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Statistical approach to dynamics of modulation instabilities in optic fibers. Classical case

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Abstract: In the present paper, we prove equality of the wave equation for Modulation Instability and Vlasov kinetic equation. On the basis of Bogolyubov-Born-Green-Kirkwood-Yvon (BBGKY) hierarchy of classical kinetic equations, the kinetic equations for any number of Modulation Instabilities are defined and the method a solution for these equations is proposed.

Keywords: Modulation Instability; Vlasov kinetic equation; BBGKY hierarchy of quantum Kinetic equation.

1 Introduction

Modulation instability (MI) is a process by which continuous radiation becomes unstable by the simultaneous action of nonlinearity and anomalous dispersion [1]. Ever since the first observation of the modulation instability effect in the mid-1970s by N. F. Piliptetskii and A. R. Rustamov [2], V. I. Bespalov and V. I. Talanov [3] and T. Brooke Benjamin and Jim E. Feir [4] remain the subject of research [5,6,7,8,9,10,11,12].

It is known that the dynamics of modulation instability in optical fibers were first described by Hasegawa in the form of a kinetic equation for waves, and dispersion relations for one and two quasi-particles were derived [5]. It was assumed that these two quasi-particles were mutually independent. That is, correlations between these quasi-particles were not taken into account when taking into account the presence of interaction between these quasi-particles. In reality, all quasi-particles interact with each other, and therefore, when solving the equation for MI, one should take into account the correlations between these quasi-particles.

On the other hand, usually during the real process of transfer of quasi-particles in optical fibers, many quasi-particles rise and therefore, the problem of taking into account all these quasi-particles and the interactions between them arises.

The present work is devoted to solving this problem. For this purpose, in the present paper, the equality of wave equations for MI to the Vlasov kinetic equation [13] is shown. Then, using a hierarchy of Bogolyubov-Born-Green-Kirkwood-Yvon (BBGKY) classical kinetic equations for a many-particle system [14, 15,16,17,18,19] and the relation between kinetic equation for one MI and kinetic equation for a many-particle system [20] we can define the classical kinetic equations for any number of MIs, describing the actual IM system. We can also define the exact solution of this kinetic equation for MI. In this case, the method of Ichimaru [21] and Liboff [22] was used to order the interaction, correlation matrices, and perturbations with respect to a small parameter.

The results can be useful for describing transport phenomena of any number of modulation instabilities in optic fibers.

2 The equality of wave kinetic equation for modulation instability to Vlasov equation

The wave equation for modulation instability is [5]:

$$\frac{\partial f(t)}{\partial t} - v_g \frac{\partial f(t)}{\partial z} - w_0 \frac{n_2}{2n_0} \frac{\partial |E|^2}{\partial z} \frac{\partial f(t)}{\partial k} = 0, \quad (1)$$

where w_0 is the frequency of an unmodulated lightwave, k is the wave number, z is distance of propagation, t is time,n_0 index of refraction, n_2 is the Kerr coefficient, $E(z,t)$ is the optical electric field and the group velocity is given by

$$v_g(k) = \frac{c}{n_0},$$

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where c is the velocity of light. Since f is the phase space density of the wave pocket, f is related to the real space density of the quasi-particles through

$$\frac{|E|^2}{w_0} = \int_{-\infty}^{\infty} f dk. \qquad (2)$$

Equations (1) and (2) give a closed set of equations that describe the phase-space dynamics of the quasi-particles.

The Vlasov kinetic equation has the form [13]:

$$\left(\frac{\partial}{\partial t} - \frac{k}{m}\frac{\partial}{\partial z}\right) f(t,z,k) = \frac{1}{v}\int \frac{\partial \Phi(|z-z'|)}{\partial z}\frac{\partial f(t,z,k)}{\partial k}\cdot$$

$$f(t,z',k')dz'dk', \qquad (3)$$

where z is the 1th particle coordinate, k - is the 1th particle impulse, v is volume per particle, θ is the potential between two particles, $f(t,z,k)$ is distribution function, t is time, $m=1$ is particle mass.

