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Some Properties and Application of Modified Jacknified Liu-Type Negative Binomial Ridge Regression

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Abstract: The aim of this paper is to introduce a new estimator for the modified Jackknified Liu Type Negative Binomial. As in the presence of multicollinearity, the Maximum Likelihood Estimator (MLE) is unable to produce valid statistical inference. So, this paper is designed to solve the problem of multicollinearity. Several ridge regression estimators are used for this purpose. Moreover, Monte Carlo simulation and real life data set are applied on the proposed and existing estimators to evaluate the performance of proposed estimator in the case of MSE. The results reveal that our proposed estimator has best performance among all other estimators $(i.e. ML, NBRR, LTNB, JNBR)$.

Keywords: Negative Binomial Regression, Jacknified, Liu Type, Multicollinearity, MSE, MLE, Negative Binomial Ridge Regression, JNBR, LTNBR.

1 Introduction

Multicollinearity arises when two or more regressors correlated with each other and cause serious disturbing for the maximum likelihood (ML) estimates. The presence of higher dependency among explanatory variables in any regression causes misleading information such as explosive estimates and bias, large inflated variance of regression coefficients, and even lessen power in prediction [1,2,3,4,5,6]. The linear regression model is given as:

$$y = X\beta + \varepsilon, \tag{1}$$

where y is observable random vector of order $(n*1)$, X is non stochastic independent variable of order $(n*p)$ with p refers to the number of independent variables, β is the regression coefficient of order $(p*1)$ and ε is the error term of $(n*1)$ with $E[\varepsilon] = 0$, $cov[\varepsilon] = \sigma^2 I$. In some real-life applications, the data observations are not independent and identically distributed (iid), such as non-negative integers or count observations. Researchers are usually interested in finding the regression model observation for real-life context. Thus, the negative binomial regression is a type of generalized linear model used when the regressor is in the form of non-negative integers or counts [7,8,9,10,11,12]. The density function of traditional negative binomial of dependent variable Y is given by

$$P(Y = y_i|m,\theta) = \frac{\Gamma(y+\theta^{-1})}{\Gamma(y+1)\Gamma\theta^{-1}}\left(\frac{\theta^{-1}}{\theta^{-1}+m_i}\right)^{\theta^{-1}} \left(\frac{m_i}{\theta^{-1}+m_i}\right)^{y_i} \tag{2}$$

The conditional mean m and variance of this distribution are given respectively as:

$$E(y_i|x_i) = m_i, \tag{3}$$

$$cov(y_i|x_i) = m_i(1+\theta m_i) \tag{4}$$

Let x_i be the ith row of X_{n*p} with p independent variables and let γ_{n*p} vector of coefficients obtained by

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ML as

$$L=\sum_{i=0}^{n}\left\{ln\left[\Gamma(y_i+\theta^{-1})\right]-ln\left[\Gamma\theta^{-1}\right]-ln\Gamma(y+1)\right.$$
$$\left.-\theta^{-1}ln\left(1+\theta m_i\right)-y_i ln\left(1+\theta m_i\right)+y_i ln\left(\theta\right)\right\}. \quad (5)$$

It gives

$$L=\sum_{i=0}^{n}\left\{\left(\sum_{j=0}^{y_i-1}ln\left(j+\theta^{-1}\right)\right)-ln\left[\Gamma\left(y_i+1\right)\right]\right.$$
$$\left.-\left(y_i+\theta^{-1}\right)ln\left(1+\theta m_i\right)+y_i ln(m_i)+y_i ln\theta\right\}. \quad (6)$$

The mean of y_i is determine using the following expression.

$$L=\sum_{i=0}^{n}\left\{\left(\sum_{j=0}^{y_i-1}ln\left(j+\theta^{-1}\right)\right)-ln\left[\Gamma\left(y_i+1\right)\right]\right.$$
$$\left.-\left(y_i+\theta^{-1}\right)ln\left(1+\theta\exp(x_i^t\gamma)\right)+y_i ln(x_i^t\gamma)+y_i ln\theta\right\} \quad (7)$$

Take derivatives to get

$$S(\gamma)=\frac{\partial L(\theta,\gamma)}{\partial\gamma}=\sum_{i=0}^{n}\frac{(y_i-m_i)}{1+\theta m_i}x_i=0. \quad (8)$$

Since the Eq. (7) is non linear in γ, we can use the IWLS algorithm as

$$\hat{\gamma}_{ML}=\left(X^T\hat{W}X\right)^{-1}X^T\hat{W}_{\hat{v}}, \quad (9)$$

where the ith element of vector $\hat{v}$ is

$$\hat{v}=log(\hat{m}_i)+\frac{y_i-\hat{m}_i}{\hat{m}_i}. \quad (10)$$

However, the matrix $X^T\hat{W}X$ is unsatisfactory in the presence of multicollinearity. To overcome this problem, Månsson [33] proposed Ridge Regression estimator for NBR as

$$\hat{\gamma}_{NBR}=\left(X^T\hat{W}X+KI\right)^{-1}X^T\hat{W}X\hat{\gamma_{ML}}. \quad (11)$$

This is shrinkage method used to minimize weighted sum of square error, where the value of 'k' lies between 0 to ∞. The most useful method to address the problem of multicollinearity is the ridge regression (RR) analysis proposed by Hoerl and Kennard [20]. The RR estimation method depends on the value of 'k', which is small positive number to the diagnol of X^TX matrix which gives biased estimator and smaller MSE than OLS. In linear regression of LR analysis,the RR estimator is defined as

$$\hat{\gamma}_{RR}=\left(Z^TZ+KI\right)^{-1}Z^Ty, K\geq 0, \quad (12)$$

where estimated parameter $\hat{\gamma}_{RR}$ is $p\times 1$ vector, Z is an $n\times p$ matrix of n observation on p explanatory variables. Many contribution has been conducted to estimate the biasing parameter k; we cite for examples, [19,27,18,15, 24,12,37,22,36]. Månsson proposed an estimator to overcome the problem of Multicollinearity in Negative Binomial Ridge Regression ($NBRR$) [33]. A two-parameter estimator for NB regression model was developed by Huang and Yang [21] to control the problem of Multicollinearity. Månsson et al. in ref. [35] constructed a restricted Liu-type for binary regression model. A new biased estimator for NB regression model was proposed by Asar et. al. [10], and later, Asar [11] constructed a restricted two parameter Liu-type estimator for binary logistic regression. For works on the same area, we cite [26], among many others. Another estimator that composed the advantage of Stein estimator and ORRE is proposed by Liu [29], which is given by,

$$\hat{\gamma_{LE}}=\left(Z^TZ+I\right)^{-1}\left(Zy+d\hat{\gamma}_{OLS}\right) \quad (13)$$
$$\hat{\gamma_{LE}}=\left(Z^TZ+I\right)^{-1}\left(Z^TZ+dI\right)\hat{\gamma}_{OLS}, \quad 0<d<1. \quad (14)$$

Here, the LE is more preferable choice than ORE for the selection of d, because LE is linear function of biasing parameter d. Kaçiranlar [23] proposed a modified version of LE, and concluded that it is superior than usual LE in terms of MSE matrix. Here, ORRE and LE are based on OLSE, which is not suitable estimator for multicollinearity. To overcome this problem, Liu [30] developed Liu-type estimator (LTE), defined by

$$\hat{\gamma}_{LT}=\left(Z^TZ+kI\right)^{-1}\left(Zy-d\hat{\gamma}^*\right), \quad (15)$$
$$k>0,\ -\infty<d<\infty.$$

The estimator in (15) contains two biasing parameter k and d, and $\hat{\gamma}^*$ is based on any estimator of γ, i.e., in case of low level of multicollinearity, $\hat{\gamma}^*=\hat{\gamma}_{OLS}$ is used, where $\hat{\gamma}^*=\hat{\gamma}_{ORR}$ used for high level of multicollinearity.
Further, Ozkale and Kaçiranlar [38] developed estimator for solving the multicollinearity problem, defined as:

$$\hat{\gamma_{TP}}=\left(Z^TZ+I\right)^{-1}\left(Zy+Kd\right)\hat{\gamma}_{OLS} \quad (16)$$
$$\hat{\gamma_{TP}}=\left(Z^TZ+I\right)^{-1}\left(\Lambda+KdI\right)\hat{\gamma}_{OLS}, \quad (17)$$
$$k>0,\ -\infty<d<\infty$$

For different value of parameters (i.e. k and d), the generalized estimator produces OREE, LE, OLSE and contraction estimators. Sakallioglu and Kaçiranlar [39] proposed a new biased estimator, known as $(K-d)$ class estimator. The main idea behind their estimator is to replaced $\hat{\gamma}_{OLS}$ by $\hat{\gamma}_{ORR}$ in LE estimator to become more consistent biased estimator. Their estimator is defined as,

$$\hat{\gamma_{TP}}=\left(Z^TZ+I\right)^{-1}\left(Zy+d\right)\hat{\gamma}_{ORR}, \quad (18)$$
$$\gamma_{\hat{(k-d)}}=\left(\Lambda+I\right)^{-1}\left(\Lambda+(d+k)I\right)\hat{\gamma}_{ORR}, \quad (19)$$
$$k>0,\ -\infty<d<\infty.$$

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The above estimator is generalized based on two biasing parameter k and d, which also produces OLSE, OREE and LE as special cases. Recently, Ahmad and Aslam [7] proposed a biased estimator by modifying NTPE; the new estimator becomes MNTPE, i.e.,

$$\gamma_{M\hat{N}TP} = (\Lambda + I)^{-1} (\Lambda + (d - kd)\hat{\gamma}^{\star}) \quad (20)$$

$$\gamma_{M\hat{N}TP} = (\Lambda + I)^{-1} (\Lambda + dI)\hat{\gamma}^{\star} k > 0, 0 < d < 1 \quad (21)$$

where $\hat{\gamma}^{\star} = (\Lambda + kdI)^{-1} Z' y$. This MNTP estimator can be reduced to LE, OLSE and OREE at different values of k and d. Based on the contribution of [7], we construct a new biased estimator. The new estimator reduces the problem of multicollinearity in linear regression, and we call it Modified Liu Ridge type estimator (MLRTE) which is the special case of OLSE, LE, OREE and MRTE, i.e.,

$$\gamma_{M\hat{L}RT} = (\Lambda + I)^{-1} \left(Z' y + (d - k(1+d)I)\hat{\gamma}^{\star}_{MRT} \right) \quad (22)$$

The rest of the article is organized as follows. The construction of Modeified Jackknified Liu Type for Negative Binomial Ridge Regression Estimator is presented In Section 2. Some Ridge parameters are presented in Section 3. In Section **??**, Monte Carlo Simulation is carried out to check the efficiency of proposed estimator. Section **??** is devoted to results and discussions. It also consider real life data set to further check the performance of proposed estimator. Finally, our conclusion and remarks are presented in Section **??**.

