Information Sciences Letters
An International Journal

http://dx.doi.org/10.18576/isl/100307

A Three Species Ecological Model with Holling Type-II Functional Response

Srinivasarao Thota

Department of Applied Mathematics, School of Applied Natural Sciences, Adama Science and Technology University, Post Box No. 1888, Adama, Ethiopia

Received: 2 Mar. 2021, Revised: 3 Apr. 2021, Accepted: 5 Jun. 2021
Published online: 1 Sep. 2021

Abstract: This paper elucidates an ecological model of a three-space food chain consists of two competing species and a third species, which is supporting the first competing species in non-linear manner of Holling type-II functional response and ingests the second competitive species in linear style, within the limited resources. This model is constituted by a system of non-linear decoupled first order ordinary differential equations. By using perturbed method, we investigate the local stability nature of the system at each possible equilibrium point. Further, the numerical illustrations at suitable parametric values to the model are presented by observing the species survivalness in nature for long time.

Keywords: Competition, Predator, Holling type-II response function, Local stability, Numerical simulations.

1 Introduction

Many applications of real world problems are expressed in ecological models. The study of ecological models has become central role of mathematics and created much interest among authors. The classical models of food chain with only two tropic levels are discussed by many researchers and scientists. In the study of interactions between the species, like Prey-Predator, Competition, Mutualism, Ammensalism, Commensalism and so on, much attention is paid to converting them mathematically so that one can study the behavior of species in both analytically and numerically. The discussion on local and global stability of ecological models in different types of interactions of species is very engrossing and demanding mathematically and biologically. Any complex interactions among the species are mixing of the above interactions. More work related to complex interactions among species and their stability nature are discussed by May [1], Naji and Balasim [2], Smith [3], and Edelstein-Keshet [4]. A general discussion on the multi-species populations models can be viewed in Kapur [5]. The comprehensive report on theoretical ecology can be found from Stiling [6]. The basic concepts in mathematical modeling are got from the treatises of Lotka [7] and Kuang [8]. For review and fruitful discussion on models with mutualism, we can refer to Addicott John [9], Dean [10], Goh [11], Wolin [12] and Zhibin [13]. The investigations on multi-interactions among species can be observed from Gyllenberg et al. [14], Suresh Kumar et al. [15] and Wang et al. [16]. The work on commensalism interaction between two species can be seen from Seshagiri Rao et al. [17]. Differential equations and their applications to dynamical systems can be viewed from the books of Brauer et al. [18], Braun [19], and Hirsch et al. [20]. The species, in environment interacts other species in different functional response types within the availability natural resources. There have been great number of generalizations in this direction. Chen et al. [21] studied the global stability of the existing positive equilibrium for the prey-predator model incorporating a constant number prey refuges. In [22], Seo et al. discussed the stability analysis of a predator-prey model with a Holling type-I functional response including a predator mutual interference. The bifurcation analysis of prey-predator system with constant harvesting of Holling type-II is investigated by Peng et al. [23]. Later Yu [24] discusses the global stability analysis of prey-predator model with modified Leslie-Gower and Holling type-II schemes. More work on a prey-predator systems with Holling type-III and Holling type-IV response functions, the reader may refer to

* Corresponding author e-mail: srinithota@ymail.com

Huang et al. [25] and Naji et al. [26]. Species epidemic food chains models in different tropical levels with various response functions can be found in [27–36].

Therefore, in this paper, we focused on the dynamics of a three-space food chain consists of two logistically growing competing species and a third species acts as a predator as well as host for the competing species which interacts linear and non-linear manner.

The applications of these type models are available in real life. For example, deer and elk are both competing each other for their food in the same region and a lion (predator) harms elk then automatically deer gets survival for time being, indirectly deer gets benefit from lion. One of demerits of the presented model is that the species with such kind of interactions will not be survival for long time in the environment due to under extreme ecological conditions such as animals (competition among themselves) and a predator which will be more interested (or) harm the second competitor for its food. On the other hand, it indirectly helps the first competitor, situations leading to one of the species growth is exponential and another is not such kind of solution exists in ecosystem.

The rest of the paper is organized as follows. In Section 2, we formulate the model in terms of the non-linear decoupled ordinary differential equations; In Section 3, we present analysis of the proposed model by finding the equilibrium points of the model in Section 3.1 and the stability analysis of the model in Section 3.2; and Section 4 discuss the numerical simulation of the proposed model.