Substituting in the Vlasov equation (3) $\frac{k}{m} = v_g$, $\Phi(|z-z'|) = v\theta(|z-z'|) = w_0\frac{n_2}{2n_0}\delta(|z-z'|)$ and

$$|E|^2(t,z) = \frac{1}{v}\int_{z',k'} \Phi(|z-z'|) f(t,z',k')dz'dk =$$

$$\int_{z',k'} \theta(|z-z'|) f(t,z',k')dz'dk' =$$

$$w_0\frac{n_2}{2n_0}\int_{z',k'} \delta(|z-z'|) f(t,z',k')dz'dk' =$$

$$w_0\frac{n_2}{2n_0}\int_{k'} f(t,z,k')dk'$$

we obtain the equation (1). This is the proof of the equality of Vlasov equation (3) and the wave kinetic equation (1).

3 Solution of linearized Vlasov equation

Let us now describe the modulation instability of quasi-particles using the (1), (2) set of the equation. We first linearize this set by writing [5]

$$f(t) = f_0(k) + \frac{1}{2}[\tilde{f}_1 exp^{i(kz-\sigma t)} + c.c.]$$

$$|E|^2 = |E|_0^2(k) + \frac{1}{2}[\tilde{E}_1^2 exp^{i(kz-\sigma t)} + c.c.]. \qquad (4)$$

in this case, linearized Vlasov equation will

$$\left(\frac{\partial}{\partial t} + \frac{k}{m}\frac{\partial}{\partial z}\right)\tilde{f}_1(t,z,k) = \frac{1}{v}\int \frac{\partial \theta(|z-z'|)}{\partial z}\frac{\partial f_0(k)}{\partial k}\cdot$$

$$\tilde{f}_1(t,z',k')dz'dk'. \qquad (5)$$

From (1),(3), (4) and (5) is followed that

$$\tilde{f}(k,\sigma) = \frac{w_0 n_2}{2n_0}\frac{k}{(\sigma_0 - kv_g)}\frac{\partial f_0}{\partial k}|\tilde{E}|_1^2. \qquad (6)$$

Substituting (6) into (2), we obtain the following dispersion relation

$$\frac{w_0^2 n_2}{2n_0}\int_{-\infty}^{\infty} \frac{k}{\sigma_0 - kv_g}\frac{\partial f_0}{\partial k}dk = 1. \qquad (7)$$

The modulational instability is a process of localization of monochromatic waves. Thus we have

$$f_0(k) = \frac{|E|_0^2}{\delta(k-k_0)}. \qquad (8)$$

If we substitute (8) into (7) and integrate the result by parts, we have

$$1 + \frac{w_0 n_2}{2n_0}\frac{\partial v_g}{\partial k^2}\frac{k}{(\sigma_0 - kv_g)^2} = 0.$$

Here

$$\frac{\partial v_g}{\partial k^2} = -k'' v_g^3.$$

That is, the quasi-particles have a monochromatic energy.

If there are two sets of quasi-particles of different wave numbers, for example, if we take the two wave numbers at k_1 and k_2, f_0 may be given by

$$f_0 = \frac{|E|_0^2}{w}[\delta(k-k_2) + \delta(k-k_2)].$$

Then the dispersion is given

$$\frac{k_1'' v_{g1}^3 w_{02} n_2}{2n_0}\frac{k^2}{(\sigma - kv_{g1})^2} + \frac{k_2'' v_{g2}^3 w_{01} n_2}{2n_0}\frac{k^2}{(\sigma - kv_{g2})^2} = 1, \qquad (9)$$

where v_{g1} and $4v_{g2}$ are the group velocities at wave numbers k_1 and k_2. This dispersion relation was first devised by Hasegawa [5].

Unfortunately, when deriving the equation (9), the contribution of correlations between the two modulation instabilities was not taken into account, considering them independent, i.e. , considering distribution function of two modulation instabilities as a product of distribution functions of one-particle modulation instabilities

$$f_2(t,z_1,z_2,r_1,k_2) = f(t,z_1,k_1)f(t,z_2,k_2)$$

It is known reality consists of many particles and these particles are interconnected. Accordingly, the two modulation instabilities are also interconnected. Therefore, an urgent task is to determine the dynamics of an arbitrary number of modulation instabilities that satisfies the equation (1) for one modulation instability and also takes into account the correlations that arise between many modulation instabilities.

Below we propose a method for generalizing the theory of modulation instabilities for an arbitrary number of particles, taking into account the correlations between the, and satisfying the equation (1) for one modulation of the instability.

