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# Fluorescence Spectrum of A Laser Driven Polar Quantum Emitter Damped By Degenerate Squeezed Vacuum With Finite Bandwidth

*Nikolai N. Bogolyubov, Jr. and Andrey V. Soldatov* *

Department of Mechanics, V.A. Steklov Mathematical Institute of the Russian Academy of Sciences, Gubkina str. 8, 119991 Moscow, Russia

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**Abstract:** A two-level quantum emitter with broken inversion symmetry driven by external monochromatic high-frequency electromagnetic (e.g. laser) field and damped by squeezed vacuum reservoir with finite bandwidth was studied. The squeezed vacuum field source is assumed to be a degenerate parametric oscillator (DPO). It was shown that low-frequency fluorescence spectrum of the emitter can be effectively shaped by controlling the effective pump amplitude, cavity damping and phase of the squeezing of the vacuum field source.

**Keywords:** Polar emitter, Fluorescence spectrum, Degenerate squeezed vacuum, Two-level atom, Broken inversion symmetry, Quantum dot, Polar molecule, Rydberg atom

## 1 Introduction

As a rule, theoretical and experimental studies of resonance fluorescence, i.e. the process in which a two-level quantum emitter of some sort is driven by the quantum electromagnetic field at a frequency near to the natural frequency of the emitter, have been undertaken with assumed inversion symmetry of the quantum emitter in question. At the same time, violation of this symmetry is rather common in such natural systems as polar molecules, highly excited Rydberg atoms placed in external asymmetric electrostatic field, atoms embedded into asymmetric crystallic environment, as well as in artificially manufactured systems, like quantum dots. The cause for this violation may be different for different systems. In polar molecules its origin is due to the parity mixing of the molecular states [1], in quantum dots it is induced by the asymmetry of the confining potential of the dot. Whatever the cause may be, this violation stipulates the existence of nonzero permanent dipole matrix elements in the ground and excited states of the emitter as its consequence. And the interaction of this permanent dipole moment with external driving electromagnetic adds new features to the resonance fluorescence phenomenon. Among them it was predicted that a simple two-level quantum system driven by high-frequency classical electromagnetic (EM) field can emit EM field of much lower frequency if its dipole operator possesses permanent non-equal diagonal matrix elements [2]. The properties of this low-frequency radiation were thoroughly investigated later for the case of a two-level system driven by external EM field and damped by a dissipative reservoir [3,4,5]. Also, it was shown that the same system can be employed for the amplification of the weak low-frequency EM field [4], which property may provide opportunities for development of useful techniques for manipulation of the low-frequency EM radiation, especially in the terahertz range of frequencies. The goal of the present research is to study the low-frequency EM fluorescence radiation phenomenon in an externally driven two-level system with broken inversion symmetry interacting with a finite bandwidth degenerate squeezed vacuum dissipative reservoir, which properties can be tuned appropriately in order to control the shape of the fluorescence radiation spectrum. The case of interaction with a broadband squeezed vacuum dissipative reservoir was already studied earlier for weak driving EM field in [6,7].

* Corresponding author e-mail: soldatov@mi-ras.ru

## 2 Model Hamiltonian

In this study we consider a two-level atom with ground state $|g\rangle$, excited state $|e\rangle$, transition frequency $\omega_0$ and the electric dipole moment $\hat{\mathbf{d}}$, driven by external classical monochromatic field $\mathbf{E}(t) = \mathbf{E}\cos(\omega_f t)$ with an amplitude $\mathbf{E}$ and frequency $\omega_f$. The atom is also coupled to a reservoir $B$ made of a plurality of modes of quantized electromagnetic field being in the squeezed vacuum state. It is assumed that the frequency Lamb shift due to interaction with the reservoir is already incorporated into the atomic transition frequency $\omega_0$. Thus, the model Hamiltonian reads

$$H = H_S(t) + \hbar\sum_k \omega_k b^+(\omega_k)b(\omega_k)$$

$$+\sum_k \left(g(\omega_k)S^+ b(\omega_k) + g^*(\omega_k)b^+(\omega_k)S^-\right). \quad (1)$$

Here $S^+ = |e\rangle\langle g|$ and $S^- = |g\rangle\langle e|$ are the usual raising and lowering atomic operators and $S^z = \frac{1}{2}(|e\rangle\langle e| - |g\rangle\langle g|)$ is the atomic population inversion operator.

