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On Analytical and Numerical Study for the Peyrard-Bishop DNA Dynamic Model

Khalid K. Ali[1], *M. Ismail Abdelrahman* [1], *K. R. Raslan* [1] *and Waleed Adel* [2,3,*]

[1]Mathematics Department, Faculty of Science, Al-Azhar University, Nasr-City, Cairo, Egypt
[2]Department of Mathematics and Engineering Physics, Faculty of Engineering, Mansoura University, Mansoura, Egypt
[3]Université Française d´Egypte, Ismailia Desert Road, El Shorouk, Cairo, Egypt

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Abstract: In this work, the Peyrard-Bishop DNA dynamic model is being investigated through three analytical and numerical techniques. The Kudryashov method and modified Kudryashov method for analytical solutions as well as the B-spline method for numerical verification are those methods that are utilized for solving the presented problem. The results obtained from these techniques are then compared through tables and an excellent agreement between them is noticed. We give some figures to show how accurate the solutions will be obtained from analytical and numerical methods. These methods have the advantages of providing accurate results besides being straightforward and docent requiring any complex computations.

Keywords: The DNA dynamics model; The Kudryashov method; The modified Kudryashov method; A cubic B-spline collocation method.

1 Introduction

Over the past few years, the search for finding approximate solutions using analytical or numerical methods for different mathematical models is an ongoing process. The importance of finding these solutions comes from the fact that these different forms of solutions may contribute greatly to better understanding the behavior of various natural phenomena. This natural phenomenon may occur in different areas of science, engineering, biology, chemistry, and physics. Recently, different effective techniques have been utilized to simulate some of these models with applications. For example, Adel et. al adapted a collocation based numerical method for solving Lienard's equation [1], CD4+T cell model [2], Lane-Emden equation [3], Hunter Saxton equation [4], fractional foam drainage equation [5] and Ambartsumian equation [6]. Other researchers recently described real-life phenomena through some numerical and analytical techniques. These studied can be found in [7–13].

In this paper, we are interested in obtaining a solitary wave solution to a famous biological model named the Peyrard-Bishop which has an important application in DNA flow [14]. The novelty of this paper lies in finding an approximate solution to the previously mentioned model using two different techniques. The first technique depends on acquiring the analytical solution while the other technique is a numerical-based technique to confirm the obtained results. A convergence between the analytical and numerical solutions can be confirmed by comparing the results through absolute error. This model was first introduced by Peyrard-Bishop in [15] and this was the motivation for beginning this continuous work.

To explore the behavior of this model, Peyrard and Bishop in [15] investigated the statistical mechanics of the nonlinear DNA denaturation model and the possibility of the appearance of the solitonic structure. Also, in [16], Abazari et. al formulated a new model to simulate the vibrational dynamics and solitary wave solution of this model. Dusuel et. al in [17], and Alverez et. al [18], both provided a detailed connection in the continuum limit, between the nonlinear terms with no power and the scattering within the DNA energetic.

There have been many analytical methods that can deal with such models and among these models are the Kudryashov and the modified Kudryashov methods. This method has been used ever since for solving different

* Corresponding author e-mail: waleedadel85@gmail.com

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forms of biological models as well as models with great engineering applications. For example, the Kudryashov method has been used for finding the exact solutions to high order nonlinear evolution equation [19], time-fractional differential equations [20], nonlinear conformable time-fractional Boussinesq equations [21], a system of some nonlinear evolution equations [22], time-fractional Burger type equation [23]and other related models. These methods prove to be reliable methods of providing accurate solitons solutions with some important physical behaviors. Also, for the purpose of numerical verification, the B-spline method uses different bases considered as a benchmark method for solving such equations. The method has been tested on different problems including Newell Whitehead Segel type equations [24], Gardner and Harry Dym equation [25], Jaulent–Miodek coupled equations [26]and PHI-four and Allen–Cahn equations [27]. Due to the above-mentioned reasons, we choose these two methods for solving the Peyrard-Bishop model.

The organization of the paper is as follows: in section 2, a short presentation of the model under study is analyzed. Section 3 illustrates the main steps of the proposed analytical technique for solving the given problem. In section 4, the Cubic B-spline method is utilized for solving the model problem. In section 5, the results obtained using the two methods are presented in tables and figures and compared to each other. Finally, the last section is devoted to the conclusion of the study.

2 Model Formulation

In this section, we shall introduce and analyze the DNA dynamic model of the Peyrard-Bishop type. First, the main structure for a molecule of a DNA model is to be of a form of the double helix and this means that it mainly consists of double polymeric chains which are wrapped around each other [28]. The Watson Crick model is a form of these double helix models that describes the basic formation of the DNA of B-shaped type which may contain a double chain. The masses of these molecules are not different since they are formed from some homogenous structure and the bond between the hydrogen toms is weak while the longitudinal length is strong [29]. The main equation for the prescribed model is in the form

$$F_m(f_n - g_n) = D[e^{-a(f_n-g_n)} - 1]^2, \quad (1)$$

in which f_n and g_n are the displacements of the nucleotides. In addition, Zdravkovı́c described the Hamiltonian for DNA's chain in [29]. Also, Dauxois [30] provided a modification to the known Peyrard Bishop model taking into account other several factors. The Hamiltonian for the system can take the following form [31]

$$G(f) = \frac{1}{2m}q_n^2 + \frac{k_1}{2}\Delta^2 f_n + \frac{k_2}{4}\Delta^4 f_n + \delta(e^{-a\sqrt{2}f_n})^2, \quad (2)$$
$$\Delta f_n = f_{n+1} - f_n.$$

where k_1 and k_2 represent the couplings of the both linear and nonlinear and $q_n = m\dot{f_n}$ defined as the momentum of the displacement. The equation of motion (2) can be in the form

$$f_{tt} - (l_1 + 3l_2 f_{xx}) - 2\sqrt{2}aDe^{-af}(e^{-af} - 1) = 0, \quad (3)$$

with $l_1 = \frac{k_1}{m}d^2, l_2 = \frac{k_2}{m}d^4, D = \frac{\delta}{m}, \alpha \equiv \sqrt{2}a$ and being d the inter-site nucleotide distance in the DNA ladder [32–34]. In this paper, consider the Peyrard-Bishop DNA dynamic model equation as follows

$$f_{tt} - (l_1 + 3l_2 f_x^2)f_{xx} - 2\alpha\omega e^{-\alpha f}(e^{-\alpha f} - 1) = 0, \quad (4)$$

where l_1, l_2, α and $\omega = D$ are constants. Next, we shall investigate the analytical solution for the main equation using the Kudryashov and the modified Kudryashov methods.

