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Analytical-Numerical Solutions of Photo-Thermal Interactions in Semiconductor Materials

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Abstract: Analytical and numerical solutions are two basic tools in the study of photothermal interactions problems in semiconductor medium. This paper is devoted to a study of the photothermal interaction in semiconductor media in the context of the coupled photo-thermal theory. The governing relations are expressed in Laplace transforms domain and solved in the domain by the eigenvalue scheme. The numerical solution is obtained by using the implicit finite difference method (IFDM), the studied fields are obtained numerically and presented graphically. A comparison between the numerical solutions and the analytical solution are obtained. It is found that the implicit finite difference method (IFDM) is applicable, simple and efficient for such problems.

Keywords: Finite difference method; Laplace transforms; Semi-conductor medium; eigenvalue approach.

Nomenclature

$T = T^* - T_o, T^*$	the variations of temperature
T_o	the reference temperature
t	the time
u_i	the displacement components
ρ	the density of material
c_e	the specific heat at constant strain,
τ	the lifetime of photo-generated carrier,
$N = n - n_o, n_o$	the carrier concentration at equilibrium,
$\gamma_n = (3\lambda + 2\mu)d_n, d_n$	the electronic deformation coefficient,
$k = \frac{\partial n_o}{\partial T}$	the coupling parameter of thermal activation
K	the thermal conductivity
$\gamma_t = (3\lambda + 2\mu)\alpha_t, \alpha_t$	the linear thermal expansion coefficients
σ_{ij}	the components of stresses,
D_e	the carrier diffusion coefficient,
λ, μ	the Lame's constants,

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T_1	the constant temperature
t_f	the final value of time
x_f	the final value of length
s_b	the speed of recombination on the surface
Ω	the exponent of the decayed heat flux

1 Introduction

The models of bodies explained the properties of the internal structure of medium when used the secondly law of thermodynamic with the development of semi-conductor integrated circuit technology and solid-state sensors technology have been widely used in several fields. The significance of semiconductor material is due to its recent uses in several interesting application, essentially in modern technology upon new energy alternative. Previously, micro-mechanical structures of the thermoelasticity and plasma field are analyzed theoretical and experimental as in Todorovic et al. [1-3]. In these investigates, the theoretically analysis to depict these two-phenomena that give information about the attributes of carriers recombination and transports in the semiconductors materials. Abd-Alla et al. [4] studied the solutions of the transient coupled thermoelastic of an annular fins by the implicit finite-difference technique. Mukhopadhyay and Kumar [5] applied the finite difference technique to study the generalized thermoelasticity problem of annular cylinders with variable material properties. Abd-Alla et al. [6] studied the effects of nonhomogeneous in an isotropic cylinder under magnetic field. Patra et al. [7] used the finite difference technique to study the computational model on thermoelastic analysis with the magnetic field in a rotating cylinder. Abd-Alla et al. [8] studied the effects in a thermoelastic annular cylinder using the finite difference method. Lotfy et al. [9] duscussed the responses of Thomson and electro-magnetic influnces of semi-conductor material casued by laser pulses under photo-thermoelastic excitation. Abbas et al. [10] presnted th solutions of photo-thermal interaction in a semiconducting materials with cylindrical hole and variable thermal conductivity. Alzahrani [11] investigated the effects of variable thermal conductivity in semi-conductor materiale. Lotfy et al. [12] investigated the influences of variable thermal conductivity in semiconductors mediums with cavity under fractional-order magneto-photothermal models. Lotfy et al. [13] studied the electro-magnetic and Thomson effects through the photo-thermal transport process of semiconductor material. Hobiny and Abbas [14] discussed the photothermoelasticity interaction in a two-dimension semiconducting plane under Green-Naghdi theory. Alzahrani and Abbas [15] discussed the photothermoelasticity interactions in a two-dimentionn semiconductors mediums without energy dissipation.
Several authors [16-30] used the venous thermoelastic theories to get the solutions of several problems.

The present work is devoted to study the photothermal interaction in a semiconductor material by using the numerical and analytical methods. Numerical outcomes for the displacement, the temperature, the carrier density and the stress distributions are presented graphically. Finally, the accuracy of the finite difference method was validated by the comparing between the numerical and the analytical solutions for all physical fields.