The operators $b(\omega_k)$ and $b^+(\omega_k)$ are the annihilation and creation operators for the vacuum modes satisfying the commutation relations

$$[b(\omega), b^+(\omega')] = \delta(\omega - \omega'), \quad (2)$$

$$[b^+(\omega), b^+(\omega')] = [b(\omega), b(\omega')] = 0, \quad (3)$$

and the term

$$H_S(t) = \hbar\omega_0 S^z + \frac{\hbar}{2}\Omega_R(S^- e^{i\omega_f t} + S^+ e^{-i\omega_f t})$$

$$+\frac{\hbar}{2}(e^{i\omega_f t} + e^{-i\omega_f t})\left[\delta_a S^z - \frac{\delta_s}{2}(|e\rangle\langle e| + |g\rangle\langle g|)\right] \quad (4)$$

contains an interaction between the driving field and the atom in the rotating wave approximation (RWA). Here $\Omega_R = -\mathbf{E}\mathbf{d_{eg}}/\hbar$ is the Rabi frequency being made real and positive by the appropriate choice of the phase factors of the states $|e\rangle$ and $|g\rangle$, and $\mathbf{d}_{eg} = e\langle e|\hat{\mathbf{r}}|g\rangle$, $\mathbf{d}_{ge} = e\langle g|\hat{\mathbf{r}}|e\rangle$, $\mathbf{d}_{ee} = e\langle e|\hat{\mathbf{r}}|e\rangle$, $\mathbf{d}_{gg} = e\langle g|\hat{\mathbf{r}}|g\rangle$ are the atomic dipole moment operator matrix elements. As a rule, it is assumed that $\mathbf{d}_{ee} = \mathbf{d}_{gg} = 0$, because typical physical systems, like atoms and molecules, possess the inversion symmetry, and each of the states $|g\rangle$ and $|e\rangle$ is either symmetric or antisymmetric. Contrary to this view, we assume that the inversion symmetry of the system in question is violated, $\mathbf{d}_{ee} \neq \mathbf{d}_{gg}$, so that $\delta_a = \mathbf{E}(\mathbf{d}_{gg} - \mathbf{d}_{ee})/\hbar$ and $\delta_s = \mathbf{E}(\mathbf{d}_{gg} + \mathbf{d}_{ee})/\hbar$. The term proportional to $\delta_s$ does not influence the dynamics of the system and can be omitted, while the term proportional to the symmetry violation parameter $\delta_a$ is retained. The squeezed vacuum reservoir source is assumed to be a degenerate parametric oscillator (DPO). The output fields from this oscillators are characterized by the following correlation functions [8]

$$\langle b^+(\omega_k)b(\omega_{k'})\rangle_{svac} = N(\omega_k)\delta(\omega_k - \omega_{k'}), \quad (5)$$

$$\langle b(\omega_k)b(\omega_{k'})\rangle_{svac} = -M(\omega_k,\theta)\delta(\omega_k + \omega_{k'} - 2\omega_s), \quad (6)$$

$$\langle b(\omega_k)b^+(\omega_{k'})\rangle_{svac} = (N(\omega_k)+1)\delta(\omega_k - \omega_{k'}), \quad (7)$$

$$\langle b^+(\omega'_k)b^+(\omega_k)\rangle_{svac} = -M^*(\omega_k,\theta)\delta(\omega_{k'} + \omega_k - 2\omega_s), \quad (8)$$

where $\omega_s$ is the carrier frequency of the squeezed field, $\theta$ is the phase of squeezing, $N(\omega)$ is related to the mean number of photons at frequency $\omega$ and $M(\omega,\theta)$ is characteristic of the squeezed vacuum field and describes the correlation between the two photons created in the down-conversion process. They satisfy the inequality $|M(\omega,\theta)| \leq \sqrt{N(\omega)(N(\omega)+1)}$, which in the case of ideal squeezed state produced by an optical parametric oscillator transforms into the equality. The frequency dependencies of $N(\omega)$ and $M(\omega,\theta)$ for an optical parametric oscillator below threshold for the ideal DPO are given by [9]

$$N(\omega) = \frac{\lambda^2 - \mu^2}{4} \times$$

$$\left[\frac{1}{(\omega - \omega_s)^2 + \mu^2} - \frac{1}{(\omega - \omega_s)^2 + \lambda^2}\right], \quad (9)$$

$$M(\omega,\theta) = e^{i\theta}\frac{\lambda^2 - \mu^2}{4} \times$$

$$\left[\frac{1}{(\omega - \omega_s)^2 + \mu^2} + \frac{1}{(\omega - \omega_s)^2 + \lambda^2}\right], \quad (10)$$