3 Analytical solutions

In this section, we give a detailed view of the Kudryashov and the modified Kudryashov methods are used to find the solution of the model in (1). First, we begin with the first method next.

3.1 The Kudryashov method

The partial differential equation (4) with the following transformation:

$$f(x,t) = h(\xi), \quad \xi = x - \beta t, \quad (5)$$

which can be reduced into the following

$$\beta^2 h'' - (l_1 + 3l_2 h'^2)h'' - 2\alpha\Omega e^{-\alpha h}(e^{-\alpha h} - 1) = 0. \quad (6)$$

The next step is to multiply the (6) by the value of h' and then apply integration to ξ once, the we reach the following

$$\frac{(\beta^2 - l_1)}{2}(h')^2 - \frac{3}{4}l_2(h')^4 + \Omega e^{-\alpha h}(e^{-\alpha h} - 2) + R = 0, \quad (7)$$

Then, by assuming the following

$$\phi(\xi) = e^{-\alpha h(\xi)}, \quad (8)$$

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Then, by substituting (8) into (7), we get the following form of nonlinear equation:

$$\frac{(\beta^2-l_1)}{2\alpha^2}\phi^2(\phi')^2-\frac{3}{4\alpha^4}l_2(\phi')^4+\Omega\phi^5(\phi-2)+R\phi^4=0, \tag{9}$$

Now, we can express for the Kudryashov method in a finite series as follows:

$$\phi(\xi)=A_0+\sum_{i=1}^{N}A_i\Omega^i(\xi), \tag{10}$$

where, $A_0, A_1, A_2, \ldots, A_N$ are constants and N is a positive integer that can be determined by using homogeneous balancing method. The function $\Omega(\xi)$ can be expressed in this form:

$$\Omega(\xi)=\frac{1}{1+de^{\xi}}, \tag{11}$$

where (11) achieve the ordinary differential equation

$$\Omega'(\xi)=\Omega(\xi)(\Omega(\xi)-1). \tag{12}$$

We can compensate by (10) with some derivatives that we need into (9) then, we get from that of a polynomial as a function in $\Omega(\xi)$

$$P(\Omega(\xi))=0. \tag{13}$$

Thus equating the coefficient of each power of $\Omega(\xi)$ in the above equation to zero gives a set of nonlinear algebraic equations with the aid of symbolic computation using Mathematica which will be used to yield the exact solutions for (9).
Now, if we make balancing between ϕ^6, $\phi^2(\phi')^2$ and $(\phi')^4$ in (9) and take the high balance we get $6N=4N+4$, thus $N=2$.
This gives a truncated form of Eq. (10) as

$$\phi(\xi)=A_0+A_1\Omega(\xi)+A_2\Omega^2(\xi). \tag{14}$$

Finally, by replacing the terms of (14) into (9), we get the following:

$$\omega A_0^6-2\omega A_0^5+RA_0^4=0,$$

$$6\omega A_1A_0^5-10\omega A_1A_0^4+4RA_1A_0^3=0,$$

$$6\omega A_2A_0^5+15\omega A_1^2A_0^4-10\omega A_2A_0^4-20\omega A_1^2A_0^3$$
$$+4RA_2A_0^3+\frac{\beta^2A_1^2A_0^2}{2\alpha^2}+6RA_1^2A_0^2-\frac{A_1^2l_1A_0^2}{2\alpha^2}=0,$$

$$30\omega A_1A_2A_0^4+20\omega A_1^3A_0^3-40\omega A_1A_2A_0^3-20\omega A_1^3A_0^2$$
$$+\frac{2\beta^2A_1A_2A_0^2}{\alpha^2}+12RA_1A_2A_0^2+\frac{A_1^2l_1A_0^2}{\alpha^2}$$
$$-\frac{\beta^2A_1^2A_0^2}{\alpha^2}-\frac{2A_1A_2l_1A_0^2}{\alpha^2}+\frac{\beta^2A_1^3A_0}{\alpha^2}$$
$$+4RA_1^3A_0-\frac{A_1^3l_1A_0}{\alpha^2}=0,$$

$$15\omega A_2^2A_0^4-20\omega A_2^2A_0^3+60\omega A_1^2A_2A_0^3+15\omega A_1^4A_0^2$$
$$+\frac{\beta^2A_1^2A_0^2}{2\alpha^2}+\frac{2\beta^2A_2^2A_0^2}{\alpha^2}+6RA_2^2A_0^2-60\omega A_1^2A_2A_0^2$$
$$+\frac{4A_1A_2l_1A_0^2}{\alpha^2}-\frac{4\beta^2A_1A_2A_0^2}{\alpha^2}-\frac{2A_2^2l_1A_0^2}{\alpha^2}$$
$$-\frac{A_1^2l_1A_0^2}{2\alpha^2}-10\omega A_1^4A_0+\frac{5\beta^2A_1^2A_2A_0}{\alpha^2}+12RA_1^2A_2A_0$$
$$+\frac{2A_1^3l_1A_0}{\alpha^2}-\frac{2\beta^2A_1^3A_0}{\alpha^2}-\frac{5A_1^2A_2l_1A_0}{\alpha^2}$$
$$+\frac{\beta^2A_1^4}{2\alpha^2}+RA_1^4-\frac{A_1^4l_1}{2\alpha^2}-\frac{3A_1^4l_2}{4\alpha^4}=0,$$

$$-2\Omega A_1^5+6\Omega A_0A_1^5+\frac{l_1A_1^4}{\alpha^2}+\frac{3l_2A_1^4}{\alpha^4}-\frac{\beta^2A_1^4}{\alpha^2}$$
$$+\frac{\beta^2A_0A_1^3}{\alpha^2}+\frac{3\beta^2A_2A_1^3}{\alpha^2}+60\Omega A_0^2A_2A_1^3+4RA_2A_1^3$$
$$-40\Omega A_0A_2A_1^3-\frac{A_0l_1A_1^3}{\alpha^2}-\frac{3A_2l_1A_1^3}{\alpha^2}$$
$$-\frac{6A_2l_2A_1^3}{\alpha^4}+\frac{10A_0A_2l_1A_1^2}{\alpha^2}-\frac{10\beta^2A_0A_2A_1^2}{\alpha^2}$$
$$+60\omega A_0^3A_2^2A_1-60\Omega A_0^2A_2^2A_1+\frac{8\beta^2A_0A_2^2A_1}{\alpha^2}$$
$$+12RA_0A_2^2A_1+\frac{2\beta^2A_0^2A_2A_1}{\alpha^2}-\frac{8A_0A_2^2l_1A_1}{\alpha^2}$$
$$-\frac{2A_0^2A_2l_1A_1}{\alpha^2}+\frac{4A_0^2A_2^2l_1}{\alpha^2}-\frac{4\beta^2A_0^2A_2^2}{\alpha^2}=0,$$

