0000							
Rn-squared statistic			405.0331		378.7980		376.9683	
Prob(Rn-squared stat.)			0.0000		0.0000		0.0000	
Non-robust Statistics								
Mean dependent var	10.2950		10.2950		10.2950		10.2950	
S.D. dependent var	0.4923		0.4923		0.4923		0.4923	
S.E. of regression	0.1304		0.1316		0.2472		0.1646	
Sum squared resid	0.4078		0.4156		1.4666		0.6499	

Source: Researcher's Computation, 2022

In the Table 5, the robust principal component regression result of the estimated parameter for the variables such as, internal debt component (PINDT), external debt component (PEXDT), interest rate (RINR), exchange rate (REXR) and trade openness (OPEN) used to drives the Nigeria's economic growth (RGDP) were presented. It must be noted that in this study that a quarterly data that contained multicollinearity (addressed) and outliers were used and based on this a robust principal component regression with M-estimator of weighted bisquare with 4.685 turning and median centered as scale, S-estimator of 1.5476 turning with 0.5 breakdown for 200 trials and MM-estimator with S turning of 1.5476 with 0.5 breakdown for 200 trials and M weighted bisquare with 4.685 turning designed to address the two identified assumptions violation was adopted. Thus, in Table 5, the results PCR estimator revealed that the impact of PINDT, PEXDT, RINR, REXR and OPEN on the RGDP were positive and significant for impact of PINDT and OPEN on the RGDP. Specifically, the contribution of PINDT, PEXDT, RINR, REXR and OPEN to the growth of the Nigeria economy during the period under study were 0.11, 0.01, 0.01, 0.07 and 2.86 percent respectively. The *P*-value < 0.05 for the estimated parameters of the economic growth drivers evidently showed their statistical significant in determining the economic growth in Nigeria.

The robust M-estimation method results revealed that the impact of PINDT, PEXDT, RINR, REXR and OPEN on the RGDP were positive and significant for PEXDT and OPEN on the RGDP. In specific, PINDT, PEXDT, RINR, REXR and OPEN increased the Nigeria's economic growth to the turn of 0.10, 0.02, 0.04, 0.06 and 3.01 percent respectively during the period under consideration. The *P*-value < 0.05 for the estimated parameters of the economic growth drivers evidently showed their statistical significant in assessing economic growth in Nigeria.

The robust S-estimation method for the estimated parameters revealed that the impact of PINDT, RINR, REXR and OPEN on the RGDP were positive except the impact of PEXDT which was negative and significant on the RGDP. In specific, PINDT, RINR, REXR and OPEN increased the Nigeria's economic growth by 0.06, 0.14, 0.09 and 2.51 percent while, PEXDT reduced the growth of the Nigeria's economy by 0.07 and 0.08 percent respectively during the period under consideration. The *P*-value < 0.05 for the estimated parameters of the economic growth drivers evidently showed their statistical significant in examining economic growth in Nigeria.

The robust MM-estimation method results output revealed that the impact of PINDT, REXR and OPEN on the RGDP were positive except the impact of PEXDT and RINR which were negative on the RGDP. In specific, PINDT, REXR and OPEN increased the Nigeria's economic growth to the turn of 0.10, 0.07 and 2.68 percent while, PEXDT and RINR reduced the growth of the Nigeria's economy to the turn of 0.15 percent during the period under consideration. The *P*-value < 0.05 for the estimated parameters of the economic growth drivers evidently showed their statistical significant in examining economic growth in Nigeria.

Also, the adjusted R-square of 0.93, 0.78, 0.82 and 0.66 for the robust principal component regression along with M-estimator, S-estimator and MM-estimator revealed that 93, 78, 82 and 66 percent of proportional changes in RGDP can be explained by changes in PINDT, PEXDT, RINR, REXR and OPEN using the respective aforementioned estimation method and as such emphasized the significance of the method in addressing the presence of multicollinearity and outlier simultaneously in fitting a linear model for this study.

The F-statistic of 77.9224 with *p*-value < 0.05 and Rn-squared statistic of 405.0331, 378.7980 and 376.9683 with *p*-value < 0.05 which measured the overall significance of the fitted robust principal component regression along with M-estimator, S-estimator and MM-estimator models which were statistically significance in examining the causal relation of INDT, EXDT, RINR, REXR and OPEN on RGDP in Nigeria in the presence of multicollinearity and outliers. The test to confirm that the multicollinearity problem had been addressed was presented in in Table 5 and to examine the most efficiency estimator among the fitted robust estimators, the predictive power of the estimators was considered and the results were presented as follows in Table 6

Table 6: Test for the Absence of Multicollinearity

Variable	Coefficient Variance	VIF
PINDT	0.0022	7.3358
PEXDT	0.0008	1.4858
RINR	0.0489	3.5388
REXR	0.0030	2.8501
OPEN	0.0862	2.8531

Source: Researcher's Computation, 2022

Table 6 showed the result of variance inflation factor (VIF) for examining the presence of multicollinearity after the extraction of component of the explanatory variables such as INDT, EXDT, RINR, REXR and OPEN. In this study the first and the second component with highest proportion of eigenvalue were selected for the values of INDT and EXDT and were test for the presence of multicollinearity. Thus, it was revealed from Table 4.6 that VIF value of 7.3358, 1.4858, 3.5388, 2.8501 and 2.8531 respectively. Hence, according to [26], the result of VIF < 10.00 indicated the absence of multicollinerity. Based on this evidence, it can be stated that multicollinearity problem as an assumptions violation for linear model had been addressed using a robust principal component analysis to obtain an estimation of the parameters of linear model that can efficiently predict the economic growth in Nigeria.

