

Dynamic Deformation Model Calculations in Gd Nuclei

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Abstract: Dynamic Deformation Model (DDM) has been employed to study the nuclear structure of ^{154–156}Gd isotopes. The energy levels, electromagnetic transition properties, and the mixing ratios for transitions $\delta(E2/M1)$ are studied; the various static and dynamic shape characteristic for these isotopes are discussed. The results obtained for ^{154–156}Gd isotopes are reasonably in a good agreement with the known experimental results.

Keywords: Dynamic deformation model; energy levels; B(E2); B(M1); mixing ratios.

1 Introduction

The dynamic deformation model has been employed to study the collective features of nuclei and this model developed over many years ago starting from the Pairing Plus Quadrupole Model (PPQ) which proposed by Kumar and Baranger [1]. The DDM is an attempt to describe the collective spherical, transitional and deformed nuclei transitions from the s-d shell light nuclei to heavy nuclei by a collective motion of microscopic theory. In this model there is no required to fitting parameters to obtaining the results for any nucleus under study.

Some preliminary results of such a study were presented earlier [2]. Detailed results for ^{154–156}Gd isotopes are presented below along with some results for ¹⁵⁴Gd, detailed results for ^{154–156}Gd will be presented in a subsequent publication.

The spherical-deformed transition in Gd isotopes, mentioned above, provides no new challenge for this model or for some other microscopic theories of collective motion (see, for instance, refs. [3,4]). But the B(E2) values for the higher 2⁺ states do provide a new challenge.

Kumar and Gupta [5] applied the dynamic deformation theory, combined with the pairing-plus-quadrupole model, to calculate the level energies and the B(E2) values of even ^{152–160}Gd isotopes. Calculated energies of the 2 _{β} , 2 _{γ} states are too high by 0.5 MeV, but the absolute B(E2) values for the excitation of these as well as several other states are reproduced

remarkably well. Various static and dynamic shape characteristics of the Gd nuclei are discussed.

Gupta [6] in 1984, studied the inter-band B(E2) values from dynamic deformation theory using the pairing plus quadrupole model (PPQ) interaction in ¹⁵²Gd isotope reproduce recent accurate data from Doppler shift recoil experiment contrary to recent claims.

Dynamic deformation theory based on the pairing-plus-quadrupole model is employed by Kumar and Gupta [7] for a detailed study of the $N = 88$ transitional nucleus ¹⁵²Gd. Comparisons with experiment and with other models are presented for the potential minimum and related properties, B(E2) values, $\rho(E0)$, $X(E0/E2)$ and $\sigma(E2/M1)$ values for transitions from seven 2⁺ states and a number of other states below 2 MeV.

2 The Dynamic Deformation Model (DDM)

The dynamic deformation model (DDM) description was given in refs. [8,9,10] and therein references. The theory of DDM can be divided into two main parts:

First: the collective Hamiltonian microscopic derivation and a numerical solution of the Hamiltonian.

Second: The Hamiltonian of microscopic is composed the normormalized Nilsson-type single particle plus pairing and given in the form:

$$H = T_{rot} + T_{vib} + V(\beta, \gamma) \quad (1)$$

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T_{rot} is the rotational kinetic energy, which is given by:

$$T_{rot} = \frac{1}{2} \sum_{k=1}^3 \Theta_k(\beta, \gamma) \omega_k^2 \quad (2)$$

and the vibrational kinetic energy T_{Vib} is written as:

$$T_{Vib} = [B_{\beta\beta}(\beta, \gamma)\beta^2 + 2B_{\beta\gamma}(\beta, \gamma)\beta \cdot \beta\gamma + B_{\gamma\gamma}(\beta, \gamma)\beta^2\gamma^2] \quad (3)$$

