Applied Mathematics & Information Sciences
An International Journal

http://dx.doi.org/10.18576/amis/160102

(R,S)-Fuzzy G*P-Closed Sets and its Applications

E. El-sanowsy and A. Atef*

Mathematics Department, Faculty of Science, Sohag University, Sohag, Egypt

Received: 2 Jul. 2021, Revised: 2 Sep. 2021, Accepted: 11 Nov. 2021
Published online: 1 Jan. 2022

Abstract: In this paper, we introduce and study the new class of (r,s)-fuzzy generalized closed sets called (r,s)-fuzzy g*p-closed and (r,s)-fuzzy g*p-open sets. Also, fuzzy g*p-continuous mappings and fuzzy-T*p axioms in double fuzzy topological spaces in Šostak sense are introduced and characterized.

Keywords: double fuzzy topology, (r,s)-fuzzy-g*p continuous mappings, fuzzy T*p-space, fuzzy T**p-space and fuzzy αT*p-space.

1 Introduction and preliminaries

In 1986, Atanassov [1] introduced the concept of intuitionistic fuzzy sets. The idea of intuitionistic fuzzy topological spaces was introduced by Coker [2]. The notion of intuitionistic gradation of openness of fuzzy sets was introduced by Samanta and Mondal [3] and it has been developed in many directions [4,5,6,7,8,9,10,11, 12,13,14,15,16]. Thakur and Chaturvedi [17] defined the intuitionistic fuzzy generalized closed set in intuitionistic fuzzy topological space. Recently, different mathematicians worked and studied in different forms of intuitionistic fuzzy-g-closed set and its topological propertiesn [18,19,20,21,22,23,24,25]. The name (intuitionistic) was replaced with the name (double) by Gutierrez Garcia and Rodabaugh [26]. In this paper, we introduce and define some new concept in fuzzy topological spaces in Šostak sense such as (r,s)-fuzzy g*p-closed sets. We also introduced the concepts of (r,s)-fuzzy g*p-open sets, and obtain some of their characterization and properties. Moreover, we introduce double fuzzy g*p-continuous mappings with some of its properties. As an application of this set we introduce double fuzzy–T*p-space, double fuzzy–T**p-space. and double fuzzy-αT*p-space.

Throughout this paper, let X be a nonempty set, $I=[0,1]$, $I_0=(0,1]$ and $I_1=[0,1)$. For $\alpha\notin I$, $\underline{\alpha}(x)=\alpha$ for each $x\in X$. The set of all fuzzy subsets of X are denoted by I^X.

Definition 1.1 .[3] A double fuzzy toplogy on X is an ordered pair (τ,τ^*) of mappings from I^X to I such that

(1) $\tau(\lambda)+\tau^*(\lambda)\leq 1, \forall\, \lambda\in I^X$
(2) $\tau(\underline{0})=\tau(\underline{1})=1,\ \tau^*(\underline{0})=\tau^*(\underline{1})=0.$
(3) $\tau(\lambda_1\wedge\lambda_2)\geq\tau(\lambda_1)\wedge\ \tau(\lambda_2)$ and $\tau^*(\lambda_1\wedge\lambda_2)\geq\tau^*(\lambda_1)\vee\ \tau^*(\lambda_2),\ \forall\,\lambda_1,\lambda_2\in I^X$
(4) $\tau(\vee_{i\in\triangle}\lambda_i)\geq\wedge_{i\in\triangle}\tau(\lambda_i)$ and $\tau^*(\vee_{i\in\triangle}\lambda_i)\geq\vee_{i\in\triangle}\tau^*(\lambda_i),\ \forall\,\lambda_i\in I^X$, $i\in\triangle$.

The trible (X,τ,τ^*) is called a double fuzzy topological space (dfts, for short).

τ and τ^* may interpreted as gradation of openness and gradation of nonopenness, respectively.

Definition 1.2. [3] The operators $cl_\tau^{\tau^*}, int_\tau^{\tau^*}: I^X\times I_0\times I_1\to I^X$ defined as , for $\lambda\in I^X$ and $r\in I_0,\ s\in I_1$,

$$cl_\tau^{\tau^*}(\lambda,r,s)=\wedge\{\mu\in I^X:\ \lambda\leq\mu,\ \tau(\underline{1}-\mu)\geq r,\ \tau^*(\underline{1}-\mu)\geq s\}.$$
$$iint_\tau^{\tau^*}(\lambda,r,s)=\vee\{\mu\in I^X:\ \mu\leq\lambda,\tau(\mu)\geq r,\ \tau^*(\mu)\geq s\}.$$

Definition 1.3. [8] Let (X,τ,τ^*) be a dfts. For $\lambda\in I^X$ and $r\in I_0,\ s\in I_1$. Then, λ is called:

(1)(r,s)-fuzzy semi-closed set if $\lambda\geq int_\tau^{\tau^*}(cl_\tau^{\tau^*}(\lambda,r,s),r,s)$.
(2)(r,s)-fuzzy regular closed set if $\lambda=cl_\tau^{\tau^*}(int_\tau^{\tau^*}(\lambda,r,s),r,s)$.
(3)(r,s)-fuzzy preclosed set if $\lambda\geq cl_\tau^{\tau^*}(int_\tau^{\tau^*}(\lambda,r,s),r,s)$.
(4)(r,s)-fuzzy α-closed set if $\lambda\geq cl_\tau^{\tau^*}(int_\tau^{\tau^*}(cl_\tau^{\tau^*}(\lambda,r,s),r,s),r,s)$.

* Corresponding author e-mail: elsanowsy@yahoo.com

(5)(r,s)-fuzzy semi-preclosed set if $\lambda \geq int_{\tau}^{\tau^*}(cl_{\tau}^{\tau^*}(int_{\tau}^{\tau^*}(\lambda,r,s),r,s),r,s)$.
The complements of the above mentioned closed set are open, respectively.

Definition 1.4. [8] Let (X,τ,τ^*) be a dfts. For $\lambda,\mu \in I^X$ and $r \in I_0,\ s \in I_1$. Then:
(1) $scl_{\tau}^{\tau^*}(\lambda,r,s) = \wedge\{\mu \in I^X : \lambda \leq \mu, \mu$ is $(r,s)-$fuzzy semi-closed set $\}$.
(2) $pcl_{\tau}^{\tau^*}(\lambda,r,s) = \wedge\{\mu \in I^X : \lambda \leq \mu, \mu$ is $(r,s)-$fuzzy preclosed set $\}$.
(3) $\alpha cl_{\tau}^{\tau^*}(\lambda,r,s) = \wedge\{\mu \in I^X : \lambda \leq \mu, \mu$ is $(r,s)-$fuzzy $\alpha-$closed set $\}$.
(4) $spcl_{\tau}^{\tau^*}(\lambda,r,s) = \wedge\{\mu \in I^X : \lambda \leq \mu, \mu$ is $(r,s)-$fuzzy semi-preclosed set $\}$.

