Water, Energy, Food and Environment Journal
An International Journal

http://dx.doi.org/10.18576/wefej/020104

An application of two- Stage Stochastic Programming for Water Resources Management Problem in Pentagonal Fuzzy Neutrosophic Environment

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Received: 25 Oct. 2020, Revised: 21 Nov. 2020, Accepted: 25 Dec. 2020.
Published online: 1 Jan. 2021.

Abstract: Water resources management models aims to utilize the availability of water resources so as to meet the demand of many users as possible as can. In this paper, a two- stage stochastic programming for optimizing water resources management problem is introduced in pentagonal fuzzy neutrosophic environment. The problem is considered by incorporating probabilistic seasonal flow and pentagonal fuzzy neutrosophic numbers for all water-allocation target, net benefit, net benefit reduction, and seasonal flow. Based on the score function, the problem under consideration becomes deterministic one and with any linear programming techniques, we obtain the solution. For illustration, an example is given for the sake of the paper.
Keywords: Water resources management; Two- stage stochastic programming; Pentagonal fuzzy neutrosophic numbers; linear programming technique.

1 Introduction

Over the past decades, the water resources allocation through municipal, industrial, and agricultural have been of increasing important (Huang and Chang; 2003; Wang et al. 2003). Among the water users, the competition due to the growing of population shifts, shrinking water availabilities, varying natural conditions, and deterioration of water resources quality (Li and Huang, 2008). Hajkowicz and Collins (2007) reviewed multiple criteria analysis for water resources management and planning. Water resources management has a wider range of case studies based on economic benefits maximizations (for instance, Cai et al., 2002; Cai et al., 2003a; Cai et al., 2003b; Letcher et al., 2004; Ringler and Cai, 2006; Ringler et al. 2006; Ward et al., 2006; Pulido- Velazquez et al., 2006 etc.). Draper et al. (2003) presented an economic- engineering optimization model comprises of California's major water supply system and demonstrates the feasibility of using economic- engineering model for the planning of California's water resources.

In literature, first of all, Zadeh (1965) proposed the philosophy of fuzzy sets. Decision making in a fuzzy environment has an improvement and a great help in the management decision problems (Bellman and Zadeh, 1970). Zimmermann (1978) introduced fuzzy programming and linear programming with multiple objective functions. Later several researchers worked in fuzzy set theory. Dubois and Prade (1980) studied the theory and applications of fuzzy sets and systems. Kaufmann and

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Gupta (1988) studied several fuzzy mathematical models with their applications to engineering and management sciences.

This paper deals with the two- stage stochastic programming for solving water resources management in pentagonal fuzzy neutrosophic environment.

The rest of the paper is: Section 2 introduces some preliminaries needed in this paper. In Section 3, water resources management is formulated as a two- stage stochastic programming. Section 4 introduced a solution procedure for obtaining the pentagonal neutrosophic solution. Section5 presented an example for illustration. Section 6 presents the discussion of the results. Finally, some concluding remarks are reported in section 7.

2 Preliminaries

This section introduces some of basic concepts and results related to fuzzy numbers, pentagonal fuzzy numbers, neutrosophic set, pentagonal fuzzy neutrosophic number and their arithmetic operations.

Definition1.(Zadeh, 1965). A fuzzy set $\widetilde{P}$ defined on the set of real numbers $\mathbb{R}$ is said to be fuzzy numbers if its membership function

$\mu_{\widetilde{P}}(x): \mathbb{R} \to [0,1]$, have the following properties:

1. $\mu_{\widetilde{P}}(x)$ is an upper semi- continuous membership function;
2. $\widetilde{P}$ is convex fuzzy set, i.e., $\mu_{\widetilde{P}}(\delta\, x + (1-\delta)\, y) \geq \min\{\mu_{\widetilde{P}}(x), \mu_{\widetilde{P}}(y)\}$ for all $x, y \in \mathbb{R}; 0 \leq \delta \leq 1$;
3. $\widetilde{P}$ is normal, i.e., $\exists\, x_0 \in \mathbb{R}$ for which $\mu_{\widetilde{P}}(x_0) = 1$;
4. $\text{Supp}\,(\widetilde{P}) = \{x \in \mathbb{R}: \mu_{\widetilde{P}}(x) > 0\}$ is the support of $\widetilde{P}$, and the closure $\text{cl}(\text{Supp}(\widetilde{P}))$ is compact set.