2 Model Formulation

The ecological setup for three species food web system with multi-intraction among themselves for the proposed model is shown in the following Figure 1.

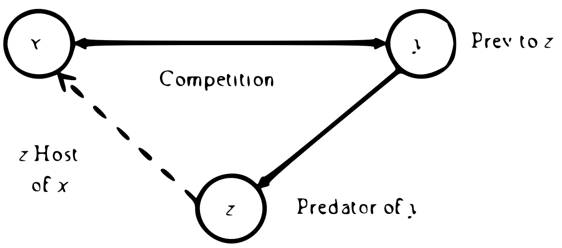

Fig. 1: Schematic diagram of three species food-chain system

In the Figure 1, x, y and z are the population densities of two competing species and a third species, which is predator for y and host for x at any time instant t respectively. The species z consumes y linearly and helps the first competing species x in Holling type-II functional response manner. The parameter α_{23} is the consumption coefficient of z on y; α_{32} is the benefit coefficient of z on y due to the interaction; α is the commensal coefficient of the species x with respected to z; r is the intrinsic growth rate of the first competing species; ω is death rate of third species z. The coefficients β_{ij} $(i \neq j; i, j = 1, 2)$ are the inter-species competition coefficients of two competing species x and y.

By employing the terminology given above, the model equations for an ecosystem consisting three species interacting in different ways, as given in Figure 1, consists of a set of non-linear decoupled first order ordinary differential equations as follows,

$$\begin{aligned} \frac{dx}{dt} &= x\left(1 - \beta_{12}y + \frac{\alpha z}{1+x}\right) \\ \frac{dy}{dt} &= ry(1 - \beta_{21}x) - \alpha_{23}yz \\ \frac{dz}{dt} &= z(-\omega + \alpha_{32}y) \end{aligned} \tag{1}$$

with the non-negative initial conditions $x(0) \geq 0, y(0) \geq 0$ and $z(0) \geq 0$.

3 Model Analysis

In this section, we present the stability analysis of the proposed model by finding the feasible equilibrium points of the dynamical system.

3.1 Equilibrium Points

The equilibrium points of the system are necessary for the purpose of studying the local stability nature of the ecological model. The system, under investigation, has the following four equilibrium points

(i) Fully washed out state or extinct state: $E_1 = (0, 0, 0)$.
(ii) Only first competitive species washed out state: $E_2 = \left(0, \frac{\omega}{\alpha_{32}}, \frac{r}{\alpha_{23}}\right)$.
(iii) Only third species washed out state: $E_3 = \left(\frac{1}{\beta_{21}}, \frac{1}{\beta_{12}}, 0\right)$.
(iv) Interior or Coexistence state: $E_4 = (x^*, y^*, z^*)$, where

$$\begin{aligned} x^* &= \frac{r\alpha - \alpha_{23}(\beta_{21}y^* - 1)}{\beta_{21}r\alpha + (\beta_{12}y^* - 1)\alpha_{23}}, \\ y^* &= \frac{\omega}{\alpha_{32}}, \\ z^* &= \frac{r(\beta_{12}y^* - 1)(\beta_{21} + 1)}{\beta_{21}r\alpha + (\beta_{12}y^* - 1)\alpha_{23}}. \end{aligned}$$

In the following section, we discuss about the local stability of the system at the above existing equilibrium points by employing perturbed technique.

3.2 Stability Analysis of Equilibrium Points

The Jacobian matrix for the system (1) at an equilibrium point $E = (\bar{x}, \bar{y}, \bar{z})$ is given by

$$J_E = \begin{pmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ 0 & J_{32} & J_{33} \end{pmatrix}, \quad (2)$$

where

$$J_{11} = 1 - \beta_{12}\bar{y} + \frac{\alpha\bar{z}}{1+\bar{x}} - \frac{\alpha\bar{x}\bar{z}}{(1+\bar{x})^2}, \quad J_{12} = -\beta_{12}\bar{x},$$
$$J_{13} = \frac{\alpha\bar{x}}{1+\bar{x}}, \quad J_{21} = -r\beta_{21}\bar{y}, \quad J_{22} = r(1-\beta_{21}\bar{x}) - \alpha_{23}\bar{z},$$
$$J_{23} = -\alpha_{23}\bar{y}, \quad J_{32} = \alpha_{32}\bar{z} \text{ and } J_{33} = -\omega + \alpha_{32}\bar{y}.$$

