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New Quantum Algorithms using Multi-Qubit Interaction

E. El-Sanousy[1,∗], *Mohamed Tolan*[2,3], *Ibrahim A. Abouelsaad*[4,5], *Ayman Abd-Elhamed*[6,7] *and H. F. Abdel-Hameed*[1,8]

[1]Department of Mathematics, Faculty of Science, Sohag University, Sohag, Egypt
[2]Mechanical Department, Faculty of Technology and Education, Suez University, Suez, Egypt
[3]Faculty of Technological Industries, King Salman International University, South Sinai, Egypt
[4]Horticulture Department, Faculty of Agriculture, Damanhour University, Damanhour, Egypt
[5]Faculty of Desert Agriculture, king Salman International University, South Sinai, Egypt
[6]Faculty of Engineering at Mataria, Helwan University, Cairo, Egypt
[7]Faculty of Engineering, King Salman International University, South Sinai, Egypt
[8]Mathematics Department, Khurma University College, Taif University, Taif, Saudi Arabia

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Abstract: Searching whether an item is stored in an unsorted database of N items is a crucial issue in nowadays real world, computer science and engineering, applications. The complexity of this problem is $O(N)$ and speed up quadratically, $O(\sqrt{N})$, when using Grover's algorithm if $N = 2^n$, where n is the number of qubits that are used store the unsorted database. This letter shows that Grover's algorithm is inefficient for unsorted quantum databases of size $N < 2^n$ items. In this letter, a novel quantum searching algorithm is proposed to check whether a given item is stored in a given unknown and unsorted database of $N \leq 2^n$ such that these states, items, are stored in a weighted/uniform superposition. The proposed search algorithm accomplishes this operation in polynomial time.

Keywords: Quantum algorithm, Computing models, Concurrence Measure

1 Introduction

Nowadays, due to the promising results of quantum mechanical algorithms [1]-[7] that are can perform computation faster than classical computers [8]-[12]. There is a race in academic and industrial research for achieving a continuous breakthroughs to ward build a models for quantum computers. Although building reliable quantum computer for our daily life computations is still far from reality [13]. Quantum computer is proposed to be universal computer by R. Feynman in 1982 when he was investigate to simulate quantum systems over classical computers. Petr shore and L. Grover are draw attentions to quantum computing when they proposed the factorization and search algorithms to solve a real world applications faster than classical computers [15,16]. Recently, it was showed that quantum computers can be used to solve novel types of quantum computing problems which cannot be achieved absolutely in classical computers [11].

Grover's algorithm[17,18,19] is the fastest and optimal to search in an unsorted database. Gover proved that it is possible to retrieve a single target object from an unsorted database quadratically faster compared with classical computers [15,16,20]. Grover co-operated with Radhakrishnan and extended his algorithm to what is known as quantum partial search algorithm. This algorithm has the potential to divide the database into several blocks and then look for particular block which contain the target object instead of searching for this target in the whole database [21]. Korepin et al. [23] proposed a version of this algorithm to search with several target items. Korepin optimized the partial search algorithm [24,25,26,13] and further generalized to what is known later as GRK [27,28,29]. Grover's algorithm and its variants toward GRK is deeply investigated in the article review [13]. There are two known techniques to mark states in a superposition which are marking by phase inverse and marking by entanglement. Grover's algorithm is proposed to search for item/s is/are stored in an unsorted database based on phase inverse of the target state.

In daily life, there are plenty of databases that it's content items are unknown. So, search to know whether

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[∗] Corresponding author e-mail: elsayed_hussien@science.sohag.edu.eg an item is stored or is exists in a given unsorted database is significantly in most of computer science applications. In this letter, we explain that this proposed search problem is beyond Grover's algorithm. Moreover, a novel search algorithm to solve the under scrutiny problem is proposed based on marking by entanglement and entanglement measurements. Then, the efficiency of the proposed algorithm is investigated for applications counting, test valid and invalid arguments, test logical equivalence and test system specification.

The letter is organized in following fashion. In Sec. 2 the abstract problem and then performance of Grover's algorithm for the problem under scrutiny is investigated. In Sec. 3 shows the proposed search algorithm based on entanglement measurement. Also, this section shows the efficiency of the proposed algorithm . Finally, Sec. 4 concludes the letter.