$$\omega A_1^6+\frac{\beta^2A_1^4}{2\alpha^2}-10\omega A_2A_1^4+30\omega A_0A_2A_1^4$$
$$-\frac{l_1A_1^4}{2\alpha^2}-\frac{9l_2A_1^4}{2\alpha^4}+\frac{6A_2l_1A_1^3}{\alpha^2}$$
$$+\frac{24A_2l_2A_1^3}{\alpha^4}-\frac{6\beta^2A_2A_1^3}{\alpha^2}$$
$$+\frac{13\beta^2A_2^2A_1^2}{2\alpha^2}+90\omega A_0^2A_2^2A_1^2+6RA_2^2A_1^2$$
$$-60\omega A_0A_2^2A_1^2+\frac{5\beta^2A_0A_2A_1^2}{\alpha^2}-\frac{5A_0A_2l_1A_1^2}{\alpha^2}$$
$$-\frac{13A_2^2l_1A_1^2}{2\alpha^2}-\frac{18A_2^2l_2A_1^2}{\alpha^4}+\frac{16A_0A_2^2l_1A_1}{\alpha^2}$$
$$-\frac{16\beta^2A_0A_2^2A_1}{\alpha^2}+20\omega A_0^3A_2^3-20\omega A_0^2A_2^3$$
$$+\frac{4\beta^2A_0A_2^3}{\alpha^2}+4RA_0A_2^3+\frac{2\beta^2A_0^2A_2^2}{\alpha^2}$$
$$-\frac{4A_0A_2^3l_1}{\alpha^2}-\frac{2A_0^2A_2^2l_1}{\alpha^2}=0,$$

$$6\omega A_2A_1^5+\frac{3l_2A_1^4}{\alpha^4}-20\omega A_2^2A_1^3+60\omega A_0A_2^2A_1^3$$
$$+\frac{3\beta^2A_2A_1^3}{\alpha^2}-\frac{3A_2l_1A_1^3}{\alpha^2}-\frac{36A_2l_2A_1^3}{\alpha^4}$$
$$+\frac{13A_2^2l_1A_1^2}{\alpha^2}+\frac{72A_2^2l_2A_1^2}{\alpha^4}-\frac{13\beta^2A_2^2A_1^2}{\alpha^2}$$
$$+\frac{6\beta^2A_2^3A_1}{\alpha^2}+60\omega A_0^2A_2^3A_1+4RA_2^3A_1-40\omega A_0A_2^3A_1$$

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$$+\frac{8\beta^2 A_0 A_2^2 A_1}{\alpha^2}-\frac{6A_2^3 l_1 A_1}{\alpha^2}-\frac{8A_0 A_2^2 l_1 A_1}{\alpha^2}-\frac{24A_2^3 l_2 A_1}{\alpha^4}+\frac{8A_0 A_2^3 l_1}{\alpha^2}-\frac{8\beta^2 A_0 A_2^3}{\alpha^2}=0,$$

$$\begin{aligned}&15\omega A_2^2 A_1^4-\frac{3l_2 A_1^4}{4\alpha^4}+\frac{24A_2 l_2 A_1^3}{\alpha^4}\\&-20\omega A_2^3 A_1^2+60\omega A_0 A_2^3 A_1^2+\frac{13\beta^2 A_2^2 A_1^2}{2\alpha^2}\\&-\frac{13A_2^2 l_1 A_1^2}{2\alpha^2}-\frac{108A_2^2 l_2 A_1^2}{\alpha^4}+\frac{12A_2^3 l_1 A_1}{\alpha^2}\\&+\frac{96A_2^3 l_2 A_1}{\alpha^4}-\frac{12\beta^2 A_2^3 A_1}{\alpha^2}+\frac{2\beta^2 A_2^4}{\alpha^2}\\&+15\omega A_0^2 A_2^4+RA_2^4-10\omega A_0 A_2^4+\frac{4\beta^2 A_0 A_2^3}{\alpha^2}\\&-\frac{2A_2^4 l_1}{\alpha^2}-\frac{4A_0 A_2^3 l_1}{\alpha^2}-\frac{12A_2^4 l_2}{\alpha^4}=0,\end{aligned}\tag{15}$$

$$\begin{aligned}&-10\omega A_1 A_2^4+30\omega A_0 A_1 A_2^4+\frac{4l_1 A_2^4}{\alpha^2}+\frac{48l_2 A_2^4}{\alpha^4}\\&-\frac{4\beta^2 A_2^4}{\alpha^2}+20\omega A_1^3 A_2^3+\frac{6\beta^2 A_1 A_2^3}{\alpha^2}-\frac{6A_1 l_1 A_2^3}{\alpha^2}\\&-\frac{144A_1 l_2 A_2^3}{\alpha^4}+\frac{72A_1^2 l_2 A_2^2}{\alpha^4}-\frac{6A_1^3 l_2 A_2}{\alpha^4}=0,\end{aligned}$$

$$\begin{aligned}&2\omega A_2^5+6\omega A_0 A_2^5+\frac{2\beta^2 A_2^4}{\alpha^2}+15\omega A_1^2 A_2^4\\&-\frac{2l_1 A_2^4}{\alpha^2}-\frac{72l_2 A_2^4}{\alpha^4}+\frac{96A_1 l_2 A_2^3}{\alpha^4}-\frac{18A_1^2 l_2 A_2^2}{\alpha^4}=0,\end{aligned}$$

$$6\omega A_1 A_2^5+\frac{48l_2 A_2^4}{\alpha^4}-\frac{24A_1 l_2 A_2^3}{\alpha^4}=0,$$

$$\omega A_2^6-\frac{12A_2^4 l_2}{\alpha^4}=0.$$

Thus, solving the above system gives

Case 1:

$$A_0=0,\ A_1=-\frac{2\sqrt{3}\sqrt{l_2}}{\alpha^2\sqrt{\omega}},\ A_2=\frac{2\sqrt{3}\sqrt{l_2}}{\alpha^2\sqrt{\omega}},$$

$$\beta=\mp\frac{\sqrt{2\sqrt{3}\alpha^2\sqrt{l_2}\sqrt{\omega}+\alpha^2 l_1+3l_2}}{\alpha},$$

$$R=\frac{-4\sqrt{3}\alpha^2\sqrt{l_2}\sqrt{\omega}-3l_2}{4\alpha^4}.$$

By using (5), (8), (11) and (14) yields the solution in the form of a bright soliton solution for (4)

$$\begin{aligned}f_{1,2}(x,t)=-\frac{1}{\alpha}ln\Bigg(&-\frac{2\sqrt{3}\sqrt{l_2}}{\alpha^2\sqrt{\omega}(1+de^{(x-\beta t)})}\\&+\frac{2\sqrt{3}\sqrt{l_2}}{\alpha^2\sqrt{\omega}(1+de^{(x-\beta t)})^2}\Bigg).\end{aligned}\tag{16}$$