The corresponding characteristic equation is $\lambda^3 + a_0\lambda^2 + a_1\lambda + a_2 = 0$, where

$$a_0 = -J_{33} - J_{22} - J_{11}$$
$$a_1 = J_{11}J_{22} + J_{11}J_{33} - J_{12}J_{21} + J_{22}J_{33} - J_{23}J_{32}$$
$$a_2 = -J_{11}J_{22}J_{33} + J_{11}J_{23}J_{32} + J_{12}J_{21}J_{33} - J_{13}J_{21}J_{32}$$

The dynamical system is stable when all three eigenvalues of (2) are negative in case of real roots or having negative real parts in case of the complex roots. In other cases, the dynamical system is unstable always.

Theorem 1. *The interior equilibrium point $E_4 = (x^*, y^*, z^*)$ would exist when $\beta_{21}y^* > 1$, $\beta_{12}y^* > 1$ and $r\alpha > \alpha_{23}(\beta_{21}y^* - 1)$.*

Proof. Let x^*, y^*, z^* be the positive solutions of the equations

$$x^*\left(1 - \beta_{12}y^* + \frac{\alpha z^*}{1+x^*}\right) = 0,$$
$$ry^*(1-\beta_{21}x^*) - \alpha_{23}y^*z^* = 0,$$
$$z^*(-\omega + \alpha_{32}y^*) = 0.$$

Solving these equations for x^*, y^* and z^*, we obtain

$$x^* = \frac{r\alpha - \alpha_{23}(\beta_{21}y^* - 1)}{\beta_{21}r\alpha + (\beta_{12}y^* - 1)\alpha_{23}},$$
$$y^* = \frac{\omega}{\alpha_{32}},$$
$$z^* = \frac{r(\beta_{12}y^* - 1)(\beta_{21} + 1)}{\beta_{21}r\alpha + (\beta_{12}y^* - 1)\alpha_{23}}.$$

These would be positive when $r\alpha > \alpha_{23}(\beta_{21}y^* - 1), \beta_{21}y^* > 1$ and $\beta_{12}y^* > 1$. So, the interior equilibrium point $E_4 = (x^*, y^*, z^*)$ for system (1) exists if $\beta_{21}y^* > 1$, $\beta_{12}y^* > 1$ and $r\alpha > \alpha_{23}(\beta_{21}y^* - 1)$.

Theorem 2. *The dynamical system (1) is always unstable at the equilibrium points E_1, E_3 and E_4.*

Proof. (i) The eigenvalues of the dynamical system at the extinct equilibrium point $E_1 = (0,0,0)$ are $1, r, -\omega$. So, the equilibrium point E_1 is a saddle point, and hence the dynamical system is unstable in general.

(ii) The corresponding Jacobian matrix at the equilibrium point $E_3 = \left(\frac{1}{\beta_{21}}, \frac{1}{\beta_{12}}, 0\right)$ is

$$J_{E_3} = \begin{pmatrix} 0 & -\frac{\beta_{12}}{\beta_{21}} & \frac{\alpha}{1+\beta_{21}} \\ -\frac{r\beta_{21}}{\beta_{12}} & 0 & -\frac{\alpha_{23}}{\beta_{12}} \\ 0 & 0 & \frac{-\omega\beta_{12}+\alpha_{32}}{\beta_{12}} \end{pmatrix}.$$

The characteristic equation of J_{E_3} is

$$\left(\frac{-\omega\beta_{12} + \alpha_{32}}{\beta_{12}} - \lambda\right)(\lambda^2 - r) = 0.$$

The eigenvalues of J_{E_3} are $\lambda_1 = \sqrt{r}$, $\lambda_2 = -\sqrt{r}$ and $\lambda_3 = \frac{\alpha_{32} - \omega\beta_{12}}{\beta_{12}}$. Therefore, the equilibrium point E_3 is a saddle point and hence the dynamical system (1) is unstable.