Definition 1.5. Let (X,τ,τ^*) be a dfts. For $\lambda \in I^X$ and $r \in I_0,\ s \in I_1$. Then, λ is called:
(1) (r,s)-fuzzy g-closed (resp., (r,s)-fuzzy g*-closed and (r,s)-fuzzy rg-closed) set if $cl_{\tau}^{\tau^*}(\lambda,r,s) \leq \mu$, whenever $\lambda \leq \mu$ and μ is (r,s)-fuzzy open (resp., (r,s)-fuzzy g- open and (r,s)-fuzzy regular open) in (X,τ,τ^*) [6,10,22].
(2) (r,s)-fuzzy gpr-closed (resp., (r,s)-fuzzy gp-closed and (r,s)-fuzzy sgp-closed) set if $pcl_{\tau}^{\tau^*}(\lambda,r,s) \leq \mu$, whenever $\lambda \leq \mu$ and μ is (r,s)-fuzzy regular (resp. (r,s)-fuzzy open and (r,s)-fuzzy semi-open) in (X,τ,τ^*)[11,18,24].
(3) (r,s)-fuzzy gsp-closed (resp., (r,s)-fuzzy gspr-closed) set if $spcl_{\tau}^{\tau^*}(\lambda,r,s) \leq \mu$, whenever $\lambda \leq \mu$ and μ is (r,s)-fuzzy open (resp., (r,s)-fuzzy regular open) in (X,τ,τ^*)[20,22].
(4) (r,s)-fuzzy αg-closed set if $\alpha cl_{\tau}^{\tau^*}(\lambda,r,s) \leq \mu$, whenever $\lambda \leq \mu$ and μ is (r,s)-fuzzy $\alpha-$open in (X,τ,τ^*).[27]
(5) (r,s)-fuzzy sg-closed set if $scl_{\tau}^{\tau^*}(\lambda,r,s) \leq \mu$, whenever $\lambda \leq \mu$ and μ is (r,s)-fuzzy semi-open in (X,τ,τ^*).[23]

Definition 1.6. Let (X,τ,τ^*) and (Y,η,η^*) be dfts's. Then the function $f : (X,\tau,\tau^*) \to (Y,\eta,\eta^*)$ is called:
(1) F-gp-continuous iff $f^{-1}(\mu)$ is (r,s)- fuzzy gp-closed set of X, for all (r,s)- fuzzy closed set μ of Y [18].
(2) F-gpr-continuous iff $f^{-1}(\mu)$ is (r,s)- fuzzy gpr-closed set of X, forall (r,s)- fuzzy closed set μ of Y [24].

(3) F-g*-continuous iff $f^{-1}(\mu)$ is (r,s)- fuzzy g*-closed set of X, for all (r,s)- fuzzy closed set μ of Y [10].

Definition 1.7. A dfts (X,τ,τ^*) is called::

(1) F-$T_{\frac{1}{2}}$-space if every (r,s)-fuzzy g-closed set is (r,s)-fuzzy closed set [17].

(2) F-preregular-$T_{\frac{1}{2}}$-space if every (r,s)-fuzzy gpr-closed is (r,s)-fuzzy closed [24].

(3) F-semi-preregular-$T_{\frac{1}{2}}$-space if every (r,s)-fuzzy gp-closed is (r,s)-fuzzy closed [18].
(4) F-$pT_{\frac{1}{2}}$ -space if every (r,s)-fuzzy gspr-closed set is (r,s)-fuzzy closed set [20].

2 (r,s)-fuzzy g*P-closed set

Definition 2.1. Let (X,τ,τ^*) be a dfts, $\lambda,\mu \in I^X$, $r \in I_0, s \in I_1$. A fuzzy set λ is called:
(1) (r,s)-fuzzy g*p-closed set if $pcl_{\tau}^{\tau^*}(\lambda,r,s) \leq \mu$, whenever $\lambda \leq \mu$ and μ is (r,s)-fuzzy g-open set.
(2) (r,s)- fuzzy g*p-open set if $\underline{1} - \lambda$ is (r,s)-fuzzy g*p-closed set.

Theorem 2.2.
(1) Every (r,s)-fuzzy preclosed set is (r,s)-fuzzy g*p-closed set.
(2) Every (r,s)-fuzzy α-closed set is (r,s)-fuzzy g*p-closed set.
(3) Every (r,s)-fuzzy closed set is (r,s)-fuzzy g*p-closed set.
(4) Every (r,s)-fuzzy regular closed set is (r,s)-fuzzy g*p-closed set.
(5) Every (r,s)-fuzzy g*-closed set is (r,s)-fuzzy g*p-closed set.
(6) Every (r,s)-fuzzy g*p-closed set is (r,s)-fuzzy gpr-closed set.
(7) Every (r,s)-fuzzy g*p-closed set is (r,s)-fuzzy gp-closed set.
(8) Every (r,s)-fuzzy g*p-closed set is (r,s)-fuzzy gspr-closed set.
(9) Every (r,s)-fuzzy g*p-closed set is (r,s)-fuzzy gsp-closed set.
(10) Every (r,s)-fuzzy g*p-closed set is (r,s)-fuzzy sgp-closed set.

Proof. (1) Let λ is (r,s)-fuzzy preclosed set and $\lambda \leq \mu$ with μ is (r,s)-fuzzy g-open set in X. Since λ is (r,s)-fuzzy preclosed set, we get that, $\lambda = pcl_{\tau}^{\tau^*}(\lambda,r,s) \leq \mu$, whenever $\lambda \leq \mu$ and μ is (r,s)-fuzzy g-open set in X. Hence, λ is (r,s)-fuzzy g*p-closed set.
(2) As in (1) and by the fact, $pcl_{\tau}^{\tau^*}(\lambda,r,s) \leq \alpha cl_{\tau}^{\tau^*}(\lambda,r,s)$.
(3) As in (1) and by the fact, $pcl_{\tau}^{\tau^*}(\lambda,r,s) \leq cl_{\tau}^{\tau^*}(\lambda,r,s)$.
(4) By (3) and the fact, every (r,s)-fuzzy regular closed set is (r,s)-fuzzy closed set.
(5) By the definition of (r,s)-fuzzy g*-closed set and the fact, $pcl_{\tau}^{\tau^*}(\lambda,r,s) \leq cl_{\tau}^{\tau^*}(\lambda,r,s)$.
(6) Let λ is (r,s)-fuzzy g*p-closed set and $\lambda \leq \mu$ with μ is (r,s)-fuzzy regular open set in X. Since every (r,s)-fuzzy regular open set is (r,s)-fuzzy g-open set, μ is

(r,s)-fuzzy g-open set. By the definition of (r,s)-fuzzy g*p-closed set, $pcl_{\tau}^{\tau^*}(\lambda,r,s) \leq \mu$. Hence, λ is (r,s)-fuzzy gpr-closed set.

(7) As in (6) and by the fact, every (r,s)-fuzzy open set is (r,s)-fuzzy g-open set.

(8) As in (6) and by facts, every (r,s)-fuzzy regular open set is (r,s)-fuzzy g-open set and $spcl_{\tau}^{\tau^*}(\lambda,r,s) \leq pcl_{\tau}^{\tau^*}(\lambda,r,s)$.

(9) As in (8) and by the fact, every (r,s)-fuzzy open set is (r,s)-fuzzy g-open set.

(10) As in (6) and by the fact, every (r,s)-fuzzy semi-open set is (r,s)-fuzzy g-open set.

