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New Wave Behaviours of the Generalized Kadomtsev-Petviashvili Modified Equal Width-Burgers Equation

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Abstract: In this article, we applied two different methods namely as the $(1/G^{'})$-expansion method and the Bernoulli sub-equation method to investigate the generalized Kadomtsev-Petviashvili modified equal width-Burgers equation, which is designated the propagation of long-wave with dissipation and dispersion in nonlinear media. To transform the given equation into a nonlinear ordinary differential equation, a traveling wave transformation has been carried out. As a result, we constructed distinct exact solutions like complex solutions, singular solutions, and complex singular solutions. Besides, 2D, 3D, and contour surfaces are illustrated to demonstrate the physical properties of the obtained solutions.

Keywords: The generalized KP-MEW-B equation, the $(1/G^{'})$-expansion method, the Bernoulli sub-equation method, analytical methods, exact solutions

1 Introduction

Nonlinear evolution equations (NLEEs) have been helpful to implement for representing the natural phenomena of sciences and engineering. NLEEs covers several areas of science, like physics, mathematics, and engineering [1,2,3]. Recently, research on the propagation of nonlinear equations among researchers has received a great deal of attention, such as Alberto *et al.* have been investigated the relation between the Langmuir wave field and the transverse electromagnetic field [1]. The behavior of ion-solitary waves in plasma was investigated by Pakzad [2]. Salahuddin *et al.* have been analyzed the ion-acoustic cover solitons in a collisionless unmagnetized electron-positron-ion plasma [3]. Various numerical and computational approaches have been formulated to handle the solutions of these types of nonlinear models, such as the exp-function scheme [4], the Homotopy perturbation scheme [5], the Adomian decomposition scheme [6], the sin-Gordon scheme [7], the $(G/G^{'})$-expansion scheme [8], the Shooting scheme [9], and Hirota's simple method [10,11,12,13,14,15]. Many of these methods are problem-dependent. Some of the methods work well for some problems but are not suitable for other different problems [16,17,18,19,20,21,22,23,24,25,26].

The Kadomtsev-Petviashvili (KP) equation was introduced by two Soviet physicists, Kadomtsev and Petviashvili in 1970 [27], this model is represented by the Korteweg-de Vries (KdV) equation. The KP equation was instantly adopted as a natural amplification of the classical KdV equation to two locative dimensions and was subsequently reproduced as a model for surface and inner water waves [28], and in nonlinear optics in [29]. The exploration of the KP equation occurred nearly meanwhile with the progress of the inverse scattering transform (IST) [30,31].

The generalized KP-MEW equation [32] is given by

$$(q_t + \alpha (q^n) x + \gamma q_{xxt})_x + \delta q_{yy} = 0, \tag{1}$$

where α, γ, and δ are constants. Recently, many authors investigated the different types of solutions of generalized (KP-MEW) equations with the help of different techniques. Wazwaz [32] explored the exact solutions with distinct physical structures of generalized (KP-MEW) equation with the aid of two approaches, the tanh method, and the sine-cosine method. Saha [33] discovered the traveling wave solutions of generalized (KPMEW) equation by using the method of bifurcation theory. Wei et al. [34] developed the solitary wave

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solutions of generalized (KP-MEW) (2,2) equation by applying the differential equations qualitative theory. Li and Song [35] constructed the kink-type wave and comaction-type wave solutions of generalized KP-MEW (2,2) equation. The Lie symmetry analysis was carried out by Adem et al. [36], and the symmetries and adjoint representations are supplied. Cai et al. [37] investigated the KP-MEW (3,2) problem and created parametric representations of periodic peakon and smooth periodic wave solutions using the approach of bifurcation theory of dynamical systems. Wei et al. [38] solved the KP-MEW (2, 2) problem using the qualitative theory of differential equations and found single peak solitary wave solutions. Zhong et al. [39] found compacton, peakon, cuspons, loop solutions and smooth solitons for the generalized KP-MEW equation. The generalized KP-MEW-Burgers equation, which was introduced for the first time [40] as

$$(q_t+\alpha\,(q^n)\,x+\beta q_{xx}-\gamma q_{xxt})_x+\delta q_{yy}=0, \tag{2}$$

where β is referred to damping parameter, the first term is represented as an evolution term, the second term expresses as a nonlinear term, the thirds one is typified as a dissipative term and the fourth term stands for dispersion term. Saha [40] obtained the bifurcation behavior of the KP-MEW-Burgers equation, also he has presented the stable oscillations of the KP-MEW-Burgers equation. Seadawy et al. [41] used the modified extended auxiliary equation mapping method, the single bright–dark solitons, the double bright–dark solitons, and traveling wave solutions have obtained. In this study, we construct distinct exact solutions like complex solutions, and complex singular solutions in terms of periodic, as well as singular solutions. These new solutions are obtained by using two different methods, which are $(1/G^{'})$-expansion method and the Bernoulli sub-equation method.

The outlines of the paper are designed as follows: In Section 2, fundamental concepts of the $(1/G^{'})$-expansion method are presented, and the Bernoulli sub-equation method are presented. In Section 3, the mathematical calculation of the given equation is addressed as well as the implementation of both methods to the governing equation is presented, and eventually, results are concluded in Section 4.