Here the parameters $\lambda$ and $\mu$ are expressed in terms of the parametric oscillator cavity damping rate $\gamma_c$ and the effective pump amplitude $\varepsilon$ of the coherent field driving the parametric oscillator

$$\lambda = \gamma_c + \varepsilon,\ \mu = \gamma_c - \varepsilon,\ \varepsilon = E_s/E_c, \quad (11)$$

where $E_s$ is the amplitude of the pump coherent field and $E_c$ is its threshold value for parametric oscillator. In optical parametric oscillator (OPO) the amplitude $E_s$ is related to the power of pumping $P$ [10,11], so the effective pump amplitude is related to the ratio of input pump power to the critical power, $r = P/P_c$, and we have $\varepsilon = \sqrt{r}\gamma_c/2$. The noise spectrum and the squeezing level of the output light from OPO is related to $\varepsilon/(\gamma_c/2)$. When this ratio goes to 1 and therefore $r \to 1$, the threshold happens in OPO. It is worth noticing that Eqs.(9,10) are only valid sufficiently below of the threshold, i.e. when $0 < \varepsilon < \gamma_c/2$, both parameters $\lambda$ and $\mu$ are positive and $\lambda > \mu$, and the squeezing values are not too large. When the parameters $\lambda$ and $\mu$ are much greater than all other relaxation rates in the problem, the frequency dependence of $N(\omega)$ and $M(\omega,\theta)$ can be neglected. This case is referred to as broadband squeezed vacuum.

## 3 Equations of Motion for Atomic Variables

In what follows, it is assumed that $\delta_a \ll \Omega_R$, so that the interaction of the driving field with the permanent dipole moment is much weaker than its interaction with the transitional dipole moment while, at the same time, the driving field is strong enough to be viewed as a dressing field for the two-level system. The master equation for the atomic system reduced density operator $\rho_S(t)$ can be written in the frame rotating with the driving field frequency $\omega_f$, under the assumption that the carrier frequency $\omega_s$ of the squeezed field coincides with the frequency $\omega_f$, as

$$\frac{\partial \rho_S^{rf}(t)}{\partial t} = i\Gamma\delta[S^z,\rho_S^{rf}(t)] - \frac{i}{2}\delta_a(e^{i\omega_f t}+e^{-i\omega_f t})[S^z,\rho_S^{rf}(t)] + \frac{1}{2}\Gamma\tilde{N}(2S^+\rho_S^{rf}(t)S^- - S^-S^+\rho_S^{rf}(t) - \rho_S^{rf}(t)S^-S^+) + \frac{1}{2}\Gamma(\tilde{N}+1)(2S^-\rho_S^{rf}(t)S^+ - S^+S^-\rho_S^{rf}(t) - \rho_S^{rf}(t)S^+S^-) - \Gamma\tilde{M}S^+\rho_S^{rf}(t)S^+ - \Gamma\tilde{M}^*S^-\rho_S^{rf}(t)S^- - \frac{1}{2}i\Omega_R[S^+ + S^-,\rho_S^{rf}(t)] + \frac{1}{2}i(\beta[S^+,[S^z,\rho_S^{rf}(t)]] - \beta^*[S^-,[S^z,\rho_S^{rf}(t)]]), \tag{12}$$

where

$$\rho_S^{rf}(t) = e^{i\omega_f S^z t}\rho_S(t)e^{-i\omega_f S^z t}, \tag{13}$$

$$\tilde{N} = N(\omega_f+\Omega') + \frac{1}{2}(1-\tilde{\Delta}^2)\gamma_n, \tag{14}$$

$$\tilde{M} = (|M(\omega_f+\Omega',\theta)| + i\tilde{\Delta}\delta_M)\exp(i\theta) - \frac{1}{2}(1-\tilde{\Delta}^2)(\gamma_n - i\delta_n), \tag{15}$$

$$\delta = \Delta/\Gamma + \tilde{\Delta}\delta_N + \frac{1}{2}(1-\tilde{\Delta}^2)\delta_n, \tag{16}$$

$$\beta = \Gamma\tilde{\Omega}[\delta_N + \delta_M\exp(i\theta) - i\tilde{\Delta}(\gamma_n - i\delta_n)], \tag{17}$$