However, the converse of above theorem is not true in general, as shown in the following examples.

Example 2.3. Let $X = \{a,b,c\}$. Define $\lambda_1,\ \lambda_2 \in I^X$ as follows, $\lambda_1 = \{0.1,0,0\}$ and $\lambda_2 = \{1,0.8,0.6\}$. Define fuzzy topologies $\tau,\ \tau^* : I^X \to I$ as follows:

$$\tau(\mu) = \begin{cases} 1 & \text{if} \quad \mu = \underline{0},\ \underline{1}, \\ \frac{1}{2} & \text{if} \quad \mu = \lambda_1,\ \lambda_2,\ , \\ 0 & \text{otherwise}, \end{cases}$$

$$\tau^*(\mu) = \begin{cases} 0 & \text{if} \quad \mu = \underline{0},\ \underline{1}, \\ \frac{1}{2} & \text{if} \quad \mu = \lambda_1,\ \lambda_2, \\ 1 & \text{otherwise}, \end{cases}$$

Then, (1) the fuzzy set $\rho = \{0.7,0.6,0\}$ is $(\frac{1}{2},\ \frac{1}{2})$-fuzzy g*p-closed set but it is neither $(\frac{1}{2},\ \frac{1}{2})$-fuzzy preclosed set nor $(\frac{1}{2},\ \frac{1}{2})$-fuzzy α-closed set.

(2) the set $v = \underline{0,6}$ is $(\frac{1}{2},\ \frac{1}{2})$-fuzzy g*p-closed set but it is neither $(\frac{1}{2},\ \frac{1}{2})$-fuzzy closed set nor $(\frac{1}{2},\ \frac{1}{2})$-fuzzy regular closed set.

Example 2.4. Let $X = \{a,b,c\}$. Define $\lambda \in I^X$ as follows, $\lambda = \{0.6,0,0\}$. Define fuzzy topologies $\tau,\ \tau^* : I^X \to I$ as follows:

$$\tau(\mu) = \begin{cases} 1 & \text{if} \quad \mu = \underline{0},\ \underline{1}, \\ \frac{2}{3} & \text{if} \quad \mu = \lambda, \\ 0 & \text{otherwise}, \end{cases} ,$$

$$\tau^*(\mu) = \begin{cases} 0 & \text{if} \quad \mu = \underline{0},\ \underline{1}, \\ \frac{1}{3} & \text{if} \quad \mu = \lambda, \\ 1 & \text{otherwise}, \end{cases}$$

Then, the fuzzy set $\rho = \{0.7,0.7,0\}$ is $(\frac{2}{3},\ \frac{1}{3})$-fuzzy gp-closed set and $(\frac{2}{3},\ \frac{1}{3})$-fuzzy gpr closed set but it is not $(\frac{2}{3},\ \frac{1}{3})$-fuzzy g*p-closed set.

Example 2.5. Let $X = \{a,b,c,d\}$. Define $\lambda_1,\ \lambda_2,\lambda_3 \in I^X$ as follows, $\lambda_1 = \{0.7,0.6,0,0\}$, $\lambda_2 = \{0,0,0.7,0.6\}$ and $\lambda_3 = \{0.7,0.6,0.7,0.6\}$. Define fuzzy topologies $\tau,\ \tau^* : I^X \to I$ as follows:

$$\tau(\mu) = \begin{cases} 1 & \text{if} \quad \mu = \underline{0},\ \underline{1}, \\ \frac{3}{5} & \text{if} \quad \mu = \lambda_1,\ \lambda_2,\lambda_3, \\ 0 & \text{otherwise}, \end{cases} ,$$

$$\tau^*(\mu) = \begin{cases} 0 & \text{if} \quad \mu = \underline{0},\ \underline{1}, \\ \frac{2}{5} & \text{if} \quad \mu = \lambda_1,\ \lambda_2,\lambda_3, \\ 1 & \text{otherwise}, \end{cases}$$

Then, the fuzzy set $\rho = \{0.7,0,0,0\}$ is $(\frac{3}{5},\ \frac{2}{5})$-fuzzy gspr-closed set but it is not $(\frac{3}{5},\ \frac{2}{5})$-fuzzy g*p-closed set.

Example 2.6. Let $X = \{a,b,c\}$. Define $\lambda_1,\ \lambda_2 \in I^X$ as follows, $\lambda_1 = \{0.9,0.8,1\}$ and $\lambda_2 = \{0.8,0.5,0\}$. Define fuzzy topologies $\tau,\ \tau^* : I^X \to I$ as follows:

$$\tau(\mu) = \begin{cases} 1 & \text{if} \quad \mu = \underline{0},\ \underline{1}, \\ \frac{5}{7} & \text{if} \quad \mu = \lambda_1,\ \lambda_2, \\ 0 & \text{otherwise}, \end{cases} ,$$

$$\tau^*(\mu) = \begin{cases} 0 & \text{if} \quad \mu = \underline{0},\ \underline{1}, \\ \frac{2}{7} & \text{if} \quad \mu = \lambda_1,\ \lambda_2, \\ 1 & \text{otherwise}, \end{cases}$$

Then, the fuzzy set $\rho = \{0,0.5,0\}$ is $(\frac{5}{7},\ \frac{2}{7})$-fuzzy g*-closed set but it is not $(\frac{5}{7},\ \frac{2}{7})$-fuzzy g*p-closed set.

Example 2.7. Let $X = \{a,b,c,d\}$. Define $\lambda_1,\ \lambda_2,\lambda_3,\lambda_4 \in I^X$ as follows, $\lambda_1 = \{0.9,0,0,0\}$, $\lambda_2 = \{0,0.8,0,0\}$, $\lambda_3 = \{0.9,0.8,0,0\}$ and $\lambda_4 = \{0.9,0.8,0.7,0\}$. Define fuzzy topologies $\tau,\ \tau^* : I^X \to I$ as follows:

$$\tau(\mu) = \begin{cases} 1 & \text{if} \quad \mu = \underline{0},\ \underline{1}, \\ \frac{4}{5} & \text{if} \quad \mu = \lambda_1,\ \lambda_2,\lambda_3,\lambda_4, \\ 0 & \text{otherwise}, \end{cases} ,$$

$$\tau^*(\mu) = \begin{cases} 0 & \text{if} \quad \mu = \underline{0},\ \underline{1}, \\ \frac{1}{5} & \text{if} \quad \mu = \lambda_1,\ \lambda_2,\lambda_3,\lambda_4, \\ 1 & \text{otherwise}, \end{cases}$$

Then, the fuzzy set $\upsilon = \{0,0,0.5,0\}$ is $(\frac{4}{5},\ \frac{1}{5})$-fuzzy sgp-closed set but it is not $(\frac{4}{5},\ \frac{1}{5})$-fuzzy g*p-closed set.