2 Construction of Modified Jacknified Liu-Type Negative Binomial Ridge Regression Estimator

In this section, a new modified jacknified Liu-type negative binomial ridge regression is proposed. Then, the Bias and MSEM properties of the proposed estimator are obtained and ridge parameter, k, is described under different estimation methods. Let $C = (c_1, c_2, ..., c_p)$ be $p \times p$ matrix whose columns are normalized eigen vector of $X^T \hat{W} X$ and $\Lambda_{NBRR} = diag(\lambda_{1NBRR}, \lambda_{2NBRR}, ...\lambda_{pNBRR})$ such that $C^T X^T \hat{W} X C = Z^T \hat{W} Z = \Lambda_{NBRR}$ and $Z = XC$. The NB estimators of $\hat{\alpha}_{ML}$ in Eq. (9) can be represented as

$$\hat{\gamma}_{ML} = \Lambda^{-1}_{NBRR} Z^T \hat{W} \hat{v}. \quad (23)$$

Then, the NBRR gives

$$\hat{\gamma}_{NBRR} = (\Lambda_{NBRR} + KI)^{-1} Z^T \hat{W} \hat{v} \quad (24)$$

$$= (I - KA)\hat{\gamma}_{ML}, \quad (25)$$

where $A = (\Lambda_{NBRR} + KI)$ and $K > 0$. Following Turkan and Ozel [40] proposed NBRR (of ith observation deleted), we define

$$\hat{\gamma}_{[-i]NBRR} = \left(Z^T_{-i} \hat{W}_{[-i]} Z_{-i} + KI\right)^{-1} (k+d) Z^T_{-i} \hat{W}_{[-i]} \hat{v}_{-i}. \quad (26)$$

The matrix $W_{[-i]}$ denotes the i^{th} row and column deleted, the vector v_{-i} and matrix Z_{-i} denote the i^{th} row deleted respectively. We also have $Z^T_{-i} \hat{W} \hat{v}_{-i} = Z^T \hat{W}_{[-i]} \hat{v} - Z^T_{-i} \hat{v}_i$ and the inverse of the matrix $\left(Z^T_{-i} \hat{W}_{[-i]} Z_{-i} + KI\right)^{-1}$ is obtained from the Sherman-Morrison Woodbury Theorem as

$$\left(Z^T_{-i} \hat{W}_{[-i]} Z_{-i} + KI\right)^{-1} = \left(Z^T \hat{W} Z + KI\right)^{-1} + \frac{\left(Z^T \hat{W} Z + KI\right)^{-1} Z_i Z_i^T \left(Z^T \hat{W} Z + KI\right)^{-1}}{1 - Z_i^T \left(Z^T \hat{W} Z + KI\right)^{-1} Z_i}. \quad (27)$$

Then, the Liu-type Negative Binomial regression is defined as

$$\hat{\gamma}_{LTNBRR} = \frac{\left(Z^T \hat{W} Z + KI\right)^{-1} (k+d) \left(\hat{v}_i - Z^T \hat{\gamma}_{NBRR}\right)}{1 - Z_i^T \left(Z^T \hat{W} Z + KI\right)^{-1} (k+d) Z_i}, \quad (28)$$

where $\hat{\gamma}_{NBRR} = \left(Z^T \hat{W} Z + KI\right)^{-1} Z^T \hat{W} \hat{v}$. Following [16], the Liu-type Jacknified Negative Binomial Ridge Regression ($LTJNBRR$) is derived as

$$\hat{\gamma}_{LTJNBRR} = \hat{\gamma}_{NBRR} + \left(Z^T \hat{W} Z + KI\right)^{-1} (k+d) \sum Z^T \frac{\hat{\mu}}{1 + \left(\frac{1}{\theta}\right) \hat{\mu}_i} \left(\hat{s}_i - Z^T \hat{\gamma}_{NBRR}\right), \quad (29)$$

$$\hat{\gamma}_{LTJNBRR} = \hat{\gamma}_{NBRR} + B^{-1} (k+d) \sum Z^T \hat{W} \left(\hat{s}_i - Z^T \hat{\gamma}_{NBRR}\right). \quad (30)$$

The above equation can be simplified as

$$\hat{\gamma}_{LTJNBRR} = (I + (k+d))\hat{\gamma}_{NBRR}, \quad (31)$$

where $\beta^{-1} = \Lambda_{NBRR} + KI$. Thus, the modified Jacknified Liu-type Negative Binomial Ridge Regression is defined as:

$$\hat{\gamma}_{MLTJNBRR} = \left(I - (k+d)^2 \beta^{-2}\right) \left(I - (K+d)\beta^{-1}\right)\hat{\gamma}_{ML}. \quad (32)$$

The expression for Bias and MSEM of modified jacknified Liu-type negative binomial regression is defined by

$$Bias = -(k+d)(\Lambda + kI)^{-1} W (\Lambda + KI)^{-1} \gamma \quad (33)$$

$$= -(k+d)\beta^{-1} W \beta^{-1} \gamma,$$

$$MSEM = \sigma^2 \phi \Lambda^{-1} \phi + (K+d)^2 \left[\beta^{-1} W \beta^{-1}\right] \gamma\gamma \left[\beta^{-1} W \beta^{-1}\right]', \quad (34)$$

where

$$W = I + (K+d)(\Lambda + KI)^{-1} - (K+d)^2 (\Lambda + KI)^{-2}$$

$$\phi = \left(2I - \left(I + (K+d)(\Lambda + KI)^{-1}\right) (K+d)^2 (\Lambda + KI)^{-2}\right)$$

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3 The Estimators of Ridge Parameter

Several contributions are being considered for estimating ridge parameters; see for examples, [18,24,37,8,9]. In the following, we present the most standard ridge estimators because they will be used later for comparison with our proposed estimator. [18] proposed estimator as

$$K_1 = \frac{\hat{\varsigma}^2}{\hat{\vartheta}^2_{max}}, \tag{35}$$

where $\hat{\varsigma}^2 = \frac{\Sigma(y_i-\hat{\mu}_i)^2}{n-p-1}$ and $\hat{\vartheta}^2_{max}$ is the i^{th} element of $\delta\hat{\alpha}_{ML}$ which is the maximum element of $\hat{\vartheta}^2$, where the δ is the eigenvector of $X^T\hat{W}X$. Hocking et al. [37] proposed two estimators for ridge parameter, given by

$$K_2 = \frac{\hat{\varsigma}^2}{\left\{\prod_{j=1}^{p}\hat{\vartheta}_j^2\right\}^{\frac{1}{p}}}, \tag{36}$$

$$K_3 = median\left(m_j^2\right), \tag{37}$$

where $m_j = \sqrt{\frac{\hat{\varsigma}^2}{\hat{\vartheta}_j^2}}$. [8] proposed the following estimator

$$K_4 = max\left(S_j\right), \tag{38}$$

where $S_j = \frac{t\hat{\vartheta}^2}{(n-p)\hat{\varsigma}^2+t_j\hat{\vartheta}_j^2}$ where t_j is eigen value of $X'X$ matrix. [35] proposed some estimators defined as:

$$K_5 = max\left(\frac{1}{m_j}\right) \tag{39}$$

$$K_6 = \left(\prod_{j=1}^{p}\frac{1}{m_j}\right) \tag{40}$$

$$K_7 = median\left(\frac{1}{m_j}\right) \tag{41}$$

4 Monte Carlo Simulation

In this section, extensive Monte Carlo simulation studies are designed to check the performance of our proposed estimator. In this simulation process, several values of ρ are considered i.e., 0.90, 0.95 and 0.99 with different sample sizes 50, 100, 150, 200. The values of θ are chosen as 1 and 2 [13,14,16]. Then, independent variables are generated by using different values of ρ, p, θ and n. The following formula is used to generate the explanatory variables with several degrees of correlation coefficient as

$$x_{ij} = \sqrt{1-\rho^2}g_{ij} + \rho g_{ip}, \tag{42}$$
$$i = 1,2,3,....,n, \; j = 1,2,.....,p,$$

where g_{ij} is a random number generated, following standard normal distribution with ρ number of paramters [20]. Then, on the basis of explanatory variable, the dependent variable are generated from Negative Binomial (μ_i, a_i) where $\mu_i = exp\left(e_i^T\beta\right)$, $i = 1,2,.....,n$. The largest value of X^TX matrix is decided as a normalized eigen vector of slope parameter β, under the constraint $\beta^T\beta = 1$. According to Jadhav and Kashid [22], it becomes minimized MSE when it is chosen as mentioned condition.

To cogitate the performance of proposed estimator, the experiment was repeated 2000 times. The value of k for the proposed estimator and the corresponding Ridge estimators, mentioned in the previous section, are computed for each iteration. Then, the estimated MSE values of the proposed estimator are calculated.

5 Results and Discussion

The results of Percentage Relative Error (PRE) are shown in Tables **??**-**??** for the different estimators: MJLTNBR, JNBR, NBRR, LTNB and ML estimators. As in Table **??**, the Ridge parameters k_4 , k_5 and k_7, all the MSE ratio of all estimators ML, NBRR, LTNB, JNBR over MJLTNBR are greater than 1. This means that proposed estimators are more efficient among others. In Table **??**, the ridge parameter k_2 , k_4 and k_5 and k_6 are greater than 1 for MSE ratios of all estimators, which shows the efficiency of proposed estimator. According to Tables **??**-**??**, the ridge parameters k_4 , k_5 and k_6 display results greater than 1, which also indicate the efficiency of proposed estimator among others. For different values of θ and p, the ridge parameters k_4 and k_5 show better result.

The results shown almost in all table by increasing the value of θ, the estimated MSE values are also increases. The MSE of MLE , NBRR are negatively affected by MJLTNBR, as the degree of correlation increases, it also increases the value of MSE of ML and NBRR estimators. So, in most of the situation considered, JNBR and LTNB with rdge parameter k_4 and k_5 showed best performance than NBRR and ML estimators. Further result showed that the negtive effect of all estimators by increasing the number of explanatory variables as in terms of MSE. As high correlation increases the value of MSE of LTNB with k_4, k_5 when $p = 4$, it become robust when $p = 6$. Therefore, k_4, k_5 showed the best performance among other estimators.

5.1 Real Life Application

To illustrate the efficiency of proposed estimator, real life data is also applied. Data is extracted from Thomson Reuters Streams. Several sectors stock indices and Dow Jones Islamic Market World Index of daily data is used over the period of Jan 1996 to May 2019. DJ Islamic Index of 58 countries covers 2578 companies and 10 main

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Table 1: Percentage Relative Error (PRE) of different estimators with $(\theta = 1, p = 2)$

n	50			100			150			200		
ρ	0.90	0.95	0.99	0.90	0.95	0.99	0.90	0.95	0.99	0.90	0.95	0.99
$\frac{ML}{k_1}$	3.26	2.64	4.01	2.67	3.87	3.92	4.41	2.23	3.21	4.46	3.89	1.37
$\frac{NBRR}{k_1}$	3.90	9.67	4.87	5.24	3.87	4.42	1.86	4.49	5.43	3.71	4.09	3.99
$\frac{JNBR}{k_1}$	2.83	1.05	2.99	2.00	1.95	1.09	2.90	1.50	2.09	3.94	2.95	1.06
$\frac{LTNBR}{k_1}$	1.89	2.09	3.86	1.98	3.87	2.59	1.50	2.15	3.99	2.40	2.95	2.18
$\frac{ML}{k_2}$	3.32	2.68	4.84	2.90	3.13	2.99	2.10	3.05	2.94	3.12	1.95	3.21
$\frac{NBRR}{k_2}$	2.89	2.71	1.70	2.83	1.95	2.38	1.80	1.95	1.00	2.46	1.59	2.04
$\frac{JNBR}{k_2}$	3.95	2.08	3.71	2.10	2.95	1.99	3.01	3.15	2.72	2.67	1.89	1.69
$\frac{LTNBR}{k_2}$	2.18	3.65	2.13	2.00	1.55	1.00	3.90	2.56	2.19	2.17	1.35	1.05
$\frac{ML}{k_3}$	2.74	2.45	1.84	2.73	2.15	1.44	1.50	1.00	0.43	0.98	0.98	1.63
$\frac{NBRR}{k_3}$	1.98	1.05	0.56	2.00	1.82	0.72	1.83	0.95	1.02	1.91	1.75	0.34
$\frac{JNBR}{k_3}$	1.00	0.93	0.40	1.83	0.95	1.24	1.72	0.52	1.59	0.73	0.92	0.87
$\frac{LTNBR}{k_3}$	3.78	2.35	0.90	2.60	2.25	1.49	2.86	1.74	5.83	0.90	0.95	0.99
$\frac{ML}{k_4}$	2.90	1.28	1.97	1.84	1.09	0.67	1.83	0.92	1.83	1.45	0.95	0.12
$\frac{NBRR}{k_4}$	0.27	1.65	1.39	1.34	2.11	1.34	1.20	0.56	1.45	1.20	0.55	0.76
$\frac{JNBR}{k_4}$	1.23	1.45	1.42	1.87	1.57	1.49	1.37	1.36	1.59	1.48	1.03	0.92
$\frac{LTNBR}{k_4}$	1.24	1.78	0.99	0.75	0.26	1.23	1.45	1.42	1.23	1.11	0.43	0.77
$\frac{ML}{k_5}$	1.86	1.86	0.88	1.94	1.23	0.57	1.22	1.43	1.54	1.38	1.39	1.25
$\frac{NBRR}{k_5}$	0.67	0.92	0.17	0.34	0.92	0.73	0.27	0.46	0.36	0.89	1.00	0.45
$\frac{JNBR}{k_5}$	1.22	0.95	0.44	0.38	0.34	0.56	0.61	0.28	0.48	0.39	0.15	0.28
$\frac{LTNBR}{k_5}$	0.55	0.45	0.39	0.90	1.00	0.19	0.23	0.67	0.19	0.32	0.95	0.37
$\frac{ML}{k_6}$	1.45	0.95	0.92	0.90	0.37	0.45	0.89	0.27	0.34	0.26	0.22	0.42
$\frac{NBRR}{k_6}$	1.76	0.45	0.67	0.10	1.95	0.11	0.91	0.25	1.00	0.90	0.11	0.32
$\frac{JNBR}{k_6}$	0.45	0.23	0.43	0.25	0.57	1.82	0.14	1.46	1.46	0.36	0.29	0.39
$\frac{LTNBR}{k_6}$	0.81	0.29	1.84	1.65	0.35	0.09	0.80	0.49	0.39	0.10	0.34	0.59
$\frac{ML}{k_7}$	2.45	1.95	1.45	1.67	1.38	1.23	1.43	1.67	0.72	1.67	1.93	1.95
$\frac{NBRR}{k_7}$	1.92	1.47	0.38	0.60	0.75	1.09	1.90	3.95	2.99	1.90	3.95	2.99
$\frac{JNBR}{k_7}$	0.90	1.05	1.65	1.60	0.47	0.73	0.28	0.83	0.47	0.56	0.75	0.47
$\frac{LTNBR}{k_7}$	1.82	1.24	0.49	0.74	0.95	1.29	1.34	1.90	1.23	0.40	0.65	0.23