Table 7: Forecasting Power of the Robust Estimators

Forecast RGDP	M-estimator	S-estimator	MM-estimator
Root Mean Square Error	0.1177	0.2211	0.1472
Mean Absolute Error	0.0896	0.1109	0.0967
Mean Abs Percent Error	0.8605	1.0583	0.9199
Theil Inequality Coefficient	0.0057	0.0107	0.0072
Bias Proportion	0.0014	0.0230	0.0339
Variance Proportion	0.0001	0.0548	0.0672
Covariance Proportion	0.9985	0.9222	0.8989

Source: Researcher's Computation, 2022

In Table 7, the forecasting power of the robust estimation techniques that simultaneously addressed the presence of multicollinearity and outliers in linear model was presented. It was revealed form the Table 7 that the root mean square error (RMSE) of M-estimator value 0.1177 was the smallest when compared with the S-estimator and MM-estimator with value 0.2211 and 0.1472 respectively. The mean absolute error (MAE) of the fitted robust estimation technique revealed that M-estimator had the smallest mean absolute error of 0.0896 when compared with the S-estimator and MM-estimator with value 0.1109 and 0.09672 respectively. Also, mean absolute percentage error (MAPE) of the fitted robust estimation method for the linear model revealed that M-estimator with the value 0.8605 had the smallest MAPE when compared with the S-estimator and MM-estimator with value 0.2211 and 0.1472 respectively with MAPE of 1.0583 and 0.9199 respectively. The results were obtained using bias proportion showed that M-estimator with the value 0.0014 was the smallest in comparison with the S-estimator and MM-estimator with value 0.0230 and 0.0339 respectively. The theil inequality coefficient and the variance proportion also revealed the same result and as such, it can be asserted that the M-estimator with principal component is the most efficient robust estimation technique for addressing the presence of multicollinearity and outlier jointly in modelling economic growth using internal debt, external debt, interest rate, exchange rate and degree of economy openness in Nigeria.

4 Conclusion

An examination of an estimation of economic growth's parameters in Nigeria among the identified determinants (INDT, EXDT, RINR, REXR, and OPEN) in the presence of multicollinearity and outliers as basic assumptions violation. An exploratory and diagnostics analysis established a relationship between the economic growth (RGDP) and the aforementioned determinants. The presence of multicollinearity and outlier were revealed by the VIF and Grubb's test carried out on the macroeconomic data set under consideration. Consequently, to jointly handling the problems and to obtain efficient parameter estimate for INDT, EXDT, RINR, REXR, and OPEN on RGDP, a robust principal component analysis technique that simultaneously addressed the problems to produce efficient estimates was adopted. Thus, a robust principal component regression along with M-estimator, S-estimator and MM-estimator technique that can produce efficient parameter estimate when multicollinearity and outliers were jointly present in data set were used to fit a linear model under investigation.

However, it was discovered that a robust principal component regression analysis with M-estimator was the most efficient and optimal estimation technique for investigating the impact of INDT, EXDT, RINR, REXR, and OPEN on economic growth in Nigeria during the period under study. This assertion was based on the root mean square error (RMSE) of M-estimator value 0.1177 as the smallest when compared with the S-estimator and MM-estimator respectively. The mean absolute error (MAE) of the M-estimator had the smallest value of 0.0896 when compared with the S-estimator and MM-estimator. Also, mean absolute percentage error (MAPE) of the M-estimator with the value 0.8605 was the minimum when compared with both S-estimator and MM-estimator. The results obtained using bias proportion showed that M-estimator with the value 0.0014 was the smallest in comparison with the S-estimator and MM-estimator respectively. Theil inequality coefficient and the variance proportion also revealed the same result and as such it can be asserted that the M-estimator with principal component is the most efficient robust estimation technique for addressing the presence of multicollinearity and outlier jointly in modelling economic growth.

Moreover, it was discovered from the results that robust principal component with M-estimator that the impact of INDT, EXDT, RINR, REXR and OPEN on the RGDP were positive. In specific, INDT, EXDT, RINR, REXR and OPEN increased the Nigeria's economic growth to the turn of 0.10, 0.02, 0.04, 0.06 and 3.01 percent respectively during the period under consideration. The P-value < 0.05 for the estimated parameters of the economic growth drivers evidently showed their statistical significant in assessing economic growth in Nigeria. Also, the Rn-squared statistic of 405.0331 with p-value of $0.0000 < 0.05$ which measured the overall significance of the fitted robust principal component regression along with M-estimator model was statistically significance in examining the relationship among the macroeconomic variables used as a determinants of economic growth in Nigeria in the presence of multicollinearity and outliers.

Therefore, we concluded economic recession, crash in crude oil price at international market, insecurity, terrorist activities and astronomical increase in naira to dollar exchange rate and above all the Covid-19 pandemic that was heavily witnessed during the aforementioned period greatly affect the identified determinants of economic growth in which the impact was translated to the Nigeria's economic growth (RGDP) during the period under investigation. Hence, this served as a great benefit to the policy makers as the study provide a better understanding of the relationship that exist between economic growth and the aforementioned determinants. Also, robust principal component regression analysis and M-estimation technique remain the best and unbiased technique for modeling and estimating the parameters of a linear model when multicollinearity and outliers are jointly present in the data set under investigation

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Conflict of interest

The authors declare that there is no conflict regarding the publication of this paper.

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