2 Methodology

In the current section, the main concepts of the $(1/G^{'})$-expansion method and the Bernoulli sub-equation method (BSEM) are addressed in this section.

2.1 The $(1/G^{'})$-expansion scheme

The main steps of the $(1/G^{'})$-expansion method are presented in this portion [42,43]:

Step 1. Consider the following nonlinear partial differential equation (NPDE):

$$P(q,q_t,q_x,q_{xt},q_{xx},\ldots)=0. \tag{3}$$

Step 2. Let the following wave transform

$$q(x,t)=H(\xi),\ \xi=a(x-bt), \tag{4}$$

where a, and b are non zero constants.

Inserting Eq. (4) into Eq. (3), we get the nonlinear ordinary differential equation (NODE)

$$N(H^{'},H^{''},H^{'''},\ldots)=0. \tag{5}$$

Step 3. Let the solution of Eq. (5) could be specified as a polynomial in $\left(1/G^{'}\right)$ as

$$H(\xi)=\sum_{i=0}^{k}A_i\left(\frac{1}{G'}\right)^{i}, \tag{6}$$

where $A_0,\ A_1,\ A_2,\ldots,A_k$ are constants to be announced later, and k is a balance terms, and $G=G(\xi)$ satisfy the following second order linear ordinary differential equation:

$$G^{''}+\lambda G^{'}+\mu=0, \tag{7}$$

where λ *and* μ are constants to be announced later.

Plugging Eq. (6) with Eq. (7) into Eq. (5), one may acquire a polynomial of $\left(\frac{1}{G'}\right)$. Setting the coefficients of $\left(\frac{1}{G'}\right)$ with likely order to zero, one cane obtained a system of equations. Solving the obtained system, we get the value of $A_i, i\geq 0$ and λ, μ scalars and then putting the founded values into Eq. (6), one can easily get the solutions of Eq. (3).

2.2 The BSEM

In the current subsection, the main concepts of the BSEM are presented [20].

Step 1. Let the NPDE

$$P(q_x,q_t,q_{xt},q_{xx},\ \ldots)=0. \tag{8}$$

Consider the following wave transformation

$$q(x,t)=H(\xi),\xi=\alpha x-\beta t, \tag{9}$$

where α, β are nonzero scalars.

Plugging Eq. (9) into Eq. (8), we obtain the NODE as follows:

$$N\left(H,H',H'',\ \ldots\right)=0. \tag{10}$$

Step 2. Take the solution of Eq. (10) of the form:

$$H=\sum_{i=0}^{k}A_iF^{i}=A_0+A_1F+A_2F^2+\ldots+A_kF^{k}, \tag{11}$$

and

$$F' = bF + dF^M, b \neq 0, d \neq 0,\ M \in \mathbb{R} - \{0,1,2\}, \quad (12)$$

where $F = F(\xi)$ is the Bernoulli differential polynomial, putting Eq. (11) along with Eq. (12) in Eq. (10), one can obtain an equation of polynomial $\Phi(F(\xi))$ of $F(\xi)$ as follows:

$$\Phi(F) = \rho_r F^r + \ldots + \rho_1 F + \rho_0 = 0. \quad (13)$$

By using the balance principle, the relationships among k and M will be found.

Step 3. Let the coefficients of $\Phi(F)$ all be zero, obtaining a system of equations as follows:

$$\rho_1 = 0,\ i = 0,\ldots,r, \quad (14)$$

solving the obtained system, the values of $A_0,\ldots,A_m$ will be found.

Step 4. By solving Eq. (12), two situations are obtained as

$$F(\xi) = \left[\frac{-d}{b} + \frac{E}{e^{b(M-1)\xi}}\right]^{\frac{1}{1-M}},\ b \neq d, \quad (15)$$

$$F(\xi) = \left[\frac{(E-1)+(E+1)\tan\left(\frac{b(1-M)\xi}{2}\right)}{1-\tan\left(\frac{b(1-M)\xi}{2}\right)}\right]^{\frac{1}{1-M}}, b = d,\ E \in \mathbf{R}, \quad (16)$$

where E is a constant of integration, with the assist of computational computer packets, the solutions of Eq. (10) can be achieved.