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Table 2: Percentage Relative Error (PRE) of different estimators with $(\theta = 1, p = 4)$

n	50			100			150			200		
ρ	0.90	0.95	0.99	0.90	0.95	0.99	0.90	0.95	0.99	0.90	0.95	0.99
$\frac{ML}{k_1}$	1.87	0.88	1.56	2.22	1.11	0.66	4.35	3.79	6.24	4.78	0.67	5.36
$\frac{NBRR}{k_1}$	1.56	0.45	0.67	12.90	10.95	3.99	0.23	5.91	3.29	0.90	2.35	3.99
$\frac{JNBR}{k_1}$	0.12	0.89	3.49	5.10	4.45	6.39	0.88	0.45	2.99	4.90	5.65	4.99
$\frac{LTNBR}{k_1}$	0.55	3.95	2.19	5.00	4.95	1.31	3.23	4.97	4.89	3.56	2.94	3.11
$\frac{ML}{k_2}$	1.23	8.96	5.66	9.34	8.34	6.23	5.56	2.43	0.99	0.90	0.95	0.99
$\frac{NBRR}{k_2}$	0.76	0.15	3.99	4.33	6.25	4.19	4.76	3.45	6.01	4.10	3.65	6.09
$\frac{JNBR}{k_2}$	0.60	0.41	3.09	7.49	3.91	6.91	1.97	2.44	2.89	3.91	2.25	1.78
$\frac{LTNBR}{k_2}$	0.45	0.59	0.34	1.50	1.95	2.34	2.90	2.15	2.34	3.90	3.33	3.19
$\frac{ML}{k_3}$	1.40	1.34	1.76	2.40	2.95	3.94	2.76	0.44	0.89	1.40	1.72	1.26
$\frac{NBRR}{k_3}$	0.10	3.25	2.65	2.46	2.55	3.09	3.11	1.65	1.66	1.34	0.44	1.09
$\frac{JNBR}{k_3}$	1.30	0.11	0.16	0.66	11.95	7.19	1.45	3.95	1.01	0.66	0.34	0.61s
$\frac{LTNBR}{k_3}$	0.90	0.95	0.99	0.90	0.95	0.99	0.90	0.95	0.99	0.90	0.95	0.99
$\frac{ML}{k_4}$	1.90	2.95	2.45	2.80	6.15	4.59	3.40	3.95	4.91	3.56	3.89	5.60
$\frac{NBRR}{k_4}$	11.40	8.95	5.99	3.90	9.55	4.45	5.67	3.56	4.67	5.34	6.67	4.56
$\frac{JNBR}{k_4}$	6.23	5.23	1.34	3.45	5.76	8.12	7.35	4.80	6.23	4.45	6.23	7.99
$\frac{LTNBR}{k_4}$	1.56	3.98	2.44	3.40	2.92	3.39	1.94	6.01	5.19	3.21	2.15	2.09
$\frac{ML}{k_5}$	1.90	2.95	2.22	2.30	2.93	3.09	3.60	3.85	2.79	3.80	2.65	4.99
$\frac{NBRR}{k_5}$	2.56	2.45	2.19	3.90	3.55	2.01	3.94	2.41	2.85	2.05	1.94	4.12
$\frac{JNBR}{k_5}$	3.76	3.83	4.29	3.56	3.35	3.05	3.20	2.95	2.46	3.56	2.95	3.94
$\frac{LTNBR}{k_5}$	2.45	3.13	2.39	6.96	5.15	4.42	3.92	3.75	3.78	3.10	3.23	3.96
$\frac{ML}{k_6}$	0.59	3.95	2.99	0.20	0.11	0.59	0.70	1.68	1.99	1.56	0.78	0.87
$\frac{NBRR}{k_6}$	0.10	0.35	0.59	4.10	4.15	3.59	3.89	3.45	3.78	2.30	3.15	4.29
$\frac{JNBR}{k_6}$	0.36	0.15	1.49	2.30	2.67	1.39	2.93	1.38	0.39	0.20	0.15	0.59
$\frac{LTNBR}{k_6}$	0.10	1.95	1.99	6.60	2.45	2.39	1.40	1.35	0.49	1.40	4.05	3.49
$\frac{ML}{k_7}$	2.00	1.95	1.38	1.30	1.87	1.59	1.95	3.05	4.56	4.94	4.32	3.19
$\frac{NBRR}{k_7}$	2.36	2.65	2.59	2.70	4.45	5.90	5.67	4.65	3.49	3.40	3.12	3.79
$\frac{JNBR}{k_7}$	2.34	2.83	2.46	1.56	4.55	3.49	4.50	1.95	4.67	3.50	7.75	5.09
$\frac{LTNBR}{k_7}$	1.91	1.85	2.45	2.60	2.65	1.69	2.70	2.78	2.09	1.45	1.55	3.49

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Table 3: Percentage Relative Error (PRE) of different estimators with ($\theta = 1, p = 6$)

n	50			100			150			200		
ρ	0.90	0.95	0.99	0.90	0.95	0.99	0.90	0.95	0.99	0.90	0.95	0.99
$\frac{ML}{k_1}$	1.35	0.55	2.95	0.35	4.95	2.49	2.36	2.15	2.25	1.80	1.86	1.69
$\frac{NBRR}{k_1}$	1.56	1.58	0.26	1.37	1.04	1.63	1.57	1.65	2.89	2.00	1.05	0.29
$\frac{JNBR}{k_1}$	0.45	0.35	2.49	2.67	2.45	0.39	0.55	1.95	1.54	1.35	0.45	0.76
$\frac{LTNBR}{k_1}$	0.46	0.35	0.56	0.34	5.19	1.45	3.95	1.01	0.66	0.99	0.82	0.34
$\frac{ML}{k_2}$	0.47	1.35	1.06	0.73	0.36	0.09	0.30	2.95	1.99	1.40	1.45	1.59
$\frac{NBRR}{k_2}$	2.00	1.85	1.09	0.30	0.65	0.39	0.80	3.95	2.99	4.90	3.95	1.45
$\frac{JNBR}{k_2}$	0.45	1.65	0.85	0.37	2.85	3.99	0.60	0.35	0.79	0.84	0.44	0.19
$\frac{LTNBR}{k_2}$	0.76	0.35	0.39	2.95	2.55	1.29	1.91	0.44	1.31	1.70	0.65	0.18
$\frac{ML}{k_3}$	0.34	0.15	0.64	0.27	0.35	1.56	1.47	1.45	1.76	1.83	0.45	0.83
$\frac{NBRR}{k_3}$	0.12	0.55	2.87	1.91	0.45	0.74	0.35	0.71	0.94	0.50	0.67	0.85
$\frac{JNBR}{k_3}$	0.45	0.73	2.59	5.16	4.42	1.67	0.53	1.95	2.80	0.90	0.73	0.56
$\frac{LTNBR}{k_3}$	0.73	0.54	2.19	1.74	1.67	0.66	1.54	1.15	2.99	2.13	0.89	0.36
$\frac{ML}{k_4}$	2.30	2.45	1.69	3.46	1.65	1.47	1.78	1.65	1.89	3.67	2.65	2.69
$\frac{NBRR}{k_4}$	1.78	2.65	2.56	2.48	2.05	1.69	1.83	1.97	1.59	1.95	2.47	2.39
$\frac{JNBR}{k_4}$	1.68	1.65	3.49	2.50	1.45	2.69	2.56	1.45	1.89	1.23	1.62	1.93
$\frac{LTNBR}{k_4}$	2.56	3.09	4.12	3.56	3.12	3.49	2.45	1.45	1.76	3.45	4.34	3.09
$\frac{ML}{k_5}$	1.45	2.05	3.97	1.96	1.66	2.49	6.00	3.95	2.09	2.56	2.25	2.69
$\frac{NBRR}{k_5}$	1.57	2.45	3.44	2.97	2.46	2.59	2.41	2.77	2.47	1.48	2.00	1.75
$\frac{JNBR}{k_5}$	1.46	1.25	1.62	1.52	1.37	1.69	1.47	1.27	1.55	1.60	5.95	3.62
$\frac{LTNBR}{k_5}$	1.47	1.42	1.36	2.57	2.45	2.76	2.69	2.38	2.55	2.38	2.95	3.20
$\frac{ML}{k_6}$	1.30	2.65	3.39	2.97	2.50	2.38	2.58	1.35	2.79	5.20	3.75	3.86
$\frac{NBRR}{k_6}$	2.13	2.58	2.58	3.70	2.26	2.99	1.38	1.69	1.29	2.40	6.66	2.78
$\frac{JNBR}{k_6}$	2.30	4.16	3.47	2.70	2.47	2.68	2.48	3.67	4.48	2.37	2.55	2.62
$\frac{LTNBR}{k_6}$	1.59	1.49	2.45	1.67	1.48	1.38	2.37	2.59	2.01	2.84	3.82	2.18
$\frac{ML}{k_7}$	1.45	1.78	0.19	2.78	0.76	1.29	1.46	0.37	0.82	0.40	0.55	0.92
$\frac{NBRR}{k_7}$	0.06	0.45	4.81	2.36	1.45	1.75	0.93	0.95	1.38	1.04	0.75	0.64
$\frac{JNBR}{k_7}$	1.30	2.76	4.01	2.70	2.05	1.87	4.90	3.00	2.47	1.03	0.90	0.04
$\frac{LTNBR}{k_7}$	1.87	1.23	0.79	0.56	0.72	0.82	0.20	3.95	2.63	0.50	0.35	1.19