Definition2. (Abbasbandy and Hajjari, 2009; Panda and Pal, 2016 and Chakraborty et al. 2019). A fuzzy number $\widetilde{A}_P = (r, s, t, u, v), r \leq s \leq t \leq u \leq v$, on $\mathbb{R}$ is said to be a pentagonal fuzzy number if its membership function is

$$\mu_{\tilde{A}_P} = \begin{cases} 0, \quad x < r, \\ w_1\left(\frac{x-r}{s-r}\right), \text{for } r \leq x \leq s, \\ 1-(1-w_1)\left(\frac{x-s}{t-s}\right), \text{for } s \leq x \leq t \\ 1, \text{for } x = t, \\ 1-(1-w_2)\left(\frac{u-x}{u-t}\right), \text{for } t \leq x \leq u, \\ w_2\left(\frac{v-x}{v-u}\right), \text{for } u \leq x \leq v, \\ 0, \text{for } x > v. \end{cases}$$

The graphical representation of the pentagonal fuzzy number is illustrated as in the following figure

$\mu_{\tilde{A}_P}$

Definition3. (Smarandache, 1998). A neutrosophic set $\widetilde{B}^N$ of non empty set X is defined as

$\widetilde{B}^N = \{\langle x;\ I_{\widetilde{B}^N}(x), J_{\widetilde{B}^N}(x), V_{\widetilde{B}^N}(x)\rangle : x \in X, I_{\widetilde{B}^N}(x), J_{\widetilde{B}^N}(x), V_{\widetilde{B}^N}(x) \in\ ]0_-, 1^+[\}$,

where $I_{\widetilde{B}^N}(x), J_{\widetilde{B}^N}(x)$and $V_{\widetilde{B}^N}(x)$ are truth membership function, an indeterminacy- membership function, and a falsity- membership function and there is no restriction on the sum of $I_{\widetilde{B}^N}(x), J_{\widetilde{B}^N}(x)$, and $V_{\widetilde{B}^N}(x)$, so $0^- \leq \text{Sup}\{I_{\widetilde{B}^N}(x)\} + \text{Sup}\{J_{\widetilde{B}^N}(x)\} + \text{Sup}\ \{V_{\widetilde{B}^N}(x)\} \leq 3^+$, and $]0^-, 1^+[$ is a nonstandard unit interval.

Definition 4. . A Single- valued neutrosophic set $\widetilde{B}^{SVN}$of a non empty set X is defined as

$\overline{B}^{SVN} = \{\langle x, I_{\widetilde{B}^N}(x), J_{\widetilde{B}^N}(x), V_{\widetilde{B}^N}(x)\rangle : x \in X\}$, where $I_{\widetilde{B}^N}(x), J_{\widetilde{B}^N}(x)$, and $V_{\widetilde{B}^N}(x) \in [0, 1]$ for each $x \in X$ and $0 \leq I_{\widetilde{B}^N}(x) + J_{\widetilde{B}^N}(x) + V_{\widetilde{B}^N}(x) \leq 3$.

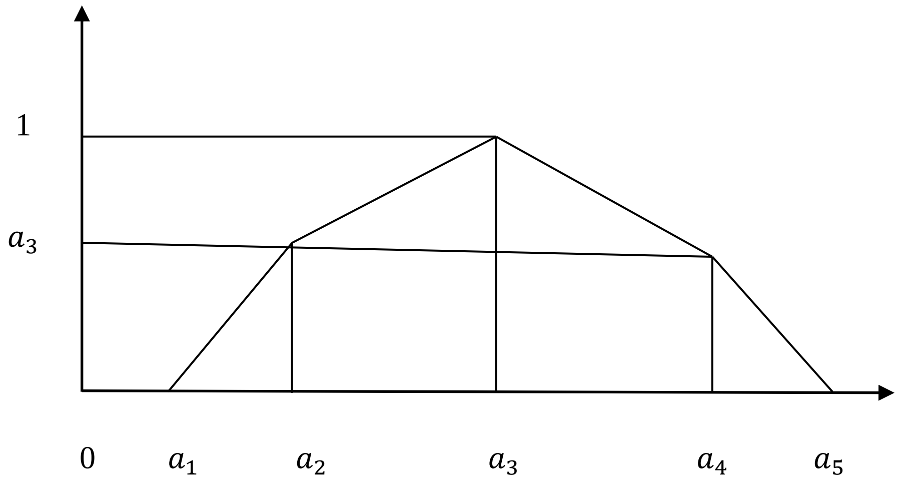

Fig.1: Graphical Representation of pentagonal Fuzzy number (Panda and Pal, 2016; Chakraborty et al. 2019).