This Hamiltonian contains seven parameters, these parameters are: the potential energy $V(\beta, \gamma)$, the moments of inertia (three moments), $\Theta_k(\beta, \gamma)$, coefficients of vibrational mass (three masses) $B_{\mu\nu}(\beta, \gamma)$. The Hamiltonian in Eq(1) is written as :

$$H = H_{av} + V_{res} \quad (4)$$

$$H_{av} = \frac{p^2}{2M} + \frac{1}{2} M \sum_{k=1}^3 \omega_k^2 \chi_k^2 + \hbar\omega_0 [v_{ls}L \cdot S + v_{ll}(L^2 - \langle L^2 \rangle_N)] \quad (5)$$

The potential energy is written as:

$$V_{coll} = V_{DM} + dU + dV_{proj} + dE_{pair} \quad (6)$$

where dV_{proj} is a nine-dimensional projection correction introduced by Kumar [9]. The generalized cranking method is employed to derive the general expression for mass parameter which is used in collective kinetic energy.

$$T_{coll} = \frac{1}{2} M \sum_{\mu\nu} B_{\mu\nu} \alpha_\mu \alpha_\nu \quad (7)$$

The program codes of dynamic deformation model used for our calculations are a dynamic deformation model version modified. This program code which was modified to use for super-heavy nuclei, this code written by Kumar [9]. In the present calculations as well as the deformation definition, are identical to those of Kumar et al. [9].

The wave function given by:

$$\Psi_{\alpha IM} = \sum_{K \geq 0, even} A_{\alpha IK}(\beta, \gamma) \Phi_{MK}^I(\varphi, \theta, \psi) \quad (8)$$

I – is the total angular momentum, M – is projection of angular momentum on Z – axis and K – is projection of angular momentum on the laboratory (intrinsic), α is the index $\alpha = 0, 1, 2, \dots$

In this work we presented to examine the applicability of the dynamic deformation model on the deformed nuclei, such as $^{154-156}\text{Gd}$ isotopes. The energy spectra of $^{154-156}\text{Gd}$ isotopes were presented. As one can see through the even-even nuclei, the beta band, gamma band and band cross in energy. While the beta band and gamma bands cross each other in ^{154}Gd isotope [11]. The band comes down quite low in ^{156}Gd ; the mixing of band is few [12].

The quadrupole interaction effects are first included directly in the single-particle basis. The intrinsic axes

attached to the nucleus are chosen such that the average field depends on two of the five quadrupole variables (i.e., the shape variables β and γ) but not on the other three variables (the orientation angles Φ , θ and ψ of the intrinsic axes with respect to the laboratory axes). (β, γ) effects on the single-particle energies and wave function are calculated exactly as in the Nilsson model calculations.

However, these isotopes are chosen to test the microscopic theory of dynamic deformation model (DDM), and compared the calculations of DDM with experimental data. These nuclei provide some of the most interesting cases for testing the microscopic DDM theory.

In view of the limitation inherent to the DDM and to the theory of the collective motion, complete agreement with the experiment is not expected. Instead our aim is to try to understand and predict trends of the nuclear properties associated with the quadrupole motion in a self-consistent, unified manner.

3 Results and Discussion

3.1 Energy Levels

Energy levels for $^{154-156}\text{Gd}$ isotopes have been calculated within DDM, and experimental data are given in Table (1), for DDM results there is no fitted parameters used here, only Z and A for the nucleus under study. The energy levels in Table (1) are tabulated in four bands grouping depending on the largest K component for the wave-function of each energy level. The comparison between DDM results and experimental values are in good agreement.

The energy ratios are presented in Table (2), one can observe from these results the energy ratios are increased with increasing neutron number ($N = 90 - 92$), far from the closed shell ($N = 82$), therefore, these nuclei appear as a deformed character (rotor character).

From the results of energy levels in Table (1), shows that the beta and gamma band-heads are larger by 0.12MeV and 0.21MeV respectively, the spread or distribution of energy levels in beta and gamma band are quite well. The shape parameters (β, γ) , β where is called the deformation parameter and γ is the deviation magnitude from the spherical shape (angle) are given in Table (1), these parameters contained the nuclear motion dynamic effects. The values of β_{rms} are shown in their similarity, the variation in β_{rms} values in three state bands (ground, beta and gamma bands) is about 19%. Whereas the variation in the values of γ_{rms} are about 20° to 180° . The variation reduced in γ_{rms} for $^{154-156}\text{Gd}$ isotopes with vibration rotation and rotation with increased the moment of inertia in the band.