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Table 4: Percentage Relative Error (PRE) of different estimators with $(\theta = 2, p = 2)$

n	50			100			150			200		
ρ	0.90	0.95	0.99	0.90	0.95	0.99	0.90	0.95	0.99	0.90	0.95	0.99
$\frac{ML}{k_1}$	1.37	3.05	2.79	1.67	0.32	0.19	0.36	1.95	2.99	1.63	1.83	1.78
$\frac{NBRR}{k_1}$	1.74	0.65	0.19	4.90	2.95	2.54	2.47	2.54	1.63	1.40	4.32	2.01
$\frac{JNBR}{k_1}$	2.74	1.38	1.99	1.67	1.35	1.85	1.69	1.74	0.88	0.20	0.92	0.66
$\frac{LTNBR}{k_1}$	2.76	2.45	1.75	1.65	1.37	2.83	2.20	1.95	0.09	0.60	0.85	0.42
$\frac{ML}{k_2}$	2.31	0.27	1.88	1.53	1.73	1.27	1.56	2.75	1.28	1.48	1.02	1.48
$\frac{NBRR}{k_2}$	1.48	1.15	0.26	0.20	2.73	2.99	1.30	1.05	0.79	2.50	2.85	1.66
$\frac{JNBR}{k_2}$	0.90	0.56	1.32	1.37	1.45	1.38	1.48	0.65	0.89	1.70	0.65	0.12
$\frac{LTNBR}{k_2}$	1.37	1.85	2.89	1.30	0.74	0.64	0.38	0.36	0.99	0.90	0.95	0.99
$\frac{ML}{k_3}$	0.90	0.95	0.99	0.90	0.95	0.99	0.90	0.95	0.99	0.90	0.95	0.99
$\frac{NBRR}{k_3}$	0.90	0.95	0.99	0.90	0.95	0.99	0.90	0.95	0.99	0.90	0.95	0.99
$\frac{JNBR}{k_3}$	0.90	0.95	0.99	0.90	0.95	0.99	0.90	0.95	0.99	0.90	0.95	0.99
$\frac{LTNBR}{k_3}$	0.90	0.95	0.99	0.90	0.95	0.99	0.90	0.95	0.99	0.90	0.95	0.99
$\frac{ML}{k_4}$	2.30	2.03	2.83	2.10	1.95	1.83	1.71	3.28	2.92	1.64	2.83	2.56
$\frac{NBRR}{k_4}$	1.34	2.65	2.47	2.47	1.58	1.38	1.37	1.38	1.92	1.38	2.45	2.78
$\frac{JNBR}{k_4}$	1.85	1.46	2.37	2.08	2.54	2.17	1.38	3.45	2.46	3.60	5.45	2.19
$\frac{LTNBR}{k_4}$	2.45	1.45	1.59	2.75	2.83	1.97	1.56	2.57	2.09	1.80	3.95	2.72
$\frac{ML}{k_5}$	1.58	2.35	2.09	2.50	2.75	2.69	1.70	4.35	2.39	1.40	2.65	1.99
$\frac{NBRR}{k_5}$	2.42	4.90	2.67	1.56	1.47	1.38	2.57	2.48	2.73	2.73	3.74	2.48
$\frac{JNBR}{k_5}$	3.56	3.45	2.72	2.93	2.70	2.36	1.83	1.27	2.58	2.37	2.58	2.38
$\frac{LTNBR}{k_5}$	2.37	3.02	1.27	5.22	3.01	2.04	6.83	2.74	1.19	2.95	2.74	2.83
$\frac{ML}{k_6}$	2.47	2.41	2.72	3.10	1.76	2.34	1.36	1.27	1.27	1.37	1.65	1.28
$\frac{NBRR}{k_6}$	2.47	2.38	1.74	1.37	3.48	1.48	1.28	2.37	2.38	2.46	2.18	2.05
$\frac{JNBR}{k_6}$	1.57	1.35	1.79	2.52	2.45	2.36	2.19	2.35	4.76	3.10	2.65	1.96
$\frac{LTNBR}{k_6}$	2.30	4.15	4.69	3.60	2.15	2.37	2.47	2.17	3.49	2.10	1.67	4.73
$\frac{ML}{k_7}$	1.26	1.75	2.46	0.50	1.96	1.48	5.38	4.37	2.28	1.28	2.37	1.36
$\frac{NBRR}{k_7}$	1.26	2.95	2.37	2.54	2.26	2.63	3.50	2.15	1.99	1.20	0.95	0.49
$\frac{JNBR}{k_7}$	0.36	2.36	2.37	2.74	2.77	3.19	1.88	1.28	1.63	1.28	1.83	0.76
$\frac{LTNBR}{k_7}$	2.36	1.95	2.99	3.43	2.95	2.43	1.91	1.23	1.26	1.26	1.26	1.84

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Table 5: Percentage Relative Error (PRE) of different estimators with $(\theta = 2, p = 4)$

n	50			100			150			200		
ρ	0.90	0.95	0.99	0.90	0.95	0.99	0.90	0.95	0.99	0.90	0.95	0.99
$\frac{ML}{k_1}$	1.23	1.26	2.37	1.90	0.34	1.76	3.46	3.37	4.45	2.35	0.25	0.11
$\frac{NBRR}{k_1}$	1.25	2.37	2.89	1.27	1.07	1.73	1.50	0.05	1.99	3.90	2.65	2.65
$\frac{JNBR}{k_1}$	2.93	1.26	1.36	1.60	0.95	1.59	2.60	1.65	0.98	1.50	1.45	1.26
$\frac{LTNBR}{k_1}$	1.35	2.54	1.39	3.50	0.76	2.45	4.00	1.32	1.73	1.25	2.61	2.91
$\frac{ML}{k_2}$	1.63	2.73	3.09	2.46	0.84	2.46	3.77	3.46	2.52	2.31	1.55	0.18
$\frac{NBRR}{k_2}$	1.26	1.37	2.46	5.63	3.26	3.84	2.44	2.98	1.35	1.80	1.36	3.99
$\frac{JNBR}{k_2}$	2.40	2.44	2.61	1.66	0.43	1.72	1.66	0.98	1.37	1.28	1.73	1.26
$\frac{LTNBR}{k_2}$	1.63	2.05	1.27	1.83	0.73	1.26	1.86	1.36	1.43	1.76	1.72	1.83
$\frac{ML}{k_3}$	1.36	1.46	1.36	1.47	0.85	1.62	2.56	5.53	3.82	1.55	2.83	2.44
$\frac{NBRR}{k_3}$	1.65	1.36	2.82	2.55	2.35	1.73	1.12	0.42	3.36	2.84	2.59	2.17
$\frac{JNBR}{k_3}$	1.25	2.67	2.35	2.16	1.26	1.36	0.19	1.93	5.26	1.53	0.58	0.66
$\frac{LTNBR}{k_3}$	1.7	2.48	2.37	2.72	4.96	1.26	1.24	1.84	1.35	1.62	1.63	0.09
$\frac{ML}{k_4}$	2.10	2.52	1.59	1.47	1.37	1.57	5.58	1.38	1.28	3.37	2.36	2.73
$\frac{NBRR}{k_4}$	2.37	2.45	2.37	2.38	2.36	1.37	6.90	1.95	1.36	1.90	2.95	2.99
$\frac{JNBR}{k_4}$	3.45	3.25	2.35	3.90	2.58	2.83	2.25	1.36	1.37	1.90	2.95	3.34
$\frac{LTNBR}{k_4}$	2.96	2.94	2.48	1.93	1.38	1.36	1.37	2.38	2.58	2.81	2.51	2.58
$\frac{ML}{k_5}$	1.87	2.95	1.34	1.36	1.36	1.63	2.67	1.46	2.59	2.67	2.15	1.46
$\frac{NBRR}{k_5}$	2.56	2.67	2.19	2.56	1.78	1.65	1.40	2.95	5.34	3.56	3.47	2.79
$\frac{JNBR}{k_5}$	2.46	5.45	2.47	2.50	4.15	3.89	2.50	2.41	2.37	2.27	2.36	2.19
$\frac{LTNBR}{k_5}$	3.40	2.26	3.29	2.76	2.38	2.38	2.76	2.17	2.82	1.00	2.55	3.09
$\frac{ML}{k_6}$	1.67	1.92	0.03	2.19	2.04	1.18	2.90	1.47	1.84	1.71	1.31	2.95
$\frac{NBRR}{k_6}$	2.62	1.74	1.26	1.46	2.63	2.37	1.44	0.26	2.47	2.12	2.66	1.25
$\frac{JNBR}{k_6}$	1.35	1.26	0.36	2.85	1.44	4.27	2.90	0.25	1.27	2.34	1.78	1.99
$\frac{LTNBR}{k_6}$	2.14	1.25	1.04	1.93	1.06	1.97	1.00	0.09	0.55	1.83	1.95	2.12
$\frac{ML}{k_7}$	2.38	2.34	2.16	4.10	0.83	1.39	0.16	1.65	1.84	1.70	1.74	1.83
$\frac{NBRR}{k_7}$	0.85	0.26	0.87	4.10	3.25	1.27	1.40	2.35	2.18	2.74	1.93	1.84
$\frac{JNBR}{k_7}$	2.35	1.95	1.74	1.64	0.25	0.49	3.14	2.37	4.19	2.70	2.95	2.04
$\frac{LTNBR}{k_7}$	2.50	4.35	5.19	4.44	2.95	2.17	2.62	1.64	1.28	1.48	1.28	1.83

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Table 6: Percentage Relative Error (PRE) of the ML with MJNBR estimators $(\theta = 2, p = 6)$

n	50			100			150			200		
ρ	0.90	0.95	0.99	0.90	0.95	0.99	0.90	0.95	0.99	0.90	0.95	0.99
$\frac{ML}{k_1}$	1.28	1.73	1.25	0.88	1.27	1.26	1.37	1.36	1.37	0.64	1.45	1.74
$\frac{NBRR}{k_1}$	2.37	2.18	0.37	1.50	2.74	1.66	2.40	1.74	2.65	1.42	1.65	2.74
$\frac{JNBR}{k_1}$	0.12	1.95	3.73	2.40	1.73	1.17	2.67	1.38	1.33	1.65	1.25	2.93
$\frac{LTNBR}{k_1}$	1.28	1.73	2.29	3.77	0.96	1.36	2.50	1.82	1.37	1.00	0.61	0.53
$\frac{ML}{k_2}$	4.56	2.25	2.44	1.38	0.58	2.28	2.27	2.73	2.46	2.53	2.15	0.69
$\frac{NBRR}{k_2}$	1.30	2.74	2.37	0.18	1.37	2.50	0.74	1.66	2.40	5.74	1.25	0.85
$\frac{JNBR}{k_2}$	1.26	1.75	0.46	4.50	2.96	1.48	3.48	1.35	1.28	1.23	2.32	1.36
$\frac{LTNBR}{k_2}$	2.47	2.38	1.74	1.37	3.48	1.48	1.28	2.37	2.38	2.46	2.18	2.05
$\frac{ML}{k_3}$	0.40	2.65	3.96	1.76	3.96	2.61	4.09	3.28	2.44	2.17	2.74	2.11
$\frac{NBRR}{k_3}$	2.67	2.17	2.23	3.76	2.85	0.28	3.17	1.22	1.65	1.26	1.27	2.36
$\frac{JNBR}{k_3}$	2.63	0.95	1.99	1.23	2.51	2.60	2.16	1.09	1.99	2.22	2.83	1.45
$\frac{LTNBR}{k_3}$	1.54	0.39	1.37	2.18	0.37	1.50	2.34	1.06	1.20	1.74	2.65	1.42
$\frac{ML}{k_4}$	2.90	1.85	1.39	1.47	1.89	1.79	1.96	1.48	1.73	1.65	2.92	1.34
$\frac{NBRR}{k_4}$	1.40	1.36	1.36	1.37	1.37	1.27	2.37	2.37	2.39	1.65	1.38	3.30
$\frac{JNBR}{k_4}$	2.45	2.57	3.10	2.37	2.87	2.38	2.84	2.38	2.74	2.93	4.01	2.83
$\frac{LTNBR}{k_4}$	2.39	2.93	2.65	2.19	2.29	2.83	2.22	1.34	1.74	1.20	1.57	1.28
$\frac{ML}{k_5}$	1.71	2.00	2.83	1.64	1.95	1.62	1.63	1.62	1.62	1.73	1.73	1.63
$\frac{NBRR}{k_5}$	2.50	2.63	2.73	2.53	2.95	2.29	2.38	1.47	1.88	1.68	1.73	2.47
$\frac{JNBR}{k_5}$	4.66	3.35	2.62	3.86	3.12	2.63	2.74	2.73	2.48	2.47	2.84	2.76
$\frac{LTNBR}{k_5}$	2.24	1.85	1.83	1.47	1.38	1.38	1.38	1.86	2.37	2.90	2.95	3.99
$\frac{NBRR}{k_6}$	2.76	1.45	0.67	1.90	4.95	0.11	3.91	0.25	2.00	0.30	0.21	3.78
$\frac{NBRR}{k_6}$	2.93	2.91	3.27	1.77	2.65	2.89	2.10	3.59	4.09	0.23	2.35	2.07
$\frac{JNBR}{k_6}$	2.12	3.05	1.87	1.50	2.34	2.85	2.76	2.78	0.98	1.91	3.95	1.99
$\frac{LTNBR}{k_6}$	2.76	1.45	3.75	0.65	1.37	2.83	2.20	1.95	0.09	0.60	0.85	0.42
$\frac{ML}{k_7}$	1.91	2.95	0.11	1.90	2.87	1.99	2.23	1.87	1.51	1.70	2.78	1.89
$\frac{NBRR}{k_7}$	2.10	3.87	1.89	0.20	4.15	3.32	2.19	1.95	0.99	1.77	2.65	2.09
$\frac{JNBR}{k_7}$	1.70	0.37	0.19	2.84	1.73	1.29	2.80	1.94	4.49	3.56	3.09	1.93
$\frac{LTNBR}{k_7}$	1.80	2.45	5.99	2.98	2.05	2.85	2.76	0.78	0.98	1.91	2.05	1.76