Definition5. Let $\tau_{\tilde{q}}, \varphi_{\tilde{q}}, \omega_{\tilde{q}} \in [0,1]$ and $r,s,t,u,v \in \mathbb{R}$ such that $r \le s \le t \le u \le v$. Then a single- valued pentagonal fuzzy neutrosophic (SVPFN), $\tilde{b}^{PN} = \langle (r,s,t,u,v): \tau_{\tilde{q}}, \varphi_{\tilde{q}}, \omega_{\tilde{q}} \rangle$ is a special neutrosophic set on $\mathbb{R}$, whose truth-membership, hesitant- membership, and falsity- membership functions are

$$\mu_{\tilde{q}^{PN}}(x) = \begin{cases} 0, \quad x < r; \\ \tau_{\tilde{q}^{PN}}\left(\frac{1}{2}\frac{1}{(s-r)^2}(x-r)^2\right), \quad r \le x \le s; \\ \tau_{\tilde{q}^{PN}}\left(\frac{1}{2}\frac{1}{(t-s)^2}(x-t)^2+1\right), s \le x \le t; \\ \tau_{\tilde{q}^{PN}}\left(\frac{1}{2}\frac{1}{(u-t)^2}(x-t)^2+1\right), \quad t \le x \le u; \\ \tau_{\tilde{q}^{PN}}\left(\frac{1}{2}\frac{1}{(v-u)^2}(x-v)^2\right), u \le x \le v; \\ 0, \quad x > v. \end{cases}$$

$$\rho_{\tilde{q}^{PN}}(x) = \begin{cases} 0, \quad x < r; \\ \varphi_{\tilde{q}^{PN}}\left(\frac{1}{2}\frac{1}{(s-r)^2}(x-r)^2\right), \quad r \le x \le s; \\ \varphi_{\tilde{q}^{PN}}\left(\frac{1}{2}\frac{1}{(t-s)^2}(x-t)^2+1\right), s \le x \le t; \\ \varphi_{\tilde{q}^{PN}}\left(\frac{1}{2}\frac{1}{(u-t)^2}(x-t)^2+1\right), \quad t \le x \le u; \\ \varphi_{\tilde{q}^{PN}}\left(\frac{1}{2}\frac{1}{(v-u)^2}(x-v)^2\right), u \le x \le v; \\ 0, \quad x > v. \end{cases}$$

$$\sigma_{\tilde{q}^{PN}}(x)=\begin{cases}0, & x<r;\\ \omega_{\tilde{q}^{PN}}\left(\frac{1}{2}\frac{1}{(s-r)^2}(x-r)^2\right), & r\le x\le s;\\ \omega_{\tilde{q}^{PN}}\left(\frac{1}{2}\frac{1}{(t-s)^2}(x-t)^2+1\right), & s\le x\le t;\\ \omega_{\tilde{q}^{PN}}\left(\frac{1}{2}\frac{1}{(u-t)^2}(x-t)^2+1\right), & t\le x\le u;\\ \omega_{\tilde{q}^{PN}}\left(\frac{1}{2}\frac{1}{(v-u)^2}(x-v)^2\right), & u\le x\le v;\\ 0, & x>v.\end{cases}$$

Where $\tau_{\tilde{q}^{PN}}, \varphi_{\tilde{q}^{PN}},$ and $\omega_{\tilde{q}^{PN}}$ denote the maximum truth, minimum- hesitant, and minimum falsity membership degrees, respectively. SVPFN$\tilde{q}^{PN}=\langle(r,s,t,u,v):\tau_{\tilde{q}^{PN}},\varphi_{\tilde{q}^{PN}},\omega_{\tilde{q}^{PN}}\rangle$ may express in ill- defined quantity about q, which is approximately equal to $[s,u]$.

Definition 6. Let$\tilde{q}^{PN}=\langle(r,s,t,u,v):\tau_{\tilde{q}^{PN}},\varphi_{\tilde{q}^{PN}},\omega_{\tilde{q}^{PN}}\rangle$, and $\tilde{d}^{PN}=\langle(r',s',t',u',v'):\tau_{\tilde{d}^{PN}},\varphi_{\tilde{d}^{PN}},\omega_{\tilde{d}^{PN}}\rangle$ be two single- valued PQFNs, The arithematic operations on $\tilde{q}^{PN}$, and $\tilde{d}^{PN}$are

