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# Optimal Control and Solving of Cellular DNA Cancer Model

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**Abstract:** In this study, we have presented a novel fractional numerical model for the breast cancer stages. We have proved the existence of a stable solution of the fractional model. Also, the optimal control of this model and numerical technique for the simulation of the control problem is discussed. We have proved the existence of the solution. We have achieved the results of the dissection with numeral emulations. The compartment diagram of the model is done. We have utilized Mathematica programming to calculate the outcomes. A lot of investigators have shown that bacterial DNA could generate and suck electromagnetic waves. One of the major experiments about bacterial waves has been completed via Montagnier et al., where they revealed that genomic DNA of highly pathogenic bacteria has certain sequences that are fit to generate electromagnetic waves. The outcomes revealed that electromagnetic waves affect the vital physicochemical processes in both Gram-positive and Gram-negative bacteria. Since infectious diseases threaten human health where billions of individuals suffer from serious diseases caused by many infectious agents so the current study restores the hope in the control of infectious agents as the numerical demonstration by the current study was of impressive significance in the study of disease transmission since it might explain the basic components which impact the spread of infection and may recommend certain control measures. The numerical style used in this study to solve the proposed investigated model has not been applied by any other authors before that.

**Keywords:** : DNA; breast cancer; fractional mathematical model; compartment diagram; optimal control; existence.

## 1 Introduction

Most fractional differential conditions do not have accurate logical arrangements, so rough and mathematical methods should be utilized. Along these lines, numerous scientists have been keen on concentrating on the properties of fragmentary math and tracking down a solid and effective strategy for the arrangement of fractional differential equations (FDEs) [1,2,3,4,5,6,7,8,9,10,11,12, 13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31, 32,33,34,35,36,37,38,39,40,41,42,43,44,45,46,47,48,49,50]. Many researchers made the discovery of the breast cancer, a malignant tumor that attacks the breast tissue [43]. Every lady has a risk for breast cancer. The World health organization (WHO) in 2004 reported that breast cancer is the second fatal malignancy after lung cancer[5,6,7].The phase of malignancy is determined by the TNM system (Tumor, lymph node, Metastasis). The TNM system claims the phase of malignancy, based on the tumor size (T), number of lymph nodes (N), and spread (metastasis) to other sites (M). the recovery from breast cancer may be achieved if the cancer was detected at early stages in which the advanced stages of malignancy indicate a poor prognosis. Many therapeutic choices can be used individually or in combination with each other such as surgery, hormonal therapy, chemotherapy, radiotherapy

We enjoy a ton of benefits to utilize fractional derivative for instance: Fractional derivatives can be utilized for displaying frameworks with memory. Fractional request differential conditions (FDEs) are normally identified with frameworks with memory which exists in most actual issues and, models in summed up thermoelasticity hypothesis, and natural frameworks.

The paper is prepared as follows: Representing the marking, preliminaries and reduced differential transforms method (RDTM) in part 2. Also, we represent the cancer model by its diagram of signal flow in part 2. In part 3, we discuss the OC for fractional cancer model, and the existence of a uniformly stable solution after control. In part 4, we put RDTM to resolve the nonlinear fractional

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cancer model. In part 5, the cancer model is numerically simulated. Conclusions are approached in part 6.

## 2 Marking and Preliminaries

Here, we put some basal simplification and advantages of the fractional calculus notion, which shall be useful in this paper:

**Definition 2.1.** The derivative fractional ( $D^q$ ) of $g(t)$ in the Caputo's sense can be defined as ([6,22]):

$$D^q g(t) = \frac{1}{\Gamma(m-q)} \int_0^t (t-\xi)^{m-q-1} g^{(n)}(\xi) d\xi, \quad \text{for } m-1 < q \le m, \quad m \in \mathrm{N}. \tag{1}$$

**Definition 2.2.** The order derivative fractional Caputo $q$ can be defined by [7]:

$${}^{c}D_{+}^{q} g(t) = \frac{1}{\Gamma(m-q)} \int_{-\infty}^{t} (t-\xi)^{m-q-1} D^m f(\xi) d\xi. \tag{2}$$

### *2.1 Reduced Differential Transform Method*

Here, we insert the basal definitions of the RDTM. Let a function of three variables $w(x,y,t)$, and suppose that [20,21]

$$h(x,y,t) = R(x,y)T(t). \tag{3}$$

Based on the characteristic of RDTM, the function $h(x.y,t)$ can be introduced as

$$h(x,y,t) = \sum_{i_1=0}^{\infty}\sum_{i_2=0}^{\infty} R(i_1,i_2) x^{i_1} y^{i_2} \sum_{j=0}^{\infty} T(j) t^j = \sum_{i_1=0}^{\infty}\sum_{i_2=0}^{\infty}\sum_{j=0}^{\infty} H(i_1,i_2) x^{i_1} y^{i_2} t^j. \tag{4}$$

$$R_D[h(x,y,t)] = H_k(x,y) = \frac{1}{\Gamma(kq+1)} \left[\frac{\partial^k}{\partial t^k} h(x,y,t)\right]_{t=t_0}, \tag{5}$$

is the RDTM of $h(x,y,t)$. The differential inverse RDTM of $H_k(x,y)$ is know as:

$$R_D^{-1}[H_k(x,y)] = h(x,y) = \sum_{k=0}^{\infty} H_k(x,y)(t-t_0)^{kq}. \tag{6}$$

By combining Eqs.(5) and (6), we find

$$h(x,y,t) = \sum_{k=0}^{\infty} \frac{1}{\Gamma(kq+1)} \left[\frac{\partial^k}{\partial t^k} h(x,y,t)\right]_{t=t_0} (t-t_0)^{kq}. \tag{7}$$

If $t=0$, Eq.(7) converts to

$$h(x,y,t) = \sum_{k=0}^{\infty} \frac{1}{\Gamma(kq+1)} \left[\frac{\partial^k}{\partial t^k} h(x,y,t)\right]_{t=t_0} t^{kq}. \tag{8}$$

From Eq.(8), it will be seen that the idea of the RDTM is derived from the power chain extension of the paper in [20,21].