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Table 7: MSE property of existing and Proposed Estimator $(\theta = 1, p = 2)$

ρ	0.90				0.95				0.99			
n	50	100	150	200	50	100	150	200	50	100	150	200
$ML1$	0.4235	0.4187	0.4080	0.4001	0.4580	0.4499	0.4240	0.4095	0.4991	0.4790	0.4945	0.4409
$NBRR1$	0.5641	0.4409	0.4034	0.3998	0.4495	0.4359	0.5409	0.4012	0.4856	0.4663	0.4593	0.4499
$JNBR1$	0.4312	0.4123	0.4108	0.3905	0.4395	0.4349	0.4245	0.4095	0.4839	0.4672	0.4525	0.4319
$LTNBR1$	0.4120	0.4191	0.4009	0.4090	0.4305	0.4299	0.4102	0.4009	0.5099	0.4610	0.4501	0.4121
$MJLTNBR1$	0.4109	0.4010	0.3993	0.3901	0.3959	0.3991	0.4102	0.3995	0.4699	0.4490	0.4432	0.4098
$ML2$	0.5098	0.5145	0.5199	0.5090	0.6795	0.6299	0.6190	0.6095	0.5599	0.5478	0.5865	0.5719
$NBRR2$	0.6140	0.6095	0.6131	0.5900	0.6195	0.6184	0.6290	0.6175	0.6039	0.6110	0.5634	0.5496
$JNBR2$	0.5921	0.5645	0.5494	0.5190	0.5693	0.5719	0.5060	0.5015	0.5573	0.5415	0.5315	0.5295
$LTNBR2$	0.4185	0.4095	0.4121	0.3968	0.4097	0.3990	0.3960	0.3895	0.4199	0.4065	0.4006	0.4019
$MJLTNBR2$	0.4128	0.4023	0.3949	0.3915	0.4135	0.4009	0.4082	0.4101	0.4169	0.4031	0.4015	0.4029
$ML3$	1.6013	0.9509	0.9193	0.9051	2.1945	1.9190	2.9007	2.1695	2.1699	3.0090	2.0595	2.0199
$NBRR3$	3.1490	2.9504	2.9469	2.8263	4.1645	2.9931	2.9472	2.9651	3.8469	3.9560	3.9275	2.9119
$JNBR3$	1.3390	1.2945	1.3099	1.2890	2.0895	2.1499	2.0450	1.9127	1.3894	1.3862	1.3321	1.2163
$LTNBR3$	1.8829	1.7298	1.6792	1.4594	1.8839	1.7614	1.7031	1.5897	1.9347	1.9076	1.7300	1.7091
$MJLTNBR3$	0.9016	0.9654	0.9129	0.9035	0.8485	0.7699	0.8040	0.7985	0.9899	0.5690	0.6057	0.6011
$ML4$	0.0765	0.0713	0.0699	0.0691	0.0595	0.0599	0.0590	0.0532	0.0329	0.0298	0.0195	0.0117
$NBRR4$	0.0658	0.0685	0.0519	0.0716	0.0576	0.0509	0.0614	0.0525	0.0474	0.0390	0.0295	0.0329
$JNBR4$	0.0690	0.0694	0.0588	0.0683	0.0535	0.0599	0.0582	0.0505	0.0374	0.0316	0.0302	0.99
$LTNBR4$	0.0426	0.0465	0.0437	0.0510	0.0542	0.04826	0.0516	0.0478	0.0419	0.0349	0.0389	0.0265
$MJLTNBR4$	0.0413	0.0385	0.0312	0.0470	0.0397	0.0443	0.0406	0.0412	0.0313	0.0214	0.0215	0.0135
$ML5$	0.0340	0.0312	0.0217	0.0100	0.0295	0.0287	0.0219	0.0225	0.0217	0.0187	0.0195	0.0112
$NBRR5$	0.0498	0.0457	0.0471	0.0412	0.0618	0.0427	0.0507	0.0472	0.0493	0.0479	0.0426	0.0391
$JNBR5$	0.0316	0.0328	0.0319	0.0370	0.0315	0.0359	0.0340	0.0351	0.0391	0.0378	0.0408	0.0369
$LTNBR5$	0.0412	0.0445	0.0428	0.0410	0.0505	0.0499	0.0490	0.0435	0.0495	0.0490	0.0315	0.0434
$MJLTNBR5$	0.0093	0.0087	0.0059	0.0030	0.0075	0.0069	0.0050	0.0051	0.0049	0.0042	0.0037	0.0025
$ML6$	0.6807	0.7094	0.6739	0.6430	1.8713	1.4373	0.5791	0.0560	0.9070	0.8689	0.8459	0.2274
$NBRR6$	1.2891	1.0988	0.8468	0.7007	1.8516	1.7323	1.6516	1.5323	2.0001	1.9002	1.3656	0.8358
$JNBR6$	1.5902	0.9785	0.6730	0.5713	1.4063	1.1793	0.4463	0.1012	1.8481	1.6467	1.1545	0.8692
$LTNBR6$	1.4421	0.7044	0.4728	0.0961	1.5637	1.5473	0.7073	0.4261	1.9254	1.5656	0.6634	0.3885
$MJLTNBR6$	0.5146	0.3964	0.1953	0.1903	0.6938	0.5429	0.4473	0.1769	0.8393	0.5852	0.2542	0.1093
$ML7$	1.7177	1.1324	1.1174	0.6650	1.6161	0.7668	0.0763	0.2538	1.2552	0.5111	0.1726	0.0165
$NBRR7$	1.9448	1.4616	1.4299	0.6607	1.6149	0.9784	0.9672	0.6846	1.8617	1.2629	0.1509	0.1089
$JNBR7$	1.2433	1.2151	1.1071	1.0046	1.7021	0.7190	0.6882	0.5063	1.7178	0.9575	0.6025	0.0673
$LTNBR7$	1.7578	1.1449	0.9516	0.7296	1.9529	0.6433	0.4114	0.2509	1.9754	0.3599	0.0899	0.0779
$MJLTNBR7$	0.6388	0.5735	0.4394	0.3734	0.6956	0.3682	0.3221	0.2235	0.9868	0.1095	0.0147	0.0138

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Table 8: MSE property of existing and Proposed Estimator ($\theta = 1, p = 4$)

ρ	0.90				0.95				0.99			
n	50	100	150	200	50	100	150	200	50	100	150	200
$ML1$	1.2742	1.5504	0.8537	0.5294	1.7239	1.5747	1.5074	0.9746	1.1632	1.2918	0.5444	0.8224
$NBRR1$	1.7224	1.6282	1.3098	0.7735	1.7289	1.0418	1.0325	0.7372	1.7963	1.2656	0.9817	0.8708
$JNBR1$	1.8040	0.9910	0.9579	0.6613	1.8664	1.0546	1.8326	1.7541	1.8789	1.6865	1.6223	0.6622
$LTNBR1$	1.7329	1.5217	0.4604	0.3007	1.5999	0.9811	0.9788	0.7078	1.6184	0.7942	0.6273	0.5535
$MJLTNBR1$	0.8542	0.7913	0.3617	0.2869	0.9100	0.6887	0.3924	0.3865	0.9530	0.7508	0.6116	0.3798
$ML2$	1.8714	1.7162	1.0388	0.3967	1.4866	1.4545	1.2534	0.4713	1.9537	0.9362	0.8400	0.6027
$NBRR2$	1.6949	1.6135	1.4683	0.2695	1.8552	1.2301	0.4975	1.2150	1.1635	1.8727	1.2208	0.9304
$JNBR2$	2.6533	1.9323	0.8068	0.4078	2.7842	2.3655	1.7398	0.8954	2.8766	2.4995	1.8284	0.4990
$LTNBR2$	1.9565	1.8202	0.4866	0.3360	1.9872	1.0570	1.5879	0.6172	2.4460	1.3394	0.8602	0.6090
$MJLTNBR2$	0.6572	0.6373	0.4170	0.3299	0.7400	0.6135	0.5482	0.5022	0.4768	0.8703	0.6202	0.3652
$ML3$	2.3243	2.0110	1.9510	1.8011	2.7101	1.9908	1.1258	0.2711	2.7158	2.2012	2.0309	1.5765
$NBRR3$	2.7097	2.0125	1.6711	0.5217	2.7448	2.5587	2.3422	0.5020	2.9489	2.6319	2.4754	0.6477
$JNBR3$	2.0245	1.7271	1.5244	1.2447	2.2487	1.9936	0.5787	0.4585	2.4175	1.9534	1.9156	0.4669
$LTNBR3$	1.6534	0.8639	0.7471	0.2934	1.8597	1.1297	0.4249	0.4157	2.1932	1.3383	0.8917	0.7928
$MJLTNBR3$	1.3447	1.0763	0.4857	0.3440	1.3630	0.7939	0.5712	0.4960	1.4114	0.9408	0.8064	0.5085
$ML4$	1.2844	1.2033	0.6216	0.3983	1.0834	1.4362	0.9452	0.6872	1.4415	1.2527	1.0145	0.6136
$NBRR4$	1.7268	1.5140	1.0452	0.8470	1.7643	1.5320	1.4850	0.3643	1.9355	1.5719	1.2604	1.2478
$JNBR4$	1.4138	1.3928	0.8854	0.6668	1.7599	1.5942	1.3818	0.6313	1.8819	1.6535	1.0345	0.2888
$LTNBR4$	1.3918	0.4806	0.2993	0.9551	1.4895	0.6029	0.5185	0.5070	1.9728	1.1888	1.2221	0.5060
$MJLTNBR4$	0.6444	0.5643	0.2283	0.0798	0.8205	0.6807	0.5064	0.5003	0.8447	0.7844	0.1101	0.0836
$ML5$	1.0374	0.7474	0.1372	0.0722	1.7053	0.5563	0.2583	0.2189	1.8381	1.2794	0.9313	0.3470
$NBRR5$	1.8122	0.6405	0.0401	0.0357	1.9714	0.6827	0.1605	0.1490	1.9784	0.8676	0.6117	0.5332
$JNBR5$	1.8515	0.1058	0.0913	0.8134	1.8889	1.7596	1.1479	0.9221	1.9848	1.4992	1.4283	1.2706
$LTNBR5$	1.5101	0.5828	0.1033	0.0993	1.5101	0.5798	0.3409	0.3255	0.2246	1.6972	1.3337	0.7295
$MJLTNBR5$	0.6071	0.5239	0.0770	0.0396	0.9038	0.4937	0.4343	0.3645	0.9990	0.3475	0.3638	0.2464
$ML6$	1.5083	1.5748	0.1797	0.1647	1.0441	0.8556	0.7766	0.7191	1.8265	1.4155	1.2604	0.8017
$NBRR6$	2.3649	1.9659	0.8118	0.5864	2.4151	1.5613	1.2968	0.9088	2.4444	2.2275	1.2094	1.2192
$JNBR6$	2.1914	1.8249	0.2391	0.0821	2.3887	1.7075	0.7740	0.4396	2.8292	1.9241	1.5594	0.8422
$LTNBR6$	1.3414	1.2107	0.3333	0.1763	1.3809	1.0758	0.8557	0.5193	1.8141	1.1866	0.9271	0.8695
$MJLTNBR6$	0.7550	0.1323	0.1105	0.0560	0.7937	0.7110	0.5323	0.3056	0.9896	0.7080	0.6599	0.1569
$ML7$	1.4934	0.8687	0.3000	0.0184	1.9143	0.7830	0.7497	0.3175	2.0118	1.3941	0.5392	0.5087
$NBRR7$	1.7149	1.5756	0.3639	0.1805	1.8039	1.1836	0.8357	0.0984	1.8413	1.1912	0.7990	0.5686
$JNBR7$	1.9597	1.8570	0.4787	0.0250	2.1788	1.5260	1.0211	0.7305	2.2272	2.0854	1.3534	1.3205
$LTNBR7$	1.4876	1.4195	0.2285	0.0065	1.8473	1.7681	0.7364	0.4403	1.8628	1.2075	1.1026	0.5899
$MJLTNBR7$	1.0924	0.3551	0.3329	0.1669	1.0948	0.9475	0.5247	0.4542	1.2267	1.0636	0.9976	0.3958