4 Consideration of the any number modulation instability

For a description of the any number modulation instability, we start from the Bogolyubov-Born-Green-Kirkwood-Yvon (BBGKY) hierarchy of classical kinetic equations [14]:

$$\frac{\partial f_s(t,z_1,z_2,...,z_s,k_1,k_2,...,k_s)}{\partial t} =$$
$$[H,f]_s(t,z_1,z_2,...,z_s,k_1,k_2,...,k_s)+$$
$$\frac{1}{v}\int_\Omega [\sum_{1\le i\le s}\Phi(|z_i-z'|),$$
$$f_{s+1}(t,z_1,z_2,...,z_s,z';k_1,k_2,...,k_s,k')]dz'dk'$$

where Ω is the infinite space of points (z,k), $m=1$ is mass, $[,]$ is the Poisson bracket, $s\in N, N$ is the number of particles, V is the volume of the system, $N\to\infty$, $V\to\infty$, $v=\frac{V}{N}=const$ is volume per particle, and H is the Hamiltonian:

$$H_s=T_s+U_s=-\sum_{1\le i\le s}\frac{\partial^2}{2\partial z_i^2}+\sum_{1\le i<j\le s}\Phi(|z_i-z_j|),$$

where

$$T_s=-\sum_{1\le i\le s}\frac{\partial^2}{2\partial z_i^2},\quad U_s=\sum_{1\le i<j\le s}\Phi(|z_i-z_j|).$$

Using the following relation [20,23]

$$f(t)=\Gamma\psi(t)=\Gamma f_0*\Gamma\tilde{f}(t),$$

where f_0 is the equilibrium state and $\tilde{f}(t)$ is the non-equilibrium perturbation of equilibrium correlation function f_0 and [20,24,25]

$$\Gamma f_0=I+f_0+\frac{f_0*f_0}{2}+...+\frac{(*f_0)^s}{s!}+...,$$

$$\Gamma\tilde{f}(t)=I+\tilde{f}(t)+\frac{\tilde{f}(t)*\tilde{f}(t)}{2}+...+\frac{(*\tilde{f}(t))^s}{s!}+...,$$

$$f_0=\{(f_0)_1(z_1;k_1),(f_0)_2(z_1,z_2;k_1,k_2),...$$
$$...,(f_0)_s(z_1,z_2,...,z_s;k_1,k_2,...,k_s),...\},$$
$$\tilde{f}(t)=\{\tilde{f}_1(t,z_1;k_1),\tilde{f}_2(t,z_1,z_2;k_1,k_2),...$$
$$...,\tilde{f}_s(t,z_1,z_2,...,z_s;k_1,k_2,...,k_s),...\},$$
$$(f_0*f_0)(X)=\sum_{Y\in X}f_0(Y)f_0(X\setminus Y),$$
$$(\tilde{f}*\tilde{f})(t,X)=\sum_{Y\in X}\tilde{f}(t,Y)\tilde{f}(t,X\setminus Y),$$

where

$$X=(x_1,x_2,...,x_s),\quad Y=(x_1,x_2,...,x_{\tilde{s}}),\quad x=(z;k)$$

$$s,\tilde{s}=1,2,..,s,\quad s'\in s,\quad I*f=f,$$
$$(*f)^s=f*f*...*f,\quad s\quad time,$$

on the basis

$$\frac{\partial}{\partial t}(\Gamma f_0*\Gamma\tilde{f}(t))=\mathscr{H}(\Gamma f_0*\Gamma\tilde{f}(t))+$$
$$\int\mathscr{A}_{z'}\mathscr{D}_{z'}(\Gamma f_0*\Gamma\tilde{f}(t))dz',\qquad(10)$$