### *2.2 Our Proposed Model*

In the current study, we tried to find an approximate analytical solutions for a novel mathematical fractional model for the breast cancer. Current work looks at the analytical approximate solution of the fractional-order cancer model [5]:

$$\begin{bmatrix} \frac{d^\alpha A}{dt^\alpha} \\ \frac{d^\alpha B}{dt^\alpha} \\ \frac{d^\alpha C}{dt^\alpha} \\ \frac{d^\alpha D}{dt^\alpha} \\ \frac{d^\alpha E}{dt^\alpha} \end{bmatrix} = \begin{bmatrix} -(\mu_{AD}+\mu_{AB}) & 0 & 0 & 0 & 0 \\ \mu_{AB} & -(\mu_{BD}+\mu_{BC}+\mu_{BC}+\gamma_2) & 0 & \mu_{BD} & 0 \\ 0 & \mu_{BC} & -(\mu_{CD}+\mu_{CE}+\gamma_3) & \mu_{DC} & 0 \\ \mu_{AD} & \mu_{BD} & \mu_{CD} & -(\mu_{DB}+\mu_{DC}+\mu_{DE}) & 0 \\ 0 & \mu_{BE} & \mu_{CE} & \mu_{DE} & -\gamma_1 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \\ D \\ E \end{bmatrix} + \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ 0 \\ 0 \end{bmatrix}. \tag{9}$$

**Definition 2.3** Let $h(x.y,t)$ be continuously differentiable and analytic, then the function spectrum,

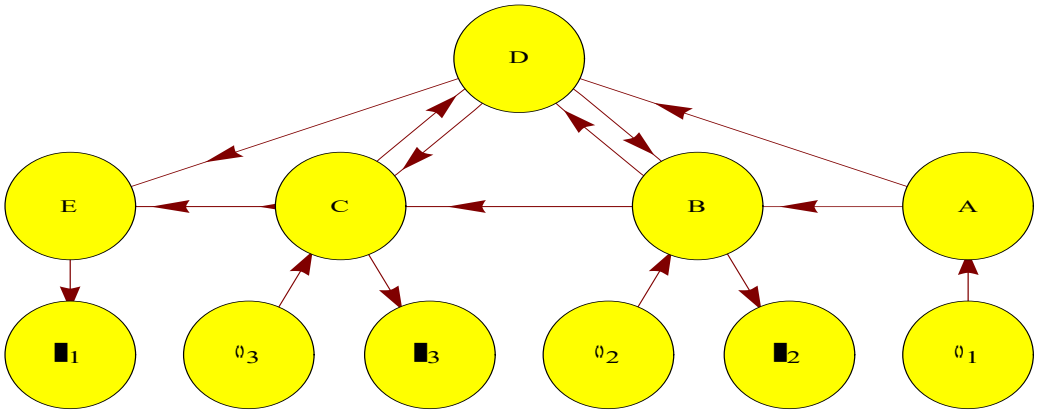


**Fig. 1:** Compartment diagram see [24,26,29,30].

**Fig. 1:** a compartment diagram (sign stream diagram) is a chart device that is utilized to show the interrelation between the framework states and empower us to utilize chart hypothetical instruments to find new elements of the system.

**Table 1:** obtain the physical meaning and description the all parameters.

| Parameter | Description |
|---|---|
| $\theta_1$ | Number of sickers analyzed to endure in phase 1 and 2 malignancy. |
| $\theta_2$ | Number of sickers analyzed to endure in phase 3 malignancy. |
| $\theta_3$ | Number of sickers analyzed to endure in phase 4 malignant growth. |
| $\mu_{AB}$ | Expanded rate from phase 1 or 2 to organize 3 (reformist ailment). |
| $\mu_{AD}$ | Pace of phase 1 or 2 sickers who experiment a total reaction. |
| $\mu_{BD}$ | Pace of phase 3 sickers who experiment a total reaction. |
| $\mu_{BC}$ | Expanded average from phase 3 to organize 4 (reformist ailment). |
| $\mu_{BE}$ | Pace of phase 3 malignant growth chemotherapy sickers who experiment cardiotoxic. |
| $\mu_{CD}$ | Pace of phase 4 sickers who experiment total reaction. |
| $\mu_{CE}$ | Pace of phase 4 malignant growth chemotherapy sickers who experiment cardiotoxic. |
| $\mu_{DB}$ | Pace of ailment release sickers who backslide back to arrange 3. |
| $\mu_{DC}$ | Pace of ailment release sickers who backslide back to arrange 4. |
| $\mu_{DE}$ | Pace of ailment release sickers who experiment cardiotoxic. |
| $\gamma_1$ | Demise pace of cardiotoxic sickers. |
| $\gamma_2$ | Demise pace of stage 3 malignant growth sickers. |
| $\gamma_3$ | Demise pace of stage 4 malignant growth sickers. |

## 3 Optimal Control for Fractional DNA Cancer Model

In this section, the optimal control for the fractional cancer system is explained:

$$
\begin{aligned}
&D^q A(t) = \theta_1 - \mu_{AD}A - \mu_{AB}A,\\
&D^q B(t) = \theta_2 + \mu_{AB}A + \mu_{DB}D - \mu_{BD}B - \mu_{BC}B - \mu_{BE}B - \gamma_2 B,\\
&D^q C(t) = \theta_3 + \mu_{BC}B + \mu_{DC}D - \mu_{CD}C - \mu_{CE}C - \gamma_3 C,\\
&D^q D(t) = \mu_{AD}A + \mu_{BD}B + \mu_{CD}C - \mu_{DB}D - \mu_{DC}D - \mu_{DE}D,\\
&D^q E(t) = \mu_{DE}D + \mu_{CE}C + \mu_{BE}B - \gamma_1 E.
\end{aligned}
\tag{10}
$$

According to references [11,12,13,14,15], [18,19,20,21, 29], written in Eqs. (9), in $R^5$, which be demonstrated as: $\Psi = \left\{ \left(\mu_{CD}(.), \mu_{BE}(.)\right) \in \left(L^\infty \times L^\infty\right) \middle| \, 0 \le \mu_{CD}(.), \mu_{BE}(.) \le 1, \quad \forall t \in [0, T_f] = [0,1] \right\}$, where $T_f$ is final time, $\mu_{CD}(.)$ is the average of phase 4 patients who experiment a full echo and $\mu_{BE}(.)$ average of phase 3 cancer chemotherapy patients who experiment cardiotoxic. $\mu_{CD}(.)$ and $\mu_{BE}(.)$ are the control functions. The space function norm $L^\infty$, $\|t\|_\infty = \max_j |t_j|$.