(iii) The Jacobian matrix at the coexistence state $E_4 = (x^*, y^*, z^*)$ is

$$J_{E_4} = \begin{pmatrix} H_{11} & H_{12} & H_{13} \\ H_{21} & 0 & H_{23} \\ 0 & H_{32} & 0 \end{pmatrix},$$

where

$$H_{11} = -\frac{\alpha x^* z^*}{(1+x^*)^2}, \quad H_{12} = -\beta_{12}x^*, \quad H_{13} = \frac{\alpha x^*}{1+x^*},$$
$$H_{21} = -r\beta_{21}y^*, \quad H_{23} = -\alpha_{23}y^* \text{ and } H_{32} = \alpha_{32}z^*.$$

The characteristic equation of J_{E_4} is

$$\lambda^3 - H_{11}\lambda^2 - (H_{23}H_{32} + H_{12}H_{21})\lambda + (H_{11}H_{23}H_{32} - H_{13}H_{21}H_{32}) = 0.$$

Denote, the characteristic equation by $\lambda^3 + \alpha_0\lambda^2 + \alpha_1\lambda + \alpha_2 = 0$, where

$$\alpha_0 = -H_{11},$$
$$\alpha_1 = -(H_{23}H_{32} + H_{12}H_{21}),$$
$$\alpha_2 = H_{11}H_{23}H_{32} - H_{13}H_{21}H_{32}.$$

According to Routh-Hurwitz criteria, the equilibrium point E_4 will be locally asymptotically stable if $\alpha_0 > 0, \alpha_2 > 0$ and $\alpha_0\alpha_1 - \alpha_2 > 0$, but we have

$$\alpha_0 = -H_{11} = \frac{\alpha x^* z^*}{(1+x^*)^2} > 0,$$
$$\alpha_2 = H_{11}H_{23}H_{32} - H_{13}H_{21}H_{32} = \frac{\alpha\alpha_{32}x^*y^*z^*}{(1+x^*)}\left(\frac{\alpha_{32}z^*}{1+x^*} + r\beta_{21}\right) > 0, \quad \text{and}$$
$$\alpha_0\alpha_1 - \alpha_2 = H_{11}H_{12}H_{21} + H_{13}H_{21}H_{32} = -\frac{r\alpha\beta_{21}}{1+x^*}x^*y^*z^*\left(\frac{\beta_{12}x^*}{1+x^*} + \alpha_{32}\right) < 0 \quad \text{always.}$$

Hence, the dynamical system (1) is unstable always at E_4, since it fails to satisfy the Routh-Hurwitz criteria.

Theorem 3. *The dynamical system* (1) *at the boundary steady state* E_2 *is*

(i) unstable, and trajectories are outward spiral, if $\alpha_{32}\alpha_{23} + \alpha_{32}\alpha r > \omega\alpha_{23}\beta_{12}$.
(ii) closed orbits, if $\alpha_{32}\alpha_{23} + \alpha_{32}\alpha r = \omega\alpha_{23}\beta_{12}$.
(iii) the trajectories are inward spiral, if $\alpha_{32}\alpha_{23} + \alpha_{32}\alpha r < \omega\alpha_{23}\beta_{12}$.

Proof. The Jacobian matrix for the system (1) at $E_2 = \left(0, \frac{\omega}{\alpha_{32}}, \frac{r}{\alpha_{23}}\right)$ is

$$J_{E_2} = \begin{pmatrix} 1 - \beta_{12}\frac{\omega}{\alpha_{32}} + \alpha\frac{r}{\alpha_{23}} & 0 & 0 \\ -r\beta_{21}\frac{\omega}{\alpha_{32}} & 0 & -\alpha_{23}\frac{\omega}{\alpha_{32}} \\ 0 & \alpha_{32}\frac{r}{\alpha_{23}} & 0 \end{pmatrix}.$$

The characteristic equation of which is

$$\left(\frac{\alpha_{32}\alpha_{23} - \omega\alpha_{23}\beta_{12} + \alpha_{32}\alpha r}{\alpha_{23}\alpha_{32}} - \lambda\right)(\lambda^2 + \omega r) = 0,$$

then the corresponding eigenvalues are

$$\lambda = \frac{\alpha_{32}\alpha_{23} - \omega\alpha_{23}\beta_{12} + \alpha_{32}\alpha r}{\alpha_{23}\alpha_{32}}, 0 \pm i\sqrt{\omega r}.$$

In particular, denote

$$\lambda_1 = \frac{\alpha_{32}\alpha_{23} - \omega\alpha_{23}\beta_{12} + \alpha_{32}\alpha r}{\alpha_{23}\alpha_{32}}, \quad \lambda_2 = i\sqrt{\omega r} \text{ and}$$
$$\lambda_3 = -i\sqrt{\omega r}.$$

We can have the following observations to the dynamical system (1) based on the eigenvalue λ_1.