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Table 9: MSE property of existing and Proposed Estimator ($\theta = 1, p = 6$)

ρ	0.90				0.95				0.99			
n	50	100	150	200	50	100	150	200	50	100	150	200
$ML1$	2.0209	0.9151	0.2091	0.2091	2.4563	2.4244	0.9140	0.6720	2.4778	1.3968	0.6085	0.4277
$NBRR1$	1.8706	1.8177	1.5538	1.3258	2.1122	1.7959	1.7143	1.6809	2.4392	2.4350	1.7934	1.6502
$JNBR1$	2.5986	1.9829	1.3238	1.0885	2.3013	1.9305	1.9043	1.6710	2.8805	2.7092	1.5264	1.3420
$LTNBR1$	1.8850	1.7791	1.3924	1.1294	1.9064	1.5435	1.4881	1.4784	1.9507	1.7401	1.6203	1.4611
$MJLTNBR1$	1.8224	1.7967	0.5737	0.3084	1.8325	1.2584	1.1460	0.6011	1.9532	1.7362	1.2649	1.0303
$ML2$	1.9865	1.8018	1.0217	1.0004	2.0875	1.2741	1.2521	1.0318	2.6923	1.6605	1.5503	1.0374
$NBRR2$	2.4786	2.4512	1.4306	1.2816	3.3246	1.7536	1.5884	1.4710	3.9621	2.2898	1.9596	1.4871
$JNBR2$	2.2432	2.0426	1.2854	1.0181	2.6661	2.0344	1.7361	1.4052	3.3838	3.2276	2.4682	2.2605
$LTNBR2$	2.7769	2.6270	1.1202	1.0856	3.1425	2.1458	1.4170	1.0719	3.1885	3.1466	2.4959	2.3431
$MJLTNBR2$	2.1474	2.1232	1.2296	1.2123	2.5834	2.5572	2.0541	1.6664	2.7824	2.7126	2.0466	1.3160
$ML3$	3.6368	3.4917	2.0901	2.0271	3.8113	3.7453	3.2471	3.1266	3.9108	3.8589	3.0057	2.3627
$NBRR3$	2.5560	2.2318	2.1288	1.3767	3.2751	3.1322	2.2304	2.1429	3.4415	3.3658	2.1287	2.0291
$JNBR3$	3.0237	2.8617	1.2831	1.0004	3.0619	3.0605	2.4967	2.0182	3.3824	3.3558	2.3013	1.9105
$LTNBR3$	2.2459	1.7098	1.2426	1.0812	2.6853	2.4868	1.4164	1.3211	2.7185	2.7073	1.3759	1.2897
$MJLTNBR3$	1.6331	1.4369	0.4089	0.1348	2.0544	2.0305	1.0264	0.9758	2.4935	2.2537	0.7627	0.7417
$ML4$	1.0452	1.0300	0.2009	0.0257	1.7198	1.4734	0.9455	0.6016	1.7701	1.7503	0.4957	0.3967
$NBRR4$	1.2472	1.0030	0.4023	0.2038	2.0781	1.2599	0.8214	0.7568	2.2240	2.1485	0.7445	0.5610
$JNBR4$	1.9617	1.4176	1.0828	1.0797	2.0513	1.9777	1.3026	1.3009	2.1142	2.0849	1.1996	1.1433
$LTNBR4$	2.1506	2.0853	1.0393	1.0119	2.2808	2.2677	2.0445	1.8927	2.4924	2.2889	1.7258	1.6575
$MJLTNBR4$	1.4384	1.2896	0.1357	0.0645	1.6089	1.5233	0.9475	0.7074	1.7080	1.6748	0.5456	0.1616
$ML5$	1.7674	1.7609	1.3093	1.0467	1.7874	1.7764	1.6876	1.6774	1.8948	1.8381	1.3687	1.3267
$NBRR5$	2.1954	2.1661	1.1110	1.0035	2.2882	2.2690	2.0839	1.8578	2.9991	2.6389	1.6836	1.5548
$JNBR5$	2.0188	1.9448	1.2200	1.1923	2.0669	2.0489	1.8979	1.7502	2.6184	2.4771	1.5583	1.2995
$LTNBR5$	2.4212	2.3058	1.5524	1.4285	2.4603	2.4373	2.2523	2.1448	2.5734	2.5309	1.6998	1.6219
$MJLTNBR5$	1.0259	0.9598	0.0611	0.0342	1.1520	1.1307	0.4263	0.3855	1.7168	1.6942	0.3196	0.1941
$ML6$	3.2672	3.1503	2.1627	2.0805	3.5971	3.5142	2.9792	2.9277	3.8917	3.7581	2.7803	2.6974
$NBRR6$	3.4782	3.0076	2.0898	2.0686	3.9155	3.7147	2.9138	2.8311	3.9718	3.9705	2.2760	2.1168
$JNBR6$	2.9245	2.4061	3.4058	2.9993	1.3450	1.1297	2.2694	1.9569	3.8887	3.8830	1.6720	1.5557
$LTNBR6$	2.1122	1.8570	1.1561	1.1437	2.5806	2.1860	1.7632	1.7494	2.6790	2.6023	1.4918	1.2080
$MJLTNBR6$	1.9503	1.8176	0.5906	0.0260	2.0443	1.9744	1.4625	1.3796	2.8416	2.8194	1.2948	1.0312
$ML7$	3.6141	3.5165	2.5575	2.2025	3.9357	3.7221	3.4518	3.2988	4.5472	4.4149	3.1223	2.5910
$NBRR7$	3.4672	3.2416	2.1828	2.0302	3.6370	3.5579	3.1264	3.0142	3.8498	3.8043	2.6649	2.6646
$JNBR7$	3.2487	3.1548	2.1885	2.1704	3.6409	3.5038	2.7618	2.5757	3.9433	3.7457	2.4034	2.3748
$LTNBR7$	2.6741	2.5812	1.5874	1.1607	3.1852	2.9174	2.2730	2.2392	3.7906	3.5668	1.7363	1.6663
$MJLTNBR7$	3.2241	3.0341	0.5178	0.3273	3.4947	3.2611	2.7038	1.3598	3.8681	3.6712	1.1170	0.7139

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Table 10: MSE property of existing and Proposed Estimator $(\theta = 2, p = 2)$

ρ	0.90				0.95				0.99			
n	50	100	150	200	50	100	150	200	50	100	150	200
$ML1$	2.8163	2.6144	0.8489	0.5505	3.3964	3.1326	2.4883	2.3253	3.8157	3.6746	1.0131	0.9626
$NBRR1$	3.3763	2.9634	1.2583	1.0200	3.6034	3.5787	2.3364	2.1616	3.8174	3.6678	1.8395	1.4338
$JNBR1$	2.9283	2.8948	1.1792	1.06	3.3081	3.2933	2.6454	2.0722	3.6353	3.4880	1.5134	1.4211
$LTNBR1$	1.9396	1.9098	1.3924	1.2050	2.5725	2.4137	1.7587	1.5980	2.6794	2.6274	1.4412	1.4141
$MJLTNBR1$	1.8597	1.3729	0.4098	0.1859	2.2645	2.2537	1.1779	0.8709	2.9392	2.2981	0.7926	0.7558
$ML2$	1.2562	1.0430	0.3392	0.2509	2.1688	1.9256	0.9363	0.9061	2.4550	2.3684	0.8179	0.5102
$NBRR2$	2.0276	1.9713	0.7283	0.3293	2.6196	2.4757	1.4698	1.3750	3.8309	3.5816	1.2132	1.1599
$JNBR2$	2.3748	1.9784	1.0175	0.4800	2.6413	2.5963	1.7445	1.5815	2.9406	2.7852	1.3043	1.2326
$LTNBR2$	1.4054	1.1139	0.6393	0.2827	2.5947	1.9020	0.9087	0.7967	2.9343	2.8429	0.7104	0.6394
$MJLTNBR2$	0.6164	0.5855	0.1879	0.1435	0.6987	0.6596	0.5635	0.4854	0.8105	0.7516	0.4479	0.3388
$ML3$	2.7317	2.6586	1.6295	1.4581	3.1886	3.1565	2.2784	2.1589	3.4628	3.2937	2.1109	2.0173
$NBRR3$	1.4374	1.4053	0.2692	0.1476	1.8768	1.7391	0.8710	0.5430	2.8790	2.5342	0.5053	0.3394
$JNBR3$	1.8255	1.5555	0.5584	0.4734	1.9592	1.8345	1.5260	0.9048	2.4250	2.2075	0.8695	0.6338
$LTNBR3$	2.2998	2.2385	1.2779	1.0863	2.4324	2.3942	2.1337	2.1282	2.6995	2.6953	1.6436	1.4825
$MJLTNBR3$	1.3278	1.3006	0.4054	0.3696	1.4070	1.3641	1.0797	0.7171	1.8711	1.6148	0.6775	0.5093
$ML4$	1.0163	0.9208	0.3533	0.2098	1.6462	1.2409	0.7945	0.7027	1.8755	1.7418	0.4747	0.4499
$NBRR4$	1.4395	1.4278	0.3508	0.0990	1.5948	1.5461	1.2324	1.1056	1.8627	1.8458	0.9299	0.8714
$JNBR4$	1.1534	1.0751	0.3319	0.2262	1.3192	1.3134	0.9557	0.7801	1.5925	1.4154	0.5326	0.3667
$LTNBR4$	0.5417	0.4685	0.1759	0.0791	0.8532	0.6324	0.4222	0.3057	0.9932	0.9807	0.2201	0.1849
$MJLTNBR4$	0.5527	0.4916	0.2864	0.1428	0.6305	0.5646	0.4489	0.4074	0.8536	0.6334	0.3081	0.2974
$ML5$	1.9510	1.9312	1.1519	1.0156	2.2324	2.1412	1.8879	1.6399	2.5745	2.3232	1.3088	1.2990
$NBRR5$	1.6236	1.5911	1.0913	1.0885	1.7235	1.7084	1.5478	1.3198	1.9568	1.7721	1.2969	1.2191
$JNBR5$	1.5171	1.5056	1.0833	1.0791	1.6875	1.6815	1.4202	1.3187	1.8300	1.7121	1.2914	1.2390
$LTNBR5$	1.6550	1.5531	1.1457	1.0536	1.8072	1.7080	1.5248	1.3756	1.9497	1.9235	1.2334	1.1632
$MJLTNBR5$	0.5635	0.5601	0.2106	0.1804	0.9468	0.8307	0.3797	0.3602	0.9747	0.9622	0.3171	0.2925
$ML6$	2.4083	2.3912	1.7359	1.7307	2.6257	2.5900	2.3429	2.3136	2.9209	2.8562	1.9420	1.9342
$NBRR6$	2.3444	2.3228	1.4434	1.1945	2.6874	2.4241	2.2652	1.8830	2.9237	2.8150	1.7152	1.5052
$JNBR6$	2.1720	1.9999	1.9654	1.9392	1.7907	1.6871	1.6145	1.4247	1.3596	1.3476	1.2432	1.1575
$LTNBR6$	2.1518	1.7393	1.3367	1.1632	2.6659	2.5767	1.4880	1.4706	2.8805	2.8237	1.4435	1.3922
$MJLTNBR6$	0.9685	0.9453	0.2769	0.1276	1.1932	1.1373	0.9013	0.6363	1.8035	1.7818	0.4546	0.4301
$ML7$	2.6640	2.6034	1.3634	1.0331	2.9120	2.8128	1.8081	1.6934	3.5452	3.1933	1.5924	1.5491
$NBRR7$	2.1616	2.0547	1.3131	1.2250	2.4059	2.2314	1.9376	1.7715	2.9649	2.9226	1.6393	1.5529
$JNBR7$	1.6143	1.5189	1.3616	1.0905	1.7644	1.6666	1.4775	1.4416	1.7941	1.7751	1.4149	1.4079
$LTNBR7$	1.6567	1.6048	1.0315	1.0315	1.9206	1.8861	1.5981	1.5759	1.9546	1.9498	1.5516	1.4606
$MJLTNBR7$	0.7742	0.5117	0.1896	0.1005	0.9559	0.7865	0.5025	0.3179	1.9974	1.8803	0.2962	0.2659

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Table 11: MSE property of existing and Proposed Estimator ($\theta = 2, p = 4$)

