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Application of Local and Non-local Kernels: The Optimal Solutions of Water-based Nanoparticles Under Ramped Conditions

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Abstract: Fractional calculus has been rising these days vastly due to its useful and exclusive properties.The classical calculus as per the fact that it is presumed that instant rate of change of the output, when the input level changes. Therefore it is unable to predict the earlier state of the process called memory effect which is absent in classical models, but Fractional Calculus(FC) famous for having memory effects. To predict the solution for fractional order derivative by using fractional calculus tools that has great importance for describing many systems. Due to this reason, we applied the modern definition of fractional derivatives (Local and non-locals kernels). In present paper the influence of energy and velocity on time dependent natural convection flow of magnetohydrodynamic water-based nanoparticles near long vertical plate of infinite length with ramped conditions nested in porous material is discussed. The fractional model of water-based nanofluid in which copper, aluminum oxide and titanium dioxide as nanoparticles using non-integer derivative operators Atangena-Baleanu, Caputo-Fabrizio and Caputo has been examined. Results are derived with the application of inversion algorithm and Laplace transformation method, for dimensionless temperature and velocity equations. The key features for different connected parameters are highlighted to display the graphs. A comparative study is accomplished for all fractional models with an ordinary model.Moreover, it is point out that ramped wall temperature and nanofluid velocity for non-integer models becomes an classical model, when the involving fractional parameter approaches to one, this reveal that fractional order models are more suitable to explain the investigational facts.

Keywords: Memory effect, nanofluids, time Fractional differential operator, nanoparticles, magnetic field, volume fraction.

1 Introduction

In modern times, nanotechnology have great attraction for scientists and researchers, due to its importance in the different fields like industrial sectors, chemical production and transportation. Simultaneously, ordinary fluids such as oil, toluene and water are generally used for heat transfer fluids. Nanoliquids are utilized to enhance the caloric characteristics and performance of conventional fluids in cooling and heating methods such as decrease the power of nuclear reactors, reduce temperature in vehicles radiators, controlled heat in different computer progressions and handling thermal flows by heat valves. In pharmacological manufacturing, detects and treatment of tumor by using nanoliquid operatives which contains different contaminations. These momentous physical features of nanofluids and their consequences are fascinating the researchers and scientists. To make nanofluids,we have to add some nano size particles in the conventional fluid, which is also called base fluid. This term suggested first time by Choi [1]. Nano-sized particles are used to raise the heating characteristics of regular fluids, like water and mineral oil. The creation of nano meter sized particles involves carbides, carbon nanotubes and metals. Nano-compounds have substantial applications in several procedures like drugs delivery, water purification, bio diesel invention and creation of carbon nanotubes, according to Sarli et. al. [2]. Masuda et al. [3] presented higher thermo physical characteristics in nanofluids due to some

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nanosized particles, undoubtedly, huge difference in the structure of nanoparticles because it has different shape and size. Das et al. [4] has studied that for different temperature range between 200 C to 500C, the thermal conductivities of TiO_2 and Al_2O_3 water base nanofluids increased at most four times.
The deep interest in flows of incompressible nanofluids is vastly substantial enhancement due to vital applications in industrial sectors, physics and chemistry. Many researchers and scientists applied various techniques to boost the rate of energy transfer of some base fluids by using nanofluids. Hamad et al. [5] has observed the characteristics of fluids flow containing nanometer sized particles over vertical plate in presence of external magnetical field. The impacts of magnetic strength on some nanofluid flow are investigated by Das and Jana [6]. Turkyilmazoglu [7] has discussed the mass with heat transmission of exact investigation of MHD flow of some fluids having nanometer sized paricles. The movement of nanofluid on a porous surface placement in revolving system are examined by Ganji and Sheikholeslami [8]. Husanan et al. [9] investigated unsteady movement of fluids containing nano particles, nested in medium which is porous, in the existence of electromagnetic field. In [10,11,12] researchers studied and discussed the influence of exothermically and radiating heat for some micro level fluid flow. Khan et al. [13] studied Casson type nanofluid movement in the occurrence of thermoradiation and heat consumption. In [14,15,16,17,18] some identical studies can be investigated. Ahmed and Dutta [19] first time float the idea of effective velocity and temperature with rapmed wall conditions at the similar time for mass transmission of Newtonian unsteady fluid flow transient through impulsively affecting long vertical plate. Generally, ramped velocity has great advantageous in the medical field specially diagnosed and treatment for heart, blood and cancer deceases are discussed and studied in [20,21,22]. Schetz [23], Hayday [24] and Malhotra et al. [25] are investigated non-uniform and time-dependent temperature with ramped conditions. Kundu [26] was highlighted important operability time-dependent conditions for temperature and also suggested five different forms of heating. Keolyar et al. [27] observed controlled heat condition on unstable MHD radiated movement of some fluids with nano partical mixtutres passing through a flat surface plate. Some researchers have discussed noteworthy facts of heat emission and mass transfer and elaborated under dissimilar physical phenomena in [28,29,30].
Over the last thirty years, non-integer order derivatives fascinated the mind of many researchers after knowing the fact that these operators are supposed to be more authentic for modeling the real world physical phenomenon's as compared to the classical derivatives which is a special case of non-integer order derivatives [31,32,33,34]. In dynamical problems, FC modeling is receiving a rapid popularity due to its exclusive properties. Experimental results are more accurate and precise derived by mathematical modeling of many engineering and physical models using the definitions, which are based on the idea of fractional calculus, corresponding to the models based on traditional integer order calculus.The non-integer operators such as Caputo, Caputo-Fabrizio and ABC that transformed the classical model to fractional model. Abro [35] studied the thermal diffusive impacts on free convective movement between integer and non-integer derivative operators, derived solution employing Fourier sine transform and Laplace transformation. It is found in the literature, several fractional differential operators exists, for instance,Riemann-Liouville, Caputo (singular and local kernel), Atangana-Baleanu (non-singular and non-local kernel), Caputo-Fabrizio (non-singular and local kernel), and few others are discussed [36,37,38,39]. For local and non-local kernel, convective flow with ramped conditions on temperature are studied by Riaz et al. [40]. Also, comparative study for MHD Maxwell fluid,the heat effect, with the application of local and non-local operators is highlighted by Riaz et al. [41]. Some other fractional associated references are investigated [42,43], dealing with non-integer differential operators, MHD Jeffrey fluid movement, heat transport and second grade fluid.
Talha Anwar et al. [44] discussed MHD natural convection flow of water base nanofluids with classical approach. They have not analyzed the behavior of fractional derivatives. It is observed that, not a single result is presented for fractional nanofluid velocity and temprature in the literature with Caputo, Caputo-Fabrizio and ABC fractional operators in this direction. Based on aforesaid literature, the object of this exploration to use Caputo, Caputo-Fabrizio and ABC fractional operators on water-based nanofluid and accomplish the comparison with the solutions Talha Anwar et al. [44] derived . Consequences of the investigation are acquired via employing Laplace transformation technique with the application of inversion algorithm for temperature and velocity. The consequences of different related physical parameters, such as solid volume fraction parameter ϕ, grashof number Gr, magnetic number M, permeability parameter K, Prandtl number Pr and fractional parameter α on non- dimensional temperature and velocity. Results are discussed in detail and demonstrated graphically via Mathcad-15 software.

2 Mathematical modeling

Let us assume that incompressible unsteady free convective MHD fluid movement through upright plate nested in a porous material with ramped temperature and energy transmission of a nanofluid. Temperature T_∞ at initial time $\tau = 0$, for fluid and the plate . At $\tau > 0$ motion is started in the plate with velocity $U_0 \frac{\tau}{\tau_0}$, also temperature raised to $T_\infty + (T_w - T_\infty)\frac{\tau}{\tau_0}$ in the long vertical plate for $0 < \tau \leq \tau_0$. But, later on, the plate maintained constant temperature T_w

and moving with uniform velocity U_0 for $\tau > \tau_0$. Suppose that the movement of fluid is considered one dimensional and unidirectional, with x-axis is assumed along the perpendicular plate , y-axis is considered in the direction, perpendicular to the palte , and plate is placed at $y = 0$ but nanofluid movement to be constrained for $y > 0$. Fig. 1 is provided the geometrical and physical interpretation of considered model. Also nanofluid is contained water as a base fluid and Cu, TiO_2 and Al_2O_3 as nanoparticles. In the light of all aforesaid suppositions, the principal governing equations for a nanofluid, the Boussinesq's approximation [44] are given as:

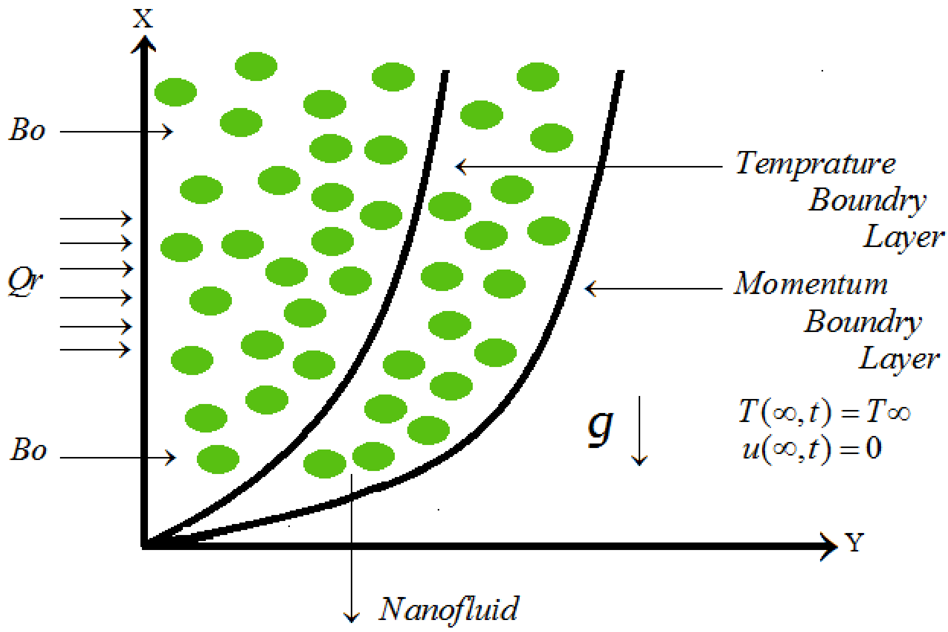

Fig. 1: Geometrical presentation of the stated problem

$$\rho_{nf}\frac{\partial u(y,\tau)}{\partial \tau} = \mu_{nf}\frac{\partial^2 u(y,\tau)}{\partial y^2} + \frac{\mu_{nf}\gamma_2}{k^*}u(y,\tau) + g(\rho\beta)_{nf}(T(y,\tau) - T_\infty) - \sigma_{nf}\beta_0^2 u(y,\tau), \tag{1}$$

$$(\rho C_p)_{nf}\frac{\partial T(y,\tau)}{\partial \tau} = (K_{nf} + \frac{16\sigma_r T_\infty^3}{3k_r})\frac{\partial^2 T(y,\tau)}{\partial y^2} - Q_0 T(y,\tau) + Q_0 T_\infty, \tag{2}$$

corresponding initial and boundary conditions for above equations are stated as:

$$u(y,i) = 0, \;\; T(y,i) = T_\infty, \;\; y \geq 0, \tag{3}$$

$$u(i,\tau) = f_1(\tau_1), \quad T(i,\tau) = f_2(\tau_2), \quad i = 0, \tag{4}$$

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$$f_1(\tau_1) = \begin{cases} U_0\frac{\tau}{\tau_0}, & 0 < \tau \leq \tau_0; \\ U_0, & \tau > \tau_0 \end{cases} \;\; and \;\; f_2(\tau_2) = \begin{cases} T_\infty + (T_w - T_\infty)\frac{\tau}{\tau_0}, & 0 < \tau \leq \tau_0; \\ T_w, & \tau > \tau_0 \end{cases}, \tag{5}$$

$$u(y,\tau) \to \;\; 0, \;\; T(y,\tau) \to \infty \;\; as \;\; y \to \infty. \tag{6}$$

The expressions for various properties of nanofluids other than conventional fluids, which are found in the literature [45] are density, coefficient of thermal expansion, electrical conductivity, specific heat capacity and dynamic viscosity, are denoted by ρ_{nf} , $(\rho\beta)_{nf}$,σ_{nf} ,$(\rho C_p)_{nf}$ and μ_{nf} respectively and defined as:

$$\mu_{nf} = \frac{\mu_w}{(1-\phi)^{2.5}}, \rho_{nf} = \rho_w\left(1-\phi+\phi\frac{\rho_{nf}}{\rho_w}\right), (\rho\beta)_{nf} = (\rho\beta)_w\left(1-\phi+\phi\frac{(\rho\beta)_{nf}}{(\rho\beta)_w}\right),$$
$$(\rho C_p)_{nf} = (\rho C_p)_w\left(1-\phi+\phi\frac{(\rho C_p)_{nf}}{(\rho C_p)_w}\right), \sigma = \frac{\sigma_{nf}}{\sigma_w}, \sigma_{nf} = \sigma_w\left(1+\frac{3\phi(\sigma-1)}{(\sigma+2)-\phi(\sigma-1)}\right), \tag{7}$$

Also Hamilton and Crosser ([46], [47]) are discussed the effective role of thermal conductivity for nanoparticles:

$$\frac{K_{nf}}{K_w} = \frac{K_{np}+2K_w-2(K_w-K_{nf})\phi}{K_{np}+2K_w-(K_w-K_{nf})\phi}, \tag{8}$$

Subscripts used in the above equations are w, nf and np are represented as base fluid, nanofluid and nano particles respectively.
To non-dimentionalize the following new variables are introduced:

$$\zeta = \frac{U_0}{\upsilon_w}y, \quad \tau^* = \frac{U_0^2}{\upsilon_w}\tau, \quad u^* = \frac{u}{u_0}, \quad T^* = \frac{T-T_\infty}{T_w-T_\infty}, \quad G_r = \frac{g(T_w-T_\infty)(\upsilon\beta)_w}{U_0^3},$$
$$M = \frac{\sigma_w\beta_0^2\upsilon_w}{\rho U_0^2}, \quad N_r = \frac{16\sigma_r T_\infty^3}{3k_w k_r}, \quad P_r = \frac{(\mu C_p)_w}{k_w}, \frac{1}{K} = \frac{\upsilon_w^2\gamma_2}{k^* u_0^2}, \quad Q = \frac{Q_0}{U_0^2}(\frac{\upsilon}{\rho C_p})_w, \tag{9}$$

After employing the dimensionless quantities and removing the asterisk $*$ notation the following partial differential equations in dimentionless form are derived as :

$$\pi_1\frac{\partial u(\zeta,\tau)}{\partial\tau} = \pi_4\frac{\partial^2 u(\zeta,\tau)}{\partial\zeta^2} + \pi_2 G_r T(\zeta,\tau) - \pi_3 M u(\zeta,\tau) - \pi_4\frac{u(\zeta,\tau)}{K}, \tag{10}$$

$$\frac{\partial T(\zeta,\tau)}{\partial\tau} = \left(\frac{\pi_5+N_r}{\pi_6 P_r}\right)\frac{\partial^2 T(\zeta,\tau)}{\partial\zeta^2} - \frac{Q}{\pi_6}T(\zeta,\tau), \tag{11}$$

where

$$\pi_1 = \left(1-\phi+\phi\frac{\rho_{nf}}{\rho_w}\right), \pi_2 = \left(1-\phi+\phi\frac{(\rho\beta)_{nf}}{(\rho\beta)_w}\right), \pi_3 = \left(1+\frac{3\phi(\sigma-1)}{(\sigma+2)-\phi(\sigma-1)}\right),$$
$$\pi_4 = \frac{1}{(1-\phi)^{2.5}}, \pi_5 = \frac{K_{np}+2K_w-2(K_w-K_{nf})\phi}{K_{np}+2K_w-(K_w-K_{nf})\phi}, \pi_6 = \left(1-\phi+\phi\frac{(\rho C_p)_{nf}}{(\rho C_p)_w}\right), \tag{12}$$

with stated conditions in dimensionless form:

$$u(\zeta,i) = 0, \quad T(\zeta,i) = 0, \quad \zeta \geq 0, \quad i = 0, \tag{13}$$

$$u(0,\tau) = f_3(\tau_3), \;\; T(0,\tau) = f_3(\tau_3) \;\; where \; f_3(\tau_3) = \begin{cases} \tau, & 0<\tau\leq 1 \\ 1, & \tau>1 \end{cases}, \tag{14}$$

$$u(\zeta,\tau)\to 0, \;\; T(\zeta,\tau)\to 0 \;\; as \;\; \zeta\to\infty, \tag{15}$$

3 Preliminaries

Caputo fractional operator having singular and local kernel is stated as:

$$^{C}D_\eta^{\wp}f(z,\eta) = \frac{1}{\Gamma(n-\wp)}\int_a^\eta\left(\frac{f^{(n)}(\tau)}{(\eta-\tau)^{\wp+1-n}}\right)d\tau, \qquad 0<\wp<1. \tag{16}$$

here $\Gamma(.)$ is represented Gamma function and Laplace transformation of Eq.(16) is written as:

$$\mathscr{L}\left({}^{C}D_{\eta}^{\wp}f(z,\eta)\right) = s^{\wp}\mathscr{L}\left(f(z,\eta)\right) - s^{\wp-1}f(z,0). \tag{17}$$

CF fractional operator having non-singular and local kernel is described as:

$$^{CF}D_{\eta}^{\wp}f(z,\eta) = \frac{1}{1-\wp}\int_0^{\eta} exp\left(-\frac{\wp(\eta-\wp)}{1-\wp}\right)\frac{\partial f(z,\tau)}{\partial\tau}d\tau, \qquad 0<\wp<1. \tag{18}$$

and Laplace transformation of Eq.(18) is obtained as:

$$\mathscr{L}\left({}^{CF}D_{\eta}^{\wp}f(z,\eta)\right) = \frac{s\mathscr{L}\left(f(z,\eta)\right) - f(z,0)}{(1-\wp)s+\wp}. \tag{19}$$

ABC fractional operator having non-singular and non-local kernel is delineated as :

$$^{ABC}D_{\eta}^{\wp}f(z,\eta) = \frac{1}{1-\wp}\int_0^{\eta} E_{\wp}\left(-\frac{\wp(\eta-\wp)^{\wp}}{1-\wp}\right)\frac{\partial f(z,\tau)}{\partial\tau}d\tau. \quad with \quad \sum_{l=0}^{\infty}\frac{(-\eta)^{\wp l}}{\Gamma(1+\wp l)} = E_{\wp}(-\eta)^{\wp}. \tag{20}$$

The Laplace Transformation of above defined ABC fractional time derivative is

$$\mathscr{L}\left({}^{ABC}D_{\eta}^{\wp}f(z,\eta)\right) = \frac{s^{\wp}\mathscr{L}\left(f(z,\eta)\right) - s^{\wp-1}f(z,0)}{(1-\wp)s^{\wp}+\wp}. \tag{21}$$

where Laplace transformation operator and fractional parameter is represented by s and $\wp$ respectively.