2 Basic Equations

The basic equations in an isotropic semi-conductor material in the absence the thermal sources and the body force are taken as in [31-33]:

$$\mu u_{i,jj} + (\lambda+\mu)u_{j,ij} - \gamma_n N_{,i} - \gamma_t T_{,i} = \rho \frac{\partial^2 u_i}{\partial t^2} \tag{1}$$

$$D_e N_{,jj} - \frac{N}{\tau} + \frac{k}{\tau} T = \frac{\partial N}{\partial t}, \tag{2}$$

$$\left(K T_{,j}\right)_j + \frac{E_g}{\tau} N - \gamma_t T_o \frac{\partial u_{j,j}}{\partial t} = \rho c_e \frac{\partial T}{\partial t}. \tag{3}$$

$$\sigma_{ij} = \left(\lambda u_{k,k} - \gamma_t T - \gamma_n N\right)\delta_{ij} + \mu\left(u_{i,j} + u_{j,i}\right), \tag{4}$$

Let us consider an unbounded isotropic semiconductor medium, whose state can be expressed as a function of the spatial variable x and time t, hence the relations (1)-(4) can be given by:

$$(\lambda+2\mu)\frac{\partial^2 u}{\partial x^2} - \gamma_t \frac{\partial T}{\partial x} - \gamma_n \frac{\partial N}{\partial x} = \rho \frac{\partial^2 u}{\partial t^2}, \tag{5}$$

$$D_e \frac{\partial^2 N}{\partial x^2} - \frac{N}{\tau} + \frac{k}{\tau} T = \frac{\partial N}{\partial t}, \tag{6}$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{E_g}{\tau} N - \gamma_t T_o \frac{\partial^2 u}{\partial t \partial x} = \rho c_e \frac{\partial \phi}{\partial t}. \tag{7}$$

$$\sigma_{xx} = (\lambda+2\mu)\frac{\partial u}{\partial x} - \gamma_t T - \gamma_n N, \tag{8}$$

3 Applications

The problem initial conditions are defined as

$$u(x,0) = 0, \frac{\partial u(x,0)}{\partial t} = 0, T(x,0) = 0, \frac{\partial T(x,0)}{\partial t} = 0, N(x,0) = 0, \frac{\partial N(x,0)}{\partial t} = 0, \tag{9}$$

While the problem boundary conditions are given as

$$u(0,t) = 0, \quad (10)$$

$$T(0,t) = T_1 e^{-\Omega t}, \quad (11)$$

$$D_e \frac{\partial N(x,t)}{\partial x}\Big|_{x=0} = s_b N(x,t) \ , \quad (12)$$

Now, for appropriatenes, the dimensionaless physical fields can be given by

$$N' = \frac{N}{n_o}, T' = \frac{T}{T_o}, \sigma'_{xx} = \frac{\sigma_{xx}}{\lambda+2\mu}, (x', u') = \eta c(x, u), (t', \tau') = \eta c^2(t, \tau), T'_1 = \frac{T_1}{T_o}, \Omega' = \frac{\Omega}{\eta c^2}, \quad (13)$$

where $c^2 = \frac{\lambda+2\mu}{\rho}$ and $\eta = \frac{\rho c_e}{K}$.

By using the variables of nondimensional forms (13), the basic relations with the neglecting of the stares can be written by:

$$\frac{\partial^2 u}{\partial x^2} - r_1\frac{\partial N}{\partial x} - r_2\frac{\partial T}{\partial x} = \frac{\partial^2 u}{\partial t^2}, \quad (14)$$

$$\frac{\partial^2 N}{\partial x^2} - \frac{r_3}{\tau}N + \frac{\beta}{\tau}T = r_3\frac{\partial N}{\partial t}, \quad (15)$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{r_4}{\tau}N - r_5\frac{\partial^2 u}{\partial t\partial x} = \frac{\partial T}{\partial t}. \quad (16)$$

$$\sigma_{xx} = \frac{\partial u}{\partial x} - r_1 N - r_2 T, \quad (17)$$

$$u(0,t) = 0, \frac{\partial N(x,t)}{\partial x}\Big|_{x=0} = x_6 N(0,t), T(0,t) = T_1 e^{-\Omega t}, \quad (18)$$

where $r_1 = \frac{n_o\gamma_n}{\lambda+2\mu}$, $r_2 = \frac{T_o\gamma_t}{\lambda+2\mu}$, $r_3 = \frac{1}{\eta D_e}$, $\beta = \frac{kT_o}{n_o\eta^2c^2D_e}$, $r_4 = \frac{n_oE_g}{\rho c_e T_o}$, $r_5 = \frac{\gamma t}{\rho c_e}$ and $r_6 = \frac{s_o}{\eta c D_e}$.