$$\gamma_n = N(\omega_f) - N(\omega_f+\Omega') - (|M(\omega_f,\theta)| - |M(\omega_f+\Omega',\theta)|)\cos(\theta), \tag{18}$$

$$\delta_n = (|M(\omega_f,\theta)| - |M(\omega_f+\Omega',\theta)|)\sin(\theta), \tag{19}$$

$$\delta_N = \frac{1}{\pi}\mathscr{P}\int_{-\infty}^{\infty} d\omega \frac{N(\omega)}{\omega-\omega_s+\Omega'}\bigg|_{\omega_s=\omega_f}, \tag{20}$$

$$\delta_M = \frac{1}{\pi}\mathscr{P}\int_{-\infty}^{\infty} d\omega \frac{|M(\omega,\theta)|}{\omega-\omega_s+\Omega'}\bigg|_{\omega_s=\omega_f}. \tag{21}$$

Here $\Gamma$ is the radiative damping constant, $\Delta = \omega_f - \omega_0$ is the detuning of the driving field frequency $\omega_f$ from the atomic frequency $\omega_0$, and

$$\tilde{\Omega} = \Omega_R/\Omega',\ \tilde{\Delta} = \Delta/\Omega',\ \Omega' = \sqrt{\Omega_R^2+\tilde{\Delta}^2} \tag{22}$$

Details on the master equation derivation can be found in [12]. The principal values of the integrals (20,21) can be evaluated by means of the contour integration [13] as

$$\delta_N = \delta_\mu - \delta_\lambda,\ \delta_M = \delta_\mu + \delta_\lambda, \tag{23}$$

where

$$\delta_\mu^{dpo} = \Omega'\frac{\lambda^2-\mu^2}{4}\frac{1}{\mu(\Omega'^2+\mu^2)}, \tag{24}$$

$$\delta_\lambda^{dpo} = \Omega'\frac{\lambda^2-\mu^2}{4}\frac{1}{\lambda(\Omega'^2+\lambda^2)}. \tag{25}$$

This equation is similar to the master equation obtained earlier in [14] for a two-level non-polar emitter without broken inversion symmetry and is derived assuming that the system-reservoir and the system-field interactions are weak and the reservoir correlation time is small compared with the time $t$ of observation. So, a closed set of equations follows from Eq.(12):

$$\frac{d}{dt}\langle\tilde{S}^-(t)\rangle = -\Gamma\left(\frac{1}{2}+\tilde{N}-i\delta+i\frac{\delta_a}{2\Gamma}(e^{i\omega_f t}+e^{-i\omega_f t})\right)\langle\tilde{S}^-(t)\rangle + \Gamma\tilde{M}\langle\tilde{S}^+(t)\rangle + \Omega_R\langle S^z(t)\rangle, \tag{26}$$

$$\frac{d}{dt}\langle\tilde{S}^+(t)\rangle = -\Gamma\left(\frac{1}{2}+\tilde{N}+i\delta-i\frac{\delta_a}{2\Gamma}(e^{i\omega_f t}+e^{-i\omega_f t})\right)\times \langle\tilde{S}^+(t)\rangle + \Gamma\tilde{M}^*\langle\tilde{S}^-(t)\rangle + \Omega_R\langle S^z(t)\rangle, \tag{27}$$

$$\frac{d}{dt}\langle S^z(t)\rangle = -\frac{1}{2}(\Omega_R+\beta^*)\langle S^-(t)\rangle - \frac{1}{2}(\Omega_R+\beta)\langle S^+(t)\rangle - \Gamma(2\tilde{N}+1)\langle S^z(t)\rangle - \Gamma/2, \tag{28}$$

where $\langle\tilde{S}^\pm(t)\rangle = \pm i\langle S^\pm(t)e^{\mp i\omega_f t}\rangle$ are slowly varying parts of the atomic operators. The system of equations (26-28) can be solved numerically by means of the technique employed earlier in [15], where the components of the vector $\mathbf{X}(t) = (\langle\tilde{S}^-(t)\rangle, \langle\tilde{S}^+(t)\rangle, \langle S^z(t)\rangle)$ are decomposed as $X_i(t) = \sum_{l=-\infty}^{+\infty} X_i^{(l)}(t)e^{il\omega_f t}$, $i = 1,2,3,$ and the slowly varying amplitudes $X_i^{(l)}(t)$ obey the system of equations