We explained the functional objective as:

$$
J(\mu_{CD}, \mu_{BE}) = \int_0^{T_f} \left[ a_1 A(t) + a_2 \mu_{CD}^2(t) + a_3 \mu_{BE}^2(t) \right] dt,
\tag{11}
$$

where $a_1$, $a_2$ and $a_3$ represent the measure of stage 4 patients who experiment a full echo and average of phase 3 cancer chemotherapy patients who experiment cardiotoxic respectively.

The minimize functions are defined

$$
J(\mu_{CD}, \mu_{BE}) = \int_0^{T_f} \phi\left[A, B, C, D, \mu_{CD}, \mu_{BE}, t\right] dt,
\tag{12}
$$

where

$$
\phi\left[A, B, C, D, \mu_{CD}, \mu_{BE}, t\right] = \left[ a_1 A(t) + a_2 \mu_{CD}^2(t) + a_3 \mu_{BE}^2(t) \right],
$$

subjected to the constraint

$$
D^q A = \xi_1, \quad D^q B = \xi_2, \quad D^q C = \xi_3, \quad D^q D = \xi_4, \quad D^q E = \xi_5,
\tag{13}
$$

where

$$
\xi_j = \xi\left(A, B, C, D, E, \mu_{CD}, \mu_{BE}, t\right), \qquad j = 1,2,3,4,5.
$$

Satisfied the initial cases:

$$
A(0) = A_1, \quad B(0) = B_1, \quad C(0) = C_1, \; D(0) = D_1, \; E(0) = E_1.
\tag{14}
$$

To purpose the fractional optimal control (FOCP), see [26,27,28,29]:

$$
J^{\upsilon} = \int_0^{T_f} \left[ \begin{array}{l} H\left(A, B, C, D, E, \mu_{CD}, \mu_{BE}, t\right) \\ -\sum_{i=1}^{5} \lambda_i \xi_i \left(A, B, C, D, E, \mu_{CD}, \mu_{BE}, t\right) \end{array} \right] dt, \; i = 1, \ldots, 5.
\tag{15}
$$

The objective Hamiltonian (15) and control equation (10) is given as:

$$
\begin{aligned}
H\left(A, B, C, D, E, \mu_{CD}, \mu_{BE}, t\right) &= \phi\left(A, B, C, D, E, \mu_{CD}, \mu_{BE}, t\right) \\
&\quad + \sum_{i=1}^{4} \lambda_i \xi_i \left(A, B, C, D, E, \mu_{CD}, \mu_{BE}, t\right),
\end{aligned}
\tag{16}
$$

then

$$
\begin{aligned}
H &= a_1 A + a_2 \mu_{CD}^2 + a_3 \mu_{BE}^2 + \lambda_1 \left[\theta_1 - \mu_{AD}A - \mu_{AB}A\right] \\
&\quad + \lambda_2 \left[\theta_2 + \mu_{AB}A + \mu_{DB}D - \mu_{BD}B - \mu_{BC}B - \mu_{BE}B - \gamma_2 B\right] \\
&\quad + \lambda_3 \left[\theta_3 + \mu_{BC}B + \mu_{DC}D - \mu_{CD}C - \mu_{CE}C - \gamma_3 C\right] \\
&\quad + \lambda_4 \left[\mu_{AD}A + \mu_{BD}B + \mu_{CD}C - \mu_{DB}D - \mu_{DC}D - \mu_{DE}D\right] \\
&\quad + \lambda_5 \left[\mu_{DE}D + \mu_{CE}C + \mu_{BE}B - \gamma_1 E\right].
\end{aligned}
\tag{17}
$$

The conditions necessary and sufficient see [26,27,28,29]

$$
D^q \lambda_1 = \frac{\partial H}{\partial A}, \quad D^q \lambda_2 = \frac{\partial H}{\partial B}, \quad D^q \lambda_3 = \frac{\partial H}{\partial C}, \quad D^q \lambda_4 = \frac{\partial H}{\partial D}, \quad D^q \lambda_5 = \frac{\partial H}{\partial E},
\tag{18}
$$

$$
\frac{\partial H}{\partial u_r} = 0, \quad r = 1, \; 2 \Rightarrow \quad \frac{\partial H}{\partial u_1} = 0, \quad \frac{\partial H}{\partial u_2} = 0,
\tag{19}
$$

$$
D^q A = \frac{\partial H}{\partial \lambda_1}, \quad D^q B = \frac{\partial H}{\partial \lambda_2}, \quad D^q C = \frac{\partial H}{\partial \lambda_3}, \quad D^q D = \frac{\partial H}{\partial \lambda_4}, \quad D^q E = \frac{\partial H}{\partial \lambda_5},
\tag{20}
$$

additionally,

$$
\lambda_i, (T_f) = 0,
\tag{21}
$$

where $\lambda_i$, $i = 1,2,3,4,5$ are the Lagrange multipliers. The following theorem fills our result:

**Theorem 1.**

If $\mu_{CD}$ and $\mu_{BE}$ are optimal controls with alike case $A^*, B^*, C^*, D^*$ and $E^*$ then there be adjoint variables $\lambda_j^*$, $j = 1,2,3,4,5$ fulfill the following:

**(i) Adjoint equations**

Putting the cases in the text theorem and putting equations (18) see, [26,27,28,29], we obtain the following five equations, that can be formed as follows:-

$$D^q\lambda_1^* = a_1 + \lambda_1^*\left[-\mu_{AD} - \mu_{AB}\right] + \lambda_2^*\left[\mu_{AB}\right] + \lambda_4^*\left[\mu_{AD}\right], \tag{22}$$

$$D^q\lambda_2^* = \lambda_2^*\left[-\mu_{BD} - \mu_{BC} - \mu_{BE} - \gamma_2\right] + \lambda_3^*\left[\mu_{BC}\right] + \lambda_4^*\left[\mu_{BD}\right] + \lambda_5^*\left[\mu_{BE}\right], \tag{23}$$

$$D^q\lambda_3^* = \lambda_3^*\left[-\mu_{CD} - \mu_{CE} - \gamma_3\right] + \lambda_4^*\left[\mu_{CD}\right] + \lambda_5^*\left[\mu_{CE}\right], \tag{24}$$

$$D^q\lambda_4^* = \lambda_2^*\left[\mu_{DB}\right] + \lambda_3^*\left[\mu_{DC}\right] + \lambda_4^*\left[-\mu_{DB} - \mu_{DC} - \mu_{DE}\right] + \lambda_5^*\left[\mu_{DE}\right], \tag{25}$$

$$D^q\lambda_4^* = \lambda_5^*\left[-\gamma_1\right]. \tag{26}$$

**(ii) Conditions transversality:**

$$\lambda_j^*(T_f) = 0, \quad j = 1,2,3,4,5. \tag{27}$$

**(iii) Conditions optimality**

$$H\left(A^*, B^*, C^*, D^*, E^*, \mu_{CD}^*, \mu_{BE}^*, \lambda_i\right) = \min_{0\le\mu_{CD}^*,\,\mu_{BE}^*\le 1} H^*\left(A^*, B^*, C^*, D^*, E^*, \mu_{CD}^*, \mu_{BE}^*, \lambda_i\right), \tag{28}$$