4 Numerical Method

The basic relations obtained are linear partial differential equations. For the solutions problem, the implicit finite difference method (IFDM) is used. The solutions domain $0 \le x \le x_f$, $0 \le t \le t_f$, are replaced by grids described by the set of nodes points (x_m, t_s), in which $x_m = mh$, $m = 0,1,2, \ldots\ldots, M$ and $t_s = sk, s = 0,1,2, \ldots\ldots, S$. Therefore, $k = \frac{t_f}{S}, h = \frac{x_f}{M}$ are taken as the time step and mess width respectively. For the time derivatives and the space derivatives, the derivatives are replaced the central differences. Thus, the approximations of finite difference method for the system of partial differential equations with respect to the independent variables:

$$\frac{\partial f}{\partial t} = \frac{f_m^{s+1} - f_m^{s-1}}{2k} + o(k^2), \frac{\partial^2 f}{\partial t^2} = \frac{f_m^{s+1} - 2f_m^s + f_m^{s-1}}{k^2} + o(k^2), \quad (19)$$

$$\frac{\partial f}{\partial x} = \frac{f_{m+1}^{s+1} - f_{m-1}^{s+1}}{2h} + o(h^2), \frac{\partial^2 f}{\partial x^2} = \frac{f_{m+1}^{s+1} - 2f_m^{s+1} + f_{m-1}^{s+1}}{h^2} + o(h^2), \quad (20)$$

The equations (14), (15), (16) and (17) are then replaced by the implicit finite difference equations by

$$\frac{u_{m+1}^{s+1} - 2u_m^{s+1} + u_{m-1}^{s+1}}{h^2} - r_1\frac{N_{m+1}^{s+1} - N_{m-1}^{s+1}}{2h} - r_2\frac{T_{m+1}^{s+1} - T_{m-1}^{s+1}}{2h} - \frac{u_m^{s+1} - 2u_m^s + u_m^{s-1}}{k^2} = 0, \quad (21)$$

$$\frac{N_{m+1}^{s+1} - 2N_m^{s+1} + N_{m-1}^{s+1}}{h^2} - \frac{r_3}{\tau}N_m^{s+1} + \frac{\beta}{\tau}T_m^{s+1} - r_3\frac{N_m^{s+1} - N_m^{s-1}}{2k} = 0, \quad (22)$$

$$\frac{T_{m+1}^{s+1} - 2T_m^{s+1} + T_{m-1}^{s+1}}{h^2} + \frac{r_4}{\tau}N_m^{s+1} - r_5\frac{f_{m+1}^{s+1} - f_{m+1}^{s-1} + f_{m-1}^{s+1} - f_{m-1}^{s-1}}{4hk} - \frac{T_m^{s+1} - T_m^{s-1}}{2k} = 0. \quad (23)$$

$$\sigma_{xx} = \frac{u_{m+1}^s - u_{m-1}^s}{2h} - r_1 N_m^s - r_2 T_m^s. \quad (24)$$

5 Analytical Method

Applying the Laplace transforms for relations (14)-(18) are defined by the formula

$$\bar{f}(x,p) = L[f(x,t)] = \int_0^\infty f(x,t)e^{-pt}dt\,, p > 0. \quad (25)$$