4 Temperature fields

4.1 *Caputo fractional operator*

Using Caputo operator provided in Eq. (16) on Eq.(11) and plugging Eqs. (13,14,15) yield

$$\frac{d^2\bar{T}_c(\zeta,p)}{d\zeta^2} - (\beta p^{\alpha}+\lambda)\bar{T}_c(\zeta,p) = 0, \tag{22}$$

where

$$\beta = \frac{\pi_6 P_r}{\pi_5+N_r}, \quad \lambda = \frac{QP_r}{\pi_5+N_r}. \tag{23}$$

For Eq. (22) the solution is obtained and written as:

$$\bar{T}_c(\zeta,p) = \left(\frac{1-e^{-p}}{p^2}\right)e^{-\zeta\sqrt{(\beta p^{\alpha}+\lambda)}} \tag{24}$$

4.2 *CF fractional operator*

Employing the definition of CF, provided in Eq.(18) on Eq.(11) and substituting Eqs. (14 and 15) yield.

$$^{CF}D_{\tau}^{\alpha}T(\zeta,\tau) = \left(\frac{\pi_5+N_r}{\pi_6 P_r}\right)\frac{\partial^2 T(\zeta,\tau)}{\partial\zeta^2} - \frac{Q}{\pi_6}T(\zeta,\tau), \tag{25}$$

Applying Laplace transformation, above equation has the form

$$\frac{\partial^2 \bar{T}_{cf}(\zeta,p)}{\partial \zeta^2} - \left(\beta\left(\frac{p}{(1-\alpha)p+\alpha}\right)+\lambda\right)\bar{T}_{cf}(\zeta,p) = 0, \tag{26}$$

with

$$\bar{T}_{cf}(0,p) = \left(\frac{1-e^{-p}}{p^2}\right), \quad \bar{T}_{cf}(\zeta,p) \to 0 \quad as \quad \zeta \to \infty. \tag{27}$$

where

$$\beta = \frac{\pi_6 P_r}{\pi_5 + N_r}, \quad \lambda = \frac{Q P_r}{\pi_5 + N_r}. \tag{28}$$

The solution of Eq.(26) by using Eq.(27) is written in the form:

$$\bar{T}_{cf}(\zeta,p) = \left(\frac{1-e^{-p}}{p^2}\right) e^{-\zeta\sqrt{(\beta\frac{p}{(1-\alpha)p+\alpha}+\lambda)}} \tag{29}$$

4.3 ABC – fractional operator

Employing the definition of ABC Fractional time derivative definition which is given in Eq.(20) on Eq.(11) and plugging Eqs. (14 and 15) yield

$$^{ABC}D^{\alpha}_{\tau} T(\zeta,\tau) = \left(\frac{\pi_5 + N_r}{\pi_6 P_r}\right)\frac{\partial^2 T(\zeta,\tau)}{\partial \zeta^2} - \frac{Q}{\pi_6}T(\zeta,\tau), \tag{30}$$

Taking Laplace transform of the above equation

$$\frac{\partial^2 \bar{T}_{abc}(\zeta,p)}{\partial \zeta^2} - \left(\beta\left(\frac{p^\alpha}{(1-\alpha)p^\alpha+\alpha}\right)+\lambda\right)\bar{T}_{abc}(\zeta,p) = 0, \tag{31}$$

Eq. (30) has solution by using given conditions which gives:

$$\bar{T}_{abc}(\zeta,p) = \left(\frac{1-e^{-p}}{p^2}\right) e^{-\zeta\sqrt{(\beta\frac{p^\alpha}{(1-\alpha)p^\alpha+\alpha}+\lambda)}} \tag{32}$$

It is noticed that when $\alpha \to 1$ the results derived by Caputo, CF and ABC in Eqs.(24),(29) and (32) for ramped wall tempreture distribution gives the same solution as acquired by Talha Anwar et al. [44], which proves the validity of derived results with the published literature.

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5 Velocity field

5.1 Caputo – fractional operator

Using Caputo operator provided in Eq. (16) on Eq.(10) and plugging Eqs. (13,14 15) yield

$$\eta^C D^{\alpha}_{\tau} u(\zeta,\tau) = \frac{\partial^2 u(\zeta,\tau)}{\partial \zeta^2} + mG_r T(\zeta,\tau) - wu(\zeta,\tau), \tag{33}$$

we employing Laplace transformation on the above equation, which gives:

$$\frac{d^2 \bar{u}_c(\zeta,p)}{d\zeta^2} - (\eta p^\alpha + w)\bar{u}_c(\zeta,p) = -mG_r \bar{T}_c(\zeta,\tau), \tag{34}$$

with

$$\bar{u}_c(0,p)=\left(\frac{1-e^{-p}}{p^2}\right),\quad \bar{u}_c(\zeta,p)\to 0\quad as\quad \zeta\to\infty. \tag{35}$$

where

$$\eta=\frac{\pi_1}{\pi_4},\quad w=M\frac{\pi_3}{\pi_4}+\frac{1}{K},\quad m=\frac{\pi_2}{\pi_4}, \tag{36}$$

The required solution of Eq.(34) has the following form

$$\bar{u_c}(\zeta,p)=c_1e^{\zeta\sqrt{\eta p^{\alpha}+w}}+c_2e^{-\zeta\sqrt{\eta p^{\alpha}+w}}-\left(\frac{mG_r(1-e^{-p})e^{-\zeta\sqrt{\beta p^{\alpha}+\lambda}}}{ap^2(p^{\alpha}-b)}\right), \tag{37}$$

Employing conditions given in Eq.(37),we get the solution

$$\bar{u_c}(\zeta,p)=\frac{1}{p^2}(1-e^{-p})e^{-\zeta\sqrt{\eta p^{\alpha}+w}}+\left(\frac{mG_r(1-e^{-p})}{ap^2(p^{\alpha}-b)}\right)\left[e^{-\zeta\sqrt{\eta p^{\alpha}+w}}-e^{-\zeta\sqrt{\beta p^{\alpha}+\lambda}}\right], \tag{38}$$

5.2 *CF – fractional operator*

Employing the definition of CF, provided in Eq.(18) on Eq.(10) and substituting Eqs. (13,14,15) yield.

$$\eta^{CF}D_{\tau}^{\alpha}u(\zeta,\tau)=\frac{\partial^2u(\zeta,\tau)}{\partial\zeta^2}+mG_rT(\zeta,\tau)-wu(\zeta,\tau), \tag{39}$$

Applying Laplace transformation, above equation has the form

$$\frac{d^2\bar{u}_{cf}(\zeta,p)}{d\zeta^2}-\left(\frac{\eta p}{(1-\alpha)p+\alpha}+w\right)\bar{u}_{cf}(\zeta,p)=-mG_r\bar{T}_{cf}(\zeta,\tau), \tag{40}$$

with

$$\bar{u}_{cf}(0,p)=\left(\frac{1-e^{-p}}{p^2}\right),\quad \bar{u}_{cf}(\zeta,p)\to 0\quad as\quad \zeta\to\infty. \tag{41}$$

The result of homogenous part of velocity Eq.(40) is,

$$\bar{u_c}(\zeta,p)=c_1e^{\zeta\sqrt{\frac{\eta p}{(1-\alpha)p+\alpha}+w}}+c_2e^{-\zeta\sqrt{\frac{\eta p}{(1-\alpha)p+\alpha}+w}}, \tag{42}$$

and the particular solution is written as

$$\begin{aligned}\bar{u_p}(\zeta,p)&=-\frac{mG_r\left(\frac{1-e^{-p}}{p^2}\right)e^{-\zeta\sqrt{\frac{\beta p}{(1-\alpha)p+\alpha}+\lambda}}}{\left(\frac{\beta p}{\alpha+(1-\alpha)p}+\lambda\right)-\left(\frac{\eta p}{(1-\alpha)p+\alpha}+w\right)},\\&=-\frac{mG_r(1-e^{-p})(a_1p+\alpha)e^{-\zeta\sqrt{\frac{\beta p}{(1-\alpha)p+\alpha}+\lambda}}}{ap^2(p-b(a_1p+\alpha))},\\&=-\frac{ma_5G_r(p+a_4)(1-e^{-p})e^{-\zeta\sqrt{\frac{\beta p}{(1-\alpha)p+\alpha}+\lambda}}}{p^2(p-a_3)},\end{aligned} \tag{43}$$

So,the required solution of Eq.(40) has the following form:

$$\bar{u_{cf}}(\zeta,p)=c_1 e^{\zeta\sqrt{\frac{\eta p}{(1-\alpha)p+\alpha}+w}}+c_2 e^{-\zeta\sqrt{\frac{\eta p}{(1-\alpha)p+\alpha}+w}}-\frac{ma_5 G_r(1-e^{-p})(p+a_4)e^{-\zeta\sqrt{\frac{\beta p}{(1-\alpha)p+\alpha}+\lambda}}}{p^2(p-a_3)}, \tag{44}$$