1. $\tilde{q}^{PN}\oplus\tilde{d}^{PN}=\langle(r+r',s+s',t-t',u+u',v+v');\ \tau_{\tilde{q}^{PN}}\wedge\tau_{\tilde{d}^{PN}},\varphi_{\tilde{q}^{PN}}\vee\ \varphi_{\tilde{d}^{PN}},\omega_{\tilde{q}^{PN}}\vee\omega_{\tilde{d}^{PN}}\rangle$,
2. $\tilde{q}^{PN}\ominus\tilde{d}^{PN}=\langle\left(r-v',s-u',t+t',u-s'',v-r'\right);\ \tau_{\tilde{q}^{PN}}\wedge\tau_{\tilde{d}^{PN}},\varphi_{\tilde{q}^{PN}}\vee\ \varphi_{\tilde{d}^{PN}},\omega_{\tilde{q}^{PN}}\vee\omega_{\tilde{d}^{PN}}\rangle$,
3. $\tilde{q}^{PN}\otimes\tilde{d}^{PN}=\frac{1}{5}\gamma_d\langle(r,s,t,u,v);\ \tau_{\tilde{q}^{PN}}\wedge\tau_{\tilde{d}^{PN}},\varphi_{\tilde{q}^{PN}}\vee\ \varphi_{\tilde{d}^{PN}},\omega_{\tilde{q}^{PN}}\vee\omega_{\tilde{d}^{PN}}\rangle,\gamma_d=\frac{1}{3}(r'+\ s'+\ t'+\ u'+v')\left(2+\tau_{\tilde{q}^{PN}}-\varphi_{\tilde{q}^{PN}}\right)\neq 0$,
4. $\tilde{q}^{N}\oslash\tilde{d}^{N}=\frac{5}{\gamma_d}\langle(r,s,t,u,v);\ \tau_{\tilde{q}^{PN}}\wedge\tau_{\tilde{d}^{PN}},\varphi_{\tilde{q}^{PN}}\vee\ \varphi_{\tilde{d}^{PN}},\omega_{\tilde{q}^{PN}}\vee\omega_{\tilde{d}^{PN}}\rangle,\gamma_d\neq 0$,
5. $k\tilde{d}^{PN}=f(x)=\begin{cases}\langle(kr,ks,kt,ku,kv):\tau_{\tilde{q}^{PN}},\varphi_{\tilde{q}^{PN}},\omega_{\tilde{q}^{PN}}\rangle,k>0,\\ \langle(kv,ku,kt,ks,kr);\ \tau_{\tau_{\tilde{d}^{PN}}},\ \varphi_{\tau_{\tilde{d}^{PN}}},\omega_{\tau_{\tilde{d}^{PN}}}\rangle,k<0,\end{cases}$
6. $\tilde{d}^{PN^{-1}}=\langle\left(\frac{1}{v},\frac{1}{u},\frac{1}{t},\frac{1}{s},\frac{1}{r}\right);\ \tau_{\tau_{\tilde{d}^{PN}}},\ \varphi_{\tau_{\tilde{d}^{PN}}},\omega_{\tau_{\tilde{d}^{PN}}}\rangle,\tilde{d}^{PN}\neq 0.$

Definition 7. (Das, 2020). Let $\tilde{q}^{PN}=\langle(r,s,t,u,v):\tau_{\tilde{q}^{PN}},\varphi_{\tilde{q}^{PN}},\omega_{\tilde{q}^{PN}}\rangle$ be a single- valued pentagonal fuzzy neutrosophic numbers, then

1. Accuracy function $AC(\tilde{q}^{PN})=\left(\frac{1}{15}\right)(r+\ s+t+u+v)*[2+\tau_{\tilde{q}^{PN}}-\varphi_{\tilde{q}^{PN}}]$.

2. Score function $SC(\tilde{q}^{PN})=\left(\frac{1}{15}\right)(r+\ s+t+u+v)*\left[2+\tau_{\tilde{q}^{PN}}-\varphi_{\tilde{q}^{PN}}-\omega_{\tilde{q}^{PN}}\right]$.

Definition 8. (Thamariselvi and Santhi, 2016). The order relations between $\tilde{q}^{PN}$and $\tilde{q}^{PN}$based on $SC(\tilde{q}^{NP})$ and $AC\left(\tilde{d}^{NP}\right)$ are defined as

1. If $SC(\tilde{q}^{PN})<SC\left(\tilde{d}^{PN}\right)$, then $q<d$
2. If $SC(\tilde{q}^{PN})=SC\left(\tilde{d}^{PN}\right)$, then $q=d$,
3. If $AC(\tilde{q}^{PN})<AC\left(\tilde{d}^{PN}\right)$, then $q<d$
4. If $AC(\tilde{q}^{PN})>AC\left(\tilde{d}^{PN}\right)$, then $q>d$,
5. If $AC(\tilde{q}^{PN})=AC\left(\tilde{d}^{PN}\right)$, then $q=d$.

3 Problem Statements

Consider the two- stage stochastic programming model for water resources management (Wang and Huang, 2011) as

$$\max \tilde{F}^{PN} = \sum_{j=1}^{n} \widetilde{NB}_j^{PN} \tilde{T}_j^{PN} \ominus E\left(\sum_{j=1}^{n} \tilde{C}_j^{PN} S_{jQ}\right) \quad (1)$$

Subject to

$$\sum_{j=1}^{n} \left(\tilde{T}_j^{PN} - S_{jQ}\right) (1 \oplus \tilde{\gamma}^{PN}) \precsim \tilde{Q}^{PN} \quad (2)$$

(Water availability constraints)

$$S_{jQ} \leq \tilde{T}_j^{PN} \precsim T_{jmax};\ \forall j \quad (3)$$

(Water- allocation target constraints)

$$S_{jQ} \geq \mathbf{0};\ \forall \mathbf{j} \quad (4)$$

(Non- negative and technical constraints).