3 Mathematical calculation

In this portion, we try to find some distinct wave solutions of the generalized KP-MEW-B equation via applying two recent analytical schemes, which is given by [38]

$$(q_t + \alpha(q^n)x + \beta q_{xx} - \gamma q_{xxt})_x + \delta q_{yy} = 0, \quad (17)$$

Consider $n = 2$, then Eq. (17) takes the form

$$\left(q_t + \alpha\left(q^2\right)x + \beta q_{xx} - \gamma q_{xxt}\right)_x + \delta q_{yy} = 0. \quad (18)$$

Let the following wave transformation

$$q(x,y,t) = H(\xi),\ \xi = kx + ly - \omega t. \quad (19)$$

Plugging Eq. (19) into Eq. (18), we get

$$-\omega kH'' + 2\alpha k^2\left(HH'\right)_\xi + \beta k^3 H''' + \gamma\omega k^3 H'''' + l^2\delta H'' = 0. \quad (20)$$

Integrating Eq. (20) twice with respect to ξ, and letting the integration constants to be zero, then Eq. (20) reduces to the following form

$$\left(l^2\delta - \omega k\right)H + \alpha k^2 H^2 + \beta k^3 H' + \gamma\omega k^3 H'' = 0. \quad (21)$$

3.1 Application of the $(1/G')$-expansion method

In this portion, implementation of the $(1/G')$-expansion method to the generalized KP-MEW-B equation is presented. Balancing H'' and H^2 in Eq. (21), we get $k = 2$. With $k = 2$ Eq. (6) reduces to the form

$$H = A_0 + A_1\left(\frac{1}{G'}\right) + A_2\left(\frac{1}{G'}\right)^2, \quad (22)$$

$$H' = -\frac{2A_2(-\mu - \lambda G')}{(G')^3} - \frac{A_1(-\mu - \lambda G')}{(G')^2}, \quad (23)$$

and

$$H'' = \frac{2A2\lambda(-\mu - \lambda G')}{(G')^3} + \frac{A1\lambda(-\mu - \lambda G')}{(G')^2} + \frac{6A2(-\mu - \lambda G')^2}{(G')^4} + \frac{2A1(-\mu - \lambda G')^2}{(G')^3}. \quad (24)$$

Plugging Eq. (22), Eq. (23) and Eq. (24) into Eq. (21), we get a polynomial in powers of $(\frac{1}{G'})$ as follows:

$$\begin{aligned}
&a_0^2k^2\alpha + a_0l^2\delta - a_0k\omega + \frac{a_2^2k^2\alpha}{(G')^4} + \frac{6a_2k^3\gamma\mu^2\omega}{(G')^4} \\
&+ \frac{2a_1a_2k^2\alpha}{(G')^3} + \frac{2a_2k^3\beta\mu}{(G')^3} + \frac{10a_2k^3\gamma\lambda\mu\omega}{(G')^3} + \\
&\frac{2a_1k^3\gamma\mu^2\omega}{(G')^3} + \frac{a_1^2k^2\alpha}{(G')^2} + \frac{2a_0a_2k^2\alpha}{(G')^2} + \frac{a_2l^2\delta}{(G')^2} + \\
&\frac{2a_2k^3\beta\lambda}{(G')^2} + \frac{a_1k^3\beta\mu}{(G')^2} - \frac{a_2k\omega}{(G')^2} + \frac{4a_2k^3\gamma\lambda^2\omega}{(G')^2} + \\
&\frac{3a_1k^3\gamma\lambda\mu\omega}{(G')^2} + \frac{2a_0a_1k^2\alpha}{G'} + \frac{a_1l^2\delta}{G'} + \frac{a_1k^3\beta\lambda}{G'} \\
&- \frac{a_1k\omega}{G'} + \frac{a_1k^3\gamma\lambda^2\omega}{G'} = 0.
\end{aligned}$$

Combining the coefficients of $(\frac{1}{G'})$ with the same powers, and setting each collection to zero, gives a system of equations as follows:

$$\begin{aligned}
&\text{Constants}: A_0^2k^2\alpha + A_0l^2\delta - A_0k\omega = 0, \\
&\frac{1}{G'}: 2A_0A_1k^2\alpha + A_1l^2\delta + A_1k^3\beta\lambda - A_1k\omega + A_1k^3\gamma\lambda^2\omega = 0, \\
&\frac{1}{(G')^2}: A_1^2k^2\alpha + 2A_0A_2k^2\alpha + A_2l^2\delta + 2A_2k^3\beta\lambda + A_1k^3\beta\mu - \\
&\qquad A_2k\omega + 4A_2k^3\gamma\lambda^2\omega + 3A_1k^3\gamma\lambda\mu\omega = 0, \\
&\frac{1}{(G')^3}: A_1A_2k^2\alpha + 2A_2k^3\beta\mu + 10A_2k^3\gamma\lambda\mu\omega + 2A_1k^3\gamma\mu^2\omega = 0, \\
&\frac{1}{(G')^4}: A_2^2k^2\alpha + 6A_2k^3\gamma\mu^2\omega = 0.
\end{aligned}$$

To achieve the solutions of Eq. (21), we solve the above system of equations.