and

$$\frac{\partial}{\partial t}(\Gamma f_0*\Gamma\tilde{f}(t))=\frac{\partial}{\partial t}\tilde{f}(t)*(\Gamma f_0*\Gamma\tilde{f}(t)),$$
$$\mathscr{T}(\Gamma f_0*\Gamma\tilde{f}(t))=\mathscr{T}f_0*\Gamma f_0*\Gamma\tilde{f}(t)+\mathscr{T}\tilde{f}(t)*\Gamma f_0*\Gamma\tilde{f}(t),$$
$$\mathscr{D}_{s'}(\Gamma f_0*\Gamma\tilde{f}(t))=\mathscr{D}_{s'}f_0*\Gamma f_0*\Gamma\tilde{f}(t)+\mathscr{D}_{s'}\tilde{f}(t)*\Gamma f_0*\Gamma\tilde{f}(t),$$
$$\mathscr{U}(\Gamma f_0*\Gamma\tilde{f}(t))=\mathscr{U}f_0*\Gamma f_0*\Gamma\tilde{f}(t)+\mathscr{U}\tilde{f}(t)*\Gamma f_0*\Gamma\tilde{f}(t)+$$
$$(\frac{1}{2}\mathscr{W}(f_0,f_0)+\frac{1}{2}\mathscr{W}(\tilde{f}(t),\tilde{f}(t))+\mathscr{W}(f_0,\tilde{f}(t)))*\Gamma f_0*\Gamma\tilde{f}(t),$$
$$\mathscr{A}_{z'}(\Gamma f_0*\Gamma\tilde{f}(t))=\mathscr{A}_{z'}f_0*\Gamma f_0*\Gamma\tilde{f}(t)+\mathscr{A}_{z'}\tilde{f}(t)*\Gamma f_0*\Gamma\tilde{f}(t),$$
$$\mathscr{A}_{z'}\mathscr{D}_{z'}(\Gamma f_0*\Gamma\tilde{f}(t))=\mathscr{A}_{z'}\mathscr{D}_{z'}f_0*\Gamma f_0*\Gamma\tilde{f}(t)+$$
$$\mathscr{A}_{z'}\mathscr{D}_{z'}\tilde{f}(t)*\Gamma f_0*\Gamma\tilde{f}(t)+$$
$$\mathscr{A}_{z'}f_0*\mathscr{D}_{z'}f_0*\Gamma f_0*\Gamma\tilde{f}(t)+\mathscr{A}_{z'}\tilde{f}(t)*\mathscr{D}_{z'}f_0*\Gamma f_0*\Gamma\tilde{f}(t)+$$
$$\mathscr{A}_{z'}f_0*\mathscr{D}_{z'}\tilde{f}(t)*\Gamma f_0*\Gamma\tilde{f}(t)+\mathscr{A}_{z'}\tilde{f}(t)*\mathscr{D}_{z'}\tilde{f}(t)*\Gamma f_0*\Gamma\tilde{f}(t),$$

where

$$(\mathscr{H}\tilde{f})_s=[H_s,\tilde{f}_s];\qquad(\mathscr{T}\tilde{f})_s=[T_s\tilde{f}_s];$$
$$(\mathscr{D}_{s'}\tilde{f})_s(t,z_1,z_2,...,z_s)=\tilde{f}(t,z_1,z_2,...,z_s,z_{s'});$$
$$(\mathscr{A}_{s'}\tilde{f})_s=\frac{1}{v}\sum_{1\le i\le s}[\Phi(|z_i-z'|),\tilde{f}_s];\qquad(\mathscr{U}\tilde{f})_s=[U_s,\tilde{f}(t)_s],$$
$$(\mathscr{W}(\tilde{f}(t),\tilde{f}(t))_s=\sum_{Y\in X}U(Y;X\setminus Y)\tilde{f}_{\tilde{s}}(t,Y)\tilde{f}_{s-\tilde{s}}(t,X\setminus Y),$$

multiplying both sides of the equation (10) by $\Gamma f_0*\Gamma\tilde{f}(t)^{-1}$ and taking into account

$$\mathscr{H}f_0+\frac{1}{2}\mathscr{W}(f_0,f_0)+\int(\mathscr{A}_{z'}\mathscr{D}_{z'}f_0+\mathscr{A}_{z'}f_0*\mathscr{D}_{z'}\varphi)dz'=0$$

we obtain quantum kinetic equations for perturbations of equilibrium correlation functions [20]:

$$\frac{\partial\tilde{f}(t)}{\partial t}=\mathscr{H}\tilde{f}(t)+\frac{1}{2}\mathscr{W}(\tilde{f}(t),\tilde{f}(t))+\mathscr{W}(f_0,\tilde{f}(t))+$$
$$+\int(\mathscr{A}_{z'}\mathscr{D}_{z'}\tilde{f}(t)+\mathscr{A}_{z'}\tilde{f}(t)*\mathscr{D}_{z'}f_0+$$
$$\mathscr{A}_{z'}f_0*\mathscr{D}_{z'}\tilde{f}(t)+\mathscr{A}_{z'}\tilde{f}(t)*\mathscr{D}_{z'}\tilde{f}(t))dz'.\qquad(11)$$