Hence, the following system are obtained

$$\frac{d^2\bar{u}}{dx^2} = p^2\bar{u} + r_1\frac{d\bar{N}}{dx} + r_2\frac{d\bar{T}}{dx}, \quad (26)$$

$$\frac{d^2\bar{N}}{dx^2} = r_3\left(p + \frac{1}{\tau}\right)\bar{N} - \frac{\beta}{\tau}\bar{T}, \quad (27)$$

$$\frac{d^2\bar{T}}{dx^2} = pT - \frac{x_4}{\tau}\bar{N} + r_5 p\frac{d\bar{u}}{dx}. \quad (28)$$

$$\bar{\sigma}_{xx} = \frac{d\bar{u}}{dx} - r_1\bar{N} - r_2\bar{T}, \quad (29)$$

$$\bar{u}(0,t) = 0, \ \frac{d\bar{N}(x,t)}{dx}\Big|_{x=0} = r_6\bar{N}(0,t), \bar{T}(0,t) = \frac{1}{p+\Omega}, \quad (30)$$

Now, we can use the eigenvalues method proposed [34-39] to get the solution of coupled differential equations (26), (27) and (28) with the boundary conditions (30). Hence, the matrices-vectors can be expressed as

$$\frac{dV}{dx} = AV, \quad (31)$$

where $V = \begin{bmatrix}\bar{u} & \bar{N} & \bar{T} & \frac{d\bar{u}}{dx} & \frac{d\bar{N}}{dx} & \frac{d\bar{T}}{dx}\end{bmatrix}^T$ and

$$A=\begin{bmatrix}0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\\a_{41}&0&0&0&a_{45}&a_{46}\\0&a_{52}&a_{53}&0&0&0\\0&a_{62}&a_{63}&a_{64}&0&0\end{bmatrix},$$

with

$a_{41}=p^2,\ \ a_{45}=r_1,\ \ a_{46}=r_2,\ \ a_{52}=r_3\left(p+\frac{1}{\tau}\right), a_{53}=-\frac{\beta}{\tau}, a_{62}=-\frac{r_4}{\tau}, a_{63}=p, a_{64}=pr_5.$

The characteristic relation of matrix A can be given by

$$\xi^6-x_3\xi^4+x_2\xi^2+x_1=0, \quad (32)$$

where

$x_1=a_{62}a_{53}a_{41}-a_{41}a_{63}a_{52},$

$x_2=a_{46}a_{52}a_{64}+a_{41}a_{52}-a_{45}a_{53}a_{64}+a_{41}a_{63}-a_{53}a_{62}+a_{52}a_{63},$

$x_3=a_{46}a_{64}+a_{52}+a_{41}+a_{63}.$

The matrix eigenvalues of A are the three roots of equation (32) which define by the forms $\pm\xi_1,\pm\xi_2,\pm\xi_3$. Thus, the eigenvectors X are computed as:

$$X_1=a_{46}(a_{52}-\xi^2)\xi-\xi a_{53}a_{45},$$

$$X_2=(a_{41}-\xi^2)a_{53},$$

$$X_3=-(a_{41}-\xi^2)(a_{52}-\xi^2),$$
$$X_4=\xi X_1,\ X_5=\xi X_2,\ X_6=\xi X_3. \quad (33)$$

Thus, the equations (31) have the solutions in the following from:

$$\mathrm{V}(x,p)=\sum_{i=1}^{3}\left(B_iX_ie^{-\xi_ix}+B_{i+1}X_{i+1}e^{\xi_ix}\right), \quad (34)$$

Due to the regularity conditions of the solution, the exponential increasing nature in the spatial variable x has been removed to infinity, therefore the final solutions of equation (31) can be written as

$$\mathrm{V}(x,p)=\sum_{i=1}^{3}B_iX_ie^{-\xi_ix}, \quad (35)$$

where B_1,B_2 and B_3 are constants which can be computed by the problem boundary conditions. Hence, the solutions of all variables can be presented the following forms:

$$\bar{u}(x,p)=\sum_{i=1}^{3}B_iU_ie^{-\xi_ix}, \quad (36)$$

$$\bar{N}(x,p)=\sum_{i=1}^{3}B_iN_ie^{-\xi_ix}, \quad (37)$$

$$\bar{T}(x,p)=\sum_{i=1}^{3}B_iT_ie^{-\xi_ix}, \quad (38)$$

6 Numerical Results and Discussions

For numerical example, magnesium material was selected for numerical estimation purposes. The values of parameters for silicon (Si) material are taken as in [40]:

$T_o=300(k),\ \mu=5.46\times10^{10}(N)(m^{-2}), d_n=-9\times10^{-31}(m^3), \lambda=3.64\times10^{10}(N)(m^{-2}),$
$\alpha_t=3\times10^{-6}(k^{-1}), E_g=1.11\ (eV), c_e=695(J)(kg^{-1})(k^{-1}),\ \rho=2330(kg)(m^{-3})x,$
$T_1=1, \tau=5\times10^{-5}(s), D_e=2.5\times10^{-3}(m^2)(s^{-1}), s_o=2\ (m)(s^{-1}), n_o=10^{20}(m^{-3}).$

The numerical inversion method adopted the final solutions of the temperature, the displacement, the carrier density and the stress distributions. The Stehfest approach [41] can be gven by

$$f(x,t)=\frac{ln(2)}{t}\sum_{n=1}^{G}V_n\bar{f}\left(x,n\frac{ln(2)}{t}\right), \quad (39)$$

With

$$V_n=(-1)^{\left(\frac{G}{2}+1\right)}\sum_{p=\frac{n+1}{2}}^{min\left(n,\frac{G}{2}\right)}\frac{(2p)!p^{\left(\frac{G}{2}+1\right)}}{p!(n-p)!\left(\frac{G}{2}-p\right)!(2n-1)!},$$

where G is the term numbers. The field quantities, carrier density, displacement, temperature and stress depend not only on the time t and space x, but also depend on the exponent of the decayed heat flux Ω. The numerical calculations are carried out for the time $t=0.4$ and $T_1=1$. Based on the above data, the variations of physical quantities along the distance x under the coupled model of thermoelastic and plasma waves are presented in figures 1–4. Figures 1 show the carrier density variation with respect to the distances x. It is observed that the carrier density begins with its maximum value on the surface $x=0.0$ then the carrier density decreases gradually with the increasing of the distance x till it reach to zero value. Figures 2 display the temperature variations along the distances x. It is observed that the temperature has maximum values on the surface $x=0.0$ then the temperature decreases with the increasing of the distance x till it closes to zero.

Figures 3 depict the displacement variations with respect to the distances x. It is observed that the displacement start from the zeros values which satisfy the problem boundary condition of on the surface $x=0.0$ after that it progressively increases up to peak values then decreases progressively with the rising of the distance x till it reach to zeros values. Figures 4 show the stress variations with respect to the distance x. It is observed that it attains some negative values then the magnitudes of stress gradually increase up to peak negative values after thhat the stress gradually increases to zeros values. The compressions between the solutions, one can conclude that considering the coupled photo-thermal model have major effects on the physical quantities distributions. The increasing of the exponent of the decayed heat flux Ω reduces to the physical quantities magnitudes. Otherwise, figures 1-4 illustrates the solutions obtained numerically by the implicit finite difference method (IFDM) overlaid onto the solutions obtained analytically. The accuracy of the implicit finite. Difference method (IFDM) formulation was validated by comparing the analytical and numerical solutions for the field quantities.