Set 1. When

$$A_0=-\frac{i\beta}{5\alpha\sqrt{\gamma}},\ A_1=-\frac{12(-1)^{1/4}l\sqrt{\beta}\gamma^{1/4}\sqrt{\delta}\mu}{5\alpha},$$
$$\omega=-\frac{6(-1)^{3/4}l\sqrt{\beta}\sqrt{\delta}}{5\gamma^{1/4}},\ k=\frac{(-1)^{1/4}l\gamma^{1/4}\sqrt{\delta}}{\sqrt{\beta}},\qquad (25)$$
$$A_2=-\frac{36l^2\gamma\delta\mu^2}{5\alpha},\lambda=\frac{(-1)^{1/4}\sqrt{\beta}}{6l\gamma^{3/4}\sqrt{\delta}},$$

we get the following complex periodic solutions (as seen in figure 1):

$$q(x,y,t)=\frac{-i\beta}{5\alpha\sqrt{\gamma}}-\frac{36l^2\gamma\delta\mu^2}{5\alpha\left(be^{-\frac{(-1)^{1/4}\sqrt{\beta}\xi}{6l\gamma^{3/4}\sqrt{\delta}}}+\frac{6(-1)^{3/4}l\gamma^{3/4}\sqrt{\delta}\mu}{\sqrt{\beta}}\right)^2}$$
$$-\frac{12(-1)^{1/4}l\sqrt{\beta}\gamma^{1/4}\sqrt{\delta}\mu}{5\alpha\left(be^{-\frac{(-1)^{1/4}\sqrt{\beta}\xi}{6l\gamma^{3/4}\sqrt{\delta}}}+\frac{6(-1)^{3/4}l\gamma^{3/4}\sqrt{\delta}\mu}{\sqrt{\beta}}\right)}.\qquad (26)$$

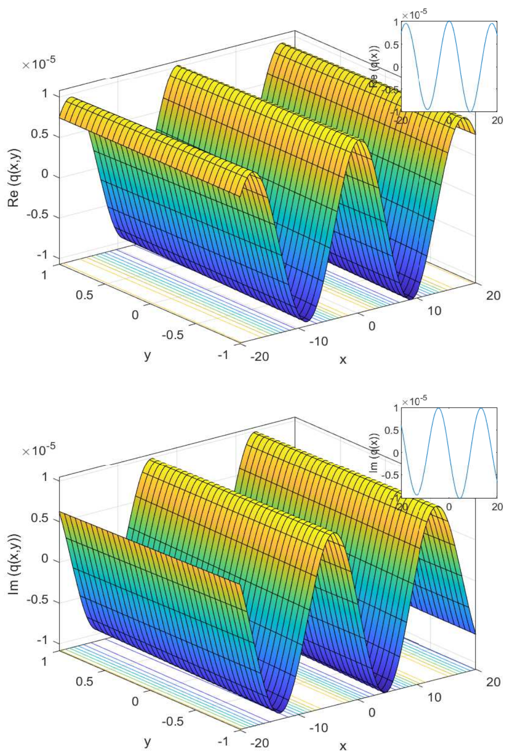

Fig. 1: The sketch of Eq. (26) where $\lambda=0.2$, $\mu=2$, $b=0.02$, $l=0.01$, $\gamma=0.9$, $\delta=5$, $\alpha=0.5$, and $\beta=0.06$.

Set 2. When

$$A_0=\frac{A_2\lambda^2}{\mu^2},\ A_1=\frac{2A_2\lambda}{\mu},\ \alpha=\frac{\mu^2\left(-l^2\delta+k\omega\right)}{a_2k^2\lambda^2},$$
$$\gamma=\frac{l^2\delta-k\omega}{6k^3\lambda^2\omega},\ \beta=\frac{5l^2\delta-5k\omega}{6k^3\lambda},\qquad (27)$$

we have the following singular solution (see figure 2):

$$q\left(x,y,t\right)=\frac{A_2\lambda^2}{\mu^2}+\frac{A_2}{\left(be^{-\lambda(kx+ly-\omega t)}-\frac{\mu}{\lambda}\right)^2}$$
$$+\frac{2Aa_2\lambda}{\mu\left(be^{-\lambda(kx+ly-\omega t)}-\frac{\mu}{\lambda}\right)}.\qquad (28)$$

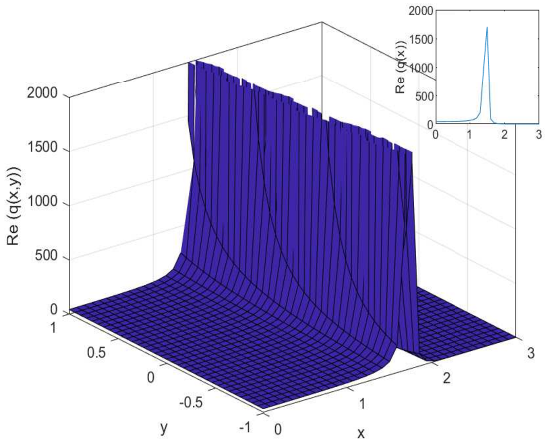

Fig. 2: The sketch of Eq. (28) where $\lambda=2.5$, $\mu=0.3$, $b=5$, $k=1.5$, $l=0.2$, $a_2=0.6$, and $\omega=0.9$.