Example 2.8. Let $X = \{a,b\}$. Define $\lambda \in I^X$ as follows, $\lambda = \{0.5,0.2\}$. Define fuzzy topologies $\tau,\ \tau^* : I^X \to I$ as follows:

$$\tau(\mu) = \begin{cases} 1 & \text{if} \quad \mu = \underline{0},\ \underline{1}, \\ \frac{1}{2} & \text{if} \quad \mu = \lambda, \\ 0 & \text{otherwise}, \end{cases} ,$$

$$\tau^*(\mu) = \begin{cases} 0 & \text{if} \quad \mu = \underline{0},\ \underline{1}, \\ \frac{1}{3} & \text{if} \quad \mu = \lambda, \\ 1 & \text{otherwise}, \end{cases}$$

Then, the fuzzy set $\rho = \{0.2,0.6\}$ is $(\frac{1}{2},\ \frac{1}{3})$-fuzzy gsp-closed set but it is neither $(\frac{1}{2},\ \frac{1}{3})$-fuzzy g*p-closed set.

Lemma 2.9. Let (X,τ,τ^*) be a dfts and $\lambda \in I^X$. Then, λ is $(r\ ,s)$-fuzzy g*p-closed set iff $\lambda \overline{q}\, \mu \Longrightarrow pcl_{\tau}^{\tau^*}(\lambda,r,s)\ \overline{q}\, \mu$ for every $(r\ ,s)$-fuzzy g-closed set μ of X.

Proof. Necessity Let μ be an $(r\ ,s)$-fuzzy g-closed set of X and $\lambda \overline{q}\, \mu$ Then, $\lambda \leq \underline{1} - \mu$

and $\underline{1}-\mu$ is $(r\ ,s)$-fuzzy g-open in X. Therefore, $pcl_{\tau}^{\tau^*}(\lambda,r,s)\leq\underline{1}-\mu$, because λ is $(r\ ,s)$-fuzzy g*p-closed. Hence, $pcl_{\tau}^{\tau^*}(\lambda,r,s)\overline{q}\ \mu$.

Sufficiency Let ρ be an (r,s)-fuzzy g-open set of X such that $\lambda\leq\rho$. Then,

$\lambda\leq\underline{1}-\rho$ and $\underline{1}-\rho$ is (r,s)-fuzzy g-closed set in X. Hence by hypothesis $pcl_{\tau}^{\tau^*}(\lambda,r,s)\ \overline{q}\ (\underline{1}-\rho)$. Therefore, $pcl_{\tau}^{\tau^*}(\lambda,r,s)\leq\rho$. Hence, λ is (r,s)-fuzzy g*p-closed set.

Remark 2.10. The intersection of two (r,s)-fuzzy g*p-closed sets in (X,τ,τ^*) may not be (r,s)-fuzzy g*p-closed set.

Example 2.11. Let $X=\{a,b,c\}$. Define $\lambda\in I^X$ as follows, $\lambda=\{1,0,0\}$. Define fuzzy topologies $\tau,\ \tau^*:I^X\rightarrow I$ as follows:

$$\tau(\mu)=\begin{cases}1 & \text{if}\quad \mu=\underline{0},\ \underline{1}\,,\\ \frac{5}{7} & \text{if}\quad \mu=\lambda,\\ 0 & \text{otherwise,}\end{cases},$$

$$\tau^*(\mu)=\begin{cases}0 & \text{if}\quad \mu=\underline{0},\ \underline{1}\,,\\ \frac{2}{7} & \text{if}\quad \mu=\lambda,\\ 1 & \text{otherwise,}\end{cases}$$

Then, the fuzzy sets $\rho=\{1,1,0\}$ and $\upsilon=\{1,0,1\}$

are $(\frac{5}{7},\ \frac{2}{7})$-fuzzy g*p-closed sets, but $\lambda=\nu\wedge\rho$ is not $(\frac{5}{7},\ \frac{2}{7})$-fuzzy g*p-closed set, because $\lambda\leq\lambda$ and λ is $(\frac{5}{7},\ \frac{2}{7})$-fuzzy g-open set, but $pcl_{\tau}^{\tau^*}(\lambda,\frac{5}{7},\ \frac{2}{7})=\underline{1}\not\leq\lambda$.

Theorem 2.12. Let λ be an (r,s)-fuzzy g*p-closed set in a dfts (X,τ,τ^*) and $\lambda\leq\mu\leq pcl_{\tau}^{\tau^*}(\lambda,r,s)$. Then, μ is (r,s)-fuzzy g*p-closed in X.

Proof Let ρ be an (r,s)-fuzzy g-open set in X such that $\mu\leq\rho$. Then, $\lambda\leq\rho$ and since λ is (r,s)-fuzzy g*p-closed, $pcl_{\tau}^{\tau^*}(\lambda,r,s)\leq\rho$. Now $\mu\leq pcl_{\tau}^{\tau^*}(\lambda,r,s)\implies pcl_{\tau}^{\tau^*}(\mu,r,s)\leq pcl_{\tau}^{\tau^*}(\lambda,r,s)\leq\rho$. Consequently, μ is (r,s)-fuzzy g*p-closed.

Theorem 2.13. Let λ be an (r,s)-fuzzy g*p-open set in a dfts (X,τ,τ^*) and $pint_{\tau}^{\tau^*}(\lambda,r,s)\leq\mu\leq\lambda$. Then μ is (r,s)-fuzzy g*p-open in X.

Proof. Suppose λ is an (r,s)-fuzzy g*p-open in X and $pint_{\tau}^{\tau^*}(\lambda,r,s)\leq\mu\leq\lambda$. Then, $pcl_{\tau}^{\tau^*}(\underline{1}-\lambda,r,s)\geq(\underline{1}-\mu)\geq(\underline{1}-\lambda)$ and $(\underline{1}-\lambda)$ is (r,s)-fuzzy g*p-closed it follows from Theorem 2.12, $(\underline{1}-\mu)$ is (r,s)-fuzzy g*p-closed. Hence, μ is (r,s)-fuzzy g*p-open.

Theorem 2.14. An (r,s)-fuzzy set λ of a dfts (X,τ,τ^*)

is (r,s)-fuzzy g*p-open if $\mu\leq pcl_{\tau}^{\tau^*}(\lambda,r,s)$. whenever, μ is (r,s)-fuzzy g-closed and $\mu\leq\lambda$.

Proof. Obvious.

Theorem 2.15. Let (X,τ,τ^*) be dfts. For $\lambda,\mu\in I^X$ and $r\in I_0, s\in I_1$. Then, a fuzzy generalized closure operator $GC^{p*}:\ I^X\times I_0\times I_1\rightarrow I^X$ defined as follows:

$$GC^{p^*}(\lambda,r,s)=\wedge\{\mu\in I^X:\ \lambda\leq\mu$$

and

$$\mu\text{ is }(r,s)\text{-fuzzy g*p-closed set}\}.$$

The operator GC^{p^*} satisfies the following properties.

(1) $GC^{p^*}(\underline{0},r,s)=\underline{0}$.

(2) $\lambda\leq GC^{p^*}(\lambda,r,s)$

(3) $GC^{p^*}(\lambda,r,s)\vee GC^{p^*}(\mu,r,s)=GC^{p^*}(\lambda\vee\mu,r,s)$.

(4)$GC^{p^*}(GC^{p^*}(\lambda,r,s),r,s)=GC^{p^*}(\lambda,r,s)$.