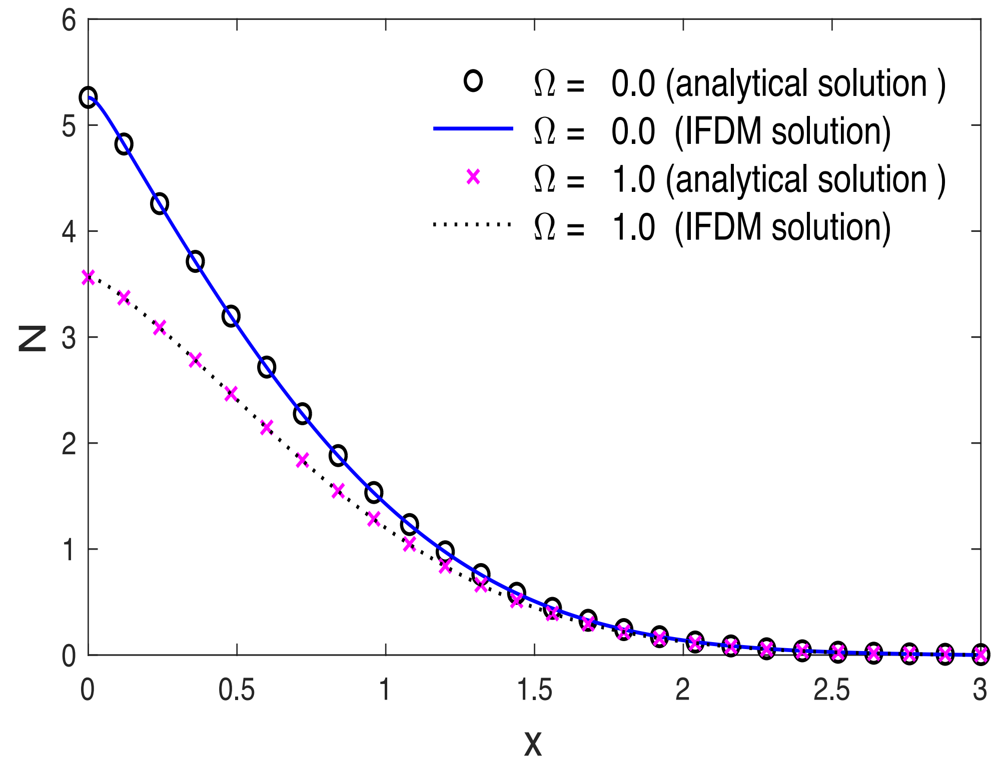

Fig. 1: The carrier density variations along the distance.

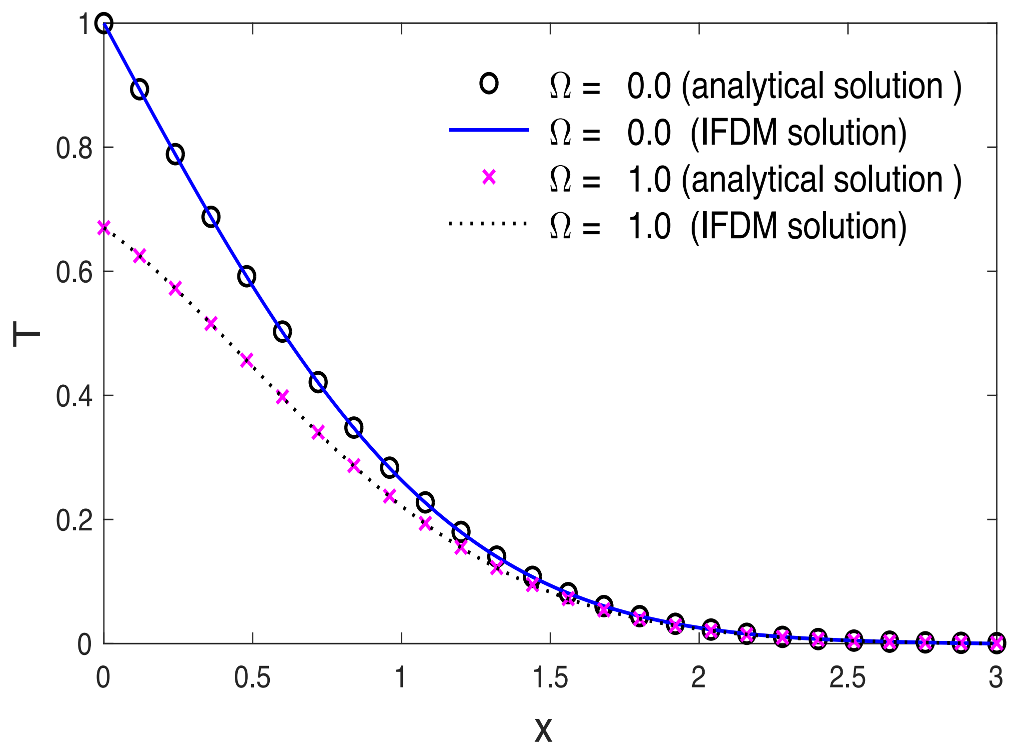

Fig. 2: The temperature variation along the distance.