Set 3. When

$$A_0=\frac{-5l^2\gamma\delta+\sqrt{\gamma(-24k^4\beta^2+25l^4\gamma\delta^2)}}{10k^2\alpha\gamma},$$
$$A_2=-\frac{3\left(5l^2\gamma\delta+\sqrt{\gamma(-24k^4\beta^2+25l^4\gamma\delta^2)}\right)\mu^2}{5\alpha},$$
$$\lambda=-\frac{-5l^2\gamma\delta+\sqrt{\gamma(-24k^4\beta^2+25l^4\gamma\delta^2)}}{12k^3\beta\gamma},$$
$$\omega=\frac{5l^2\gamma\delta+\sqrt{\gamma(-24k^4\beta^2+25l^4\gamma\delta^2)}}{10k\gamma},$$
$$A_1=-\frac{12k\beta\mu}{5\alpha},\qquad (29)$$

we construct the following singular solution (as shown in figure 3):

$$q(x,y,t) = \frac{D}{10k^2\alpha\gamma} + \left(\frac{3D\mu^2}{5\alpha\left(be^{\frac{(D)\xi}{12k^3\beta\gamma}} + \frac{12k^3\beta\gamma\mu}{D}\right)^2} \right) - \left(\frac{12k\beta\mu}{5\alpha\left(be^{\frac{(D)\xi}{12k^3\beta\gamma}} + \frac{12k^3\beta\gamma\mu}{D}\right)} \right), \tag{30}$$

where $D = -5l^2\gamma\delta + \sqrt{\gamma(-24k^4\beta^2 + 25l^4\gamma\delta^2)}$.

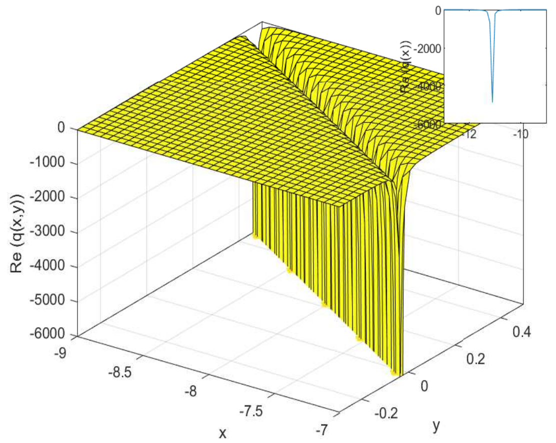

Fig. 3: The sketch of Eq. (30) using $\lambda = 2.5,\ \mu = 0.3,\ b = 5,\ k = 1.5,\ l = 0.2,\ a_2 = 0.6,$ and $\omega = 0.9$.

Set 4. When

$$A_0 = -\frac{6l^2\delta}{5k^2\alpha},\ A_1 = \frac{12il^2\sqrt{\gamma}\delta\mu}{5k\alpha},\ A_2 = \frac{6l^2\gamma\delta\mu^2}{5\alpha},$$
$$\beta = -\frac{il^2\sqrt{\gamma}\delta}{k^2},\ \lambda = \frac{i}{k\sqrt{\gamma}},\ \omega = -\frac{l^2\delta}{5k}, \tag{31}$$

we obtain the following complex periodic solutions (as seen in figure 4):

$$q(x,y,t) = -\frac{6l^2\delta}{5k^2\alpha} + \frac{6l^2\gamma\delta\mu^2}{5\alpha\left(be^{-\frac{i\xi}{k\sqrt{\gamma}}} + ik\sqrt{\gamma}\mu\right)^2} + \frac{12il^2\sqrt{\gamma}\delta\mu}{5k\alpha\left(be^{-\frac{i\xi}{k\sqrt{\gamma}}} + ik\sqrt{\gamma}\mu\right)}. \tag{32}$$

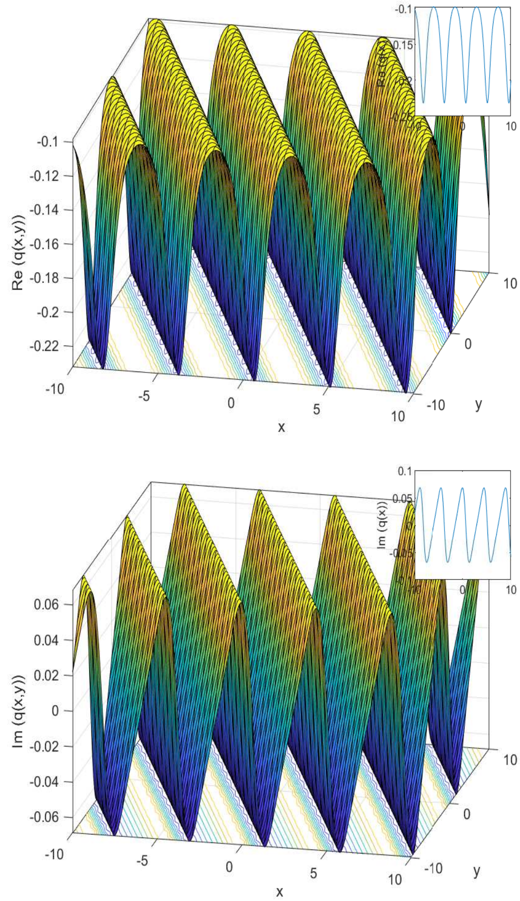

Fig. 4: The sketch of Eq. (32) using $\lambda = 0.2,\ \mu = 0.3,\ b = 0.5,\ k = 0.5,\ l = 0.2,\ \gamma = 0.5,\ \delta = 0.3,$ and $\alpha = 0.4$.