To find constants c_1 and c_2 by using conditions given in Eq.(41),the solution is written as:a

$$\begin{aligned}\bar{u_{cf}}(\zeta,p)&=\frac{(1-e^{-p})}{p^2}e^{-\zeta\sqrt{\frac{\eta p}{(1-\alpha)p+\alpha}+w}}+\frac{ma_5 G_r(p+a_4)(1-e^{-p})}{p^2(p-a_3)}\left[e^{-\zeta\sqrt{\frac{\eta p}{(1-\alpha)p+\alpha}+w}}-e^{-\zeta\sqrt{\frac{\beta p}{(1-\alpha)p+\alpha}+\lambda}}\right],\\ &=\left(\frac{1-e^{-p}}{p^2}\right)e^{-\zeta\sqrt{\frac{\eta p}{(1-\alpha)p+\alpha}+w}}+ma_5 G_r(1-e^{-p})\left(\frac{H_1}{p}+\frac{H_2}{p^2}+\frac{H_3}{p-a_3}\right)\left[e^{-\zeta\sqrt{\frac{\eta p}{(1-\alpha)p+\alpha}+w}}-e^{-\zeta\sqrt{\frac{\beta p}{(1-\alpha)p+\alpha}+\lambda}}\right],\end{aligned} \tag{45}$$

where

$$\begin{aligned}&a=\beta-\eta,\quad b=\frac{w-\lambda}{\beta-\eta},\quad a_1=1-\alpha,\quad a_2=1-ba_1,\quad a_3=\frac{\alpha b}{1-ba_1},\quad a_4=\frac{\alpha}{1-\alpha},\\ &a_5=\frac{a_1}{(1-b)(1-\alpha)(\beta-\eta)},\quad H_1=-\frac{a_3+a_4}{a_3^2},\quad H_2=-\frac{a_4}{a_3},\quad H_3=\frac{a_3+a_4}{a_3^2},\end{aligned} \tag{46}$$

5.3 ABC – fractional operator

Applying ABC operator as mentioned in Eq.(20) on Eq.(10) gives

$$\eta^{ABC}D_\tau^\alpha u(\zeta,\tau)=\frac{\partial^2 u(\zeta,\tau)}{\partial\zeta^2}+mG_rT(\zeta,\tau)-wu(\zeta,\tau), \tag{47}$$

Employing Laplace transformation on the above equation which gives:

$$\frac{d^2\bar{u}_{abc}(\zeta,p)}{d\zeta^2}-\left(\frac{\eta p^\alpha}{(1-\alpha)p^\alpha+\alpha}+w\right)\bar{u}_{abc}(\zeta,p)=-mG_r\bar{T}_{abc}(\zeta,\tau), \tag{48}$$

The required solution is obtained after lengthy calculation is written as:

$$\bar{u_{abc}}(\zeta,p)=\left(\frac{1-e^{-p}}{p^2}\right)e^{-\zeta\sqrt{\frac{\eta p^\alpha}{(1-\alpha)p^\alpha+\alpha}+w}}+\frac{ma_5 G_r(1-e^{-p})(p^\alpha+a_4)}{p^2(p^\alpha-a_3)}\left[e^{-\zeta\sqrt{\frac{\eta p^\alpha}{(1-\alpha)p^\alpha+\alpha}+w}}-e^{-\zeta\sqrt{\frac{\beta p^\alpha}{(1-\alpha)p^\alpha+\alpha}+\lambda}}\right], \tag{49}$$

It is detected that when $\alpha\to 1$ the fractionalized models Caputo, CF and ABC in Eqs. (38), (45) and (49) for ramped wall velocity profile are converted into integer order model as calculated by Talha Anwar et al. [44]. Also, we achieved the same results of Seth et al. [29] for ramped wall temperature with $\phi=0$. Moreover, we derived the similar solutions as obtained by Chandran et al. [28] when porosity parameter $\frac{1}{K}\to 0$ and magnetic parameter $M\to 0$. This authenticates the current acquired results with the published literature.Employing Stehfest's and Tzou's numerical algorithms [36,37] to find the inverse Laplace transformations to obtained the solutions for temperature and velocity . Mathematically, Stehfest's algorithm is written as:

$$\omega(\zeta,\tau)=\frac{ln(2)}{\tau}\Sigma_{z=1}^{2n}d_j\bar{\omega}(\zeta,z.\frac{ln(2)}{\tau}), \tag{50}$$

where n is a non-negative integer and

$$d_j=(-1)^{z+n}\Sigma_{i=[\frac{z+1}{2}]}^{min(z,n)}\frac{i^n(2i)!}{(n-i)!(i)!(i-1)!(z-i)!(2i-z)!}, \tag{51}$$

where integer part is a real number and described by [.].
The Tzou's numerical algorithm has the following form:

$$\bar{w}(\gamma,t)=\frac{e^{4.7}}{t}\left[\frac{1}{2}\bar{w}(\gamma,\frac{4.7}{t})+Re\left\{\Sigma_{i=1}^{M}(-1)^i\bar{w}(\gamma,\frac{4.7+z\pi i}{t})\right\}\right], \tag{52}$$

where $Re(.)$ represent a real part, the imaginary part is described as i and $M>>1$ is represented the natural number.

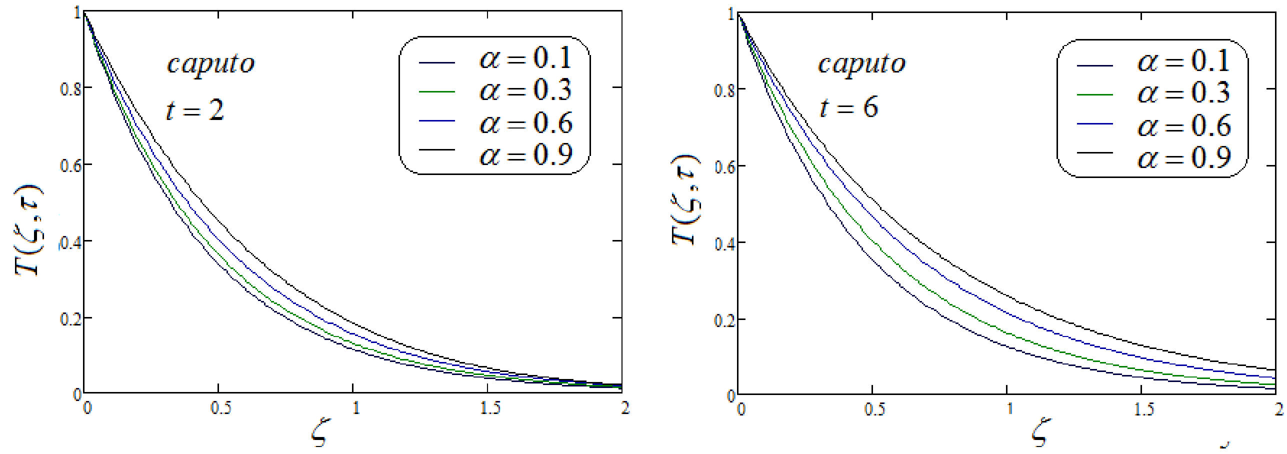

Fig. 2: Temprature behavior for two different values of t via caputo when Pr=6.2,ϕ=0.1,Q=0.4,Nr=0.5

6 Results and discussion

In this section the influence of many pertinent parameters such as ϕ, Gr, M, K, Pr and fractional parameter α on obtained solution of water base nanoparticles with ramped conditions subject to the application of different fractional operators $Caputo, CF$ and ABC are studied. The physical geometry of the stated problem is described in figure (1). Figures (2 to 17) are plotted to highlight the impact of α, time fractional parameter, via Caputo, CF and ABC fractional operators, where $0 < \alpha < 1$ and various embedded parameters on temperature and fluid velocity by MATHCAD-15 software.

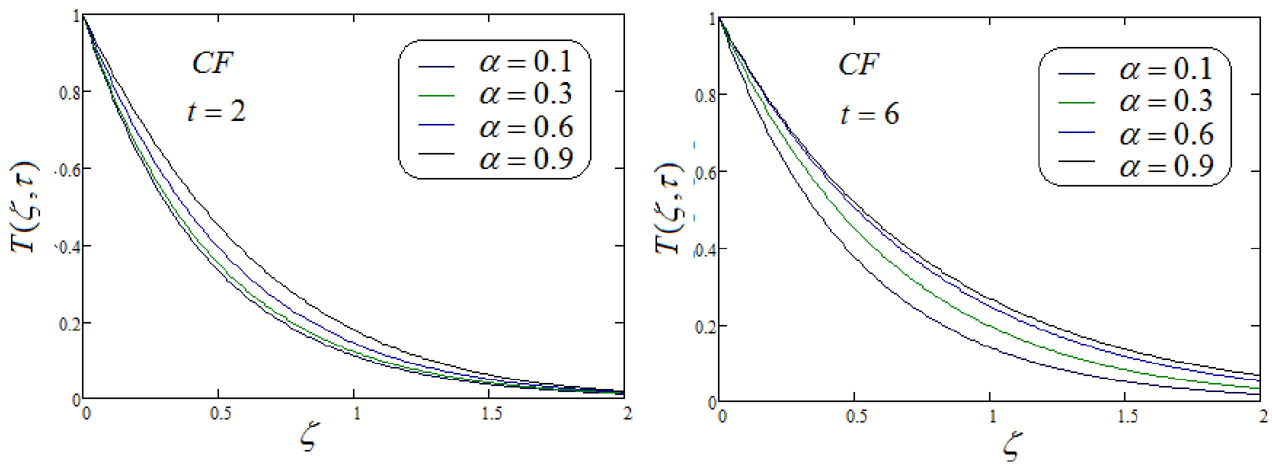

Fig. 3: Temprature behavior for two distinct values of t via CF when Pr=6.2,ϕ=0.1,Q=0.4,Nr=0.5