(i) if $\alpha_{32}\alpha_{23} + \alpha_{32}\alpha r > \omega\alpha_{23}\beta_{12}$, then $\lambda_1 > 0$. Thus, the system (1) is unstable in x direction and trajectories are circular orbits out in yz direction.
(ii) if $\alpha_{32}\alpha_{23} + \alpha_{32}\alpha r = \omega\alpha_{23}\beta_{12}$, then $\lambda_1 = 0$. Hence, the system (1) admits neutrally stable in yz direction.
(iii) if $\alpha_{32}\alpha_{23} + \alpha_{32}\alpha r < \omega\alpha_{23}\beta_{12}$, then $\lambda_1 < 0$. So, the system (1) is stable in x direction and trajectories are circular orbits in yz direction.

4 Numerical Simulations

The system of equations in (1) are solved numerically by applying Rungue-Kutta fourth order method to observe the species behavior in environment at suitable selected model characterizing parameters satisfying the conditions in Theorem 3. The software MATLAB (ode45) program has been used in finding the accurate approximate solutions for the species. The conclusions are given case-wise as follows.

Case I

By selecting the parameter values given below in the dynamical system (1), the condition (*i*) of Theorem 3 satisfied:

$$\beta_{12} = 1, \alpha = 0.9, r = 0.3, \beta_{21} = 0.05,$$
$$\alpha_{23} = 0.01, \alpha_{32} = 1, \omega = 1.5,$$

and with initial conditions $x(0) = y(0) = z(0) = 10$. The corresponding eigenvalues and equilibrium points in this case are $\lambda_1 = 26.500, \lambda_2 = 0.671i, \lambda_3 = -0.671i$, and $E_2(0, 1.5, 30)$. So, the system (1) is always unstable by observing the species growth rates in Figure 2. Only, the first competitive species (x) survives in nature long time and the remaining two species y, z gradually extinct as time goes on, this can be evidenced in Figure 2.

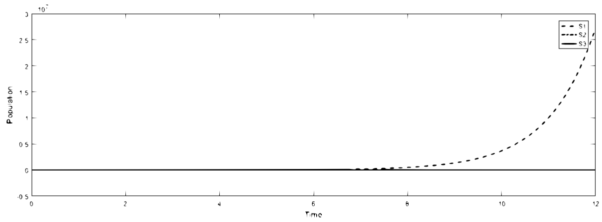

Fig. 2: Time series shows the behavior of three species.

Case II

By selecting the parameter values as $\beta_{12} = 3, \alpha = 0.5, r = 0.1, \beta_{21} = 0.05, \alpha_{23} = 0.01, \alpha_{32} = 1, \omega = 2$, it is clear that condition (*ii*) of Theorem 3 holds. In this case, the eigenvalues are $\lambda_1 = 0, \lambda_2 = 0.447i, \lambda_3 = -0.447i$ and the corresponding equilibrium point is $E_2 = (0, 2, 10)$. Initially, all three species will exist in nature but, after some time the second competitive species y gradually decreasing in its growth rate and then extinct further, whereas the first competitive species x and third species z growing with respect to time but, after some time due to death rate of z, it decreases, that can be observe from Figure 3. So, the dynamical system gets bifurcated between the time interval [0.5,1].

Case III

Now, at the parameter values, $\beta_{12} = 6, \alpha = 0.5, r = 0.1, \beta_{21} = 0.05, \alpha_{23} = 0.01, \alpha_{32} = 1, \omega = 2$, the condition (*iii*) of Theorem 3 satisfied. As discussed above, here $\lambda_1 < 0$, is real eigenvalue and λ_2, λ_3 are purely

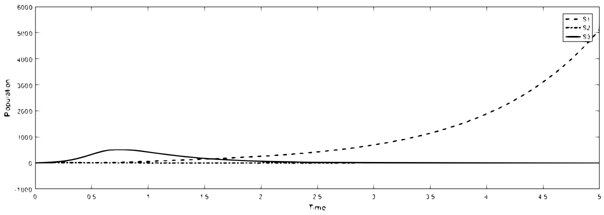

Fig. 3: Time series shows the behavior of three species.

imaginary complex eigenvalues. So, the dynamical system is unstable always that can be evidenced in Figure 4.