3.2 Implementation of the BSEM

In this portion, application of the BSEM to the generalized KP-MEW-B equation is presented.
Balancing H'' and H^2 in Eq. (21), yields the relation $M = \frac{k}{2} + 1$, by choosing $k = 4$, we get $M = 3$. With $k = 4$ Eq. (11) takes the form

$$H = A_0 + A_1F + A_2F^2 + A_3F^3 + A_4F^4, \tag{33}$$

$$H' = A_1\left(bF + dF^3\right) + 2A_2F\left(bF + dF^3\right) + 3A_3F^2\left(bF + dF^3\right) + 4A_4F^3\left(bF + dF^3\right), \tag{34}$$

and

$$H'' = 2A_2\left(bF+dF^3\right)^2+6A_3F\left(bF+dF^3\right)^2+ A_1\left(b\left(bF+dF^3\right)+3dF^2\left(bF+dF^3\right)\right) +2A_2F\left(b\left(bF+dF^3\right)+3dF^2\left(bF+dF^3\right)\right) +3A_3F^2\left(b\left(bF+dF^3\right)+3dF^2\left(bF+dF^3\right)\right) +4A_4F^3\left(b\left(bF+dF^3\right)+3dF^2\left(bF+dF^3\right)\right) +12A_4F^2\left(bF+dF^3\right)^2. \tag{35}$$

Plugging Eq. (33), Eq. (34) and Eq. (35) into Eq. (21), we have the following polynomial in powers of F:

$$A_0^2k^2\alpha+A_0l^2\delta-A_0k\omega+2A_0A_1k^2\alpha F+A_1bk^3\beta F+A_1l^2\delta F -A_1k\omega F+A_1b^2k^3\gamma\omega F+A_1^2k^2\alpha F^2+2A_0A_2k^2\alpha F^2+ 2A_2bk^3\beta F^2+A_2l^2\delta F^2-A_2k\omega F^2+4A_2b^2k^3\gamma\omega F^2+ 2A_1A_2k^2\alpha F^3+2A_0A_3k^2\alpha F^3+3A_3bk^3\beta F^3+A_1dk^3\beta F^3+ A_3l^2\delta F^3-A_3k\omega F^3+9A_3b^2k^3\gamma\omega F^3+4A_1bdk^3\gamma\omega F^3+ A_2^2k^2\alpha F^4+2A_1A_3k^2\alpha F^4+2A_0A_4k^2\alpha F^4+4A_4bk^3\beta F^4+ 2A_2dk^3\beta F^4+A_4l^2\delta F^4-A_4k\omega F^4+16A_4b^2k^3\gamma\omega F^4+ 12A_2bdk^3\gamma\omega F^4+2A_2A_3k^2\alpha F^5+2A_1A_4k^2\alpha F^5+3A_3dk^3\beta F^5 +24A_3bdk^3\gamma\omega F^5+3A_1d^2k^3\gamma\omega F^5+A_3^2k^2\alpha F^6+2A_2A_4k^2\alpha F^6 +4A_4dk^3\beta F^6+40A_4bdk^3\gamma\omega F^6+8A_2d^2k^3\gamma\omega F^6+2A_3A_4k^2\alpha F^7 +15A_3d^2k^3\gamma\omega F^7+A_4^2k^2\alpha F^8+24A_4d^2k^3\gamma\omega F^8=0.$$

Summing the coefficients of F with like powers, then equating each summation to zero, yields the following system of equations:

$$\begin{aligned}
&\text{Constant}: A_0^2k^2\alpha+A_0l^2\delta-A_0k\omega,\\
&F: 2A_0A_1k^2\alpha+A_1bk^3\beta+A_1l^2\delta-A_1k\omega+A_1b^2k^3\gamma\omega,\\
&F^2: A_1^2k^2\alpha+2A_0A_2k^2\alpha+2A_2bk^3\beta+A_2l^2\delta-A_2k\omega+\\
&\quad 4A_2b^2k^3\gamma\omega,\\
&F^3: 2A_1A_2k^2\alpha+2A_0A_3k^2\alpha+3A_3bk^3\beta+A_1dk^3\beta+A_3l^2\delta\\
&\quad -A_3k\omega+9A_3b^2k^3\gamma\omega+4A_1bdk^3\gamma\omega,\\
&F^4: A_2^2k^2\alpha+2A_1A_3k^2\alpha+2A_0A_4k^2\alpha+4A_4bk^3\beta+2A_2dk^3\beta\\
&\quad +A_4l^2\delta-A_4k\omega+16A_4b^2k^3\gamma\omega+12A_2bdk^3\gamma\omega,\\
&F^5: 2A_2A_3k^2\alpha+2A_1A_4k^2\alpha+3A_3dk^3\beta+24A_3bdk^3\gamma\omega+\\
&\quad 3A_1d^2k^3\gamma\omega,\\
&F^6: A_3^2k^2\alpha+2A_2A_4k^2\alpha+4A_4dk^3\beta+40A_4bdk^3\gamma\omega+\\
&\quad 8A_2d^2k^3\gamma\omega,\\
&F^7: 2A_3A_4k^2\alpha+15A_3d^2k^3\gamma\omega,\\
&F^8: A_4^2k^2\alpha+24A_4d^2k^3\gamma\omega,
\end{aligned}$$

To achieve the solutions of Eq. (21), we solve the above system of equations.