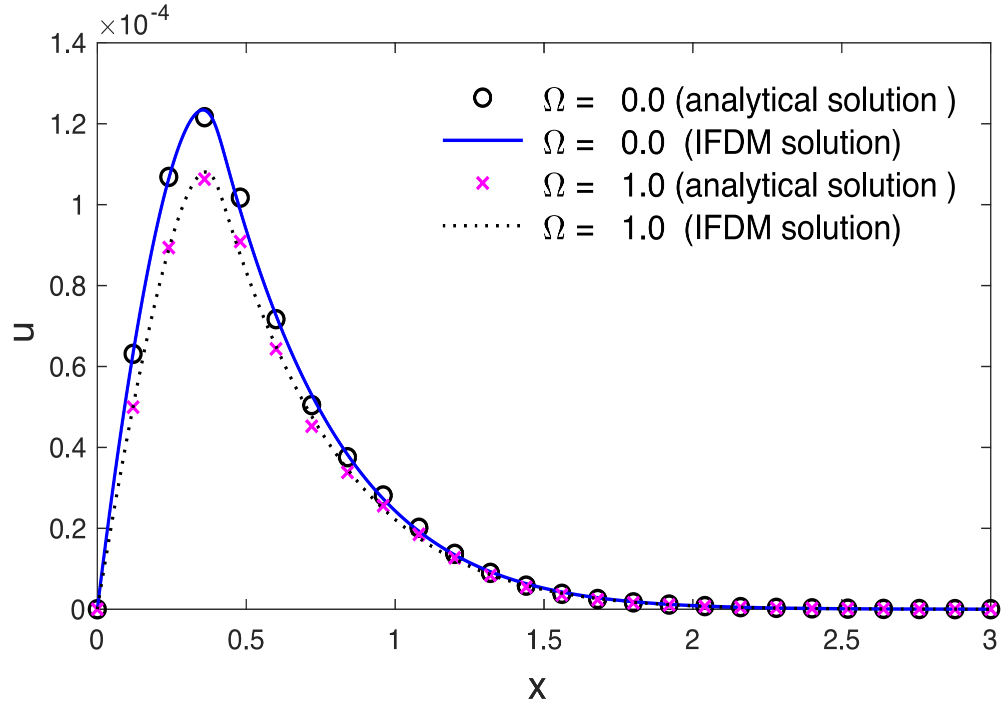

Fig. 3 The displacement variation along the distances.

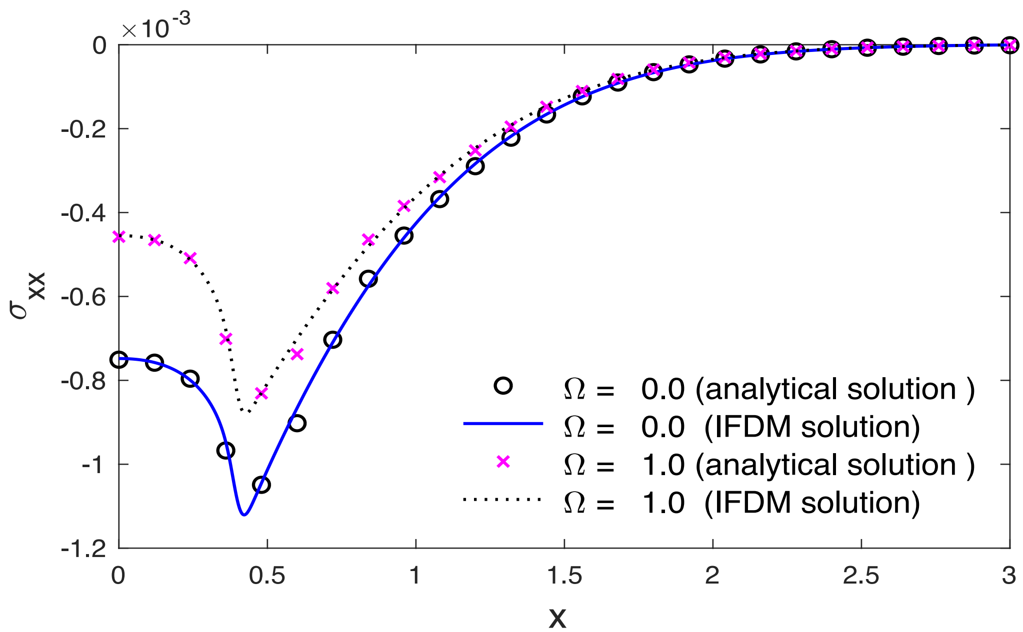

Fig. 4: the stress variations with respect to the distance.

Conflict of interest

The authors have no conflicts of interest to disclose.

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