Case 2:

$$A_0=0,\ A_1=\frac{2\sqrt{3}\sqrt{l_2}}{\alpha^2\sqrt{\omega}},\ A_2=-\frac{2\sqrt{3}\sqrt{l_2}}{\alpha^2\sqrt{\omega}},$$

$$\beta=\mp\frac{\sqrt{-2\sqrt{3}\alpha^2\sqrt{l_2}\sqrt{\Omega}+\alpha^2 l_1+3l_2}}{\alpha},$$

$$R=\frac{4\sqrt{3}\alpha^2\sqrt{l_2}\sqrt{\omega}-3l_2}{4\alpha^4}.$$

By using (5), (8), (11) and (14) yields the following bright soliton solution for (4)

$$\begin{aligned}f_{3,4}(x,t)=-\frac{1}{\alpha}ln\Bigg(&\frac{2\sqrt{3}\sqrt{l_2}}{\alpha^2\sqrt{\omega}(1+de^{(x-\beta t)})}\\&-\frac{2\sqrt{3}\sqrt{l_2}}{\alpha^2\sqrt{\omega}(1+de^{(x-\beta t)})^2}\Bigg).\end{aligned}\tag{17}$$

3.2 The modified Kudryashov method

We illustrate the modified Kudryashov method in this section by suppose a solution of (9) given in a series take shape

$$\phi(\xi)=\sum_{n=0}^{N}A_n Q^n(\xi),\tag{18}$$

where, $A_0, A_1, A_2, ..., A_N$ are constants aht need to be calculated and N is a whole number to be obtained, while $Q(\xi)$ is

$$Q(\xi)=\frac{1}{1+bm^{\xi}},\tag{19}$$

which accept the below differential equation:

$$Q'(\xi)=(Q^2(\xi)-Q(\xi))\ln(m).\tag{20}$$

Putting (18) and its possible derivatives like:

$$\begin{aligned}\phi'(\xi)&=\sum_{n=0}^{N}A_n nQ^n(Q-1)\ln(m),\\\phi''(\xi)&=\sum_{n=0}^{N}A_n nQ^n(Q-1)((1+n)(Q-n)\ln(m))^2,\end{aligned}\tag{21}$$

in (9) yields a polynomial in $Q(\xi)$;

$$P(Q(\xi))=0.\tag{22}$$

Thus equating the coefficient of each power of $Q(\xi)$ in the above equation to zero gives a set of nonlinear algebraic equations which will be used to yield the exact solutions for (9). This offers a truncated series form (18) of the form

$$\phi(\xi)=A_0+A_1 Q(\xi)+A_2 Q^2(\xi).\tag{23}$$

Then, substituting (23) into (9), we get the following system of algebraic equations:

$$A_0^4 R+A_0^6\omega-2A_0^5\omega=0,$$

$$4A_1 A_0^3 R+6A_1 A_0^5\omega-10A_1 A_0^4\omega=0,$$

$$-\frac{A_1^2 A_0^2 l_1\log^2(m)}{2\alpha^2}+\frac{A_1^2 A_0^2\beta^2\log^2(m)}{2\alpha^2}+4A_2 A_0^3 R+6A_1^2 A_0^2 R$$

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\begin{aligned}
&+6A_2A_0^5\omega+15A_1^2A_0^4\omega-10A_2A_0^4\omega-20A_1^2A_0^3\omega=0,\\
&\frac{A_1^2A_0^2l_1\log^2(m)}{\alpha^2}-\frac{2A_1A_2A_0^2l_1\log^2(m)}{\alpha^2}\\
&-\frac{A_1^3A_0l_1\log^2(m)}{\alpha^2}+\frac{2A_1A_2A_0^2\beta^2\log^2(m)}{\alpha^2}\\
&-\frac{A_1^2A_0^2\beta^2\log^2(m)}{\alpha^2}+\frac{A_1^3A_0\beta^2\log^2(m)}{\alpha^2}+12A_1A_2A_0^2R\\
&+4A_1^3A_0R+30A_1A_2A_0^4\omega\\
&+20A_1^3A_0^3\omega-40A_1A_2A_0^3\omega-20A_1^3A_0^2\omega=0,\\
&\frac{4A_1A_2A_0^2l_1\log^2(m)}{\alpha^2}-\frac{2A_2^2A_0^2l_1\log^2(m)}{\alpha^2}\\
&-\frac{A_1^2A_0^2l_1\log^2(m)}{2\alpha^2}+\frac{2A_1^3A_0l_1\log^2(m)}{\alpha^2}\\
&-\frac{5A_1^2A_2A_0l_1\log^2(m)}{\alpha^2}-\frac{A_1^4l_1\log^2(m)}{2\alpha^2}\\
&-\frac{3A_1^4l_2\log^4(m)}{4\alpha^4}+\frac{A_1^2A_0^2\beta^2\log^2(m)}{2\alpha^2}\\
&+\frac{2A_2^2A_0^2\beta^2\log^2(m)}{\alpha^2}-\frac{4A_1A_2A_0^2\beta^2\log^2(m)}{\alpha^2}\\
&+\frac{5A_1^2A_2A_0\beta^2\log^2(m)}{\alpha^2}-\frac{2A_1^3A_0\beta^2\log^2(m)}{\alpha^2}\\
&+\frac{A_1^4\beta^2\log^2(m)}{2\alpha^2}+6A_2^2A_0^2R+12A_1^2A_2A_0R+A_1^4R\\
&+15A_2^2A_0^4\omega-20A_2^2A_0^3\omega+60A_1^2A_2A_0^3\omega\\
&+15A_1^4A_0^2\omega-60A_1^2A_2A_0^2\omega-10A_1^4A_0\omega=0,\\
&\frac{A_1^4l_1\log^2(m)}{\alpha^2}+\frac{3A_1^4l_2\log^4(m)}{\alpha^4}-\frac{A_0A_1^3l_1\log^2(m)}{\alpha^2}\\
&-\frac{3A_2A_1^3l_1\log^2(m)}{\alpha^2}-\frac{6A_2A_1^3l_2\log^4(m)}{\alpha^4}\\
&+\frac{10A_0A_2A_1^2l_1\log^2(m)}{\alpha^2}-\frac{8A_0A_2^2A_1l_1\log^2(m)}{\alpha^2}\\
&-\frac{2A_0^2A_2A_1l_1\log^2(m)}{\alpha^2}+\frac{4A_0^2A_2^2l_1\log^2(m)}{\alpha^2}\\
&-\frac{A_1^4\beta^2\log^2(m)}{\alpha^2}+\frac{A_0A_1^3\beta^2\log^2(m)}{\alpha^2}\\
&+\frac{3A_2A_1^3\beta^2\log^2(m)}{\alpha^2}-\frac{10A_0A_2A_1^2\beta^2\log^2(m)}{\alpha^2}\\
&+\frac{8A_0A_2^2A_1\beta^2\log^2(m)}{\alpha^2}+\frac{2A_0^2A_2A_1\beta^2\log^2(m)}{\alpha^2}\\
&-\frac{4A_0^2A_2^2\beta^2\log^2(m)}{\alpha^2}+4A_2A_1^3R\\
&+12A_0A_2^2A_1R-2A_1^5\omega+6A_0A_1^5\omega\\
&+60A_0^2A_2A_1^3\omega-40A_0A_2A_1^3\omega
\end{aligned}
$$