Set 1. When

$$A_0=-\frac{12bk\beta}{5\alpha},\ A_1=0,\ A_2=-\frac{24dk\beta}{5\alpha},\ A_3=0,\ A_4=-\frac{12d^2k\beta}{5b\alpha},\ \omega=\frac{\beta}{10b\gamma},\ l=-\frac{\sqrt{k}\sqrt{\beta}\sqrt{1+24b^2k^2\gamma}}{\sqrt{10}\sqrt{b}\sqrt{\gamma}\sqrt{\delta}}, \tag{36}$$

we gain the following singular solution (as shown in figure 5):

$$q(x,y,t)=\left(-\frac{12bk\beta}{5\alpha}\right)-\left(\frac{12d^2k\beta}{5b\left(-\frac{d}{b}+e^BE\right)^2\alpha}\right)\left(\frac{24dk\beta}{5\left(-\frac{d}{b}+e^BE\right)\alpha}\right), \tag{37}$$

where $B=-2b\left(kx-\frac{t\beta}{10b\gamma}-\frac{\sqrt{k}y\sqrt{\beta}\sqrt{1+24b^2k^2\gamma}}{\sqrt{10}\sqrt{b}\sqrt{\gamma}\sqrt{\delta}}\right)$.

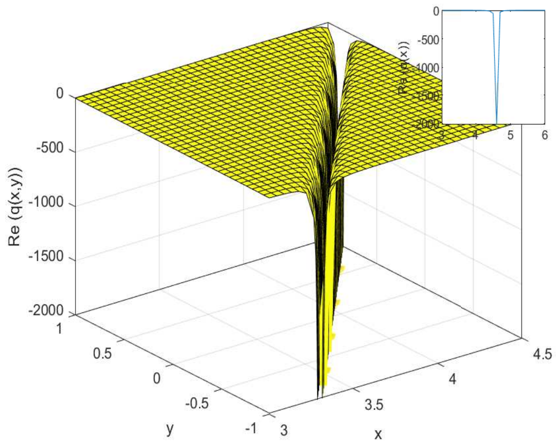

Fig. 5: The sketch of Eq. (37) using $k=0.3$, $E=2$, $\alpha=4$, $b=0.5$, $d=0.4$, $\beta=0.4$, and $\delta=2.5$.

Set 2. When

$$A_0=-\frac{6l^2\delta}{5k^2\alpha},\ A_1=0,\ A_2=\frac{24idl^2\sqrt{\gamma}\delta}{5k\alpha},\ \beta=-\frac{il^2\sqrt{\gamma}\delta}{k^2},\ A_3=0,\ A_4=\frac{24d^2l^2\gamma\delta}{5\alpha},\ b=\frac{i}{2k\sqrt{\gamma}},\ \omega=-\frac{l^2\delta}{5k}, \tag{38}$$

we get the following complex periodic singular solutions (see figure 6):

$$q(x,y,t)=\left(\frac{6l^2\delta}{5k^2\alpha}\right)+\left(\frac{24\mathrm{i}dl^2\sqrt{\gamma}\delta}{5k\alpha\left(\mathrm{e}^{-\frac{\mathrm{i}\left(kx+ly+\frac{l^2t\delta}{5k}\right)}{k\sqrt{\gamma}}}E+2\mathrm{i}dk\sqrt{\gamma}\right)}\right)+\left(\frac{24d^2l^2\gamma\delta}{5\alpha\left(\mathrm{e}^{-\frac{\mathrm{i}\left(kx+ly+\frac{l^2t\delta}{5k}\right)}{k\sqrt{\gamma}}}E+2\mathrm{i}dk\sqrt{\gamma}\right)^2}\right). \tag{39}$$

Set 3. When

$$\begin{aligned}&A_0=\frac{24b^2l^2\gamma\delta}{5\alpha},\ A_1=0,\ A_2=\frac{48bdl^2\gamma\delta}{5\alpha},\ A_3=0,\\ &A_4=\frac{24d^2l^2\gamma\delta}{5\alpha},\ \omega=-\frac{2}{5}\mathrm{i}bl^2\sqrt{\gamma}\delta,\ k=-\frac{\mathrm{i}}{2b\sqrt{\gamma}},\\ &\beta=-4\mathrm{i}b^2l^2\gamma^{3/2}\delta,\end{aligned} \tag{40}$$

we construct the following periodic singular solution (as shown in figure 7):

$$q(x,y,t)=\left(\frac{24b^2l^2\gamma\delta}{5\alpha}\right)+\left(\frac{24d^2l^2\gamma\delta}{5\left(-\frac{d}{b}+\mathrm{e}^AE\right)^2\alpha}\right)+\left(\frac{48bdl^2\gamma\delta}{5\left(-\frac{d}{b}+\mathrm{e}^AE\right)\alpha}\right), \tag{41}$$

where $A=-2b\left(ly-\frac{\mathrm{i}x}{2b\sqrt{\gamma}}+\frac{2}{5}\mathrm{i}bl^2t\sqrt{\gamma}\delta\right)$.