$$\frac{d}{dt}X_1^{(l)}(t) = -\Gamma\left(\frac{1}{2}+\tilde{N}-i\delta+il\frac{\omega_f}{\Gamma}\right)X_1^{(l)}(t) -$$

$$i\frac{\delta_a}{2}(X_1^{(l-1)}(t)+X_1^{(l+1)}(t))+\Gamma\tilde{M}X_2^{(l)}(t)+\Omega_R X_3^{(l)}(t), \quad (29)$$

$$\frac{d}{dt}X_2^{(l)}(t)=-\Gamma\left(\frac{1}{2}+\tilde{N}-i\delta+il\frac{\omega_f}{\Gamma}\right)X_2^{(l)}(t)+$$

$$i\frac{\delta_a}{2}(X_2^{(l-1)}(t)+X_2^{(l+1)}(t))+\Gamma\tilde{M}^*X_1^{(l)}(t)+\Omega_R X_3^{(l)}(t), \quad (30)$$

$$\frac{d}{dt}X_3^{(l)}(t)=-\frac{\Gamma}{2}\delta_{l,0}-(\Gamma(2\tilde{N}+1)+il\omega_f)X_3^{(l)}(t)-$$

$$\frac{\Omega_R+\beta^*}{2}X_1^{(l)}(t)-\frac{\Omega_R+\beta}{2}X_2^{(l)}(t). \quad (31)$$

## Fluorescence spectrum

The incoherent part of the steady-state fluorescence spectrum is given by[16]

$$F_{inc}(\omega)=\frac{\Gamma}{\pi}\mathrm{Re}\int_0^\infty d\tau\lim_{t\to\infty}\left[\langle\tilde{S}^+(t)\tilde{S}^-(t+\tau)\rangle-\right.$$

$$\left.\langle\tilde{S}^+(t)\rangle\langle\tilde{S}^-(t+\tau)\rangle\right]e^{i(\omega-\omega_f)\tau}, \quad (32)$$

where the coherent contribution from the incident driving field scattered by the atom is subtracted, as usual. In accordance with the so-called quantum regression hypothesis [17,18], the fluctuation correlation functions $Y_1(t,t+\tau)=\langle\tilde{S}^+(t)\tilde{S}^-(t+\tau)\rangle-\langle\tilde{S}^+(t)\rangle\langle\tilde{S}^-(t+\tau)\rangle$, $Y_2(t,t+\tau)=\langle\tilde{S}^+(t)\tilde{S}^+(t+\tau)\rangle-\langle\tilde{S}^+(t)\rangle\langle\tilde{S}^+(t+\tau)\rangle$, $Y_3(t,t+\tau)=\langle\tilde{S}^+(t)\tilde{S}^z(t+\tau)\rangle-\langle\tilde{S}^+(t)\rangle\langle\tilde{S}^z(t+\tau)\rangle$, satisfy virtually the same set of equations of motion (26-28) for the correspondent averages $\langle\tilde{S}^-(\tau)\rangle$, $\langle\tilde{S}^+(\tau)\rangle$ and $\langle\tilde{S}^z(\tau)\rangle$ with the only difference that the inhomogeneity $-\Gamma/2$ disappears due to the subtraction of the mean. These correlation functions can be decomposed as $Y_i(t,t+\tau)=\sum_{l=-\infty}^{+\infty}Y_i^{(l)}(t,\tau)e^{il\omega_f(t+\tau)}$, $i=1,2,3$, so that

$$\frac{d}{d\tau}Y_1^{(l)}(t,\tau)=-\Gamma\left(\frac{1}{2}+\tilde{N}-i\delta+il\frac{\omega_f}{\Gamma}\right)Y_1^{(l)}(t,\tau)-$$

$$-i\frac{\delta_a}{2}(Y_1^{(l-1)}(t,\tau)+Y_1^{(l+1)}(t,\tau))+$$

$$\Gamma\tilde{M}Y_2^{(l)}(t,\tau)+\Omega_R Y_3^{(l)}(t,\tau), \quad (33)$$

$$\frac{d}{d\tau}Y_2^{(l)}(t,\tau)=-\Gamma\left(\frac{1}{2}+\tilde{N}-i\delta+il\frac{\omega_f}{\Gamma}\right)Y_2^{(l)}(t,\tau)+$$