(5) If λ is (r,s)-fuzzy g*p-closed set, then $GC^{p^*}(\lambda,r,s)=\lambda$.

(6) $GC^{p^*}(\lambda,r,s)\leq cl_{\tau}^{\tau^*}(\lambda,r,s)$.

(7) $GC^{p^*}(cl_{\tau}^{\tau^*}(\lambda,r,s),r,s)=cl_{\tau}^{\tau^*}(GC^{p^*}(\lambda,r,s),r,s)=cl_{\tau}^{\tau^*}(\lambda,r,s)$.

Proof. (1), (2) and (5) are easily proved from the definition of GC^{p^*}.

(3) Since $\lambda\leq\lambda\vee\mu$ and $\mu\leq\lambda\vee\mu$,therefore, $GC^{p^*}(\lambda,r,s)\vee GC^{p^*}(\mu,r,s)\leq GC^{p^*}(\lambda\vee\mu,r,s)$.

Suppose, $GC^{p^*}(\lambda,r,s)\vee GC^{p^*}(\mu,r,s)\not\geq GC^{p^*}(\lambda\vee\mu,r,s)$. There are $x\in X$ and $t\in(0,1)$ such that:

$$GC^{p^*}(\lambda,r,s)(x)\vee GC^{p^*}(\mu,r,s)(x)<t<GC^{p^*}(\lambda\vee\mu,r,s)(x).$$

Since $GC^{p^*}(\lambda,r,s)(x)<t$ and $GC^{p^*}(\lambda,r,s)(x)<t$, there are (r,s)-fuzzy-g*p-closed sets ρ,ν with $\lambda\leq\rho$ and $\mu\leq\nu$ such that, $\rho(x)<t$ and $\nu(x)<t$.

Since $\lambda\vee\mu\leq\rho\vee\nu$ and $\rho\vee\nu$ is (r,s)-fuzzy-g*p-closed set, therefore, $GC^{p^*}(\lambda\vee\mu,r,s)(x)\leq(\rho\vee\nu)(x)<t$. It is a contradiction.

(4) From (2), we only show $GC^{p^*}(\lambda,r,s)\geq GC^{p^*}(GC^{p^*}(\lambda,r,s),r,s)$.

Suppose $GC^{p^*}(\lambda,r,s)\not\geq GC^{p^*}(GC^{p^*}(\lambda,r,s),r,s)$

There are $x\in X$ and $t\in(0,1)$ such that:

$$GC^{p^*}(\lambda,r,s)(x)<t<GC^{p^*}(GC^{p^*}(\lambda,r,s),r,s)(x).$$

Since $GC^{p^*}(\lambda,r,s)(x)<t$, there is (r,s)-fuzzy-g*p-closed set ρ with $\lambda\leq\rho$ such that

$GC^{p^*}(\lambda,r,s)(x)<\rho(x)<t$, then, $GC^{p^*}(\lambda,r,s)\leq GC^{p^*}(\rho,r,s)=\rho$.

Again, $GC^{p^*}(GC^{p^*}(\lambda,r,s),r,s)\leq GC^{p^*}(\rho,r,s)=\rho$. Hence, $GC^{p^*}(GC^{p^*}(\lambda,r,s),r,s)(x)\leq\rho(x)<t$. It is a contradiction.

(6) Since $cl_{\tau}^{\tau^*}(\lambda,r,s)$ is (r,s)-fuzzy closed set, we have $cl_{\tau}^{\tau^*}(\lambda,r,s)$ is (r,s)-fuzzy-g*p-closed set. Hence, $GC^{p^*}(\lambda,r,s)\leq cl_{\tau}^{\tau^*}(\lambda,r,s)$.

(7) $GC^{p^*}(cl_{\tau}^{\tau^*}(\lambda,r,s),r,s)=cl_{\tau}^{\tau^*}(\lambda,r,s)$, it's a trivial case.

We only show that, $cl_{\tau}^{\tau^*}(GC^{p^*}(\lambda,r,s),r,s)=cl_{\tau}^{\tau^*}(\lambda,r,s)$.

Since, $\lambda \leq GC^{p^*}(cl_\tau^{\tau^*}(\lambda,r,s),r,s)$, therefore, $cl_\tau^{\tau^*}(GC^{p^*}(\lambda,r,s),r,s) \geq cl_\tau^{\tau^*}(\lambda,r,s)$.

Suppose, $cl_\tau^{\tau^*}(GC^{p^*}(\lambda,r,s),r,s) \nleq cl_\tau^{\tau^*}(\lambda,r,s)$. There are $x \in X$ and $t \in (0,1)$ such that:

$cl_\tau^{\tau^*}(GC^{p^*}(\lambda,r,s),r,s)(x) > t > cl_\tau^{\tau^*}(\lambda,r,s)(x)$.

Since $cl_\tau^{\tau^*}(\lambda,r,s)(x) < t$, by the definition of $cl_\tau^{\tau^*}$, there exists an (r,s)-fuzzy closed set, $\rho \in I^X$ with $\lambda \leq \rho$ such that,

$cl_\tau^{\tau^*}(GC^{p^*}(\lambda,r,s),r,s)(x)$
$> t > \rho(x) \geq .cl_\tau^{\tau^*}(\lambda,r,s)(x)$

On the other hand, since $\rho = cl_\tau^{\tau^*}(\rho,r,s)$ is (r,s)-fuzzy-g*p-closed set, $\lambda \leq \rho$ implies,

$$\begin{aligned} GC^{p^*}(\lambda,r,s) &\leq GC^{p^*}(\rho,r,s) \\ &= GC^{p^*}(cl_\tau^{\tau^*}(\rho,r,s),r,s) \\ &= cl_\tau^{\tau^*}(\rho,r,s) = \rho. \end{aligned}$$

Thus, $cl_\tau^{\tau^*}(GC^{p^*}(\lambda,r,s),r,s) \leq \rho$. It is a contradiction.

Theorem 2.16. Let (X,τ,τ^*) be dfts. For $\lambda,\mu \in I^X$ and $r \in I_0, s \in I_1$. Then, a fuzzy generalized interior operator $GI^{p*}: I^X \times I_0 \times I_1 \to I^X$ defined as follows:

$$GI^{p^*}(\lambda,r,s) = \vee\{\mu \in I^X :\ \mu \leq \lambda$$

and

$$\mu \text{ is } (r,s)\text{-fuzzy g*p-open set}\}.$$

Then, $GI^{p^*}(\underline{1}-\lambda,r,s) = \underline{1} - GC^{p^*}(\lambda,r,s)$.