$$
\begin{aligned}
&+60A_0^3A_2^2A_1\omega-60A_0^2A_2^2A_1\omega=0,\\
&-\frac{A_1^4l_1\log^2(m)}{2\alpha^2}-\frac{9A_1^4l_2\log^4(m)}{2\alpha^4}\\
&+\frac{6A_2A_1^3l_1\log^2(m)}{\alpha^2}+\frac{24A_2A_1^3l_2\log^4(m)}{\alpha^4}\\
&-\frac{5A_0A_2A_1^2l_1\log^2(m)}{\alpha^2}-\frac{13A_2^2A_1^2l_1\log^2(m)}{2\alpha^2}\\
&-\frac{18A_2^2A_1^2l_2\log^4(m)}{\alpha^4}+\frac{16A_0A_2^2A_1l_1\log^2(m)}{\alpha^2}\\
&-\frac{4A_0A_2^3l_1\log^2(m)}{\alpha^2}-\frac{2A_0^2A_2^2l_1\log^2(m)}{\alpha^2}\\
&+\frac{A_1^4\beta^2\log^2(m)}{2\alpha^2}-\frac{6A_2A_1^3\beta^2\log^2(m)}{\alpha^2}\\
&+\frac{13A_2^2A_1^2\beta^2\log^2(m)}{2\alpha^2}+\frac{5A_0A_2A_1^2\beta^2\log^2(m)}{\alpha^2}\\
&-\frac{16A_0A_2^2A_1\beta^2\log^2(m)}{\alpha^2}+\frac{4A_0A_2^3\beta^2\log^2(m)}{\alpha^2}\\
&+\frac{2A_0^2A_2^2\beta^2\log^2(m)}{\alpha^2}+6A_2^2A_1^2R+4A_0A_2^3R\\
&+A_1^6\omega-10A_2A_1^4\omega+30A_0A_2A_1^4\omega+90A_0^2A_2^2A_1^2\omega\\
&-60A_0A_2^2A_1^2\omega+20A_0^3A_2^3\omega-20A_0^2A_2^3\omega=0,\\
&\frac{3A_1^4l_2\log^4(m)}{\alpha^4}-\frac{3A_2A_1^3l_1\log^2(m)}{\alpha^2}\\
&-\frac{36A_2A_1^3l_2\log^4(m)}{\alpha^4}+\frac{13A_2^2A_1^2l_1\log^2(m)}{\alpha^2}\\
&+\frac{72A_2^2A_1^2l_2\log^4(m)}{\alpha^4}-\frac{6A_2^3A_1l_1\log^2(m)}{\alpha^2}\\
&-\frac{8A_0A_2^2A_1l_1\log^2(m)}{\alpha^2}-\frac{24A_2^3A_1l_2\log^4(m)}{\alpha^4}\\
&+\frac{8A_0A_2^3l_1\log^2(m)}{\alpha^2}+\frac{3A_2A_1^3\beta^2\log^2(m)}{\alpha^2}\\
&-\frac{13A_2^2A_1^2\beta^2\log^2(m)}{\alpha^2}+\frac{6A_2^3A_1\beta^2\log^2(m)}{\alpha^2}\\
&+\frac{8A_0A_2^2A_1\beta^2\log^2(m)}{\alpha^2}-\frac{8A_0A_2^3\beta^2\log^2(m)}{\alpha^2}\\
&+4A_2^3A_1R+6A_2A_1^5\omega-20A_2^2A_1^3\omega+60A_0A_2^2A_1^3\omega\\
&+60A_0^2A_2^3A_1\omega-40A_0A_2^3A_1\omega=0,\\
&\frac{96A_1A_2^3l_2\log^4(m)}{\alpha^4}+\frac{24A_1^3A_2l_2\log^4(m)}{\alpha^4}\\
&-\frac{12A_2^4l_2\log^4(m)}{\alpha^4}-\frac{108A_1^2A_2^2l_2\log^4(m)}{\alpha^4}\\
&-\frac{3A_1^4l_2\log^4(m)}{4\alpha^4}+\frac{12A_1A_2^3l_1\log^2(m)}{\alpha^2}
\end{aligned}
$$

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$$-\frac{2A_2^4 l_1 \log^2(m)}{\alpha^2}-\frac{4A_0A_2^3 l_1 \log^2(m)}{\alpha^2}$$
$$-\frac{13A_1^2A_2^2 l_1 \log^2(m)}{2\alpha^2}+\frac{2A_2^4\beta^2 \log^2(m)}{\alpha^2}$$
$$+\frac{4A_0A_2^3\beta^2 \log^2(m)}{\alpha^2}+\frac{13A_1^2A_2^2\beta^2 \log^2(m)}{2\alpha^2}$$
$$-\frac{12A_1A_2^3\beta^2 \log^2(m)}{\alpha^2}+A_2^4R+15A_0^2A_2^4\omega-10A_0A_2^4\omega$$
$$-20A_1^2A_2^3\omega+60A_0A_1^2A_2^3\omega+15A_1^4A_2^2\omega=0,$$
$$\frac{48A_2^4 l_2 \log^4(m)}{\alpha^4}+\frac{72A_1^2A_2^2 l_2 \log^4(m)}{\alpha^4}$$
$$-\frac{144A_1A_2^3 l_2 \log^4(m)}{\alpha^4}-\frac{6A_1^3A_2 l_2 \log^4(m)}{\alpha^4}$$
$$+\frac{4A_2^4 l_1 \log^2(m)}{\alpha^2}-\frac{6A_1A_2^3 l_1 \log^2(m)}{\alpha^2}$$
$$+\frac{6A_1A_2^3\beta^2 \log^2(m)}{\alpha^2}-\frac{4A_2^4\beta^2 \log^2(m)}{\alpha^2}$$
$$-10A_1A_2^4\omega+30A_0A_1A_2^4\omega+20A_1^3A_2^3\omega=0,$$
$$-\frac{2A_2^4 l_1 \log^2(m)}{\alpha^2}-\frac{72A_2^4 l_2 \log^4(m)}{\alpha^4}$$
$$+\frac{96A_1A_2^3 l_2 \log^4(m)}{\alpha^4}-\frac{18A_1^2A_2^2 l_2 \log^4(m)}{\alpha^4}$$
$$+\frac{2A_2^4\beta^2 \log^2(m)}{\alpha^2}-2A_2^5\omega$$
$$+6A_0A_2^5\omega+15A_1^2A_2^4\omega=0,$$
$$\frac{48A_2^4 l_2 \log^4(m)}{\alpha^4}-\frac{24A_1A_2^3 l_2 \log^4(m)}{\alpha^4}$$
$$+6A_1A_2^5\omega=0,$$
$$A_2^6\omega-\frac{12A_2^4 l_2 \log^4(m)}{\alpha^4}=0.$$