$$+i\frac{\delta_a}{2}(Y_2^{(l-1)}(t,\tau)+Y_2^{(l+1)}(t,\tau))+$$

$$\Gamma\tilde{M}^*Y_1^{(l)}(t,\tau)+\Omega_R Y_3^{(l)}(t,\tau), \quad (34)$$

$$\frac{d}{d\tau}Y_3^{(l)}(t,\tau)=-(\Gamma(2\tilde{N}+1)+il\omega_f)Y_3^{(l)}(t,\tau)-$$

$$\frac{\Omega_R+\beta^*}{2}Y_1^{(l)}(t,\tau)-\frac{\Omega_R+\beta}{2}Y_2^{(l)}(t,\tau), \quad (35)$$

and the Laplace transforms

$$\bar{Y}_i^{(l)}(t,z)=\int_0^\infty e^{-z\tau}Y_i^{(l)}(t,\tau)d\tau \quad (36)$$

of the components $Y_i^{(l)}(t,\tau)$ will satisfy the following set of equations:

$$z\bar{Y}_1^{(l)}(t,z)+\Gamma\left(\frac{1}{2}+\tilde{N}-i\delta+il\frac{\omega_f}{\Gamma}\right)\bar{Y}_1^{(l)}(t,z)+$$

$$+i\frac{\delta_a}{2}\left(\bar{Y}_1^{(l-1)}(t,z)+\bar{Y}_1^{(l+1)}(t,z)\right)-$$

$$\Gamma\tilde{M}\bar{Y}_2^{(l)}(t,z)-\Omega_R\bar{Y}_3^{(l)}(t,z)=$$

$$\frac{1}{2}\delta_{l,0}+X_3^{(l)}(t)-\sum_{r=-\infty}^{\infty}X_1^{(l-r)}(t)X_2^{(r)}(t), \quad (37)$$

$$z\bar{Y}_2^{(l)}(t,z)+\Gamma\left(\frac{1}{2}+\tilde{N}-i\delta+il\frac{\omega_f}{\Gamma}\right)\bar{Y}_2^{(l)}(t,z)-$$

$$-i\frac{\delta_a}{2}\left(\bar{Y}_2^{(l-1)}(t,z)+\bar{Y}_2^{(l+1)}(t,z)\right)-$$

$$\Gamma\tilde{M}^*Y_1^{(l)}(t,z)-\Omega_R\bar{Y}_3^{(l)}(t,z)=-\sum_{r=-\infty}^{\infty}X_2^{(l-r)}(t)X_2^{(r)}(t), \quad (38)$$

$$z\bar{Y}_3^{(l)}(t,z)+(\Gamma(2\tilde{N}+1)+il\omega_f)\bar{Y}_3^{(l)}(t,z)+$$

$$\frac{\Omega_R+\beta^*}{2}\bar{Y}_1^{(l)}(t,z)+\frac{\Omega_R+\beta}{2}\bar{Y}_2^{(l)}(t,z)=$$

$$=-\sum_{r=-\infty}^{\infty}\left(\frac{1}{2}\delta_{r,0}+X_3^{(r)}(t)\right)X_2^{(l-r)}(t). \quad (39)$$

In the steady state limit $(t\to\infty)$ only the zero-order component $\bar{Y}_1^{(0)}(t,z)$ contributes to $F_{inc}(\omega)$, and the incoherent part of the spectrum (32) reads as

$$F_{inc}(\omega)=\frac{\Gamma}{\pi}\mathrm{Re}\lim_{t\to\infty}\bar{Y}_1^{(0)}(t,z)\Big|_{z=-i(\omega-\omega_f)}. \quad (40)$$