ρ	0.90				0.95				0.99			
n	50	100	150	200	50	100	150	200	50	100	150	200
$ML1$	4.2472	3.6753	2.4131	2.3887	5.0401	4.6432	3.4909	2.7944	5.2330	5.2162	2.6656	2.6284
$NBRR1$	3.0559	2.4567	1.6844	1.2035	4.0000	3.1137	2.3595	2.2340	4.7963	4.6267	2.2110	2.0883
$JNBR1$	3.1894	2.5815	1.2209	1.0136	4.3168	3.5423	2.4390	2.2679	4.9179	4.7335	2.0309	2.0104
$LTNBR1$	2.3719	2.2391	1.1258	1.0562	2.6559	2.4664	2.0290	1.9301	3.9123	3.5351	1.7568	1.6846
$MJLTNBR1$	2.5780	2.5301	1.5012	0.4638	3.0417	2.8018	2.3550	1.8326	3.6778	3.2509	1.7497	1.7207
$ML2$	4.5922	3.9731	1.4798	1.3930	5.2167	4.7093	2.8836	2.2812	5.9961	5.8382	1.7868	1.5738
$NBRR2$	3.8235	3.7917	1.8033	1.5675	4.7879	4.4132	3.6380	3.5688	5.9306	4.9040	3.3060	3.0347
$JNBR2$	3.0973	2.7148	1.0621	1.0298	3.5601	3.5352	2.6133	2.5193	3.7433	3.5924	1.8381	1.3026
$LTNBR2$	2.3627	2.0146	1.5547	1.2068	2.7430	2.6561	1.9520	1.8442	2.9911	2.8948	1.7935	1.6936
$MJLTNBR2$	2.3574	1.8902	0.3982	0.0775	2.7498	2.7402	1.6367	1.4689	2.9793	2.7846	1.2683	1.1216
$ML3$	3.0267	2.8812	1.0983	1.0714	3.5578	3.0315	2.2016	2.1977	3.9359	3.9098	2.0936	1.6053
$NBRR3$	2.7765	2.6578	1.7313	1.3274	3.2643	2.8144	2.4904	2.3982	3.7417	3.4458	2.3016	1.9183
$JNBR3$	2.3545	2.2515	1.1878	1.1478	2.8735	2.5323	1.7601	1.5407	3.6221	3.4179	1.4942	1.3893s
$LTNBR3$	2.5049	2.2362	1.4587	1.0413	2.6167	2.5198	2.2360	1.8381	2.8310	2.6377	1.7208	1.4846
$MJLTNBR3$	1.6851	1.5949	0.8045	0.6215	1.9391	1.7038	1.4321	1.3640	2.9271	2.2667	1.3326	1.2158
$ML4$	2.3586	1.7700	1.0913	1.0889	2.5277	2.4049	1.6406	1.5739	2.8296	2.7445	1.5602	1.2907
$NBRR4$	2.3794	2.2000	1.2756	1.1200	2.4278	2.4090	1.9280	1.9056	2.9453	2.8346	1.6476	1.6233
$JNBR4$	1.7097	1.6789	0.8910	0.6853	1.9118	1.8052	1.4291	1.4233	2.9132	2.7184	1.1500	0.9835
$LTNBR4$	1.9970	1.9279	0.2338	0.0659	2.3728	2.3039	1.7637	1.6752	2.6482	2.4693	0.7915	0.7474
$MJLTNBR4$	2.1156	1.9266	0.7661	0.2905	2.4483	2.3091	1.8508	1.8139	2.5774	2.4617	1.1660	1.0196
$ML5$	3.7342	3.1053	1.6750	1.0571	3.8902	3.8051	2.6499	2.3812	3.9716	3.9244	1.9634	1.8862
$NBRR5$	3.2908	3.2574	1.8664	1.3377	3.9593	3.3450	3.0684	2.7717	4.6349	4.2243	2.7559	2.6205
$JNBR5$	2.8618	2.7602	1.1285	1.1074	3.5966	3.3568	2.5130	2.0546	3.8165	3.6694	2.0036	1.7024
$LTNBR5$	2.5049	2.2362	1.4587	1.0413	2.6167	2.5198	2.2360	1.8381	2.8310	2.6377	1.7208	1.4846
$MJLTNBR5$	1.1482	1.0766	0.6392	0.2089	2.6300	2.3354	0.7314	0.7256	2.9818	2.9271	0.7121	0.7018
$ML6$	4.8083	4.6103	3.3160	1.1634	5.1064	5.0335	4.3720	4.3046	5.6223	5.6034	3.5840	3.5484
$NBRR6$	2.9904	2.6646	1.0944	1.0747	3.6724	3.4121	2.0955	1.9471	4.8564	4.7110	1.3382	1.1593
$JNBR6$	2.9266	2.8362	2.1492	1.8722	3.6905	3.3012	2.5668	2.5324	3.8774	3.8707	2.4869	2.4823
$LTNBR6$	2.7945	2.7646	1.9312	1.4919	3.5331	2.8702	2.2945	2.2536	3.8441	3.7747	2.1107	2.0867
$MJLTNBR6$	1.3552	1.2462	0.2032	0.0945	2.2483	1.6457	1.0032	0.9266	2.8427	2.7186	0.5919	0.3010
$ML7$	6.1046	5.7268	3.8186	3.6869	7.4442	7.0338	4.6666	4.4178	9.3928	8.8097	4.3055	3.8231
$NBRR7$	6.1790	5.1815	3.4518	3.2544	7.2721	7.2005	4.3813	3.8575	7.6147	7.4615	3.6574	3.4665
$JNBR7$	4.8872	4.7851	3.2966	2.6392	5.0729	4.9442	3.9954	3.8426	5.8918	5.2463	3.4748	3.3600
$LTNBR7$	4.2782	4.1629	2.1966	2.1098	4.6923	4.3992	3.7012	3.1484	4.9266	4.7892	3.0559	2.2395
$MJLTNBR7$	2.7232	2.6971	2.1063	2.0976	2.8026	2.7938	2.6706	2.6616	2.9222	2.8721	2.5478	2.1181

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Table 12: MSE property of existing and Proposed Estimator $(\theta = 2, p = 6)$

ρ	0.90				0.95				0.99			
n	50	100	150	200	50	100	150	200	50	100	150	200
$ML1$	0.90	0.95	0.99	0.90	0.95	0.99	0.90	0.95	0.99	0.90	0.95	0.99
$NBRR1$	3.9698	3.8362	1.3545	1.2527	4.7320	4.1158	3.4359	1.6759	4.9103	4.8412	1.4036	1.3577
$JNBR1$	3.1815	3.0127	1.6693	1.6045	3.3793	3.3092	2.8043	2.8022	3.6421	3.6317	2.6434	2.5020
$LTNBR1$	3.1505	2.7998	1.5101	1.2090	3.4138	3.3315	2.6390	2.1857	3.8445	3.6249	2.0891	1.6509
$MJLTNBR1$	1.4820	1.4626	0.0923	0.0270	1.5244	1.5106	0.9080	0.7782	1.9583	1.6331	0.6090	0.1730
$ML2$	4.4455	4.1644	1.7216	1.4334	4.7805	4.5363	3.4574	2.6945	5.7304	5.6647	2.3199	2.1916
$NBRR2$	2.6305	2.1340	1.1158	1.0996	3.2117	2.9792	2.0598	1.9409	3.7762	3.5318	1.7538	1.3681
$JNBR2$	2.8246	2.2032	1.2632	1.0718	2.8973	2.8260	1.9916	1.9585	3.4257	3.0235	1.7189	1.4601
$LTNBR2$	3.4533	2.7538	1.9533	1.0752	3.7646	3.4563	2.6694	2.3377	3.7885	3.7747	2.2135	2.1229
$MJLTNBR2$	1.6907	1.6538	1.0747	1.0222	1.8217	1.7442	1.6365	1.3696	1.9394	1.9302	1.2784	1.1999
$ML3$	4.0178	2.2249	0.7963	0.5724	6.1878	4.9801	1.9028	1.8630	6.7296	6.3159	1.7734	1.3095
$NBRR3$	3.6973	3.3129	0.5574	0.0907	3.9109	3.7660	2.5740	2.5181	4.0548	3.9520	2.1975	1.1072
$JNBR3$	2.6844	2.4353	1.5256	1.1243	4.0129	3.3307	2.2645	1.8213	4.9197	4.4317	1.7473	1.6664
$LTNBR3$	2.7936	2.7182	1.3093	1.0270	4.1816	4.1200	2.2584	2.2061	4.9961	4.5871	1.4966	1.4614
$MJLTNBR3$	1.3698	1.2565	0.4509	0.4281	1.5475	1.4390	1.1606	1.0684	1.9313	1.7082	0.9105	0.5651
$ML4$	2.3288	2.2305	1.0749	1.0170	2.6111	2.4345	2.2137	1.2478	2.9293	2.8006	1.2285	1.1073
$NBRR4$	2.5805	2.3700	1.5953	1.5654	2.8894	2.8046	2.3539	2.2911	2.9797	2.9058	2.2519	1.8514
$JNBR4$	1.5338	1.4648	0.3585	0.0751	2.5478	1.5656	1.4631	1.3572	2.6911	2.6524	0.8613	0.7003
$LTNBR4$	2.2955	1.9196	0.8070	0.0797	2.7234	2.7185	1.6345	1.0065	2.8267	2.8025	0.9790	0.8169
$MJLTNBR4$	1.3033	1.2997	0.3331	0.2970	1.8198	1.7572	0.9727	0.6659	1.8819	1.8739	0.5279	0.4313s
$ML5$	1.9808	1.8677	1.2504	1.1741	2.2330	2.1426	1.7022	1.3656	2.8713	2.5456	1.3613	1.3398
$NBRR5$	1.8562	1.8530	0.8353	0.1039	2.7155	2.0792	1.6171	1.3761	2.9254	2.8278	1.3049	1.0890
$JNBR5$	2.1343	1.7239	0.3671	0.0741	2.1694	2.1410	1.1642	1.0757	2.6104	2.4223	0.5298	0.4939
$LTNBR5$	1.9122	1.9116	1.4198	1.1047	2.2982	2.0629	1.5865	1.5339	2.7035	2.3095	1.5102	1.4707
$MJLTNBR5$	1.0593	1.0282	0.4295	0.0859	1.5174	1.4728	0.9858	0.9290	1.8294	1.7901	0.7290	0.4737
$ML6$	4.4282	3.5174	2.0913	2.0070	5.0720	4.9215	2.7744	2.7301	5.5369	5.0828	2.7057	2.6047
$NBRR6$	4.3465	4.1686	2.3182	2.1536	5.0933	4.8898	4.0021	3.4249	5.7877	5.3743	3.2682	2.4403
$JNBR6$	3.1201	2.9977	2.0733	2.0686	3.9622	3.2582	2.9261	2.5982	4.8201	4.0197	2.3592	2.2363
$LTNBR6$	3.8810	3.2136	0.5680	0.0571	4.0171	3.9411	3.1074	2.9264	4.9092	4.0580	2.0630	1.9019
$MJLTNBR6$	1.0478	0.8382	0.4920	0.0331	1.5090	1.2376	0.8024	0.6746	1.9944	1.6672	0.6508	0.5388
$ML7$	5.7975	5.5444	3.0760	1.0773	7.1613	4.0508	3.7145	2.0295	7.8364	1.9031	1.8889	1.3137
$NBRR7$	4.4097	4.2658	1.7236	1.6584	5.0685	4.6298	3.1851	2.9359	5.4313	5.4075	2.4741	2.0105
$JNBR7$	4.6292	4.3324	1.3036	1.0495	4.8531	4.6725	3.7497	3.6682	4.9879	4.8924	1.4576	1.3666
$LTNBR7$	3.9287	3.5133	3.3605	1.0119	4.0996	2.5961	2.2773	1.9743	4.8838	1.3496	1.3114	1.2090
$MJLTNBR7$	1.3467	1.3335	0.0483	0.0110	1.5412	1.4898	1.1585	0.9865	1.7637	1.5769	0.8332	0.5980

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Table 13: Results of correlation