Where

F: System benefit ($),

$\widetilde{NB}_j^{PN}$: Net benefit to user $\boldsymbol{j}$ per $\boldsymbol{m^3}$ of water allocation $(\$/\mathbf{m^3})$ (First- stage revenue parameters),

$\tilde{T}_j^{PN}$ Allocation target for water that is promised to user $\mathbf{j}$ $(\boldsymbol{m^3})$

(First- stage decision variables),

$\mathbf{E}[.]$: Expected value of a random variable,

$\tilde{C}_j^{PN}$: Loss to user $\mathbf{j}$ per $\mathbf{m^3}$ of water not delivered, $\tilde{C}_j^{PN} \gtrsim \widetilde{NB}_j^{PN}$ $(\$/\boldsymbol{m^3})$

(Second- stage cost parameters)

S_{jQ}: Shortage of water to user when the seasonal flow is $\tilde{Q}^{PN}$ $(\mathbf{m^3})$

(Second- stage decision variables)

$\tilde{Q}^{PN}$: Total amount of seasonal flow $(\boldsymbol{m^3})$ (random variables),

$\tilde{\gamma}^{PN}$: Rate of water loss during transportation,

T_{jmax}: Maximum allowable allocation amount for user $\mathbf{j}(\mathbf{m^3})$,

$\mathbf{n}$: Total number of water users,

$\mathbf{j}$: Water users, $\mathbf{j} = \mathbf{1}, \mathbf{2}, \mathbf{3}$:

$\mathbf{j} = \mathbf{1}$: Municipality;

$\mathbf{j} = \mathbf{2}$: Industrial user;

$\mathbf{j} = \mathbf{3}$: Agricultural sector.

In order to solve the problem (1)- (4) through linear programming technique, Huang and Loucks (2000) reformulated the problem as

$$\max \tilde{F}^{PN} = \sum_{j=1}^{n} \widetilde{NB}_j^{PN} \tilde{T}_j^{PN} \ominus \sum_{j=1}^{n} \sum_{j=1}^{m} p_i \tilde{C}_j^{PN} S_{ji}$$

Subject to (5)

$\sum_{j=1}^{n}\left(\tilde{T}_j^{PN} - S_{ji}\right)(1+\tilde{\gamma}^{PN}) \precsim \tilde{q}_i^{PN};\forall i,$

$S_{ji} \le \tilde{T}_j^{PN} \precsim T_{jmax};\ \forall j, i,$
$S_{ji} \ge \mathbf{0};\ \forall \mathbf{j}, \mathbf{i}.$

Where, S_{ji} is the amount by which the target of water allocation $\left(\tilde{T}_j^{PN}\right)$ is not met at the seasonal flow is $\left(\tilde{q}_i^{PN}\right)$ occurs with probability p_i.

For the model (5), let us define:

$\widetilde{NB}_j^{PN} = \left(\mathbf{NB_j^{PN1}}, \mathbf{NB_j^{PN2}}, \mathbf{NB_j^{PN3}}, \mathbf{NB_j^{PN4}}, \mathbf{NB_j^{PN5}};\ \boldsymbol{\tau_j^{PN}}, \boldsymbol{\varphi_j^{PN}}, \boldsymbol{\omega_j^{PN}}\right),$

$\tilde{C}_j^{PN} = \left(\mathbf{C_j^{PN1}}, \mathbf{C_j^{PN2}}, \mathbf{C_j^{PN3}}, \mathbf{C_j^{PN4}}, \mathbf{C_j^{PN5}};\ \tau_j^{PN}, \varphi_j^{PN}, \omega_j^{PN}\right)$

$\tilde{T}_j^{PN} = \left(\mathbf{T_j^{PN1}}, \mathbf{T_j^{PN2}}, \mathbf{T_j^{PN3}}, \mathbf{T_j^{PN4}}, \mathbf{T_j^{PN5}};\ \tau_j^{PN}, \varphi_j^{PN}, \omega_j^{PN}\right),$

$\tilde{q}_i^{PN} = \left(\mathbf{q_i^{PN1}}, \mathbf{q_i^{PN1}}, \mathbf{q_i^{PN1}}, \mathbf{q_i^{PN1}}, \mathbf{q_i^{PN1}};\ \tau_i^{PN}, \varphi_i^{PN}, \omega_i^{PN}\right),$