Furthermore, by putting equations (19), the functions control $u_1^*, u_2^*$ are obtained as:

$$\frac{\partial H}{\partial \mu_{CD}} = 0 \Rightarrow \mu_{CD} = \frac{C^*\left[\lambda_3^* - \lambda_4^*\right]}{2a_2}, \tag{29}$$

$$\frac{\partial H}{\partial \mu_{BE}} = 0 \Rightarrow \mu_{BE} = \frac{B^*\left[\lambda_2^* - \lambda_5^*\right]}{2a_2}, \tag{30}$$

$$\mu_{CD}^* = \min\left\{1, \quad \max\left\{0, \frac{C^*\left[\lambda_3^* - \lambda_4^*\right]}{2a_2}\right\}\right\}, \tag{31}$$

$$\mu_{BE}^* = \min\left\{1, \quad \max\left\{0, \frac{B^*\left[\lambda_2^* - \lambda_5^*\right]}{2a_2}\right\}\right\}. \tag{32}$$

**Proof.** The co-state system Eqs. (22)-(25) are found from Eq. (20) where the Hamiltonian $H^*$ is given by

$$H^* = a_1A^* + a_2\mu_{CD}^{*2} + a_3\mu_{BE}^{*2} + \lambda_1^*D^qA^* + \lambda_2^*D^qB^* + \lambda_3^*D^qC^* + \lambda_4^*D^qD^* + \lambda_4^*D^qE^*. \tag{33}$$

Then, the case in Eq. (21), and the optimal control written in Eqs. (31)–(32) can be derived from Eq. (19).

Putting $u_k^*$, $k = 1, 2$ in (10), then

$$\begin{aligned}
&D^qA^*(t) = \theta_1 - \mu_{AD}A^* - \mu_{AB}A^*,\\
&D^qB^*(t) = \theta_2 + \mu_{AB}A^* + \mu_{DB}D^* - \mu_{BD}B^* - \mu_{BC}B^* - \mu_{BE}^*B^* - \gamma_2B^*,\\
&D^qC^*(t) = \theta_3 + \mu_{BC}B^* + \mu_{DC}D^* - \mu_{CD}^*C^* - \mu_{CE}C^* - \gamma_3C^*,\\
&D^qD^*(t) = \mu_{AD}A^* + \mu_{BD}B^* + \mu_{CD}^*C^* - \mu_{DB}D^* - \mu_{DC}D^* - \mu_{DE}D^*,\\
&D^qE^*(t) = \mu_{DE}D^* + \mu_{CE}C^* + \mu_{BE}^*B^* - \gamma_1E^*.
\end{aligned} \tag{34}$$

For extra details on fractional optimal control, see [11,12,13, 14,15,18,19,20,21,29].

***Existence Solution (After Control):***

The existence solution of system (22)-(25) is to be shown here; it will be found in ([24], [26], [28]-[30]) as follows:

Let

$$\begin{aligned}
&f_1\left(\lambda_1^*, \lambda_2^*, \lambda_3^*, \lambda_4^*, \lambda_5^*\right) = a_1 + \lambda_1^*\left[-\mu_{AD} - \mu_{AB}\right] + \lambda_2^*\left[\mu_{AB}\right] + \lambda_4^*\left[\mu_{AD}\right],\\
&f_2\left(\lambda_1^*, \lambda_2^*, \lambda_3^*, \lambda_4^*, \lambda_5^*\right) = \lambda_2^*\left[-\mu_{BD} - \mu_{BC} - \mu_{BE} - \gamma_2\right] + \lambda_3^*\left[\mu_{BC}\right] + \lambda_4^*\left[\mu_{BD}\right] + \lambda_5^*\left[\mu_{BE}\right],\\
&f_3\left(\lambda_1^*, \lambda_2^*, \lambda_3^*, \lambda_4^*, \lambda_5^*\right) = \lambda_3^*\left[-\mu_{CD} - \mu_{CE} - \gamma_3\right] + \lambda_4^*\left[\mu_{CD}\right] + \lambda_5^*\left[\mu_{CE}\right],\\
&f_4\left(\lambda_1^*, \lambda_2^*, \lambda_3^*, \lambda_4^*, \lambda_5^*\right) = \lambda_2^*\left[\mu_{DB}\right] + \lambda_3^*\left[\mu_{DC}\right] + \lambda_4^*\left[-\mu_{DB} - \mu_{DC} - \mu_{DE}\right] + \lambda_5^*\left[\mu_{DE}\right],\\
&f_5\left(\lambda_1^*, \lambda_2^*, \lambda_3^*, \lambda_4^*, \lambda_5^*\right) = \lambda_5^*\left[-\gamma_1\right].
\end{aligned}$$

Let

$$\Phi = \left\{\lambda_j^{\#} \in R : \left|\lambda_j^{\#}\right| \le c, \quad j = 1,2,3,4,5, \quad t \in [0,T\right\}.$$

This proposes that all of the five functions $f_1, f_2, f_3, f_4$ and $f_5$ coincides accept or except with the Lipschitz case. For extra details on the existence and uniqueness, see [24, 26,28,29,30].

## 4 Implementation of the RDTM

Here, using RDTM to build an approximate analytic solution of model (10).