To study our system based on similar arguments to [21, 22] we can select an extension parameter v, setting:

$$\Phi=v\theta.$$

The smallness of the perturbation from equilibrium can be taken into account by setting

$$f_s(t) = (f_0)_s + \nu \tilde{f}_s(t),$$

and thus regarding $\tilde{f}(t)$ as the first approximation parameter ν. In this case the assumption for correlation functions [21,22]:

$$\psi_s(t) \sim \nu^{s-1}\tilde{\psi}_s(t),$$

can be expressed in terms of $\psi_s(t)$

$$\psi_s(t) \sim \nu^{s-1}\tilde{\psi}(t) = \nu^{s-1}(f_0)_s + \nu(\tilde{f}_1)_s(t)). \quad (12)$$

Under the assumptions (12) equation (11) for s Modulation Instability takes this form

$$\frac{\partial(\tilde{f}_1)_s(t,X)}{\partial t} = (\mathcal{T}\tilde{f}_1)_s(t,X) + \nu(\mathcal{U}\tilde{f}_1)_s(t,X) +$$
$$(\mathcal{W}(f_0,\tilde{f}_1(t)))_s(X) + \frac{\nu}{2}\mathcal{W}(\tilde{f}_1(t),\tilde{f}_1(t))_s(X) +$$
$$+\nu^2\int(\mathcal{A}_{z'}\mathcal{D}_{z'}\tilde{f}_1(t))_s(X)dz' +$$
$$\nu\int(\mathcal{A}_{z'}\tilde{f}_1(t)*\mathcal{D}_{z'}f_0)_s(X)dz' +$$
$$\nu\int(\mathcal{A}_{z'}f_0*\mathcal{D}_{z'}\tilde{f}_1(t))_s(X)dz' +$$
$$\nu^2\int(\mathcal{A}_{z'}\tilde{f}_1(t)*\mathcal{D}_{z'}\tilde{f}_1(t))_s(X))dz'. \quad (13)$$

Here and also in what follows in the symbols $\mathcal{U}$, $\mathcal{W}$, $\mathcal{A}$ the interaction Φ is replaced by θ and $\int \frac{\partial\theta(|z_i-z|)}{\partial z_i}dz = 0$.

5 Solution of equation (13)

To solve equation (13), we apply the perturbation theory. We shall seek a solution in the terms of series [20]

$$(\tilde{f}_1)_s(t,X) = \sum_\mu \nu^\mu(\tilde{f}_1)_s^\mu(t,X), \quad (14)$$

$$s = 1,2,3,...., \quad \mu = 0,1,2,,....$$

Substituting the series (14) in equation (13) and equalizing the coefficients of equal powers of $\tilde{\nu}$, we obtain the set of homogeneous and inhomogeneous equations

$$(\frac{\partial}{\partial t} + \mathcal{L}_1)(\tilde{f}_1)_1^0(t) = 0, \quad (15)$$

$$(\frac{\partial}{\partial t} + \mathcal{L}_1 + \mathcal{L}_2)(\tilde{f}_1)_2^0(t) = S_2^0(t), \quad (16)$$

....

$$(\frac{\partial}{\partial t} + \sum_{i=1}^{s}\mathcal{L}_i)(\tilde{f}_1)_s^\mu(t) = S_s^\mu(t) \quad (17)$$

where

$$\mathcal{L}_1(\tilde{f}_1)_1^0(t,z_1,k_1) = \frac{k_1}{m_1}\frac{\partial}{\partial z_1}(\tilde{f}_1)_1^0(t,z_1,k_1) -$$
$$\int\frac{\partial\theta(|z_1-z|)}{\partial z_1}\frac{\partial f_0(k_1)}{\partial k_1}\tilde{f}_1^0(t,z',k')dz'dk',$$
$$(\mathcal{L}_i\tilde{f}_1)_s^\mu(t,X) = \frac{k_i}{m_i}\frac{\partial}{\partial z_1}(\tilde{f}_1)_s^\mu(t,X) +$$
$$\int(\mathcal{A}_{z'}f_0(k_i))(\mathcal{D}_{z'}(\tilde{f}_1)_{s-1}^\mu(t,X\setminus x_i)dz',$$
$$S_s^\mu(t,X) = (\mathcal{U}\tilde{f}_1^{\mu-1}(t))_s(X) + (\mathcal{W}(f_0,\tilde{f}_1^\mu(t)))_s(X) +$$
$$\frac{1}{2}\sum_{\nu_1+\nu_2=\mu-1}(\mathcal{W}(\tilde{f}_1^{\nu_1}(t),\tilde{f}_1^{\nu_2}(t)))_s(X) +$$
$$\int(\mathcal{A}_{z'}\mathcal{D}_{z'}\tilde{f}_1^{\mu-1}(t))_s(X)dz'$$
$$+\int(\mathcal{A}_{z'}\tilde{f}_1^\mu(t)*\mathcal{D}_{z'}f_0)_s(X)dz' +$$
$$\int(\mathcal{A}_{z'}f_0*\mathcal{D}_{z'}\tilde{f}_1^\mu(t))_s(X)dz' +$$
$$\int\sum_{\nu_1+\nu_2=\mu-1}(\mathcal{A}_{z'}\tilde{f}_1^{\nu_1}(t)*\mathcal{D}_{z'}\tilde{f}_1^{\nu_2}(t))_s(X)dz'.$$