$\tilde{\gamma}^{PN} = (\boldsymbol{\gamma^{PN1}}, \boldsymbol{\gamma^{PN2}}, \boldsymbol{\gamma^{PN3}}, \boldsymbol{\gamma^{PN4}}, \boldsymbol{\gamma^{PN4}};\ \tau^{PN}, \varphi^{PN}, \omega^{PN}).$

6 Solution procedure

The steps of the proposed solution procedure are as:

Step1: Consider the pentagonal neutrosophic coefficients and decision variables, problem (5) can be rewritten as

$$\max F = \sum_{j=1}^{n}\left\{\begin{matrix}\left(\mathbf{NB_j^{PN1}}, \mathbf{NB_j^{PN2}}, \mathbf{NB_j^{PN3}}, \mathbf{NB_j^{PN4}}, \mathbf{NB_j^{PN5}};\ \tau_j^{PN}, \varphi_j^{PN}, \omega_j^{PN}\right) \otimes \\ \left(\mathbf{T_j^{PN1}}, \mathbf{T_j^{PN2}}, \mathbf{T_j^{PN3}}, \mathbf{T_j^{PN4}}, \mathbf{T_j^{PN5}};\ \tau_j^{PN}, \varphi_j^{PN}, \omega_j^{PN}\right)\end{matrix}\right\} \ominus \sum_{j=1}^{n}\sum_{j=1}^{m} p_i\left(\mathbf{C_j^{PN1}}, \mathbf{C_j^{PN2}}, \mathbf{C_j^{PN3}}, \mathbf{C_j^{PN4}}, \mathbf{C_j^{PN5}};\ \tau_j^{PN}, \varphi_j^{PN}, \omega_j^{PN}\right)S_{ji}$$

Subject to (6)

$$\sum_{j=1}^{n}\left\{\begin{matrix}\left\{\left(\mathbf{T_j^{PN1}}, \mathbf{T_j^{PN2}}, \mathbf{T_j^{PN3}}, \mathbf{T_j^{PN4}}, \mathbf{T_j^{PN5}};\ \tau_j^{PN}, \varphi_j^{PN}, \omega_j^{PN}\right) - S_{ji}\right\} \\ \otimes\left(1 \oplus (\boldsymbol{\gamma^{PN1}}, \boldsymbol{\gamma^{PN2}}, \boldsymbol{\gamma^{PN3}}, \boldsymbol{\gamma^{PN4}}, \boldsymbol{\gamma^{PN4}};\ \tau^{PN}, \varphi^{PN}, \omega^{PN})\right)\end{matrix}\right\} \le \left(\mathbf{q_i^{PN1}}, \mathbf{q_i^{PN1}}, \mathbf{q_i^{PN1}}, \mathbf{q_i^{PN1}}, \mathbf{q_i^{PN1}};\ \tau_i^{PN}, \varphi_i^{PN}, \omega_i^{PN}\right);\forall i,$$

$$S_{ji} \le \left(\mathbf{T_j^{PN1}}, \mathbf{T_j^{PN2}}, \mathbf{T_j^{PN3}}, \mathbf{T_j^{PN4}}, \mathbf{T_j^{PN5}};\ \tau_j^{PN}, \varphi_j^{PN}, \omega_j^{PN}\right) \le T_{jmax};\ \forall j, i,$$
$$S_{ji} \ge \mathbf{0};\ \forall \mathbf{j}, \mathbf{i}.$$

Step2: Applying the score function definition, problem (6) becomes

$$\max F = \Re\left\{\sum_{j=1}^{n}\left\{\begin{matrix}\left(\mathbf{NB_j^{PN1}}, \mathbf{NB_j^{PN2}}, \mathbf{NB_j^{PN3}}, \mathbf{NB_j^{PN4}}, \mathbf{NB_j^{PN5}};\ \tau_j^{PN}, \varphi_j^{PN}, \omega_j^{PN}\right) \otimes \\ \left(\mathbf{T_j^{PN1}}, \mathbf{T_j^{PN2}}, \mathbf{T_j^{PN3}}, \mathbf{T_j^{PN4}}, \mathbf{T_j^{PN5}};\ \tau_j^{PN}, \varphi_j^{PN}, \omega_j^{PN}\right)\end{matrix}\right\}\right\} \ominus \Re\left\{\sum_{j=1}^{n}\sum_{j=1}^{m} p_i\left(\mathbf{C_j^{PN1}}, \mathbf{C_j^{PN2}}, \mathbf{C_j^{PN3}}, \mathbf{C_j^{PN4}}, \mathbf{C_j^{PN5}};\ \tau_j^{PN}, \varphi_j^{PN}, \omega_j^{PN}\right)S_{ji}\right\}$$