Proof. For each, $\lambda,\mu \in I^X$ and $r \in I_0, s \in I_1$ we have

$$\begin{aligned} &GI^{p^*}(\underline{1}-\lambda,r,s) \\ &= \vee\{\mu \in I^X :\ \mu \leq (\underline{1}-\lambda) \\ &\quad \text{and } \mu \text{ is } (r,s)\text{-fuzzy g*p-open }\} \\ &= \underline{1} - \wedge\{\underline{1}-\mu :\ \lambda \leq \underline{1}-\mu \\ &\quad \text{and } \underline{1}-\mu \text{ is } (r,s)\text{-fuzzy g*p-closed set}\} \\ &= \underline{1} - \wedge\{\rho :\ \lambda \leq \rho \\ &\quad \text{and } \rho \text{ is } (r,s)\text{-fuzzy g*p-closed set}\} \\ &= \underline{1} - GC^{p^*}(\lambda,r,s). \end{aligned}$$

3 Double fuzzy g*p-continuous mapping

Definition 3.1. A mapping $f:(X,\ \tau,\tau^*) \to (Y,\eta,\eta^*)$ is called DF-g*p-continuous iff $f^{-1}(\mu)$ is (r,s)-fuzzy g*p-closed set, $\forall\ \mu \in I^Y, r \in I_0, s \in I_1$ with $\eta(\underline{1}-\mu) \geq r$ and $\eta^*(\underline{1}-\mu) \leq s$.

Remark 3.2. From the above definition and knowns results we have the following diagram of implications:

DF-continuous
$\Downarrow$
DF-g*-continuous $\Longrightarrow$ DF-g*p-continuous
$\Downarrow$ $\Downarrow$
DF-gpr-continuous $\Longleftarrow$ DF-gr-continuous

However, converses of the above implications are not true in general as following examples show.

Example 3.3. Let $X = \{a,b\}$ and $Y = \{x,y\}$. Define $\lambda \in I^X$ and $\mu \in I^Y$ as follows, $\lambda = \{0.5,0.6\}, \mu = \{0.7,0.8\}$ Define fuzzy topologies $\tau,\ \tau^*: I^X \to I$ and $\eta,\ \eta^*: I^Y \to I$ as follows:

$$\tau(\rho) = \begin{cases} 1 & \text{if } \rho = \underline{0},\ \underline{1}, \\ \frac{1}{2} & \text{if } \rho = \lambda, \\ 0 & \text{otherwise}, \end{cases},$$

$$\tau^*(\rho) = \begin{cases} 0 & \text{if } \rho = \underline{0},\ \underline{1}, \\ \frac{1}{2} & \text{if } \rho = \lambda, \\ 1 & \text{otherwise}, \end{cases}$$

$$\eta(\nu) = \begin{cases} 1 & \text{if } \nu = \underline{0},\ \underline{1}, \\ \frac{1}{2} & \text{if } \nu = \mu, \\ 0 & \text{otherwise}, \end{cases},$$

$$\eta^*(\nu) = \begin{cases} 0 & \text{if } \nu = \underline{0},\ \underline{1}, \\ \frac{1}{2} & \text{if } \nu = \mu, \\ 1 & \text{otherwise}, \end{cases}$$

Then the mapping $f:(X,\ \tau,\tau^*) \to (Y,\eta,\eta^*)$ defined by $f(a) = x$ and $f(b) = y$ is DF-g*p- continuous but not DF-continuous.

Example 3.4.. Let $X = \{a,b\}$ and $Y = \{x,y\}$. Define $\lambda \in I^X$ and $\mu \in I^Y$ as follows, $\lambda = \{0.5,0.4\}, \mu = \{0.5,0.3\}$ Define fuzzy topologies $\tau,\ \tau^*: I^X \to I$ and $\eta,\ \eta^*: I^Y \to I$ as follows:

$$\tau(\rho) = \begin{cases} 1 & \text{if } \rho = \underline{0},\ \underline{1}, \\ \frac{7}{12} & \text{if } \rho = \lambda, \\ 0 & \text{otherwise}, \end{cases},$$

$$\tau^*(\rho) = \begin{cases} 0 & \text{if } \rho = \underline{0},\ \underline{1}, \\ \frac{5}{12} & \text{if } \rho = \lambda, \\ 1 & \text{otherwise}, \end{cases}$$

$$\eta(\nu) = \begin{cases} 1 & \text{if } \nu = \underline{0},\ \underline{1}, \\ \frac{7}{12} & \text{if } \nu = \mu, \\ 0 & \text{otherwise}, \end{cases},$$

$$\eta^*(\nu) = \begin{cases} 0 & \text{if } \nu = \underline{0},\ \underline{1}, \\ \frac{5}{12} & \text{if } \nu = \mu, \\ 1 & \text{otherwise}, \end{cases}$$

Then the mapping $f:(X,\ \tau,\tau^*) \to (Y,\eta,\eta^*)$ defined by $f(a) = x$ and $f(b) = y$ is DF-g*p- continuous but not DF-g*-continuous.

Example 3.5.. Let $X = \{a,b,c,d,e\}$ and $Y = \{p,q,r,s,t\}$. Define $\lambda_1,\ \lambda_2,\ \lambda_3, \in I^X$ and $\mu \in I^Y$ as follows, $\lambda_1 = \{0.9,0.8,0,0,0\}$,

$\lambda_2 = \{0,0,0.8,0.7,0\}$, $\lambda_3 = \{0.9,0.8,0.8,0.7,0\}$ $\mu = \{0.9,0,0,0,0\}$ Define fuzzy topologies $\tau,\ \tau^* : I^X \to I$ and $\eta,\ \eta^* : I^Y \to I$ as follows:

$$\tau(\rho) = \begin{cases} 1 & \text{if} \quad \rho = \underline{0},\ \underline{1}, \\ \frac{1}{2} & \text{if} \quad \rho = \lambda_1,\ \lambda_2,\ \lambda_3, \\ 0 & \text{otherwise,} \end{cases},$$

$$\tau^*(\rho) = \begin{cases} 0 & \text{if} \quad \rho = \underline{0},\ \underline{1}, \\ \frac{1}{2} & \text{if} \quad \rho = \lambda_1,\ \lambda_2,\ \lambda_3, \\ 1 & \text{otherwise,} \end{cases}$$

$$\eta(\nu) = \begin{cases} 1 & \text{if} \quad \nu = \underline{0},\ \underline{1}, \\ \frac{1}{2} & \text{if} \quad \nu = \mu, \\ 0 & \text{otherwise,} \end{cases},$$

$$\eta^*(\nu) = \begin{cases} 0 & \text{if} \quad \nu = \underline{0},\ \underline{1}, \\ \frac{1}{2} & \text{if} \quad \nu = \mu, \\ 1 & \text{otherwise,} \end{cases}$$

Then, the mapping $f : (X,\ \tau,\tau^*) \to (Y,\eta,\eta^*)$ defined by $f(a) = p,\ f(b) = q,\ f(c) = r,\ f(d) = s$ and $f(e) = t$ is DF-gpr- continuous but not DF-g*p-continuous.