Assume that the initial conditions are

$$A(0) = A_0, \; B(0) = B_0, \; C(0) = C_0, \; D(0) = D_0, \; E(0) = E_0.$$

(35)

Effecting RDTM to Eq. (10), we get recurrence relations as:

$$A_{k+1}(t)=\frac{\Gamma(k\alpha+1)}{\Gamma[\alpha(k+1)+1]}\left[\theta_1\delta(\mathrm{k})-\mu_{AD}A_k(\mathrm{t})-\mu_{AB}A_k(\mathrm{t})\right],$$

$$B_{k+1}(t)=\frac{\Gamma(k\alpha+1)}{\Gamma[\alpha(k+1)+1]}\left[\theta_2\delta(\mathrm{k})+\mu_{AB}A_k(\mathrm{t})+\mu_{DB}D_k(\mathrm{t})-\mu_{BD}B_k(\mathrm{t})-\mu_{BC}B_k(\mathrm{t})-\mu_{BE}B_k(\mathrm{t})-\gamma_2B_k(\mathrm{t})\right],$$

$$C_{k+1}(t)=\frac{\Gamma(k\alpha+1)}{\Gamma[\alpha(k+1)+1]}[\theta_3\delta(\mathrm{k})+\mu_{BC}B_k(\mathrm{t})+\mu_{DC}D_k(\mathrm{t})-\mu_{CD}C_k(\mathrm{t})-\mu_{CE}C_k(\mathrm{t})-\gamma_3C_k(\mathrm{t})],$$

$$D_{k+1}(t)=\frac{\Gamma(k\alpha+1)}{\Gamma[\alpha(k+1)+1]}\left[\mu_{AD}A_k(\mathrm{t})+\mu_{BD}B_k(\mathrm{t})+\mu_{CD}C_k(\mathrm{t})-\mu_{DB}D_k(\mathrm{t})-\mu_{DC}D_k(\mathrm{t})-\mu_{DE}D_k(\mathrm{t})\right],$$

$$E_{k+1}(t)=\frac{\Gamma(k\alpha+1)}{\Gamma[\alpha(k+1)+1]}\left[\mu_{DE}D_k(\mathrm{t})+\mu_{CE}C_k(\mathrm{t})+\mu_{BE}B_k(\mathrm{t})-\gamma_1E_k(\mathrm{t})\right]. \tag{36}$$

Case 1, when

$\mu_{AD}=0.63,\ \mu_{BD}=0.35,\mu_{BC}=0.62,\mu_{BE}=0.30,\mu_{DC}=0.42,\mu_{DB}=0.36,\mu_{DE}=0.30,\mu_{CD}=0.10,$
$A(0)=14,\ B(0)=30,\ C(0)=20,\ D(0)=10,\ E(0)=10,\theta_1=5,\theta_2=20,\theta_3=11,\mu_{AB}=0.56,$
$\mu_{CE}=0.30,\gamma_1=0.4,\gamma_2=0.5,\text{and}\,\gamma_3=0.8.$

Equations (36) and initial conditions (35), provide

$$\mathrm{A}_1=\frac{-11.66}{\Gamma(\alpha+1)},\ B_1=\frac{21.6600}{\Gamma(\alpha+1)},\ C_1=\frac{9.8}{\Gamma(\alpha+1)},\ D_1=\frac{10.52}{\Gamma(\alpha+1)},\ E_1=\frac{14}{\Gamma(\alpha+1)},\mathrm{A}_2=\frac{13.8754}{\Gamma(2\alpha+1)},$$
$$B_2=\frac{35.595800}{\Gamma(2\alpha+1)},\ C_2=-\frac{20.770800}{\Gamma(2\alpha+1)},\ D_2=-\frac{25.3083999}{\Gamma(2\alpha+1)},\ E_2=-\frac{6.00200}{\Gamma(2\alpha+1)}, \tag{37}$$

and so on.

Mathematic 9.0 program, the strain of A(t), B(t), C(t), D(t), and E(t) are gained to the tenth refinement but two only are formed here for naivety. Then, in the following solutions the differential inverse transforms to create:

$$A=\sum_{k=0}^{\infty}A_k(t)\gamma^{k\alpha},\ B=\sum_{k=0}^{\infty}B_k(t)\gamma^{k\alpha},\ C=\sum_{k=0}^{\infty}C_k(t)\gamma^{k\alpha},\ D=\sum_{k=0}^{\infty}D_k(t)\gamma^{k\alpha},\ E=\sum_{k=0}^{\infty}E_k(t)\gamma^{k\alpha} \tag{38}$$

In Eqs. (37) and (38), the analytic approximate solutions will be written as:

$$A(t)=A_0(\mathrm{t})+A_1(\mathrm{t})t^{\alpha}+A_2(\mathrm{t})t^{2\alpha}+\ldots$$
$$B(t)=B_0(\mathrm{t})+B_1(\mathrm{t})t^{\alpha}+B_2(\mathrm{t})t^{2\alpha}+\ldots$$
$$C(t)=C_0(\mathrm{t})+C_1(\mathrm{t})t^{\alpha}+C_2(\mathrm{t})t^{2\alpha}+\ldots$$
$$D(t)=D_0(\mathrm{t})+D_1(\mathrm{t})t^{\alpha}+D_2(\mathrm{t})t^{2\alpha}+\ldots$$
$$E(t)=E_0(\mathrm{t})+E_1(\mathrm{t})t^{\alpha}+E_2(\mathrm{t})t^{2\alpha}+\ldots \tag{39}$$

**5 Numerical Simulations**

Here, we proved the conclusions of the dissection with numerical simulations

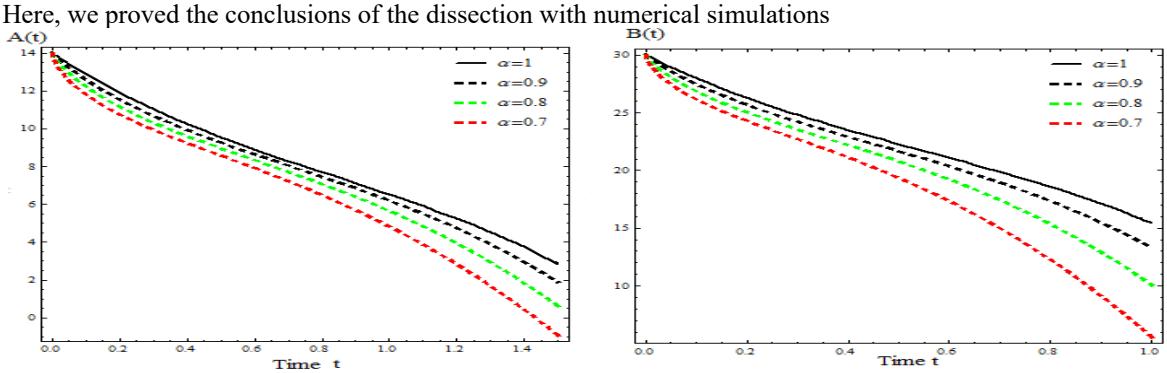


**Fig. 2**: The link between A(t), B(t) and t with different $\alpha$ .

Fig. 2, shows that when $\alpha = 0.7$ the patients with Carcinoma Ductal In Situ cancer in stage 1,2 from the initial case dropped from 14 patients to zero patients after 1.4 times period, when $\alpha = 0.8$ the patients with Ductal Carcinoma In Situ cancer from the primary case dropped 14 patients to 1 patients after 1.4 times and when $\alpha = 0.9,1$.