Subject to (7)

$$\Re\left(\sum_{j=1}^{n}\left\{\begin{matrix}\left\{\left(\mathbf{T_j^{PN1}}, \mathbf{T_j^{PN2}}, \mathbf{T_j^{PN3}}, \mathbf{T_j^{PN4}}, \mathbf{T_j^{PN5}};\ \tau_j^{PN}, \varphi_j^{PN}, \omega_j^{PN}\right) - S_{ji}\right\} \\ \otimes\left(1 \oplus (\boldsymbol{\gamma^{PN1}}, \boldsymbol{\gamma^{PN2}}, \boldsymbol{\gamma^{PN3}}, \boldsymbol{\gamma^{PN4}}, \boldsymbol{\gamma^{PN4}};\ \tau^{PN}, \varphi^{PN}, \omega^{PN})\right)\end{matrix}\right\}\right)$$

$$\le \Re\left(\mathbf{q_i^{PN1}}, \mathbf{q_i^{PN1}}, \mathbf{q_i^{PN1}}, \mathbf{q_i^{PN1}}, \mathbf{q_i^{PN1}}; \tau_i^{PN}, \varphi_i^{PN}, \omega_i^{PN}\right); \forall i,$$

$$S_{ji} \le \Re\left(\mathbf{T_j^{PN1}}, \mathbf{T_j^{PN2}}, \mathbf{T_j^{PN3}}, \mathbf{T_j^{PN4}}, \mathbf{T_j^{PN5}}; \tau_j^{PN}, \varphi_j^{PN}, \omega_j^{PN}\right) \le T_{jmax}; \forall j, i,$$
$$S_{ji} \ge \mathbf{0}; \forall \mathbf{j}, \mathbf{i}.$$

Step3: With the help of MATALB, the solution of problem (7) is obtained.

Step4: Referring to problem (5), to get the pentagonal neutrosophic solution.

7 Numerical example

Consider problem (6) with the following data
Table1 illustrated the Economic data ($\$/\mathbf{m^3}$) and seasonal flows (in $\mathbf{10^6\ m^3}$) in the case of different probabilities

Table 1: Economic data ($\$/\mathbf{m^3}$) and seasonal flows (in $\mathbf{10^6\ m^3}$).

Activity	User		
	$j=1$	$j=2$	$j=3$
Maximum allowable allocation (T_{jmax})	$\mathbf{8.0}$	$\mathbf{8.0}$	$\mathbf{8.0}$
Water- allocation target($\widetilde{T}_j^{PN}$)	For $j=1$: $\widetilde{T}_1^{PN} = \langle(1.5, 1.7, 2, 2.2, 2.5); 1, 0, 0\rangle$	For $j=2$: $\widetilde{T}_2^{PN} = \langle(2, 2.5, 3, 3.5, 4); 1, 0, 0\rangle$	For $j=3$: $\widetilde{T}_3^{PN} = \langle(3.5, 4, 4.5, 5, 6.5); 1, 0, 0\rangle$
Net benefit when water demand is satisfied ($\widetilde{NB}_j^{PN}$)	For $j=1$: $\widetilde{NB}_1^{PN} = \langle(85, 90, 105, 110, 115); 1, 0, 0\rangle$	For $j=2$: $\widetilde{NB}_2^{PN} = \langle(40, 45, 60, 65, 75); 1, 0, 0\rangle$	For $j=3$: $\widetilde{NB}_3^{PN} = \langle(27, 29, 31, 32, 33); 1, 0, 0\rangle$
Reduction of net benefit when demand is not delivered ($\tilde{C}_j^{PN}$)	For $j=1$: $\tilde{C}_1^{PN} = \langle(215, 225, 275, 280, 285); 1, 0, 0\rangle$	For $j=2$: $\tilde{C}_2^{PN} = \langle(55, 60, 85, 90, 95); 1, 0, 0\rangle$	For $j=3$: $\tilde{C}_3^{PN} = \langle(45, 55, 65, 70, 75); 1, 0, 0\rangle$

Flow level	Probability (%)	Seasonal flow ($\tilde{q}_i^{PN}$)
Low ($j=1$)	0.2	$\langle(3.3, 3.5, 4.3, 4.5, 4.7); 1, 0, 0\rangle$
Medium ($j=1$)	0.6	$\langle(7, 9, 11, 12, 13); 1, 0, 0\rangle$
High ($j=1$)	0.2	$\langle(14, 16, 18, 19, 20); 1, 0, 0\rangle$
Water loss ($\tilde{\gamma}^{PN}$)		$\langle(0.10, 0.2, 0.3, 0.35, 0.4); 1, 0, 0\rangle$

Using the ranking function definition, The issue of the problem is converted into I crisp problem. The availability of the model is as
Table2. Defuzzified Economic data ($\$/\mathbf{m^3}$) and seasonal flows (in $\mathbf{10^6\ m^3}$) in the case of different probabilities

Table 2: Defuzzified Economic data ($\$/\mathbf{m^3}$) and seasonal flows (in $\mathbf{10^6\ m^3}$).