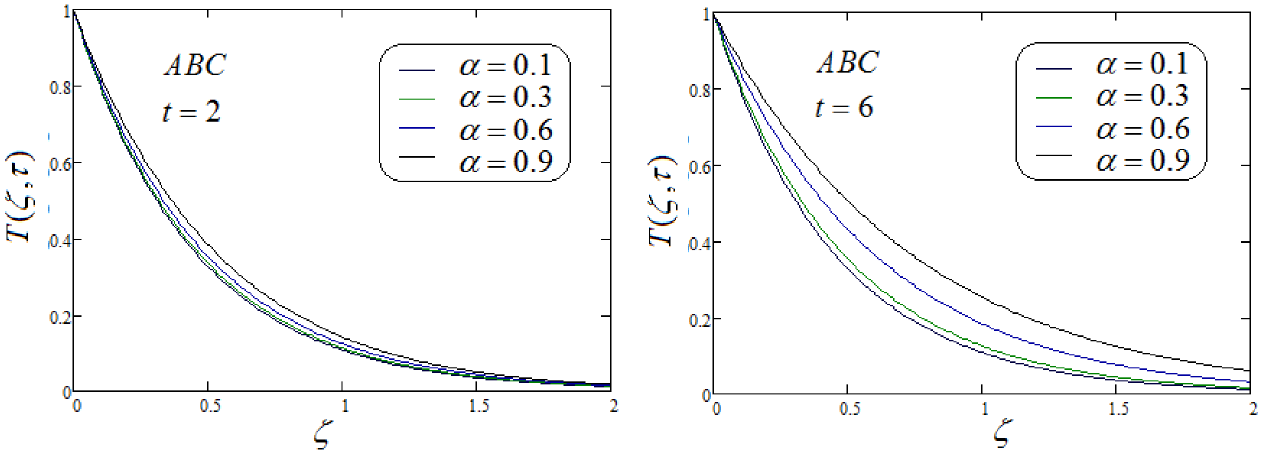

Fig. 4: Temprature behavior for two dissimilar values of t via ABC when Pr=6.2,ϕ=0.1,Q=0.4,Nr=0.5

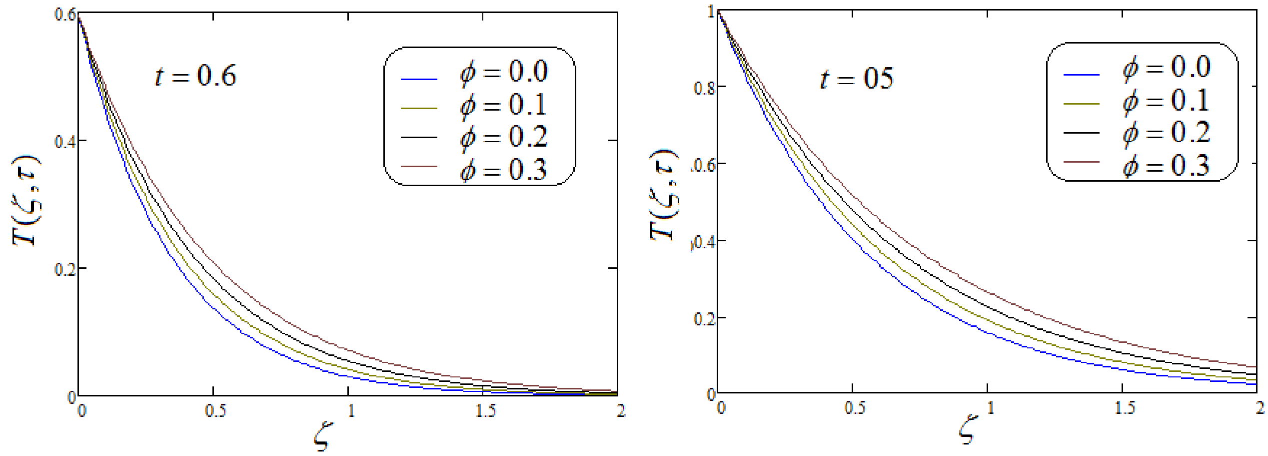

Fig. 5: Temprature behavior for two distinct values of t when Pr=6.2,α=0.5,Q=0.4,Nr=0.5

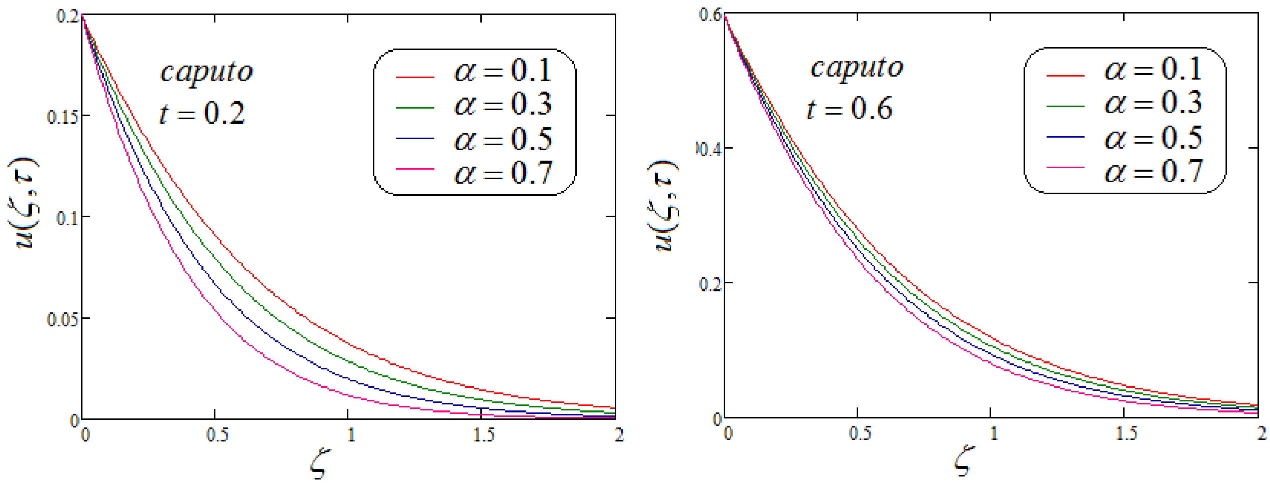

Fig. 6: Velocity behavior for two different values of t via caputo when ϕ=0.1,Gr=5,M=2,K=0.5,Q=0.5,Pr=6.2,Nr=0.5

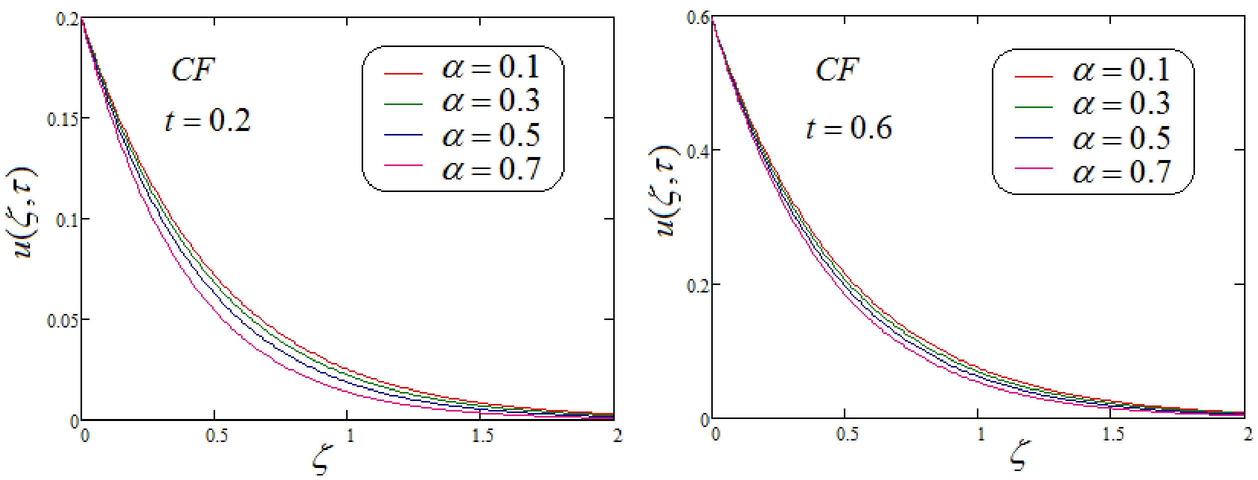

Fig. 7: Velocity behavior for two different values of t via CF when ϕ=0.3,Gr=5,M=2, ,Nr=0.5,K=0.5,Q=0.5,Pr=6.2

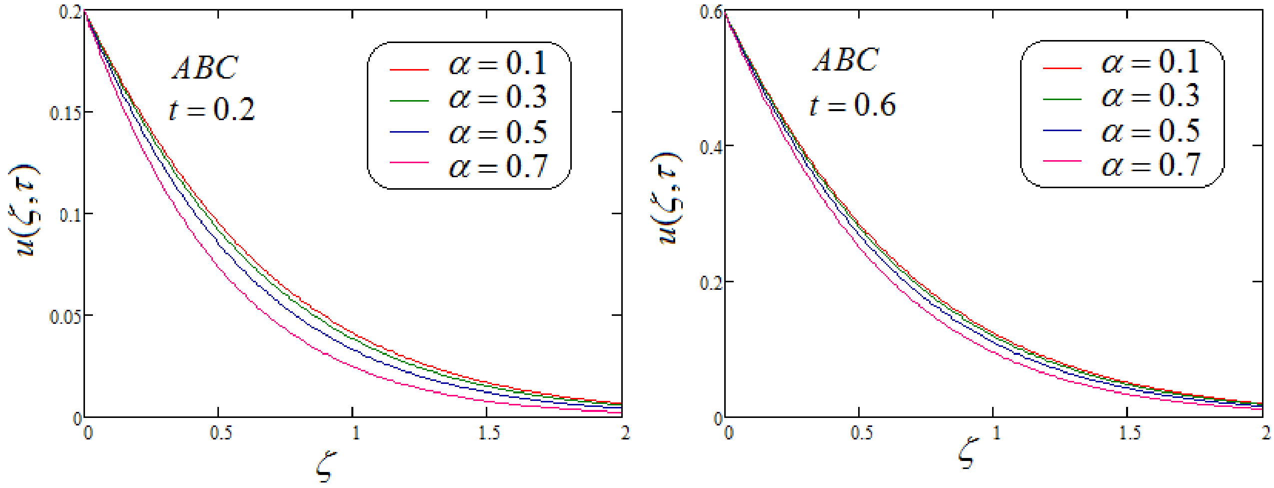

Fig. 8: Velocity behavior for two different values of t via ABC when ϕ=0.1,Gr=5,M=2,Pr=6.2, Q=0.5,Nr=0.5,K=0.5