Fig. 3, shows that when $\alpha = 0.7$ the cancer patients in stage 4 from the initial case increased from 20 patients to 110 patients after 30 times, whereas when $\alpha = 0.8$ the cancer patients in stage 4 from the initial case the number of patients increased from 20 to 112 patients after 30 times and when $\alpha = 0.9$ the cancer patients from the initial case increased from 20 patients to 120 patients after 30 times. Finally when $\alpha = 1$ the cancer patients in stage 4 from the initial case increased from 20 patients to 130 patients after 3 times.

Fig. 4, shows that when $\alpha = 0.7$ cancer patients who have cardiotoxicity from the initial case increased from 10 patients to 21 patients after 1.5 times period, then decreased from 21 patients to 18 patient after 1.5 minutes when $\alpha = 0.8$ cancer patients who have cardiotoxicity from the initial case increased from 10 patients to 22 patients in equilibrium cases after 1.5 times. Then it decreased from 22 patients to 19 patient after 1.5 minutes. When $\alpha = 0.9$ cancer patients who have cardiotoxicity from the initial case increased from 10 patients to 23 patients after 1.5 times period, then decreased from 23 patients to 20 patients after 1.5 minutes. Finally, when $\alpha = 1$ cancer patients who are cardiotoxicity from the initial case increased from 10 patients to 26 patients after 1.5 times period, and then decreased from 21 patients to 22 patients after 1.5 times.

Fig. 5, shows that when $\alpha = 0.95$, the patients with Carcinoma Ductal. In Situ cancer from the initial case dropped from 14 patients to 4 patients cases after 1.5 times. The cancer patients from the initial case dropped from 30 patients to zero patients in equilibrium cases after 1.5 times. Also, the cancer patients in stage 4 from the initial case increased from 20 patients to 40 patients in equilibrium cases after 1.5 times period, cancer patients who are cardiotoxicity from the initial case increased from 10 patients to 25 patients in equilibrium conditions after 1.5 times period, cancer patients who are cardiotoxicity from the initial case increased from 10 patients to 24 patients case after 1.5 times.

Fig. 6, shows that when $\alpha = 0.85$ the patients with Carcinoma Ductal. In cancer Situ from the initial case dropped from 14 patients to 2 patients after 1.2 times period, the cancer patients from the initial case dropped from 30 patients to 4 patients 1.2 times. Moreover, the cancer patients in stage 4 from the initial case increased from 20 patients to 30 patients after 1.2 times period, cancer patients who have cardiotoxicity from the initial case increased from 10 patients to 20 patients s after 1.2 times, cancer patients who have cardiotoxicity from the initial case increased from 10 patients to 20 patients after 1.2 times.