Activity	User		
	$j = 1$	$j = 2$	$j = 3$
Maximum allowable allocation (T_{jmax})	$\mathbf{8.0}$	$\mathbf{8.0}$	$\mathbf{8.0}$
Water- allocation target(T_j)	For $j = 1$: $T_1 = 1.98$	For $j = 2$: $T_2 = 3$	For $j = 3$: $T_3 = 4.7$
Net benefit when water demand is satisfied (NB_j)	For $j = 1$: $NB_1 = 101$	For $j = 2$: $NB_2 = 57$	For $j = 3$: $NB_3 = 30.4$
Reduction of net benefit when demand is not delivered (C_j)	For $j = 1$: $C_1 = 205.6$	For $j = 2$: $C_2 = 77$	For $j = 3$: $C_3 = 62$

Flow level	Probability (%)	Seasonal flow (q_i)
Low ($i = 1$)	0.2	4.06
Medium ($i = 2$)	0.6	10.4
High ($i = 3$)	0.2	17.4
Water loss (γ)		0.27

Problem (7) becomes

$$\max F = 513.89 - \begin{pmatrix} 41.12S_{11} + 15.4S_{21} + 12.4S_{31} + 123.36S_{12} + 46.2S_{22} + \\ 37.2S_{32} + 41.12S_{13} + 15.4S_{23} + 12.4S_{33} \end{pmatrix}$$

Subject to (8)

$$(9.68 - S_{11} - S_{21} - S_{31})(1 + \gamma) \leq 4.06,$$

$$(9.68 - S_{12} - S_{22} - S_{32})(1 + \gamma) \leq 10.4,$$

$$(9.68 - S_{13} - S_{23} - S_{33})(1 + \gamma) \leq 17.4,$$

$$0.8 \leq S_{11} \leq 1.98,$$

$$0.8 \leq S_{12} \leq 1.98,$$

$$0.8 \leq S_{13} \leq 1.98,$$

$$0.8 \le S_{21} \le 3,$$

$$0.8 \le S_{22} \le 3,$$

$$0.8 \le S_{23} \le 3,$$

$$0.8 \le S_{31} \le 4.7,$$

$$0.8 \le S_{32} \le 4.7,$$

$$0.8 \le S_{33} \le 4.7,$$

$$\mathrm{S_{ji}} \ge \mathbf{0};\ \forall \mathbf{j}, \mathbf{i}.$$

The solution is
$\boldsymbol{S_{11} = S_{21} = S_{12} = S_{22} = S_{32} = S_{13} = S_{23} = S_{33} = 0.8 \times 10^6, S_{31} = 4.02 \times 10^6, \gamma = 0}$, and $\boldsymbol{F = 198.2820 \times 10^6}$. In pentagonal fuzzy neutrosophic, we deduce that $\boldsymbol{F} = \langle(-45.4, 15.1, 142.8, 220.36, 297.84)\rangle \times \mathbf{10^6}$.

4 Discussions of the Results

In the optimum solution, the total maximum benefit is greater than $15.1 \times 10^6$and less than 220.36×10^6. The level is 100% as the total maximum lies between 15.1×10^6 and 220.36×10^6, the degree of truthfulness is

$$\mu_{\tilde{q}^{PN}}(x) = \begin{cases} 0, \quad x < r; \\ 1\left(\frac{1}{2}\frac{1}{(15.1 + -45.4)^2}(x + 45.4)^2\right), \quad -45.4 \le x \le 15.1; \\ 1\left(\frac{1}{2}\frac{1}{(142.8 - 15.1)^2}(x - 142.8)^2 + 1\right), s \le x \le 142.8; \\ 1\left(\frac{1}{2}\frac{1}{(220.36 - 142.8)^2}(x - 142.8)^2 + 1\right), \quad 142.8 \le x \le 220.36; \\ 1\left(\frac{1}{2}\frac{1}{(297.84 - 220.36)^2}(x - 297.84)^2\right), \quad 220.36 \le x \le 297.84; \\ 0, \quad x > v. \end{cases}$$

Hence, the decision maker concludes that the total benefit lies in between $\$15.1 \times 10^6$ and $\$220.36 \times 10^6$. It is clear that the obtained results is better comparing with the one obtained with Huang and Louck, (2000).

5 Conclusions

In this paper, pentagonal fuzzy neutrosophic water management problem has solved, where we developed the score function and using the MATLAB for obtaining the solution. For point of view this method is very easier for real- world problems, also it will be extended to many others real world problems as transportation problem, assignment problem, job scheduling problem as so on.

Data Availability

The data used to support the findings of the study are available for the corresponding author upon request.

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