2 The Abstract Problem and Grover's Algoritm

Grover's algorithm is a search algorithm to find the solution within a complete superposition in unsorted database after $O(\sqrt{N})$ iterations. So, in most of computer applications it is required to test if a specific record or a part of record is stored in the database or not. For example, in most of computer applications it is required to check if the typed user name and password match correctly to a record in the user database or not. The problem under scrutiny is this; assume that a given unsorted quantum database $|\xi\rangle$ has even or weighted complete/incomplete superposition. As long as, a given constraint $|\kappa\rangle$ is required to be checked whether it is stored in $|\xi\rangle$ or not. The abstract problem is:

Given: (i) A n_1-qubits unsorted database $|\xi\rangle$ of $M \leq 2^{n_1}$ states as

$$|\xi\rangle = \sum_{k=1}^{M} \alpha_k |\zeta_k\rangle, \text{ such that } \sum_{k=1}^{M} |\alpha_k|^2 = 1.$$

(ii) Some constraints one given via state $|\kappa\rangle$ of n_2-qubits, such that $n_2 \leq n_1$sss.

Goal: Check if the state $|\kappa\rangle$ is stored in $|\xi\rangle$ or not.
In other words, the goal of the proposed algorithm is to check if there are records in the given quantum database $|\xi\rangle$ satisfy the criteria defined by the state $|\kappa\rangle$ is stored or not. To investigate the performance of Grover's algorithm on a set of database of $n = 2, 3, 4, \ldots, 12$ qubits. Such that each database have incomplete superposition of $2^n - 1$ states. For each database, two experiments are conducted. In the first experiment, Grover' algorithm is used to search for the missing state only. In the second experiment, Grover' algorithm is used to search for a single state stored in the database. The probability of retrieving the missing state and the stored state is depicted in Fig. 1 by the red and the blue curves, respectively.

There is three important points to note from Fig. 1. Firstly, the probability of finding the missing state is unlike to our expectation to be attend to zero, but in the range $[75\%, 99.99\%]$. Secondly, the probability of finding a stored state from the databases stored by using $3, 4, 5, or$ 6-qubits is less than the probability of finding the missing state. Finally, the probabilities of finding both an unstored state and a stored state convergence to certainty. In addition, D. Ventura and T. Martinez proved that pesudo-states are not inevitable using Grover's algorithm when search over incomplete superposition quantum system. These four drawbacks makes Grover's algorithm inefficient to search within incomplete superposition of unsorted databases. Finally, as long as Grover's algorithm fails to solve the proposed problem, it is required to propose a novel algorithm to solve the proposed search problem.

3 Methodology

In a quantum computer, the logic circuitry and time steps are essentially classical, only the memory bits that hold the variables are in quantum superposition [**?**]. The proposed verification algorithm is based on two essential operators. The first one is called the overlapping operator ZOV, it marks the overlapped states between two given quantum systems via entanglement.The second operator is called M_Z operator, it is determine the degree of entanglement between two qubits.

3.1 Entanglement Measure computing model

An arbitrary composite system of two-qubit described by a state $|\chi\rangle$ is called two-qubit entangled system if and only if it cannot always be written mathematically in the product form $|\chi\rangle \neq |\chi_1\rangle \otimes |\chi_2\rangle$. Although this mathematical definition is simple but it can not quantify the degree of entanglement in a two-qubit system. Concurrence measures the degree of entanglement between arbitrary two-body systems.

$$|as\rangle = a_0 \, |00\rangle + b_0 \, |10\rangle, \; |a_0|^2 + |b_0|^2 = 1. \tag{1}$$

M_z operator receives two decoupled replica of this state. The first input is the two qubits $|as\rangle$ which has a state described by Eq. (1). Then M_z is applied on these two replica as $M_z(|as\rangle \otimes |as\rangle) = Measure(CNOT \otimes CNOT(|as\rangle \otimes |as\rangle))$. Then, the concurrence value C between the two qubits $|a\rangle$ and $|s\rangle$ is quantified on a quantum chip as follows [22].

$$C = \sqrt{2(P_{0011} + P_{1100})}, \tag{2}$$

where P_{0011}, P_{1100} are the probabilities of the states $|0011\rangle$, $|1100\rangle$, respectively. In this work, the second technique of the degree of entanglement computing model [22] is used