Thus, by solving the given system will result in
Case 1:

$$A_0=0,\ A_1=-\frac{2\sqrt{3}\sqrt{l_2}\log^2(m)}{\alpha^2\sqrt{\omega}},$$
$$A_2=\frac{2\sqrt{3}\sqrt{l_2}\log^2(m)}{\alpha^2\sqrt{\omega}}$$
$$\beta=\mp\frac{\sqrt{2\sqrt{3}\alpha^2\sqrt{l_2}\sqrt{\omega}+\alpha^2 l_1+3l_2\log^2(m)}}{\alpha},$$
$$R=\frac{-4\sqrt{3}\alpha^2\sqrt{l_2}\sqrt{\omega}\log^2(m)-3l_2\log^4(m)}{4\alpha^4}.$$

By using (5), (8), (19) and (23) yields the following bright soliton solution for (4)

$$f_{1,2}(x,t)=-\frac{1}{\alpha}ln\left(-\frac{2\sqrt{3}\sqrt{l_2}\log^2(m)}{\alpha^2\sqrt{\omega}(1+bm^{(x-\beta t)})}+\frac{2\sqrt{3}\sqrt{l_2}\log^2(m)}{\alpha^2\sqrt{\omega}(1+bm^{(x-\beta t)})^2}\right). \tag{24}$$

Case 2:

$$A_0=0,\ A_1=\frac{2\sqrt{3}\sqrt{l_2}\log^2(m)}{\alpha^2\sqrt{\omega}},$$
$$A_2=-\frac{2\sqrt{3}\sqrt{l_2}\log^2(m)}{\alpha^2\sqrt{\omega}}$$
$$\beta=\mp\frac{\sqrt{2\sqrt{3}\alpha^2\sqrt{l_2}\sqrt{\omega}+\alpha^2 l_1+3l_2\log^2(m)}}{\alpha},$$
$$R=\frac{4\sqrt{3}\alpha^2\sqrt{l_2}\sqrt{\omega}\log^2(m)-3l_2\log^4(m)}{4\alpha^4}.$$

By using (5), (8), (19) and (23) yields the following bright soliton solution for (4)

$$f_{3,4}(x,t)=-\frac{1}{\alpha}ln\left(\frac{2\sqrt{3}\sqrt{l_2}\log^2(m)}{\alpha^2\sqrt{\omega}(1+bm^{(x-\beta t)})}-\frac{2\sqrt{3}\sqrt{l_2}\log^2(m)}{\alpha^2\sqrt{\omega}(1+bm^{(x-\beta t)})^2}\right). \tag{25}$$

Next, we shall verify the obtained solution in the previous sections using a numerical method based on cubic B-spline technique.

4 Cubic B-spline collocation method

In this section, we shall verify the results obtained in the last section using the cubic B-spline method. First, we approximate the variables of the space an time which are x and t with their derivatives as in [?]. Next, assuming that the value of the function $f(x,t)$ which is the exact solution of the model at the points on the grid (x_i,t_j) and $f_{i,j}$ to be the same as the approximate solution at these points. The required values of f_i and its first and the second derivatives, f_i' and f_i'', at nodal points x_i are identified in terms of c_i as

$$\begin{aligned} f_{i,j}&=c_{i,j_1}+4c_{i,j}+c_{i,j-1},\\ f_x=f'_{i,j}&=\frac{3}{h}(c_{i+1,j}-c_{i-1,j}),\\ f_{xx}=f''_{i,j}&=\frac{6}{h^2}(c_{i,j-1}+c_{i,j+1}-2c_{i,j}),\end{aligned} \tag{26}$$

and if the time derivative is discretized using finite differences, we have where

$$f_{tt}=\frac{c_{i,j-1}+c_{i,j+1}-2c_{i,j}}{k^2}. \tag{27}$$

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Substituting (26) into (4) a we get

$$\frac{c_{i,j-1}+c_{i,j+1}-2c_{i,j}}{k^2}-(l_1+3l_2(\frac{3}{h}(c_{i+1,j}-c_{i-1,j}))^2) \quad (28)$$

Now we can be solved the (28) by many methods.

4.1 Numerical Simulation

Now, we shall introduce the results obtained using the previous illustrated technique for solving the model (1). A comparison is made in Table. 1 in the form of absolute error between the obtained numerical results and the results in (16) at $\alpha=1,\Omega=0.1,l_2=0.01,l_1=0.1$, $t=1,k=0.01,h=0.1,d=1$. Figure 5 illustrate the behavior of the solutions obtained by the numerical solutions for (4) at $\alpha = 1,\Omega = 0.1,l_2 = 0.01$, $l_1=0.1,d=1$.

Table 1: A comparison between the exact and numerical solutions along with the absolute error.

x	Numerical solution	Exact solution	Absolute error
-5	5.44008	5.44008	8.94098 E-7
-2	3.62989	3.62989	3.12923 E-6
-1	3.42318	3.42318	4.82165 E-6
0	3.42113	3.42112	9.77842 E-6
1	3.62319	3.62319	1.48434 E-6
5	6.25290	6.25290	6.63292 E-7

In Table 2 we introduce a comparison of the results of the numerical technique and solution in (17) at $\alpha=1,\Omega=0.1,l_2=0.01,l_1=0.1,t=1,k=0.01,h=0.1,d=1$. Also, in Fig. 6 the numerical and analytical behaviour of Eq. (4) for $\alpha=1,\Omega=0.1,l_2=0.01,l_1=0.1,d=1$. is introduced.