Example 3.6.. Let $X = \{a,b,c\}$ and $Y = \{x,y,z\}$. Define $\lambda_1,\ \lambda_2, \in I^X$ and $\mu \in I^Y$ as follows, $\lambda_1 = \{0.9,0,0\}$, $\lambda_2 = \{0.9,0.8,0\}$, $\mu = \{0,0.8,0\}$ Define fuzzy topologies $\tau,\ \tau^* : I^X \to I$ and $\eta,\ \eta^* : I^Y \to I$ as follows:

$$\tau(\rho) = \begin{cases} 1 & \text{if} \quad \rho = \underline{0},\ \underline{1}, \\ \frac{2}{3} & \text{if} \quad \rho = \lambda_1,\ \lambda_2, \\ 0 & \text{otherwise,} \end{cases},$$

$$\tau^*(\rho) = \begin{cases} 0 & \text{if} \quad \rho = \underline{0},\ \underline{1}, \\ \frac{1}{3} & \text{if} \quad \rho = \lambda_1,\ \lambda_2, \\ 1 & \text{otherwise,} \end{cases}$$

$$\eta(\nu) = \begin{cases} 1 & \text{if} \quad \nu = \underline{0},\ \underline{1}, \\ \frac{2}{3} & \text{if} \quad \nu = \mu, \\ 0 & \text{otherwise,} \end{cases},$$

$$\eta^*(\nu) = \begin{cases} 0 & \text{if} \quad \nu = \underline{0},\ \underline{1}, \\ \frac{1}{3} & \text{if} \quad \nu = \mu, \\ 1 & \text{otherwise,} \end{cases}$$

Then, the mapping $f : (X,\ \tau,\tau^*) \to (Y,\eta,\eta^*)$ defined by $f(a) = x,\ f(b) = y$ and $f(c) = z$ is DF-gp- continuous but not DF-g*p-continuous.

Theorem 3.7. Let $f : (X,\ \tau,\tau^*) \to (Y,\eta,\eta^*)$ be DF-g*p-continuous. Then, the following statements are hold.:

(1) $f(GC^{p^*}(\lambda,r,s)) \leq cl_\tau^{\tau^*}(f(\lambda),r,s)$, $\forall\ \lambda \in I^X$ and $r \in I_0, s \in I_1$.

(2) $GC^{p^*}(f^{-1}(\mu),r,s) \leq f^{-1}(cl_\tau^{\tau^*}(\mu,r,s))$, $\forall\ \mu \in I^Y$ and $r \in I_0, s \in I_1$.

(3) $GI^{p^*}(f^{-1}(\mu),r,s) \geq f^{-1}(int_\tau^{\tau^*}(\mu,r,s))$, $\forall\ \mu \in I^Y$ and $r \in I_0, s \in I_1$.

Proof. (1) Since f is a DF-g*p-continuous, $f^{-1}(cl_\tau^{\tau^*}(\mu,r,s))$ is $(r.s)$-fuzzy-g*p-closed set and $\lambda \leq f^{-1}(f(\lambda)) \leq f^{-1}(cl_\tau^{\tau^*}(f(\lambda),r,s))$, therefore, $GC^{p^*}(\lambda,r,s) \leq GC^{p^*}(f^{-1}(cl_\tau^{\tau^*}(f(\lambda),r,s)),r,s) = f^{-1}(cl_\tau^{\tau^*}(f(\lambda),r,s))$. Hence, $f(GC^{p^*}(\lambda,r,s)) \leq cl_\tau^{\tau^*}(f(\lambda),r,s)$,

(2) For each $\mu \in I^Y$. Let $\lambda = f^{-1}(\mu)$. By (1), $f(GC^{p^*}(f^{-1}(\mu),r,s)) \leq cl_\tau^{\tau^*}(f(f^{-1}(\mu)),r,s) \leq cl_\tau^{\tau^*}(\mu,r,s)$. Then, $GC^{p^*}(f^{-1}(\mu),r,s) \leq f^{-1}(cl_\tau^{\tau^*}(\mu,r,s))$,

(3) Let $\mu = \underline{1} - \nu$ By (2) we have, $GC^{p^*}(f^{-1}(\underline{1} - \nu),r,s) \leq f^{-1}(cl_\tau^{\tau^*}(\underline{1} - \nu,r,s))$. Then, $GC^{p^*}(\underline{1} - f^{-1}(\nu),r,s) \leq f^{-1}(\underline{1} - int_\tau^{\tau^*}(\nu,r,s))$. Hence, $GI^{p^*}(f^{-1}(\mu),r,s) \geq f^{-1}(int_\tau^{\tau^*}(\mu,r,s))$.

Theorem 3.8. If $f : (X,\ \tau,\tau^*) \to (Y,\eta,\eta^*)$ is DF-g*p-continuous, then for each (r,s)-fuzzy open set μ of Y and each fuzzy point x_t of X such that $f(x_t)q\mu$, there is an (r,s)-fuzzy g*p-open set λ of X such that $x_t q\lambda$ and $f(\lambda) \leq \mu$

Proof. Let x_t be a fuzzy point of X and μ be an (r,s)-fuzzy open set of Y such that $f(x_t)q\mu$. Put $\lambda = f^{-1}(\mu)$, then by hypothesis λ is (r,s)-fuzzy g*p-open set of X such that $x_t q\lambda$ and $f(\lambda) = f(f^{-1}(\mu)) \leq \mu$.

Theorem 3.9. Let $f : (X,\ \tau,\tau^*) \to (Y,\eta,\eta^*)$ be a DF-g*p-continuous and let $g : (Y,\eta,\eta^*) \to (Z,\sigma,\sigma^*)$,

(1) If g is a DF-continuous, then $g \circ f : (X,\ \tau,\tau^*) \to (Z,\sigma,\sigma^*)$ is a DF-g*p-continuous.

(2) If g is a DF-g-continuous and (Y,η,η^*) is a DF-$T_{\frac{1}{2}}$-space, then $g \circ f$ is a DF-g*p-continuous.

Proof. (1) Let λ be an (r,s)-fuzzy closed set of Z and g is a DF-continuous. Then, $g^{-1}(\lambda)$ is (r,s)-fuzzy closed set of Y. Therefore, $(g \circ f)^{-1}(\lambda) = f^{-1}(g^{-1}(\lambda))$ is $(r.s)$-fuzzy g*p–closed set in X. Hence, $g \circ f$ is a DF-g*p-continuous.

(2) Let λ be an (r,s)-fuzzy closed set of $.Z$ and g is a DF-g*p-continuous. Then, $g^{-1}(\lambda)$ is (r,s)-fuzzy g-closed set of Y . Since, (Y,η,η^*) is a DF-$T_{\frac{1}{2}}$-space, we have, $g^{-1}(\lambda)$ is (r,s)-fuzzy closed set. Therefore, $(g \circ f)^{-1}(\lambda)$ is (r,s)-fuzzy g*p-closed set in X. Hence, $g \circ f$ is a DF-g*p-continuous.

4 Applications of (r,s)-fuzzy g*p-closed sets

In this section we introduce DF-T*p-space, DF-αT*p and DF-αT**p as an application of (r,s)-fuzzy g*p-closed set. We have derived some characterizations of (r,s)-fuzzy g*p-closed sets.

Definition 4.1. A dfts $(X,\ \tau,\tau^*)$ is called:

(1) DF-T*p-space if every (r,s)-fuzzy g*p-closet set is (r,s)-fuzzy closed.

(2) DF-αT*p-space if every (r,s)-fuzzy g*p-closet set is (r,s)-fuzzy preclosed.

(3) DF-αT**p-space if every (r,s)-fuzzy g*p-closet set is (r,s)- fuzzy α-closed.

Remark 4.2. (1) Every DF-αT**p-space is

DF-αT*p-space.

(2) Every DF-T*p-space is DF-αT*p-space.

(3) Every DF-T*p-space is DF-αT**p-space.