## 4 Numerical Results

Equations (29)-(31) and (37)-(39) were solved numerically, as usual [15], in the steady state limit ($t \to \infty$) by truncation of the number of the harmonic amplitudes $X_i^{(l)}(t)$ and $\bar{Y}_i^{(l)}(t,z)$ involved, and the case of the driving laser field frequency $\omega_f$ and the carrier frequency of the squeezed field $\omega_s$ being simultaneously in resonance with the atomic transition frequency $\omega_0$ was studied. It was already found [3] in the case of a two-level system with broken symmetry interacting with non-squeezed vacuum reservoir that for $\delta_a \neq 0$ a low-frequency radiation peak centered nearly exactly at the frequency $\omega = \Omega_R$ appears in the fluorescence spectrum. The amplitude of the peak increases steadily with the increase of the symmetry violation parameter $\delta_a$ and decreases with the increase of the driving field frequency $\omega_f$. The central frequency of the peak is defined for the most part by the Rabi frequency $\Omega_R$ and depends weakly on the symmetry violation parameter $\delta_a$, so that it drifts very slowly toward $\omega = 0$ along with the increase of this parameter. Here it was found that the interaction with the finite bandwidth squeezed vacuum reservoir does not change these aspects of the spectral peak behavior. At the same time, the amplitude of this peak strongly depends on the squeezed vacuum source DPO parameters, such as the cavity damping $\gamma_c$ and effective pump amplitude $\varepsilon$. Actually, the amplitude of the peak decreases with the increase of the cavity damping while the width of the peak increases, see Fig.1. And vice versa, the amplitude of the peak increases with the increase of the effective pump amplitude while the width of the peak decreases, see Fig.3. At the same time, the position of the peak is not significantly affected by these parameters and, consequently, the DPO cavity damping and effective pump amplitude can be employed to control the fluorescence intensity output of the polar emitter at the fixed frequency $\Omega_R$. The same goal of the fluorescence intensity output control can be achieved for fixed parameters $\gamma_c$ and $\varepsilon$ by varying the phase of squeezing $\theta$ instead, see Fig.2 and Fig.4. It is also observable that the fluorescence peak is most pronounced for the case of the DPO at the threshold when $2\varepsilon/\gamma_c = 1$. Below the threshold the amplitude of the peak decreases while the position of the peak is not shifted. Therefore, the fluorescence intensity output is actually controlled by the value of the ratio $2\varepsilon/\gamma_c$, which effectively defines the degree of the squeezing.

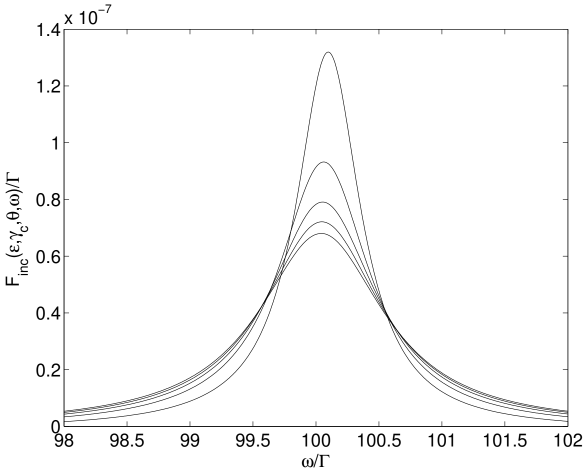


**Fig. 1:** Fluorescence spectrum at $\omega = \Omega_R$ for various values of the cavity damping $\gamma_c$. $\Gamma = 1, \gamma_c = 10:10:50, \varepsilon = 5, \theta = 0$, $\omega_f = \omega_s = \omega_0 = 5000, \Omega_R = 100, \delta_a = 10$.

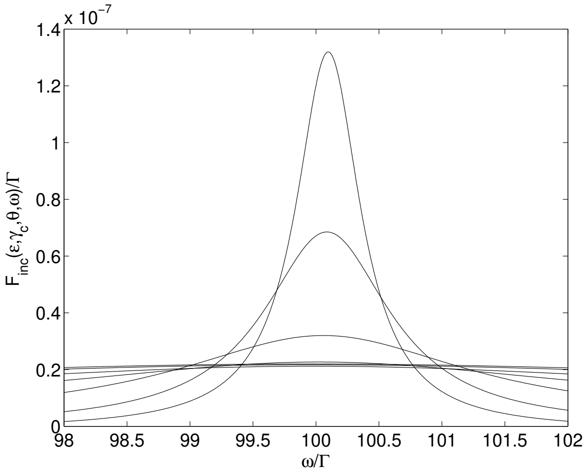


**Fig. 2:** Fluorescence spectrum at $\omega = \Omega_R$ for various values of $\theta$ at the DPO threshold. $\Gamma = 1, \gamma_c = 10, \varepsilon = 5$, $\theta = 0 : \pi/6 : \pi$, $\omega_f = \omega_s = \omega_0 = 5000, \Omega_R = 100, \delta_a = 10$.

## 5 Conclusion

In conclusion, we studied the effect of the vacuum dissipative reservoir finite bandwidth squeezing on the phenomenon of the low-frequency fluorescence by a damped quantum two-level polar system with broken inversion symmetry driven by external high-frequency classical EM (laser) field. The source of the squeezed vacuum was represented by the OPO being in the DPO mode of operation. It was shown that the parameters of the DPO, such as the effective pump amplitude, cavity damping and phase of the squeezing, provide efficient means for the control of the intensity and spectral width of the fluorescence radiation output.