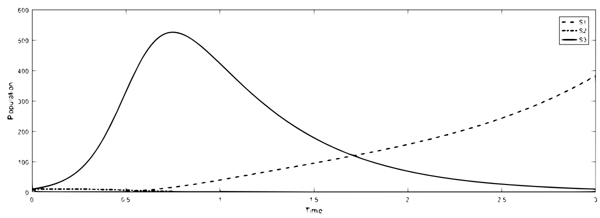

Fig. 4: Time series shows the behavior of three species.

Remark: When time goes on, the first competitive species x survival in nature due to the extended support from host z for a while, whereas second competitive species y extinct faster due to computation effect from x, z.

5 Conclusion

In this paper, we study an ecology model of three species food web system in that two species are competing each other and a third species, which is a predator as well as a host to the second and first competing species. In this multi interaction, the third species helps the first competing species in non-linear Holling type-II functional response manner and consumes the second competitive species in linear manner within the availability of natural resources. The local stability analysis of the dynamical system is discussed using well known method. At the end, the numerical illustrations are given to support the analytical findings. Finally, based on this analytical investigation such kind of ecological models exists in nature, but because of under extreme ecological conditions, the system of species with such multi interactions among themselves, may not survive in nature for long time. If the host species z survives, then both competitive species x, y may survive due to competition among them.

References

[1] R. M. May "Stability and Complexity in Model Ecosystems", Princeton University Press, Princeton, 1973.
[2] R.K. Naji, A.T. Balasim "On the dynamical behavior of three species food web model", Chaos Solitons Fractals, Vol. 34, 2007, PP: 1636–1648.
[3] J.M. Smith "Models in Ecology", Cambridge University Press, Cambridge, 1974.
[4] L. Edelstein-Keshet "Mathematical Models in Biology", SIAM, New York, 2005.
[5] J.N. Kapur "Mathematical Models in Biology and Medicine", Affiliated East-West Private Ltd, New Delhi, 1985.
[6] P. Stiling "Ecology-Theories and Applications", PHI Learning Private Limited, New Delhi, 2009.
[7] A. J. Lotka "Elements of Physical Biology", Williams and Wilkins Company, Baltimore, MD, 1925.
[8] Y. Kuang "Basic properties of mathematical population models", Biomathematics, Vol. 17, 2002, PP: 129–142.
[9] F. Addicott John "Stability properties of two species models of mutualism: simulation studies", Oecologia, Vol. 49 (1), 1981, PP: 42–49.
[10] A. M. Dean "A simple model of mutualism", Am. Nat., Vol. 121(3), 1983, PP: 409–417.
[11] B.S. Goh "Stability in models of mutualism", Am. Nat., Vol.113(2), 1979, PP: 261–275.
[12] C. Wolin "The Population Dynamics of Mutualistic System. In: Boucher, D.H. (ed.) The Biology of Mutualis", Oxford University Press, New York, 1985, PP: 248–269.
[13] Z. Zhibin: "Mutualism or cooperation among competitors promotes coexistence and competitive ability", Ecol. Model, Vol.164, 2003, PP: 271–282.
[14] M. Gyllenberg, P. Yan, Y. Wang "Limit cycles for competitor-competitor-mutualist Lotka-Volterra systems", Physica- D, Vol. 221, 2006, PP: 135–145.
[15] Y. Suresh Kumar, N. Seshagiri Rao, K. Kalyani "An Ecological Model of Mutualism Interaction between Two Species and a Predator", International Journal of Mathematics & Computation, Vol. 28(4), 2017, PP: 48–57.
[16] Y. Wang, H. Wu "A mutualism-competition model characterizing competitors with mutualism at low density", Math. Comput. Model, Vol. 53, 2011, PP: 1654–1663.
[17] N. Seshagiri Rao, K.V.L.N. Acharyulu, K. Kalyani "A Host - Mortal Commensal Species Pair with Limited Resources - A Numerical Study", International Journal of Bio-Science and Bio-Technology, Vol. 7(1), 2015, PP: 133–140.
[18] F. Brauer, C. Castillo-Chavez "Mathematical Models in Population Biology and Epidemiology", Springer, New York, 2001.
[19] M. Braun "Differential Equations and Their Applications-Applied Mathematical Sciences (15)", Springer, New York, 1983.
[20] M.W. Hirsch, S. Smale, R.L Devaney "Differential Equations, Dynamical Systems and an Introduction to Chaos", Elsevier, New York, 2004.
[21] F. Chen, Z. Ma, H. Zhang: Global asymptotical stability of the positive equilibrium of the Lotka-Volterra prey-predator model incorporating a constant number of prey refuges, Nonlinear Anal. Real World Appl., 13 (2012), 2790–2793.
[22] G. Seo, D. L. DeAngelis: A predator-prey model with a Holling type i functional response including a predator mutual interference, J. Nonlinear Sci., 21 (2011), 811–833.