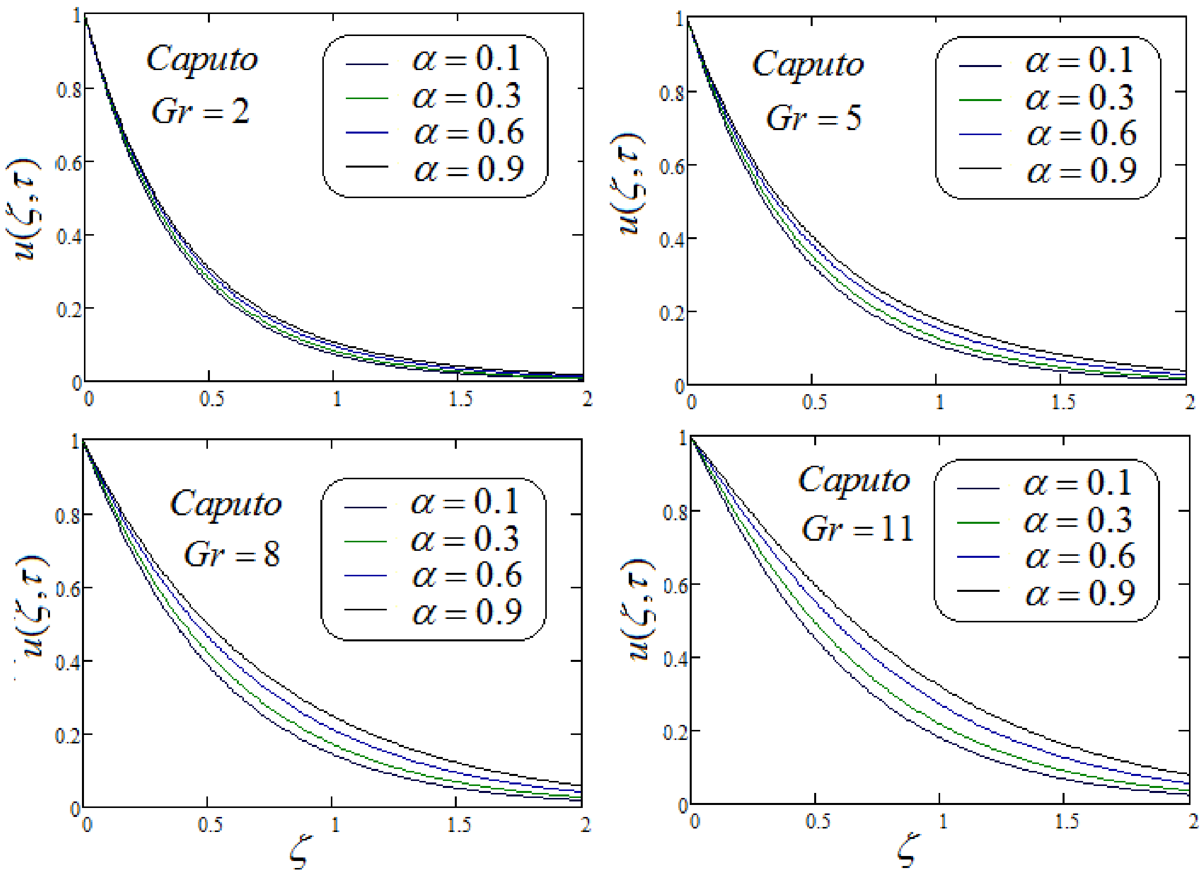

Fig. 9: Velocity behavior for four dissimilar values of Gr via caputo when ϕ=0.3,Pr=6.2,t=5,M=2, Q=0.7,Nr=0.5,K=0.5

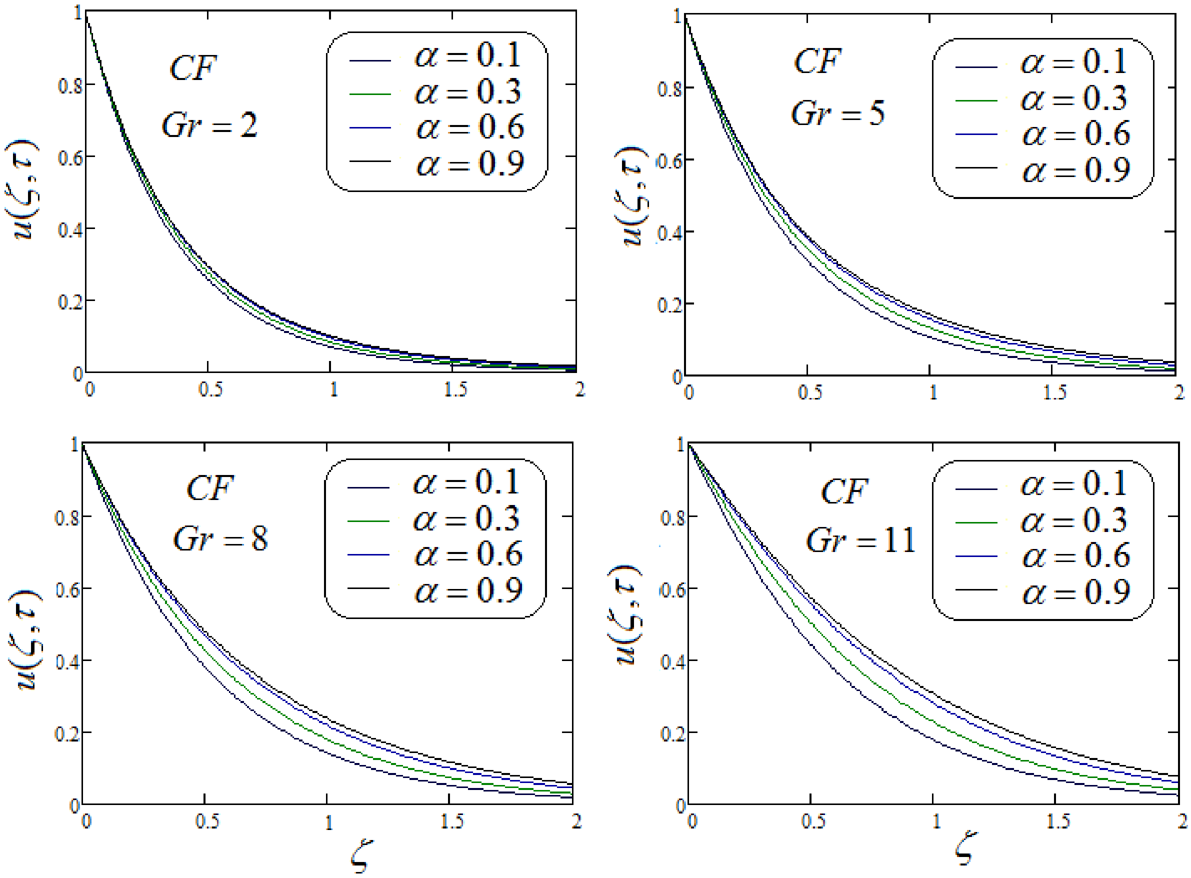

Fig. 10: Velocity behavior for four distinct values of Gr via CF when ϕ=0.3,t=5,M=2,Q=0.5, Pr=6.2,Nr=0.5,K=0.5

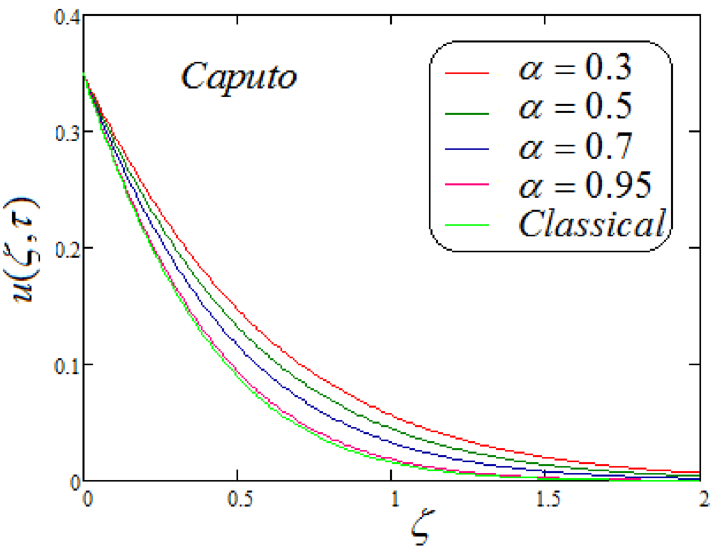

Fig. 11: Velocity comparison between Caputo and ordinary model when ϕ=0.1,t=0.35,M=2, Pr=6.2,Q=0.5