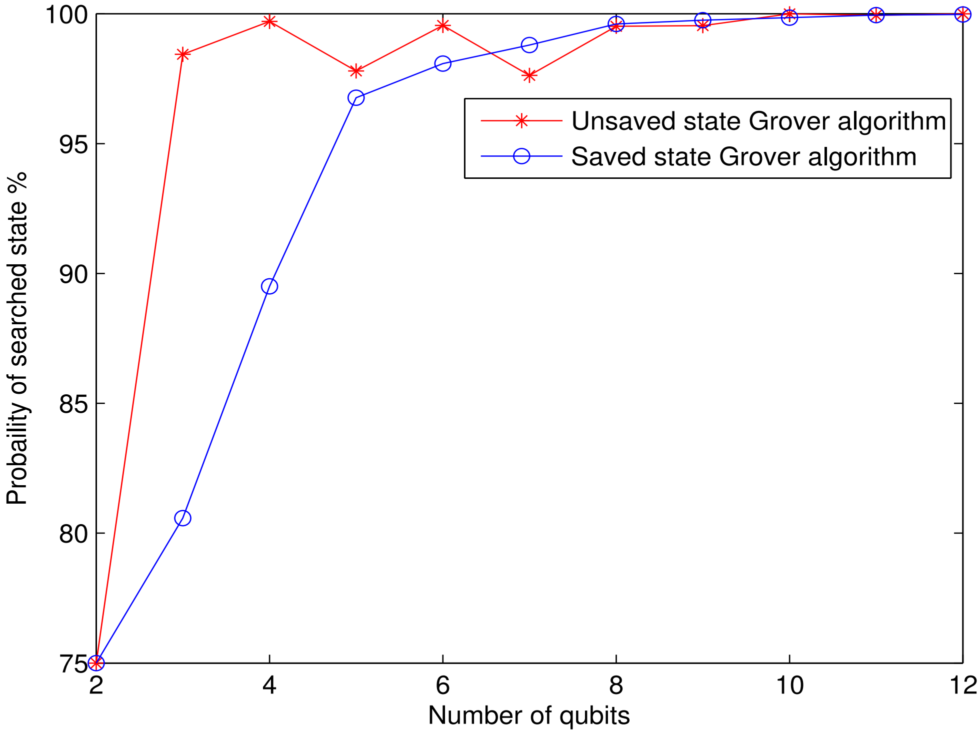

Fig. 1: Grover algorithm performance for searching within a database of a uniform superposition of $(2^n - 1)$ states, where n is the number of the qubits.

which reports the solution of the problem at hand based on the presence or absence the entanglement $C > 0$, or $C = 0$, respectively, between the two qubits $|as\rangle$.

3.2 The Proposed Quantum Algorithm

The proposed search algorithm is as follows:

1. Register Preparation: Concatenate the quantum system $|\phi\rangle$ of $k = n_1 + n_2$ qubits with the extra two qubits $|a\rangle \otimes |s\rangle$, are initialized in the state $|00\rangle$ as

$$|\psi_0\rangle = |\phi\rangle \otimes |\chi\rangle,$$

where the quantum system $|\phi\rangle$ is described as $|\phi\rangle = |\xi\rangle \otimes |\kappa\rangle$, and $|\chi\rangle = |t\rangle \otimes |d\rangle$.

2. Apply the operator ZOV on $|\phi\rangle$ and the detection qubit:

$$|\psi_1\rangle = ZOV^{\otimes k+1}|\phi, t\rangle \otimes I|d\rangle,$$

where I is 2x2 Identity operator.

3. Repeat Steps 1-2 to get another a decoupled replica of the state $|\psi_1\rangle$.

4. Apply the Operator M_z as follows:

$$|\psi_2\rangle = I^{\otimes k}|\phi\rangle \otimes M_z|as\rangle \otimes I^{\otimes k}|\phi\rangle,$$

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Then estimate $C$ using Eq. (2)
If $C > 0$ then
    $|\kappa\rangle \in |\xi\rangle$, in other words the state $|\kappa\rangle$ is stored $|\xi\rangle$.
Else
    $|\kappa\rangle \notin |\xi\rangle$.
End.
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3.3 Analysis of the Proposed Algorithm and Discussion

In the first step, we initialize the proposed algorithm with three registers. The first register is $|\kappa\rangle$ of n_2 qubits which is initialized with the designated search state. The second register is the given unsorted quantum database $|\xi\rangle$. Finally, The third register $|\chi\rangle$ contains two ancilla qubits which are called the test and the detection qubits (see [22]), are initialized by state $|11\rangle$.

In the second step of the proposed algorithm applies ZOV operator, hence there are two possible cases:

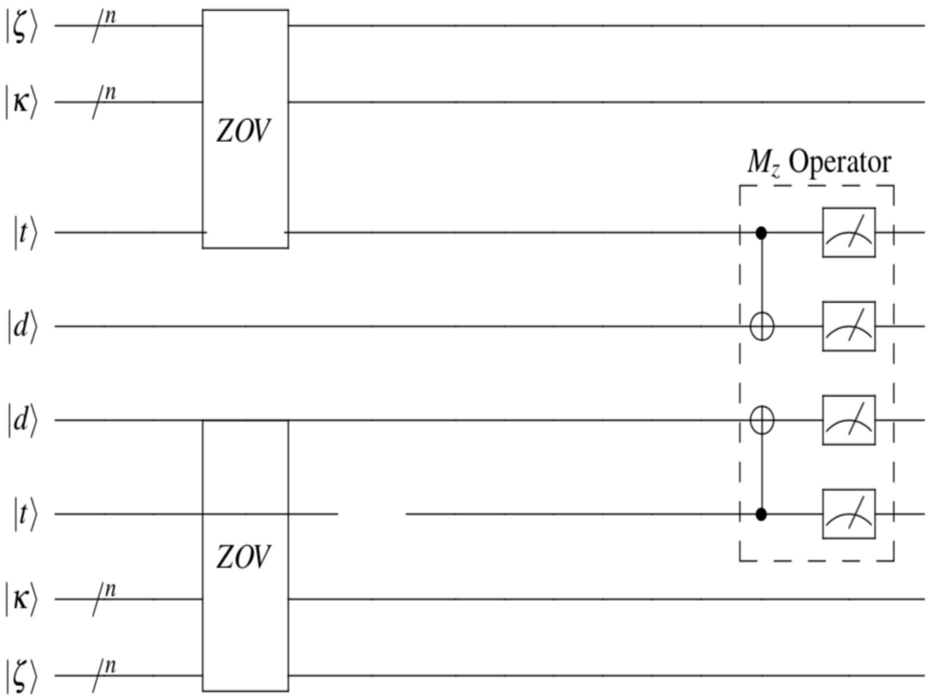

Fig. 2: The quantum circuit of the proposed algorithm

(i) If there exits a set of m_0 states $\{|\zeta_j\rangle\} \in |\xi\rangle$ overlap with the state $|\kappa\rangle$, then ZOV operator marks those records by entanglement with the test qubit $|t\rangle$. In other words, ZOV evolves the state of $|t\rangle$ to be correlated with $\{|\zeta_j\rangle\}$ to mark the overlapped states between the state $|\kappa\rangle$ and the $|\xi\rangle$ as follows:

$$|\psi_1\rangle = \sum_{j\in m_0} |\kappa,\xi_j,0\rangle \otimes |1\rangle + \sum_{i\neq j, i\in k} |\kappa,\xi_i,1\rangle \otimes |1\rangle. \quad (3)$$

Then from Eq. (3) the state of $|\chi\rangle$ is:

$$|\chi\rangle = \sqrt{\frac{m_0}{N}}|\kappa',\xi_j'\rangle\ |01\rangle + \sqrt{\frac{N-m_0}{N}}|\kappa'',\xi_j''\rangle\ |11\rangle.$$

This means that there are m_0 states of $|\xi\rangle$ match the state $|\kappa\rangle$ and other $N-m_0$ are not.

In this case, after step 3 is applied by applying M_z. This operator, firstly evolves the state of the test and detection qubits $|td\rangle$ to be entangled in between by applying CNOT gate as follows:

$$|\chi\rangle = \sqrt{\frac{m_0}{N}}|\kappa',\xi_j'\rangle\ |01\rangle + \sqrt{\frac{N-m_0}{N}}|\kappa'',\xi_j''\rangle\ |10\rangle. \quad (4)$$

Secondly, it measures the concurrence $0 < C \leq 1$ in between. This indicates that the state $|\kappa\rangle$ is stored in $|\xi\rangle$.

(ii) If there is no one state in $|\xi\rangle$ overlaps with the state $|\kappa\rangle$, then the state of $|t\rangle$ is unchanged after applying *ZOV* operator.

$$|\psi_1\rangle = \sum_k |\kappa,\xi_i,1\rangle \otimes |1\rangle. \quad (5)$$

So, there is no-correlation between the states of $|\xi\rangle$ and $|t\rangle$ therefor there is no states marked in $|\xi\rangle$. In this case, the state of the register $|\chi\rangle$ is as follows:

$$|\chi\rangle = |11\rangle. \quad (6)$$

In this case, after step 3 is applied by applying M_z. Firstly, it maintains the state of $|td\rangle$ to be separable as follows

$$|\chi\rangle = |11\rangle = |1\rangle \otimes |0\rangle,$$

when CNOT gate is applied on. Secondly, it announce that the entanglement is missed in between, $C = 0$. Which indicates that the state $|\kappa\rangle$ is not stored in $|\xi\rangle$.

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4 Perspective

In this letter, we used degree of entanglement quantum computing model for proposing a new searching algorithm that decides whether an item is stored in an unsorted database of N items. This letter explained that Grover's algorithm is inefficient for unsorted quantum databases of size $N < 2^n$ items. But on the other hand, the proposed search algorithm tackles this issue in polynomial time.

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