[23] G. Peng, Y. Jiang, C. Li: Bifurcations of a Holling-type II predator-prey system with constant rate harvesting, Int. J. Bifurc. Chaos, 19 (2009), 2499–2514.
[24] S. Yu: Global asymptotic stability of a predator-prey model with modified Leslie-Gower and Holling-type II schemes, Discret. Dyn. Nat. Soc., 2012 (2012), 8.
[25] Y. Huang, F. Chen, L. Zhong: Stability analysis of a prey-predator model with Holling type III response function incorporating a prey refuge, Appl. Math. Comput., 182 (2006), 672–683.
[26] R. K. Naji, R. N. Shalan: The dynamics of Holling type IV prey-predator model with intra-specific competitions, Iraqi J. Sci., 54 (2013), 386–396.
[27] S. Thota: "Prey-Predator Model for Awash National Park, Oromia, Ethiopia and Its Stability Analysis with Simulations", Journal of Science and Sustainable Development, 7 (2) (2019), 15–21.
[28] S. Thota: "A Mathematical Study on a Diseased Prey-Predator Model with Predator Harvesting", Asian Journal of Fuzzy and Applied Mathematics, 8 (2), 16–21.
[29] S. Thota: "On an Ecological Model of Mutualisim Between Two Species With a Mortal Predator", Applications and Applied Mathematics, 15 (2) (2020) 1309–1322.
[30] A. De Rossi, F. Lisa, L. Rubini, A. Zappavigna, E. Venturino,: "A food chain ecoepidemic model: infection at the bottom trophic level", Ecological Complexity 21 (2015) 233–245.
[31] S. Campion, R. Cena, A. Gallo, E. Venturino: "Effects of diseased top predators in food chains", Advances in Research, vol. 2, (12) AIR, Article no. AIR.2014.12.012, 833–845, 2014. doi: 10.9734/AIR/2014/11349.
[32] L. Bramante, S. Maiolo, E. Venturino: "An ecoepidemic food chain with the disease at the intermediate trophic level", Bulletin of the Polytechnic Institute of Iaşi, LX (LXIV), (4), section Mathematics, Theoretical Mechanics, Physics, 9–22, 2014.
[33] L. Assis, M. Banerjee, M. Cecconello, E. Venturino: "Lotka-Volterra Type Predator-Prey Models: Comparison of Hidden and Explicit Resources with a Transmissible Disease in the Predator Species", Applications of Mathematics 63(5), 2018, 1–32·
[34] R. A. de Assis, M. dos S. Cecconello, L. P. Casacci, F. Barbero, L. M. E. de Assis, E. Venturino, S. Bonelli: "A theory and a mathematical model for the evolution of single and multiple host behavior in a parasite-host system (Maculinea-Myrmica)", Ecological Complexity, Vol-34 (2018), 178–187.
[35] Bob W.Kooi, E. Venturino: "Ecoepidemic predator-prey model with feeding satiation, prey herd behavior and abandoned infected prey", Mathematical Biosciences, Vil-274 (2016), 58–72.
[36] C. Viberti, E. Venturino: "An ecosystem with Holling type II response and predators' genetic variability", arXiv:1403.4408 [math.DS] (2014).