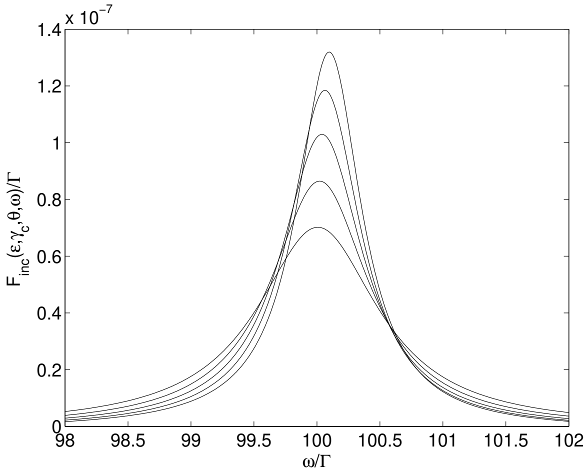


**Fig. 3:** Fluorescence spectrum at $\omega = \Omega_R$ for various values of the DPO effective pump amplitude $\varepsilon$. $\Gamma = 1$, $\gamma_c = 10$, $\varepsilon = 1:1:5, \theta = 0$, $\omega_f = \omega_s = \omega_0 = 5000, \Omega_R = 100, \delta_a = 10$.

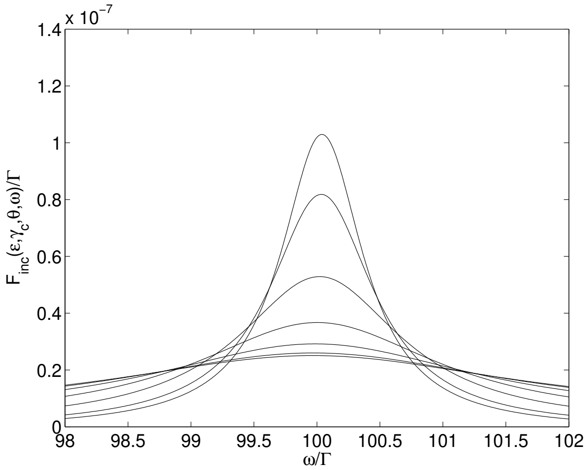


**Fig. 4:** Fluorescence spectrum at $\omega = \Omega_R$ for various values of $\theta$ below the DPO threshold. $\Gamma = 1, \gamma_c = 10, \varepsilon = 3, \theta = 0:\pi/6:\pi$, $\omega_f = \omega_s = \omega_0 = 5000, \Omega_R = 100, \delta_a = 10$.

## Conflict of Interest

The authors declare that they have no conflict of interest

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**Nikolai N. Bogolyubov, Jr.** is a Chief Scientific Researcher at the V.A. Steklov Mathematical Institute of the Russian Academy of Sciences,Corresponding member of Russian Academy of Sciences, (received his Ph.D. in Theoretical Physics from the Moscow State University in 1966 and his D.Sc. in Theoretical Physics from the Academy of Sciences of the USSR in 1970). His scientific interests are in general mathematical problems of equilibrium and nonequilibrium statistical mechanics and applications of modern mathematical methods of quantum and classical statistical mechanics to the problems of the polaron theory, superradiance theory and the theory of superconductivity. His main works belong to the field of Theoretical and Mathematical Physics, Classical and Quantum Statistical Mechanics, Kinetic theory. Many results have become a part of the modern Mathematical Physics toolbox, namely: the fundamental theorem in the theory of Model Systems of Statistical Mechanics, inequalities for thermodynamical potentials, minimax principle in problems of Statistical Mechanics. He has published more than 200 in the field of Statistical Mechanics, Theoretical and Mathematical Physics.

**Andrey V. Soldatov** is a Senior Scientific Researcher at the V.A. Steklov Mathematical Institute of the RAS. His scientific interests are in the domain of general problems of equilibrium and non-equilibrium statistical mechanics and applications of modern mathematical methods of classical and quantum statistical mechanics to the problems of the polaron theory, quantum optics, quantum systems of reduced dimensionality and the theory of open systems. His main works belong to the field of Theoretical and Mathematical Physics and Quantum Statistical Mechanics. He has more than 50 scientific publications in the field of Statistical Mechanics, Theoretical and Mathematical Physics.