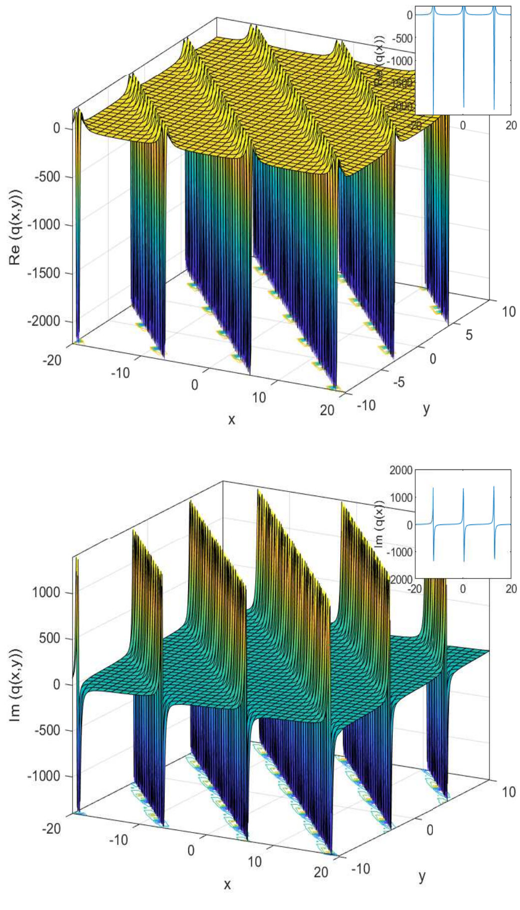

Fig. 6: The sketch of Eq. (39) where $k=0.3$, $E=4$, $\alpha=0.3$, $d=3$, $\beta=0.5$, $\delta=2$, $\gamma=4$, and $l=0.5$.

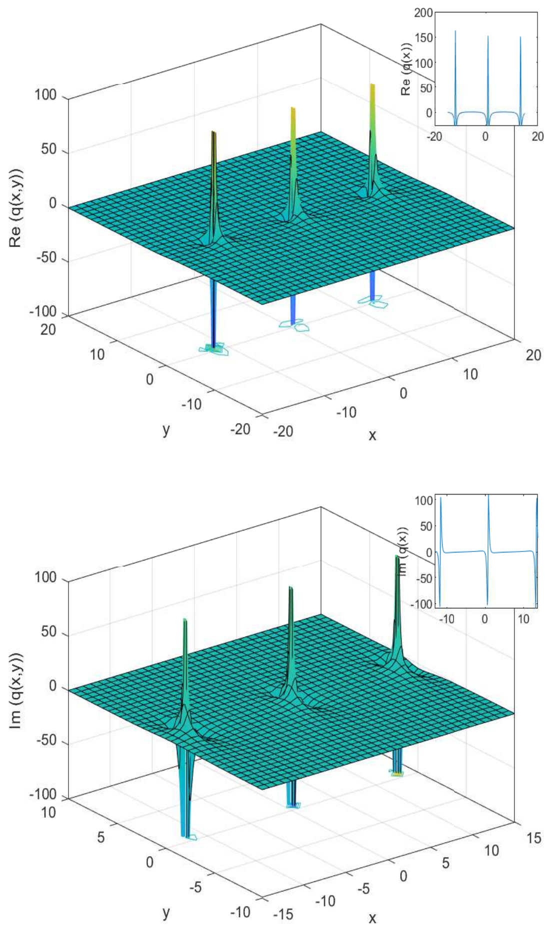

Fig. 7: The sketch of Eq. (41) using $E=2$, $\alpha=2$, $b=2$, $d=3$, $l=0.1$, $\gamma=1.2$, and $\delta=6$.

4 Conclusion

This study developed a new technique to investigate the generalized Kadomtsev–Petviashvili modified equal width-Burgers equation that describes the propagation of long-wave with dissipation and dispersion in nonlinear media. The $(1/G')$-expansion method and the Bernoulli sub-equation method. Several new distinct analytical solutions have been obtained, such as complex periodic solutions as presented in figures 1, 4, singular solutions are addressed in figures 2, 3, and 5, as well as complex periodic singular solutions as shown in figures 6 and 7. In order to understand the physical properties of the obtained solutions, all of them are drawn in 2D, 3D, and contour plots according to the convenient values of the parameters. Both methods are efficient in achieving analytical solutions for nonlinear partial differential equations. All results are new when compared to other soliton solutions reported in refs [34,35] and also all solutions satisfy the main generalized KP-MEW-B equation. We think these results can help to explain the dynamic behaviors of the studied equation.

Conflicts of interests

The author declare that they have no conflict of interest.

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