(4) Every DF-pre regular T$_{\frac{1}{2}}$-space is DF-αT*p-space.

(5) Every DF-semi-preregular T$_{\frac{1}{2}}$-space is DF-αT*p-space.

(6) Every DF-pT$_{\frac{1}{2}}$-space is DF-αT*p-space.

In general, the converse of the above remark is not true as shown in the following examples.

Example 4.3. Let $X = \{a,b\}$. Define $\lambda \in I^X$ as follows, $\lambda = \{0.5,0.4\}$. Define fuzzy topologies $\tau,\ \tau^* : I^X \to I$ as follows:

$$\tau(\mu) = \begin{cases} 1 & \text{if} \quad \mu = \underline{0},\ \underline{1}\,, \\ \frac{1}{2} & \text{if} \quad \mu = \lambda, \\ 0 & \text{otherwise,} \end{cases},$$

$$\tau^*(\mu) = \begin{cases} 0 & \text{if} \quad \mu = \underline{0},\ \underline{1}\,, \\ \frac{1}{3} & \text{if} \quad \mu = \lambda, \\ 1 & \text{otherwise,} \end{cases}$$

Then, $(X,\ \tau,\tau^*)$ is DF-αT*p-space but not DF-αT**p–space.

Example 4.4. Let $X = \{a,b,c\}$. Define $\lambda,\rho \in I^X$ as follows, $\lambda = \{0.7,0.3,1.0\}$ and $\rho = \{0.7,0,0\}$ Define fuzzy topologies $\tau,\ \tau^* : I^X \to I$ as follows:

$$\tau(\mu) = \begin{cases} 1 & \text{if} \quad \mu = \underline{0},\ \underline{1}\,, \\ \frac{5}{6} & \text{if} \quad \mu = \lambda,\rho, \\ 0 & \text{otherwise,} \end{cases},$$

$$\tau^*(\mu) = \begin{cases} 0 & \text{if} \quad \mu = \underline{0},\ \underline{1}\,, \\ \frac{1}{6} & \text{if} \quad \mu = \lambda,\rho, \\ 1 & \text{otherwise,} \end{cases}$$

Then, $(X,\ \tau,\tau^*)$ is DF-αT*p-space but not DF-T*p–space.

Example 4.5. Let $X = \{a,b,c\}$. Define $\lambda \in I^X$ as follows, $\lambda = \{0.6,0.3,1.0\}$. Define fuzzy topologies $\tau,\ \tau^* : I^X \to I$ as follows:

$$\tau(\mu) = \begin{cases} 1 & \text{if} \quad \mu = \underline{0},\ \underline{1}\,, \\ \frac{3}{4} & \text{if} \quad \mu = \lambda, \\ 0 & \text{otherwise,} \end{cases},$$

$$\tau^*(\mu) = \begin{cases} 0 & \text{if} \quad \mu = \underline{0},\ \underline{1}\,, \\ \frac{1}{4} & \text{if} \quad \mu = \lambda, \\ 1 & \text{otherwise,} \end{cases}$$

Then, $(X,\ \tau,\tau^*)$ is DF-αT**p-space but not DF-T*p–space.

Example 4.6. Let $X = \{a,b\}$. Define $\lambda,\rho \in I^X$ as follows, $\lambda = \{0.7,0.3\}$ and $\rho = \{0.7,0,0\}$ Define fuzzy topologies $\tau,\ \tau^* : I^X \to I$ as follows:

$$\tau(\mu) = \begin{cases} 1 & \text{if} \quad \mu = \underline{0},\ \underline{1}\,, \\ \frac{5}{8} & \text{if} \quad \mu = \lambda,\rho, \\ 0 & \text{otherwise,} \end{cases},$$

$$\tau^*(\mu) = \begin{cases} 0 & \text{if} \quad \mu = \underline{0},\ \underline{1}\,, \\ \frac{3}{8} & \text{if} \quad \mu = \lambda,\rho, \\ 1 & \text{otherwise,} \end{cases}$$

Then, $(X,\ \tau,\tau^*)$ is DF-αT*p-space but not DF-pre regular T$_{\frac{1}{2}}$-space.

Example 4.7. Let $X = \{a,b,c,d\}$. Define $\lambda,\rho \in I^X$ as follows, $\lambda = \{0.7,0.3,1.0,0\}$ and $\rho = \{0.7,0,0,0.5\}$ Define fuzzy topologies $\tau,\ \tau^* : I^X \to I$ as follows:

$$\tau(\mu) = \begin{cases} 1 & \text{if} \quad \mu = \underline{0},\ \underline{1}\,, \\ \frac{1}{2} & \text{if} \quad \mu = \lambda,\rho, \\ 0 & \text{otherwise,} \end{cases},$$

$$\tau^*(\mu) = \begin{cases} 0 & \text{if} \quad \mu = \underline{0},\ \underline{1}\,, \\ \frac{1}{2} & \text{if} \quad \mu = \lambda,\rho, \\ 1 & \text{otherwise,} \end{cases}$$

Then, $(X,\ \tau,\tau^*)$ is DF-αT*p-space but not DF-T*p-space..

Example 4.8. Let $X = \{a,b,c\}$. Define $\lambda \in I^X$ as follows, $\lambda = \{0.5,0.4,1.0\}$. Define fuzzy topologies $\tau,\ \tau^* : I^X \to I$ as follows:

$$\tau(\mu) = \begin{cases} 1 & \text{if} \quad \mu = \underline{0},\ \underline{1}\,, \\ \frac{6}{11} & \text{if} \quad \mu = \lambda, \\ 0 & \text{otherwise,} \end{cases},$$

$$\tau^*(\mu) = \begin{cases} 0 & \text{if} \quad \mu = \underline{0},\ \underline{1}\,, \\ \frac{5}{11} & \text{if} \quad \mu = \lambda, \\ 1 & \text{otherwise,} \end{cases}$$

Then, $(X,\ \tau,\tau^*)$ is DF-αT*p-space but not DF-pT$_{\frac{1}{2}}$-space.

5 Conclusion

The theory of fuzzy sets has several applications in different directions. In our Theoretical work we introduced the concepts of (r,s)-fuzzy g*p-closed sets and (r,s)-fuzzy g*p-open sets. Also, we investigate and studied some of their characterization and properties. Moreover, we introduced DF-g*p-continuous mappings with some of its properties. As an application of this set we introduced DF-T*p-space, DF-T**p-space and DF-αT*p-space.

Conflict of interest

The authors declare that they have no conflict of interest.

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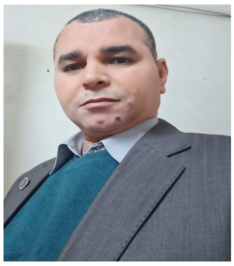

E. El-sanowsy is currently employed as associate professor at mathematics department, faculty of science, sohag university. He received his M.Sc. and Ph.D. degrees in mathematics from Egypt, in 2001 and 2008, respectively. He has published research articles in reputed international journals of mathematical and engineering sciences. He is referee and editor of mathematical journals.

A. Atef received the PhD degree in Topology at Sohag University. His research interests are in the areas of Topology. He has published research articles in reputed international journals of mathematical.