	DJ	BM	CG	CS	F	HC	I	OG	T	U	OP
DJ	1.0000	0.2678	-0.0223	0.2431	0.3722	0.2980	0.1665	0.2317	0.1416	-0.0915	0.1716
BM	0.2678	1.0000	0.1406	0.0571	0.3728	0.2080	0.1305	0.0332	0.0087	0.1065	0.1832
CG	-0.0223	0.1406	1.0000	0.0815	-0.1735	0.3782	0.1657	0.1284	0.2032	0.0673	0.2426
CS	0.2431	0.0571	0.0815	1.0000	-0.0989	0.1654	0.1436	0.2078	0.2385	0.2720	-0.1644
F	0.3722	0.3728	-0.1735	-0.0989	1.0000	0.2310	0.1294	0.1097	-0.1711	0.2876	0.1395
HC	0.2980	0.2080	0.3782	0.1654	0.2310	1.0000	0.3374	0.1329	0.4046	0.5015	0.2731
I	0.1665	0.1305	0.1657	0.1436	0.1294	0.3374	1.0000	0.3082	0.1967	0.0774	0.1386
OG	0.2317	0.0332	0.1284	0.2078	0.1097	0.1329	0.3082	1.0000	0.3745	0.1392	0.1242
T	0.1416	0.0087	0.2032	0.2385	-01711	0.4046	0.1967	0.3745	1.0000	0.1980	-0.0746
U	-0.0915	0.1065	0.0673	0.2720	0.2876	0.5015	0.0774	0.1392	0.1980	1.0000	0.4635
OP	0.1716	0.1832	0.2426	-0.1644	0.1395	0.2731	0.1386	0.1242	-0.0746	0.4635	1.0000

Table 14: Coefficients and MSE of proposed and existing estimators

	Coefficients					
	ML	NBRR	JNBR	LTNBR	MJLTNBRR(K_4)	MJLTNBRR(K_5)
DJ	0.4363	0.1764	0.0056	0.2431	0.3722	0.0256
BM	0.0989	1.0368	0.1442	0.0571	0.2983	0.0184
CG	-0.1375	0.1989	-0.9033	1.8970	0.0476	0.0963
CS	-0.1497	0.1632	0.1265	0.9183	0.0913	0.1627
F	0.02355	-0.0676	0.0987	0.0980	-1.0326	0.0089
HC	0.1363	-0.0820	-0.1472	1.0342	1.1403	1.0020
I	-0.1665	0.1142	0.1177	-0.4591	0.1165	0.0005
OG	0.0797	0.1736	0.2184	0.1989	0.1937	0.0376
T	-0.1251	0.1395	-0.2120	0.1462	-0.0784	0.1739
U	0.0031	0.1187	0.01325	-0.1317	-0.0360	0.1029
OP	0.1004	-0.1323	0.1874	0.0483	0.2137	0.4350
	MSE					
	10.9098	5.3001	2.7611	0.0269	0.0002	0.0003

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economic sectors of these countries are under considered in this study. The dependent variable Y = DJ Islamic Stock (DJ) depends on the following independent variables X_1 = Basi Materials (BM), X_2 = Consumer Goods (CG), X_3 = Consumer Sevices (CS), X_4 = Financial (F), x_5 = Health Care (HC), x_6 = Industrial (I), x_7 = Oil and Gas (OG), x_8 = Technology (T), x_9 = Utilities (U), x_{10} = Oil Prices (OP). This index excluded those companies whose business involves weapon and defenses, alcohol, entertainment, conventional financial services and pork related products. The related correlation matrix of these mentioned variables are shown is Table **??**. From this pairwise correlation, it is unable to detect multicollinearity. Therefore, condition number is used to detect this as

$$\text{Conditioned Number} = \sqrt{\frac{\lambda_{max}(X'X)}{.}\lambda_{min}(X'X)}$$

We obtained 532.467 which indicate multicollinearity among variables. Further, in Table **??**, it is observed that proposed estimator with parameter K_4 and K_5 have much smaller values of MSE as compared to other estimators. Thus, the proposed estimator has preferable choice among other estimators.

6 Conclusion

The aim of this study is to address the problem of Multicollinearity that leads to invalid statistical Inference when it arises. In this study, a new modified Jackknifed Liu-type Estimator for negative Binomial regression is proposed on the basis of Liu-type and Jackknifed estimator. Then, some ridge parameters are used to check the performance of proposed estimator. Moreover, to check the effect of degree of correlation among Independent variables, a Monte Carlo simulation is designed using values of correlation coefficients and sample sizes. further, real life data set is also used to check the performance of proposed estimator. Thus, the results reveal that proposed estimator showed better performance than other estimators in case of MSE.

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Conflict of Interest

The authors declare that there is no conflict of interest regarding the publication of this article.

References

[1] S. Guindon, F. Delsuc, J. F. Dufayard, O. Gascuel, Estimating Maximum Likelihood Phylogenies with PhyML (2009). In: Posada D. (eds) Bioinformatics for DNA Sequence Analysis. Methods in Molecular Biology (Methods and Protocols), vol 537. Humana Press. https://doi.org/10.1007/978-1-59745-251-9_6.

[2] M. Chen, F. Ding, R. Lin, T. Y. Ng, Y. Zhang, W. Wei, Maximum likelihood least squares-based iterative methods for output-error bilinear-parameter models with colored noises, Int J Robust Nonlinear Control, **30**, 6262 - 6280 (2020).

[3] K. Sahu and R. K. Srivastava, Needs and Importance of Reliability Prediction: An Industrial Perspective, Information Sciences Letters, **9**, 33-37 (2020).

[4] M. Mahmoud, M. M. Nassar, M. A. Aefa, Parameter Estimation for a Mixture of Inverse Chen and Inverse Compound Rayleigh Distributions Based on Type-II Hybrid Censoring Scheme," Journal of Statistics Applications & Probability, **10**, 467-485 (2021).

[5] S. Kumar, S. Bhougal, V. Sharma, R. Gupta, J. P. S. Joorel, Estimating the Problem of Non-Response and Measurement Error in Sample Survey," Journal of Statistics Applications & Probability, **10**, 665-675 (2021).

[6] M. H. Abu-Moussa, A. M. Abd-Elfattah, E. H. Hafez, Estimation of Stress-Strength Parameter for Rayleigh Distribution based on Progressive Type-II Censoring, Information Sciences Letters, **10**, 101-110 (2021).

[7] S. Ahmad and M. Aslam, The modified Liu-ridge-type estimator: a new class of biased estimators to address multicollinearity, Communication in Statistics - Simulation and Computation, (2020), https://doi.org/10.1080/03610918.2020.1806324.

[8] M. Alkhamisi, G. Khalaf, G. Shukur, Some modifications for choosing ridge parameters, Communications in Statistics: Theory and Methods, **35**, 2005–2020 (2006).

[9] M. Alkhamisi, G. Shukur, Developing ridge parameters for SUR model, Communications in Statistics: Theory and Methods, **37**, 544–564 (2008).

[10] Y. Asar, M. Erişoǧlu, M. Arashi, Developing a restricted two-parameter Liu-type estimator: A comparison of restricted estimators in the binary logistic regression model, Communications in Statistics-Theory and Methods, **46**, 6864-6873 (2016).

[11] Y. Asar, Liu-Type Negative Binomial Regression: A Comparison of Recent Estimators and Applications (2018). In: Tez M., von Rosen D. (eds) Trends and Perspectives in Linear Statistical Inference. Contributions to Statistics. Springer, Cham. https://doi.org/10.1007/978-3-319-73241-1_2.

[12] F. S. M. Batah, T. V. Ramanathan, S. D. Gore, The efficiency of modified Jackknife and ridge type regression estimators: A comparison, Surveys in Mathematics and its Applications, **3**, 111–122 (2008).

[13] C. Cameron, P. K. Trivedi, Regression Analysis of Count Data, Cambridge, University Press-UK (1998).

[14] W. Green, Econometric Analysis. New Jersey, NJ: Pearson Prentice Hall (2003).

[15] M. S. Haq,and B. M. G. Kibria, A shrinkage estimator for the restricted linear regression model: Ridge regression

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approach, Journal of Applied Statistical Science, **3**, 301–16 (1996).
[16] D. V. Hinkley, Jackknifing in unbalanced situations, Technometrics, **19**, 285–292 (1977).
[17] J. M. Hilbe, Negative Binomial Regression, Cambridge University Press(2011).
[18] R. R. Hocking, F. M. Speed, and M. J. Lynn, A class of biased estimators in linear regression, Technometrics, **18**, 425–37 (1976).
[19] A. E. Hoerl, R. W. Kennard, Ridge regression: biased estimation for non-orthogonal problems. Technometrics, **12**, 55–67 (1970).
[20] A. E. Hoerl, R. W. Kennard, Ridge regression: application to non-orthogonal problems, Technometrics, **12**, 69–82 (1970).
[21] J. Huang, H. Yang, A two-parameter estimator in the negative binomial regression model. Journal of Statistical Computation and Simulation, **84**, 124–134 (2014).
[22] N. H. Jadhav, D. N. Kashid, A Jackknifed ridge M-estimator for regression model with multicollinearity and outliers, Journal of Statistical Theory and Practice, **5** 659–673(2011).
[23] S. Sakallıoǧlu Kaçiranlar, S. Akdeniz, F. Styan, G. P. H. and H. J. Werner, A new biased estimator in linear regression and a detailed analysis of the widely-analysed dataset on Portland cement, SANKHYA, **61**, 443-459 (1999).
[24] B. M. G. Kibria, Performance of some new ridge regression estimators, Communications in Statistics: Theory and Methods, **32**, 419–435 (2003).
[25] B. M. G. Kibria, K. Månsson, G. Shukur, A simulation study of some biasing parameters for the ridge type estimation of Poisson regression, Communications in Statistics: Simulation and Computation, **44**, 943–957 (2013).
[26] B. M. G.Kibria, , and A. F. Lukman, A new ridge-type estimator for the linear regression model: Simulations and applications, Scientifica, **4**, 1-16 (2020).
[27] J. F. Lawless and P. Wang, Simulation study of ridge and other regression estimators, Communications in Statistics - Theory and Methods, **5**, 307–23 (1976).
[28] A. F. Lukman, K. Ayinde, Review and classifications of the ridge parameter estimation techniques, Hacettepe Journal of Mathematics and Statistics, **46**, 953 (2017).
[29] K. Liu, A new class of blased estimate in linear regression. Communications in Statistics - Theory and Methods, **22**, 393–402 (1993).
[30] K. Liu, Using Liu-type estimator to combat collinearity, Communications in Statistics - Theory and Methods, **32**, 1009–1020 (2003).
[31] A. F. Lukman, K. Ayinde, S. Binuomote, and O. A. Clement, Modified ridge-type estimator to combat multicollinearity: Application to chemical data, Journal of Chemometrics, **33**, e3125 (2019).
[32] A. F. Lukman, K. Ayinde, S. K. Sek, and E. Adewuyi, A modified new two-parameter estimator in a linear regression model, Modelling and Simulation in Engineering, **2019**, 6342702 (2019).
[33] K. Månsson, On ridge estimators for the negative binomial regression model, Economic Modelling, **29**, 178–184 (2012).
[34] K. Månsson, Developing a Liu estimator for the negative binomial regression model: method and application, Journal of Statistical Computation and Simulation, **83**, 1773–1780 (2013).
[35] K. Månsson, B. M. Golam Kibria and G. Shukur, A restricted Liu estimator for binary regression models and its application to an applied demand system, Journal of Applied Statistics, **43**, 1119–1127 (2016).
[36] K. Månsson, and G. Shukur, A Poisson ridge regression estimator. Economic Modelling, **28**, 1475-1481 (2011).
[37] G. Muniz and B. M. G.Kibria, On some ridge regression estimators: An empirical comparisons, Communications in Statistics: Simulation and Computation, **38**, 621–630 (2009).
[38] M. R. Ozkale and S. Kaçiranlar, The restricted and unrestricted two-parameter estimators, Communications in Statistics - Theory and Methods, **36**, 2707-2725 (2007).
[39] S. Sakallıoglu, and S. Kaçiranlar, A new biased estimator based on ridge estimation, Statistical Papers, **49**, 669–89 (2008).
[40] S. Turkan and G. Ozel, A Jackknifed estimators for the Negative Binomial regression model, Communications in Statistics - Simulation and Computations, **47**, 1845-1865 (2018).

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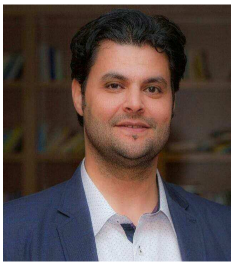

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