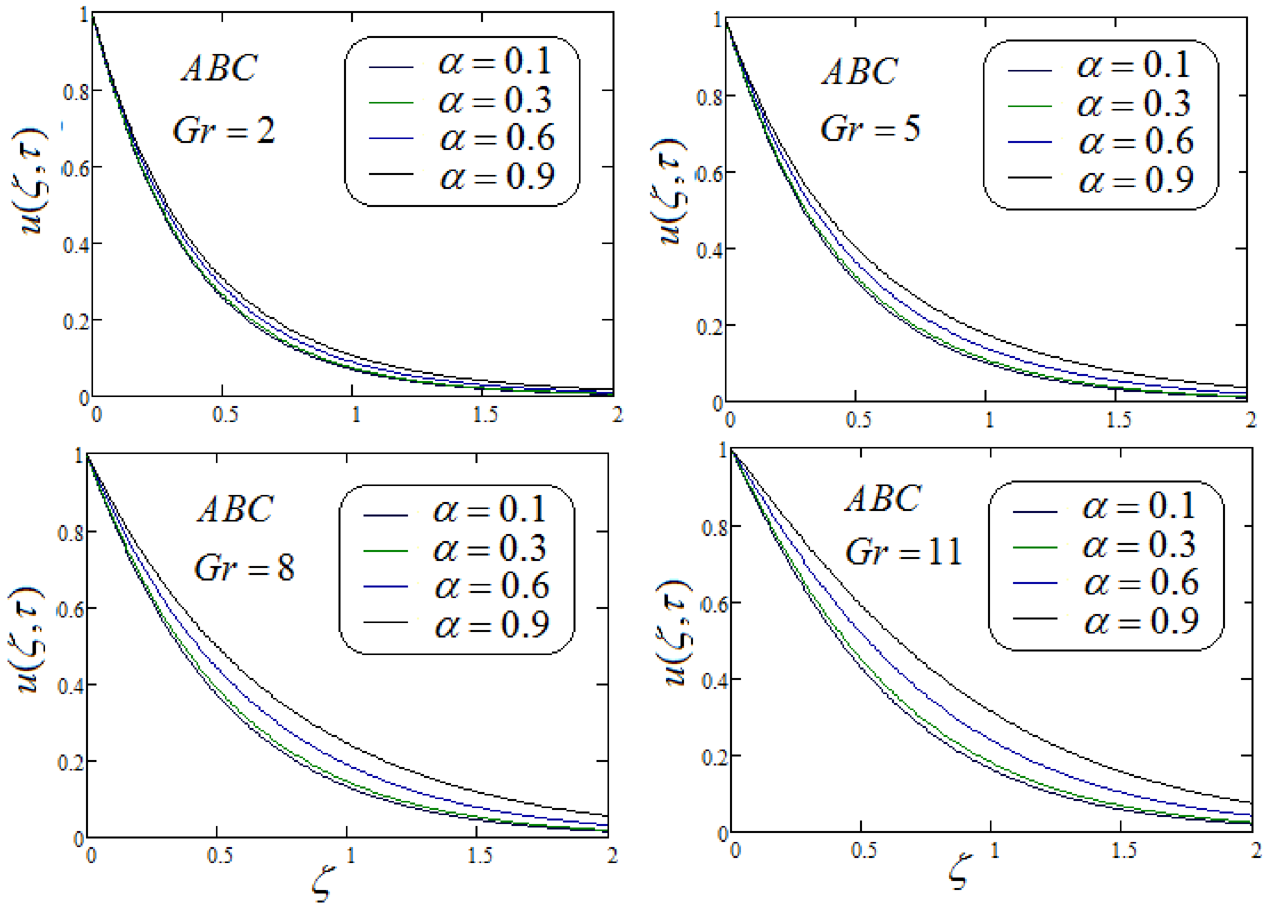

Fig. 12: Velocity behavior for four different values of Gr via ABC when ϕ=0.3,t=5,M=2, Q=0.5,Nr=0.5, Pr=6.2,K=0.5

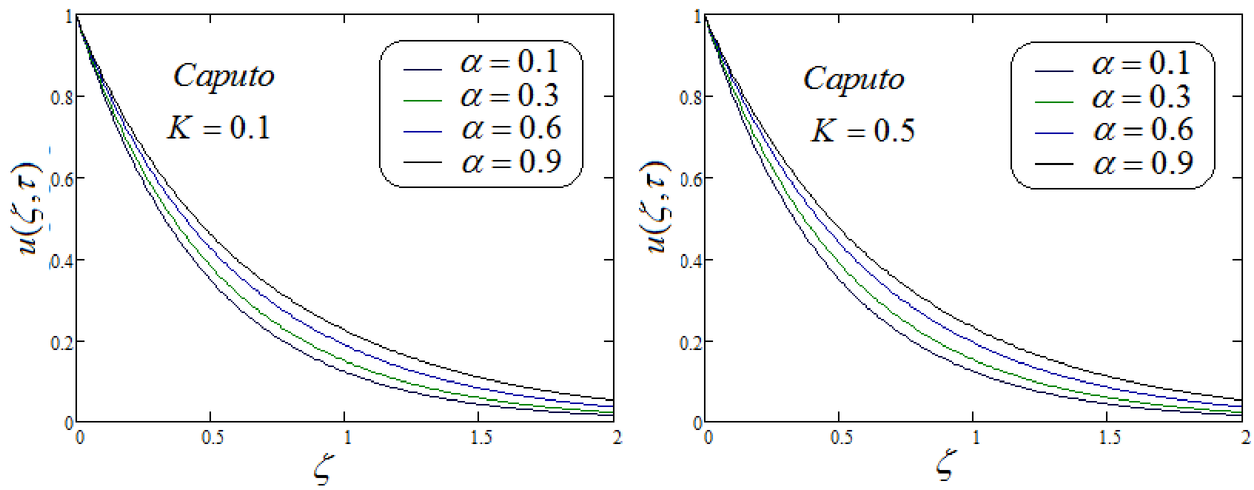

Fig. 13: Velocity behavior for two different values of K via Caputo when ϕ=0.1,t=5,M=7,Pr=6.2, Q=0.5,Nr=0.5,Gr=15

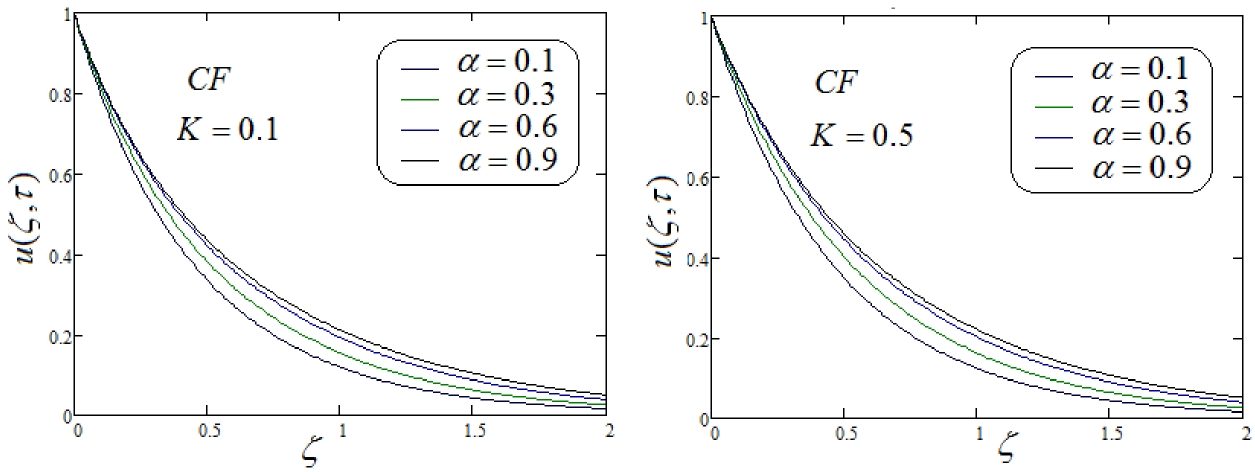

Fig. 14: Velocity behavior for two different values of K via CF when ϕ=0.1,t=5,M=7, Pr=6.2,Q=0.4,Nr=0.5,Gr=15

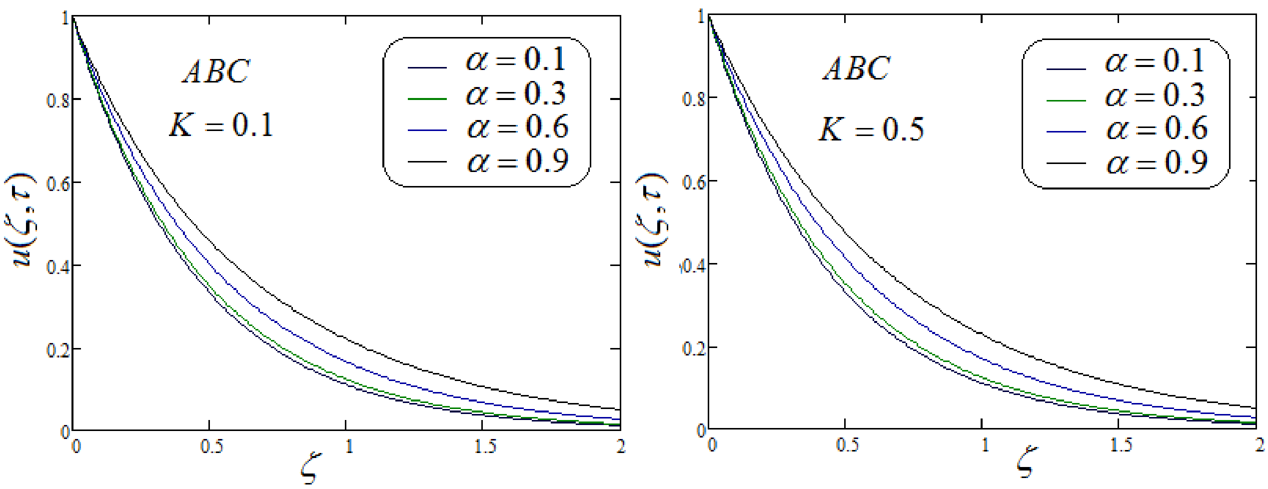

Fig. 15: Velocity behavior for two different values of K via CF when ϕ=0.1,t=5,M=7, Q=0.4,Nr=0.5,Gr=15,Pr=6.2