The solution of the equation (13) is reduced to solving the homogeneous equation (15) for $(\tilde{f}_1)_1^0(t)$ and non-homogeneous equations (16), (17) for $(\tilde{f}_1)_2^0(t)$ and $(\tilde{f}_1)_s^0(t)$ respectively. The linearised Vlasov equation depends on both one and the second particles of the system and thus serves as a link between the interrelated equations (15) and (16). Similarly, each equation of the chain of equations is mutually related recurrently and therefore the definition of the solution of the previous equation serves to determine the next equation. As defined above, the solution of the linearized Vlasov equation has the form (7). Substituting

$$(\tilde{f}_1)_2^0(t,x_1,x_2) = \int dx_1'\int dx_2'\int_\infty^t dt' S_2^0(t'x_1',x_2')\cdot$$
$$\mathcal{G}(t-t',x_1,x_1')\mathcal{G}(t-t',x_2,x_2') \quad (18)$$

in (16), you can make sure that (18) is a solution to the equation (16), if

$$S_2^0(t,z_1,z_2;k_1,k_2) = [\theta(|z_1-z_2|),f_0(k_1)\tilde{f}_1^0(t,z_2,k_2) +$$
$$\tilde{f}_1^0(t,z_1,k_1)f_0(k_2)] +$$
$$\int\sum_{1\leq i\leq 2}[\theta(|z_i-z'|),f_0(k_1,k_2)\tilde{f}_1^0(t,z',k')]dz'dk' +$$
$$\int[\theta(|z_1-z'|),\tilde{f}_1^0(t,z_1,k_1)f_0(k_2,k')]dz'dk' +$$
$$\int[\theta(|z_2-z'|),\tilde{f}_1^0(t,z_2,k_2)f_0(k_1,k')]dz'dk'$$

and if $\mathscr{G}(t-t',x_1,x_1')$ satisfies an equation similar to the linearized Vlasov equation (5):

$$(\frac{\partial}{\partial t}+\frac{k_1}{m_1}\frac{\partial}{\partial z_1})\mathscr{G}(t-t',z_1,z_1'';k_1,k_1'')-$$

$$\int\frac{\partial\theta(|z_1-z'|)}{\partial z_1}\frac{\partial f_0(k_1)}{\partial k_1}\mathscr{G}(t-t',z',z_1'';k',k_1'')dz'dk'=0, \quad (19)$$

with the initial condition

$$\mathscr{G}(z_1,z_1'';k_1,k_1'')=\delta(z_1-z_1'')\delta(k_1-k_i''). \quad (20)$$

The solution of equation (17) for s modulation instability $(\tilde{f}_1)^{\mu}_s(t)$ reduces to the solution of homogeneous (15) linearized Vlasov equation for function distribution $(\tilde{f}_1)^0_i(t,z_i,k_i)$ and inhomogeneous linear equations (17) for $(\tilde{f}_1)^{\mu}_s(t)$ on the basis of a formula

$$(\tilde{f}_1)^{\mu}_s(t,X)=\int dx_1'...\int dx_s'\int_{-\infty}^{t}dt'S^{\mu}_s(X')\cdot$$

$$\cap_{1\leq i\leq s}\mathscr{G}(t-t',x_i,x_i'). \quad (21)$$

In (21) the Green function $\mathscr{G}(t-t',x_i,x_i'')$ is the solution to the Cauchy problem (19),(20).

6 Conclusion

1. The classical analogue of kinetic equation for the modulation instability process (4.84) of [5] coincides with the Vlasov equation with potential in the form of delta function.

2. The process of transfer in optic fiber any number of modulation instability can be described by BBGKY's chain of classical kinetic equations.

Competing interests

The author declares that they have no competing interests.

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