Table 2: A comparison between the exact and numerical solutions along with the absolute error.

x	Numerical solution	Exact solution	Absolute error
-5	5.06351	5.06351	7.31236 E-7
-2	2.27369	2.27369	3.79312 E-6
-1	1.60539	1.60539	2.62002 E-6
0	1.30025	1.30024	1.01515 E-5
1	1.47338	1.47338	3.62099 E-6
5	4.78129	4.78129	7.97996 E-7

In addition, in Table 3 a comparison between the numerical and analytical results for Eq. (24) at $\alpha=1,\Omega=0.1,l_2=0.01,l_1=0.1,t=1,k=0.01,h=0.1,b=1,m=0.1$ is presented. In Figure 7 we introduce the absolute value of analytical and the absolute value of numerical solutions for (4) at $\alpha=1,\Omega=0.1,l_2=0.01,l_1=0.1,b=1,m=0.1$. It can be seen form these tables and figures that our two proposed techniques are in good agreement with each other and produce accurate results.

Table 3: Comparison between numerical results and analytical solution

x	Numerical solution	Exact solution	Absolute error
-5	8.92689	8.92689	1.48864 E-7
-2	3.49315	3.49318	8.73023 E-5
-1	3.14647	3.14646	1.04599 E-4
0	3.14263	3.14262	1.93872 E-4
1	3.71882	3.71889	1.67069 E-4
5	11.5857	11.5857	6.41029 E-8

In Table 4 we introduce comparison between the numerical results with the analytical solution (25) at $\alpha=1,\Omega=0.1,l_2=0.01,l_1=0.1,t=1,k=0.01$, $h=0.1,b=1,m=0.1$. In Figure 2 we introduce analytical and numerical solutions for (4) at $\alpha=1,\Omega=0.1,l_2=0.01,l_1=0.1,b=1,m=0.1$.

Table 4: Comparison between the numerical results with the analytical solution

x	Numerical solution	Exact solution	Absolute error
-5	8.86341	8.86341	1.23149 E-7
-2	2.00367	2.00374	7.52585 E-5
-1	0.08889	0.08903	1.44369 E-4
0	0.18073	0.18097	2.37149 E-4
1	1.51398	1.51416	1.75912 E-4
5	10.6439	10.6439	7.18443 E-8

5 Graphical results and discussion

Now that we have completed the analytical and numerical calculations of the model under study using the analytical and numerical methods described above. Next, in this section we shall provide a graphical representation of the obtained solutions through the two proposed techniques. It is clear from the presented Figures 1-8 that the provided solutions are accurate for various values of the parameters.

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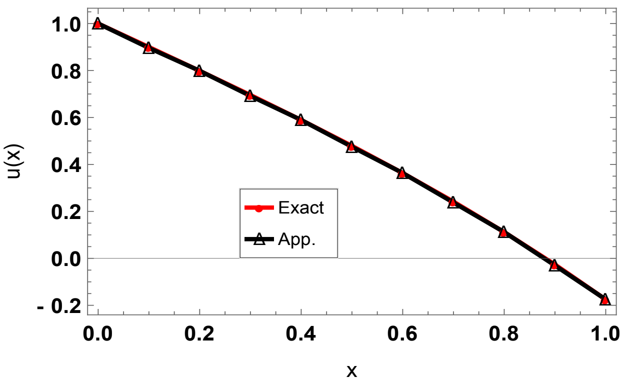

Fig. 1: Analytical solution (16) at $\alpha = 1, \Omega = 0.1, l_2 = 0.01, l_1 = 0.1, d = 1$.

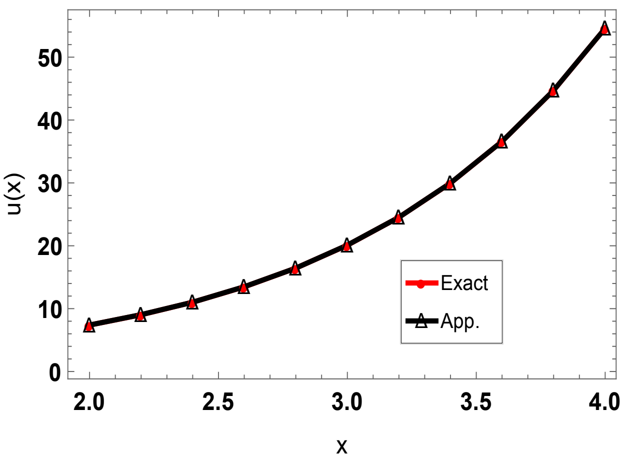

Fig. 2: Graphical representation of (17) at $\alpha = 1, \Omega = 0.1, l_2 = 0.01, l_1 = 0.1, d = 1$

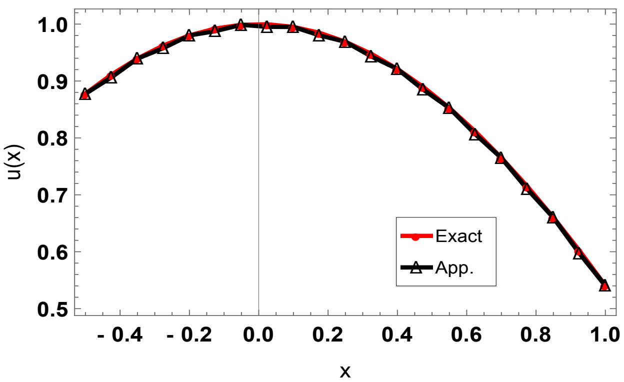

Fig. 3: Graphical representation of (24) at $\alpha = 1, \Omega = 0.1, l_2 = 0.01, l_1 = 0.1, m = 0.1, b = 1$.

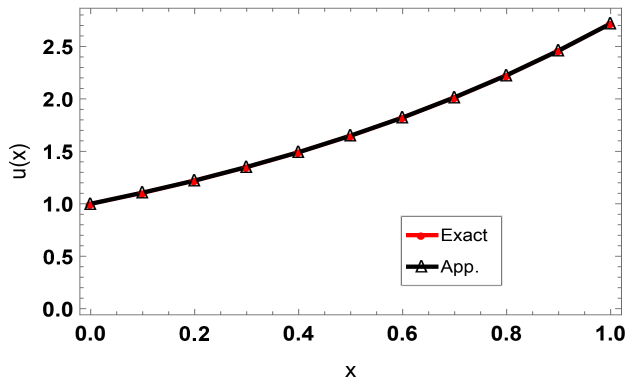

Fig. 4: Graphical representation of (25) at $\alpha = 1, \Omega = 0.1, l_2 = 0.01, l_1 = 0.1, m = 0.1, b = 1$

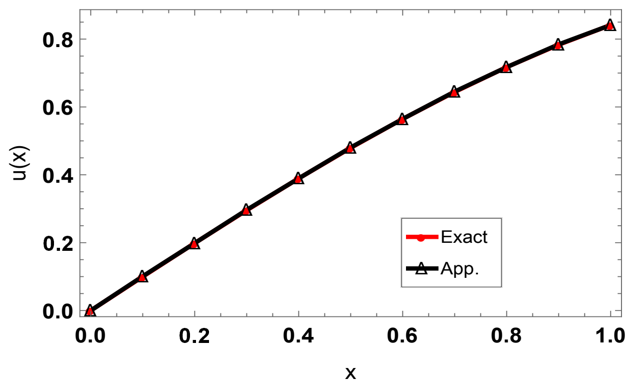

Fig. 5: Analytical and numerical solutions for (4) at $\alpha = 1, \Omega = 0.1, l_2 = 0.01, l_1 = 0.1, d = 1$.