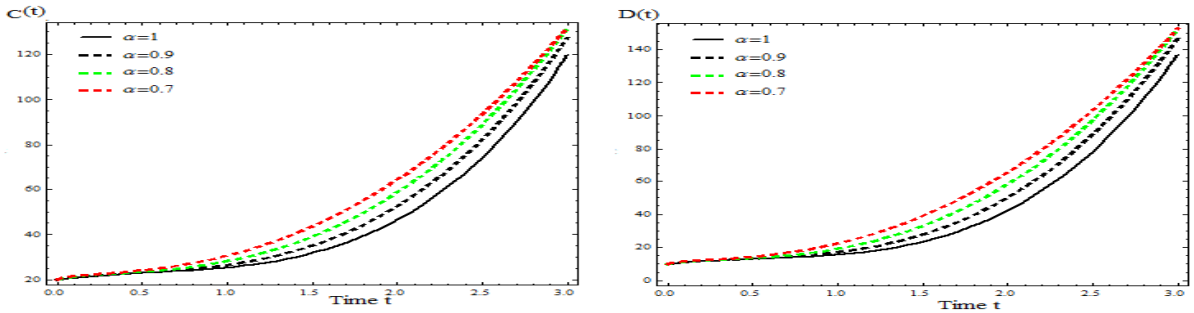


**Fig. 3:** The link between C(t), D(t) and t with different $\alpha$.

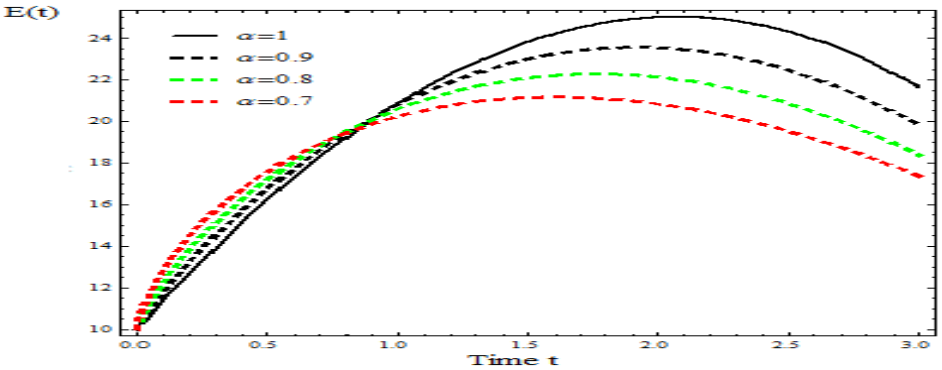


**Fig. 4:** The link between E(t), and t with different $\alpha$.

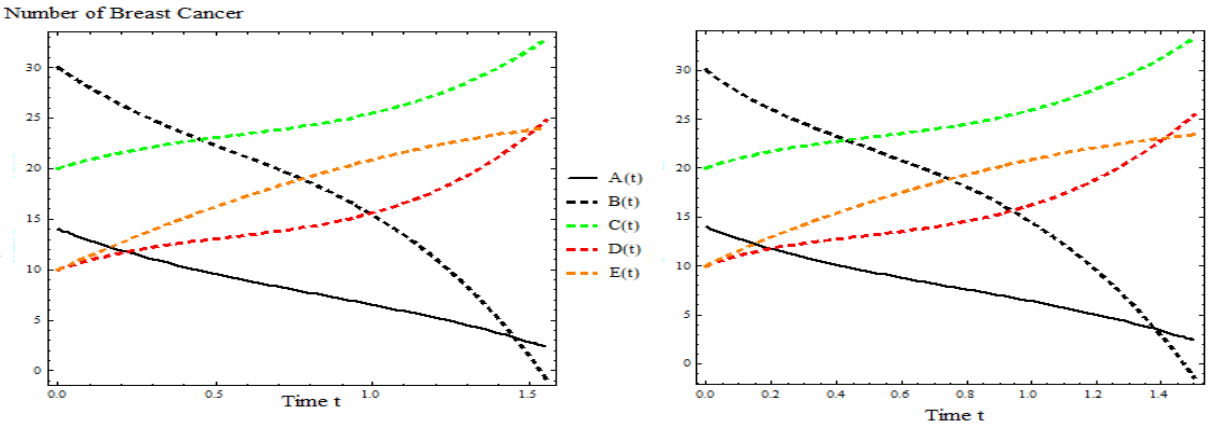


**Fig.5:** The relation between A(t), B(t), C(t), D(t) and E(t) at $\alpha = 1,\ 0.95.$

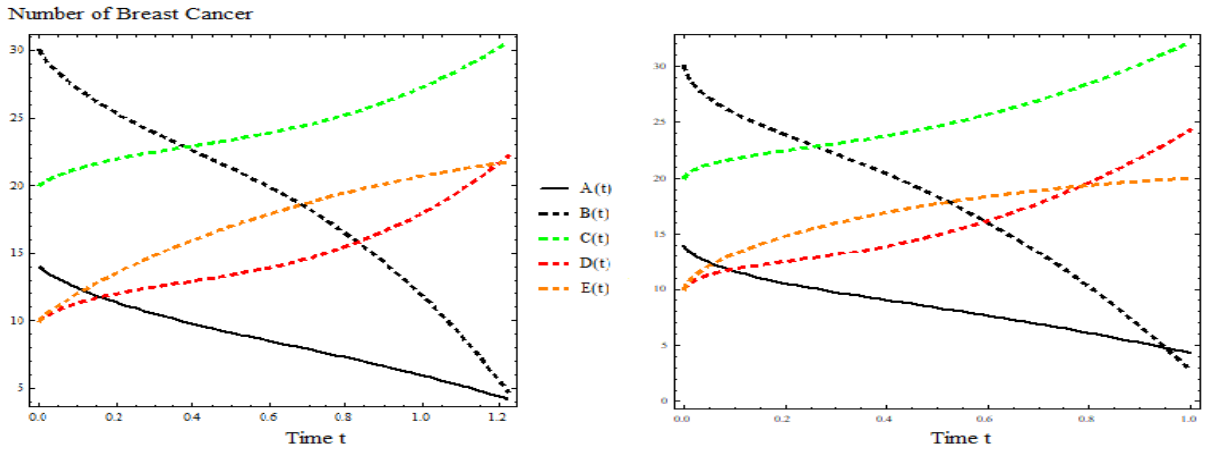


**Fig. 6:** The relation between A(t), B(t), C(t), D(t) and E(t) at $\alpha = 0.85$ and $\alpha = 0.85$.

**Case 2, when**

$A(0) = 14,\ B(0) = 30,\ C(0) = 20,\ D(0) = 10,\ E(0) = 10, \theta_1 = 5, \theta_2 = 20, \theta_3 = 11, \mu_{AB} = 0.56,$
$\mu_{AD} = 0.63, \mu_{BD} = 0.35, \mu_{BC} = 0.62, \mu_{BE} = 0.30, \mu_{DC} = 0.1, \mu_{DB} = 0.1, \mu_{DE} = 0.30, \mu_{CD} = 0.10,$
$\mu_{CE} = 0.30, \gamma_1 = 0.4, \gamma_2 = 0.5, \text{and } \gamma_3 = 0.8.$

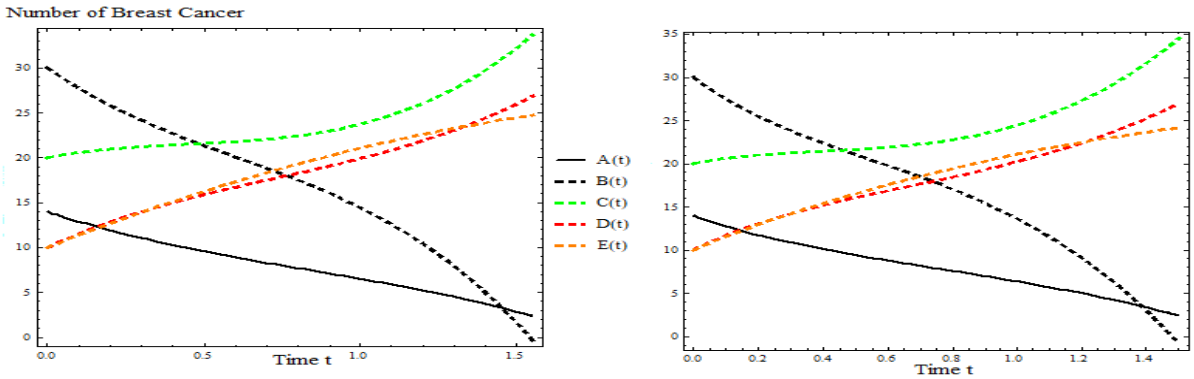


**Fig. 7:** The link between A(t), B(t), C(t), D(t) and E(t) at $\mu_{DC} = \mu_{DB} = 0.1,\ \alpha = 1,\ \alpha$=0.95.

Fig.7, shows that when $\mu_{DC} = \mu_{DB} = 0.1, \alpha = 1,$ $\alpha$=0.95. Ductal Carcinoma patients with cancer Situ of the initial case dropped from 15 patients to 4 patients after 1.5 times period, the cancer patients from the initial case dropped from 30 patients to 0 patients in equilibrium cases after 1.5 times period, the cancer patients in stage 4 from the initial case increased from 20 patients to 40 patients after 1.5 times. Additionally, cancer patients who have cardiotoxicity from the initial case increased from 10 patients to 25 patients after 1.5 times. Cancer patients who are cardiotoxicity from the initial case increased from 10 patients to 23 patients 1.5 times.