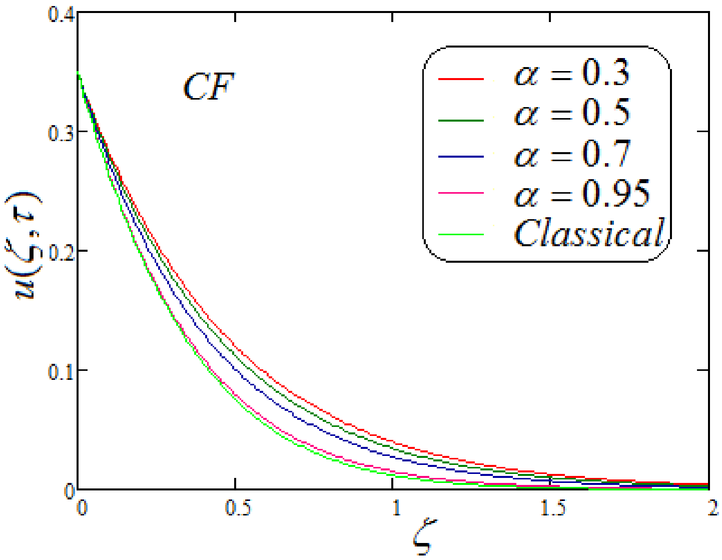

Fig. 16: Velocity comparison between CF and ordinary model when ϕ=0.1,t=0.35,M=2, Pr=6.2,Q=0.5,Nr=0.5,Gr=5

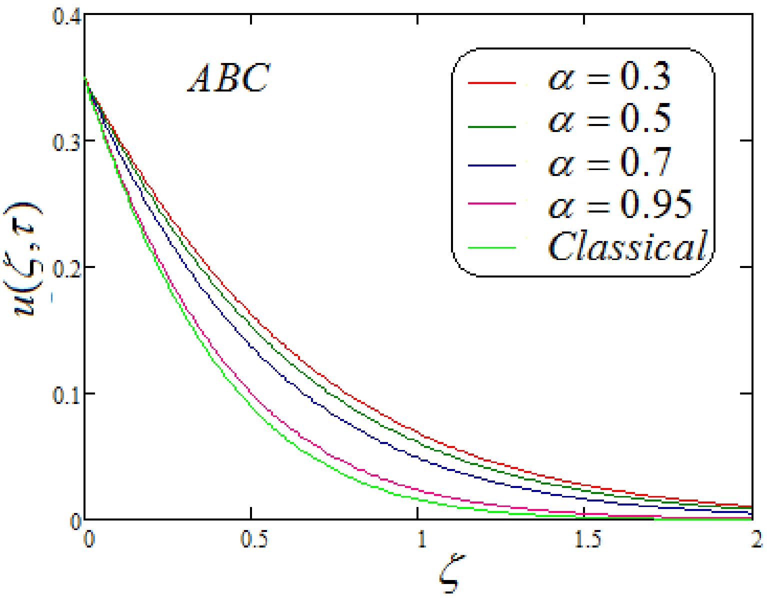

Fig. 17: Velocity comparison between ABC and ordinary model when ϕ=0.1,t=0.35,Pr=6.2,M=2, Q=0.5,Nr=0.5,Gr=5

It is observed that temperature as well as velocity profile are controlled by fractional parameter α. Figure (2, 3 and 4) illustrate for different time level the effect of α on temperature. For large and small values of τ and using distinct values of α, it is depicted from figure (2) the behavior of temperature is enlarge via Caputo time fractional derivative. Also figure (3) describes the same effect of the temperature for several values of α via CF and in figure (4) displays the temperature also increasing via ABC for different time levels and various values of α. Thus in all cases increase in temperature is examined, away from the plate in the main stream region, and finally decay for larger values of y, and then asymptotically approaches to zero as y approaches to infinity. The control of nanoparticle solid volume fraction ϕ are analysed on temperature distribution for varying time in figure (5). It is examine that raise in ϕ, solid volume fraction parameter, elevate the temperature contour. It is true that increase in the value of ϕ which increased fluid density and cause to rise the stickiness of boundary layer which helps to improve the thermic conductivity, and heat transfer rate on surface is also enhanced. In figures (6,7 and 8) investigated the influence of α for numerous values of time on velocity distribution for water based fluid. Considering different values of time and varying values of α, from figure (6) it is anlyzed that the behavior of velocity is not increasing via Caputo time fractional derivative. Also figure (7) described the same behavior of the velocity for several values of fractional parameter α via CF and in figure (8) displays the velocity also decreasing via ABC for two time levels (small and large) and various values of fractional parameter α. Thus in all cases, it shows that velocity is a decreasing function, near the plate for large values of α. This rapid decay in velocity is due to enlargement in momentum frontier layer for increasing α. It is inspected that temperature and velocity profile are controlled by time fractional parameter.

In figures (9,10 and 12) displays graphs of nanofluid velocity for varied values of Gr with same time via Caputo,CF and ABC. In figure (9),it is analysed that four different graphs are displayed for numerous values of Gr and various values of fractional parameter α via Caputo time fractional derivative the velocity profile is elevated. In figure (10), it is remarked that four different graphs are drawn for dissimilar values of Gr and numerous values of fractional parameter α via CF the velocity is increasing. In figure (12), it is analysed that four different graphs are traced for varying values of Gr and differing values of fractional parameter α via ABC the velocity is also elevated. It is declared that velocity in all cases and Gr has direct relation. Generally, Gr deals with buoyancy and viscous forces, strength of viscous force is decreased due to raise in Gr. Consequently, nanofluid velocity is elevated when it is close to oscillating plate then nanofluid flow is apart from the plate, forces becomes delicate and gradually the motion of fluid retarted to zero.

In figure (13), it is analysed that velocity is elevated for differing values of K and dissimilar values of α via Caputo time fractional derivative. In figure (14), it is observed that two different graphs are drawn for dissimilar values of K and numerous values of fractional parameter α via CF the velocity is increasing. In figure (15), it is analysed that two different graphs are traced for varied values of K and various values of fractional parameter α via ABC the velocity is also elevated. Hence, Figures (13,14 and 15) shows the relationship between dimensionless velocity and permeability parameter K which highlights the fact that nanofluid velocity is accelerated corresponding to increase the values of K for all fractional operators varying fractional parameter α. Influence of α on the nanofluids velocity and draw comparison among the different fractional operators Caputo, CF and ABC models with classical model is discussed in figures (11,16 and 17) respectively. Decay in velocity profile is perceived for large values of α. Finally, it is remarked that when $\alpha \to 1$, different fractional models Caputo, CF and ABC becomes ordinary model.

7 Conclusion

The fascinating features of this knowledgeable study is to analyzed the physical impressions of numerous relevant parameters on unsteady, MHD convective flow of non-dimentional governing equation of water base nanoparticles with ramped conditions with application of different fractional operators Caputo, CF and ABC. Laplace transformation technique with the application of inversion algorithm are employed to derived the results for temperature and velocity. The bearings of different related physical parameters like, grashof number Gr, volume fraction parameter ϕ, magnetic number M, permeability parameter K, Prandtl number Pr and fractional parameter α on non- dimensional temperature and velocity, results are demonstrated graphically. The following noteworthy points are concluded and summarized as:

• The main temperature component increases due to enlargement in nanoparticle, solid volume fraction ϕ .

• It is noticed that temperature increasing corresponding the larger values of fractional parameter α in all cases of Caputo,CF and ABC for varying time.

• It is detect that decay in velocity, when the values of α is increasing for all cases of Caputo,CF and ABC for varying time.

• For greater value of K velocity profile is elevated for varying fractional parameter α for all cases of fractional operators applied on nanofluid velocity.

• Velocity profile uplift corresponding to enlargement in the values of Gr for fixed time and different values of α.

• Caputo, CF and ABC fractional operators converges to classical model when $\alpha \to 1$.

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Nomenclature

υ	Kinematic viscosity $[m^2 s^{-1}]$	p	Laplace transform
g	Acceleration due to gravity $[ms^{-2}]$	τ	Time (s)
ρ	Fluid denisty $[kgm^{-3}]$	Pr	Prandtl number
C_p	Specific heat $[Jkg^{-1}K^{-1}]$	Sc	Schmidt number
k	Thermal conductivity $[Wm^{-1}K^{-1}]$	λ	Relaxation time (s)
β	Thermal expansion coefficient $[K^{-1}]$	k_1	Permeability $[m^2]$
μ	Dynamic viscosity $[kgms^{-1}]$	M	Magnetic field

Conflict of Interest

The authors declare that they have no conflict of interest.

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