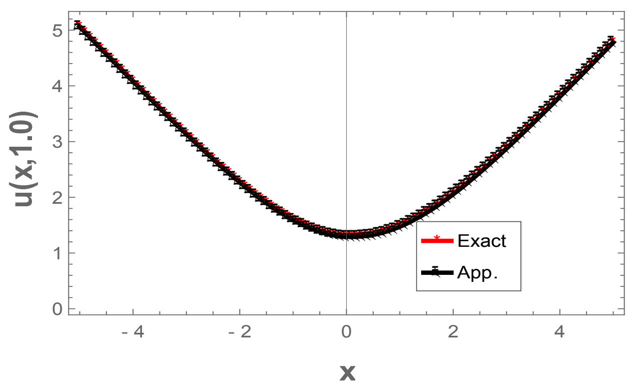

Fig. 6: Analytical and numerical solutions for (4) at $\alpha = 1, \Omega = 0.1, l_2 = 0.01, l_1 = 0.1, d = 1$

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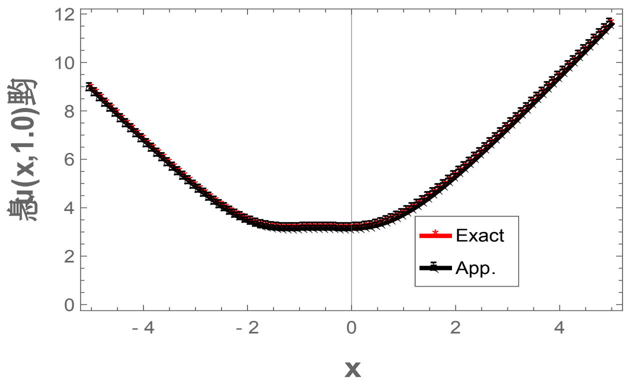

Fig. 7: Analytical and numerical solutions for (4) at $\alpha = 1, \Omega = 0.1, l_2 = 0.01, l_1 = 0.1, m = 0.1, b = 1$.

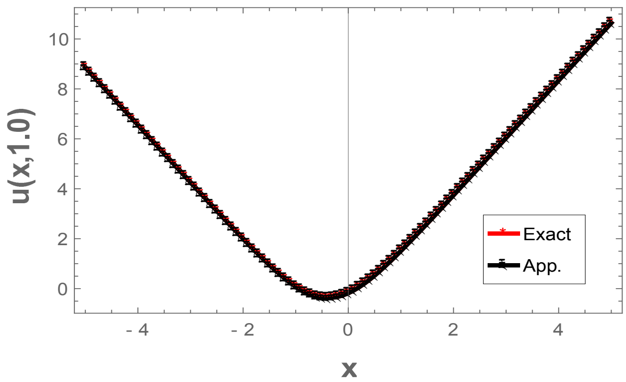

Fig. 8: Analytical and numerical solutions for (4) at $\alpha = 1, \Omega = 0.1, l_2 = 0.01, l_1 = 0.1, m = 0.1, b = 1$.

6 Conclusion

In this paper, we have investigated the solution of the Peyrard-Bishop DNA dynamic model equation using two proposed techniques named the kudryashov, modified kudryashov methods, and B-spline collocation technique. The kudryashov method and its modified form are then used to find an analytical solution to the problem. The B-spline technique is also been used to solve the same problem numerically to verify the results obtained analytically by the other two methods. Various solutions to the equation have been realized in this study and a comparison is made between the obtained solutions. A graphical behavior of the obtained solutions is being introduced through tables and figures. These methods proved to be reliable, accurate, efficient, and versatile in mathematical physics for solving similar problems. It is interesting in the future to investigate the application of these techniques for solving other NLEEs with more complex structures.

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Conflict of Interest The authors declare that they have no conflict of interest.

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Khalid K. Ali MSc in pure Mathematics, Mathematics Department 2015 from Faculty of Science, Al-Azhar University, Cairo, Egypt. Ph.D. in pure Mathematics, Mathematics Department entitled 2018 from Faculty of Science, Al-Azhar University, Cairo, Egypt. He has authored/co-authored over 50 scientific papers in top-ranked International Journals and Conference Proceedings. His research interests include Numerical Analysis, Finite Difference Methods, Finite Element Methods, and Computational Mathematics.

Kamal R. Raslan received the M.Sc. and Ph.D. degrees from the Faculty of Science, Menoufia University and Al-Azhar University, Egypt, in 1996 and 1999, respectively. He is currently a full Professor of Mathematics with the Faculty of Science, Al-Azhar University, Egypt. He has authored/coauthored over 114 scientific papers in top-ranked International Journals and Conference Proceedings. His research interests include Numerical Analysis, Finite Difference Methods, Finite Element Methods, Approximation Theory, and Computational Mathematics.

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M. Ismail Abdelrahman M. Ismail Abdelrahman received the M.Sc. and Ph.D. degrees from the faculty of Science, Al-Azhar University, Egypt, in 1998 and 2001, respectively. He is currently a Lecturer of Pure Mathematics, Mathematics Department with the Faculty of Science, Al-Azhar University, Egypt. His research interests include Functional Delay Differential Equations, Biomathematical Modelling, Stability, Oscillations, Time Lags, Delay Differential Equations.

Waleed Adel received the B.Sc. degree civil engineering and theM.Sc. and Ph.D. degrees in engineering mathematics from Mansoura University, Egypt, in 2007, 2014, and 2019, respectively. He works now as an Assistant Professor with the department of mathematics and engineering physics at the Faculty of Engineering, Mansoura University since 2019. He published more than 60 papers in prestigious journals in the field of applied mathematics and a reviewer for several international reputed journals. His current research interests include but not limited to numerical analysis for differential equations, developing novel solutions using spectral methods, computational mathematics, stability analysis for some biological models.

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