Fig. 8, shows that when $\mu_{DC} = \mu_{DB} = 0.1,$ $\alpha$=0.85. The patients with Ductal Carcinoma In Situ malignant growth from the underlying condition dropped from 15 patients to 3 patients after 1.2 times. The disease patients from the underlying condition dropped from 30 patients to 6 patients in harmony conditions after 1.2 times. The malignant growth patients in stage 4 from the underlying condition expanded from 20 patients to 35 patients after 1.2 times. Malignant growth patients who have cardiotoxicity from the underlying condition expanded from 10 patients to 25 patients after 1.2 times. Also, malignant growth patients who have cardiotoxicity from the underlying condition expanded from 10 patients to 21 patients in balance conditions after 1.2 times.

This current approach's effectiveness is significantly improved by counting further expressions of A(t), B(t), C(t), D(t), and E(t) by utilizing RDTM. Fig. 2 to Fig. 9, shows the reactions are various. The exactness of the RDTM is cleared in Fig. 2 to Fig. 8. Likewise, the time-partial attributes of bosom malignant growth stages with results on the heart are shown by different estimations of alpha. Using this arrangement of starting qualities, the impact of cancer on various sorts of bosom malignant growth stages with results on the heart has plainly revealed. These figures show the end mentality of weighty cancer if not ended. All mathematical reenactments are executed with the assistance of Mathematica 9.0.

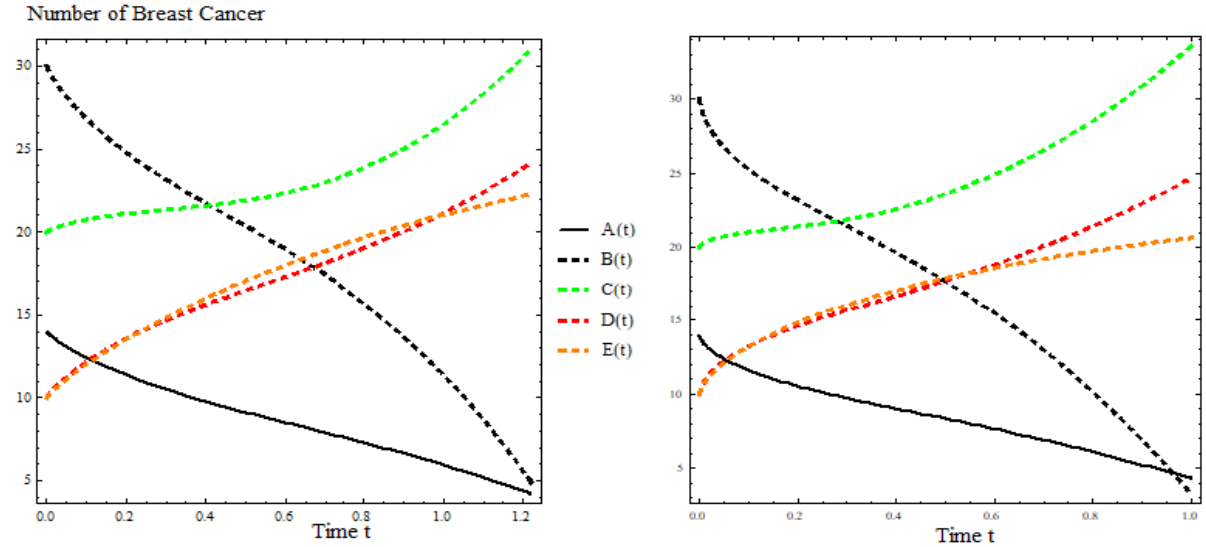


**Fig. 8:** The link between A(t), B(t), C(t), D(t) and E(t) at $\mu_{DC} = \mu_{DB} = 0.1, \alpha = 0.85,$ $\alpha$=1.

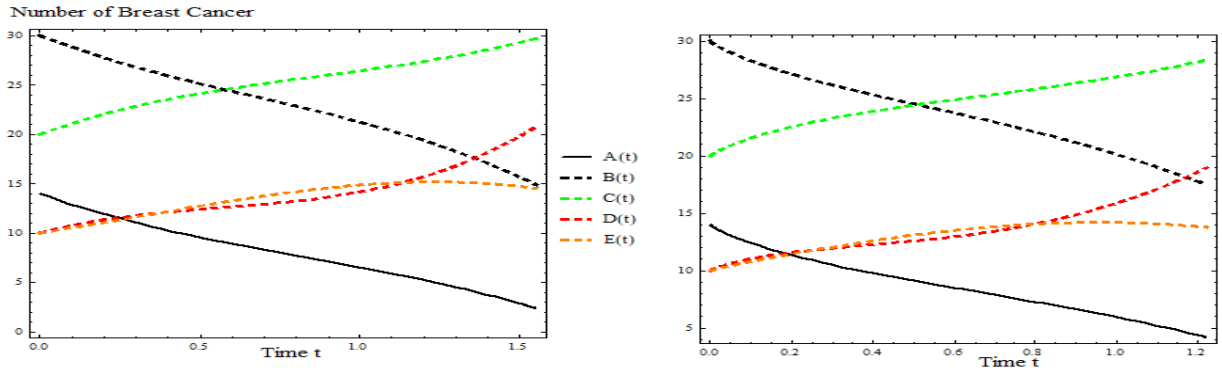


**Fig. 9:** The relation between A, B, C, D and E after control at $\mu_{CD} = \mu_{BE} = 0,$ $\alpha = 0.85,$ 1.

## 6 Conclusions

In this essay, a novel fractional numerical model of breast cancer was studied. The optimal control of this model is discussed. The reduced transform method is used to get the solutions approximate of the model. The equilibrium point stability of this model is studied. Formulation of optimal control. A numerical technique for the simulation of the control problem is introduced. Moreover, a numerical method is discussed with its stability analysis. Numerical simulations can explain the suggested idea. The compartment diagram of the model is done. The mathematical strategy utilized in this original copy to tackle this model has not been used by any researcher before that. Additionally, this model with fractional derivatives characterized in this manner has not been examined before that. The procedures used are not difficult to impact, regardless of whether scientific or mathematical and give great results.

**Conflicts of interest**: The authors declare that they have no conflicts of interest to report regarding the present study.

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