Table 1: Energy levels in (MeV) Units and shape parameters (β, γ) for $^{154-156}\text{Gd}$ isotopes.

| Band | J_i^+ | ^{154}Gd | | ^{156}Gd | | ^{154}Gd | | ^{156}Gd | |
|----------------|----------------|-------------------|-------|-------------------|-------|-------------------|------------------|-------------------|------------------|
| | | Exp.[13] | DDM | Exp.[14] | DDM | β (rms) | γ^0 (rms) | β (rms) | γ^0 (rms) |
| gsb | 0 ₁ | 0.0 | 0.0 | 0.0 | 0.0 | 0.272 | 13.2 | 0.274 | 13 |
| | 2 ₁ | 0.123 | 0.122 | 0.088 | 0.089 | 0.274 | 13 | 0.277 | 12.9 |
| | 4 ₁ | 0.370 | 0.374 | 0.288 | 0.283 | 0.280 | 12.8 | 0.283 | 12.5 |
| | 6 ₁ | 0.717 | 0.722 | 0.585 | 0.590 | 0.284 | 12.5 | 0.288 | 12.2 |
| | 8 ₁ | 1.144 | 1.212 | 0.965 | 0.298 | 0.295 | 12.3 | 0.294 | 12.0 |
| β -band | 0 ₂ | 0.680 | 0.685 | 1.094 | 1.101 | 0.292 | 11.5 | 0.283 | 9.5 |
| | 2 ₂ | 0.815 | 0.843 | 1.129 | 1.133 | 0.299 | 10 | 0.302 | 9.3 |
| | 4 ₂ | 1.047 | 1.154 | 1.298 | 1.320 | 0.310 | 10.3 | 0.303 | 10.2 |
| | 6 ₂ | 1.365 | 1.368 | 1.540 | 1.658 | 0.312 | 10.6 | 0.305 | 10.4 |
| γ -band | 2 ₃ | 0.996 | 1.112 | 1.154 | 1.351 | 0.283 | 18.3 | 0.277 | 19.9 |
| | 0 ₃ | 1.182 | 1.234 | 1.168 | 1.341 | 0.281 | 18.5 | 0.280 | 18.5 |
| | 3 ₁ | 1.127 | 1.303 | 1.248 | 1.432 | 0.285 | 19.4 | 0.284 | 19.5 |
| | 4 ₃ | 1.263 | 1.356 | 1.355 | 1.648 | 0.290 | 19.2 | 0.287 | 19.3 |
| | 6 ₃ | 1.606 | 1.897 | 1.643 | 1.65 | 0.293 | 19.6 | 0.290 | 19.4 |

Table 2: Energy ratios for $^{154-156}\text{Gd}$ isotopes.

| Isotopes | $E(4_1^+)/E(2_1^+)$ | | $E(6_1^+)/E(2_1^+)$ | | $E(8_1^+)/E(2_1^+)$ | |
|----------|---------------------|-------|---------------------|-------|---------------------|--------|
| | Exp. | DDM | Exp. | DDM | Exp. | DDM |
| Gd-154 | 3.008 | 3.065 | 5.589 | 5.918 | 9.300 | 9.934 |
| Gd-156 | 3.727 | 3.179 | 6.647 | 6.629 | 10.965 | 11.213 |

3.2 Quadrupole Moments and Magnetic Dipole Moments for $^{154-156}\text{Gd}$ isotopes

To estimate the quadrupole moments for states in three bands, we depend on the following Equation [15]:

$$Q(I_i) = Q_0 \frac{3K^2 - I(I+1)}{(I+1)(2I+3)} \quad (9)$$

Where Q_0 is the intrinsic quadrupole moment.

The DDM values of quadrupole moments $Q(I_i^+)$ are given in Table (3). On can see the variation in signs and magnitudes for calculated and experimental values, this implies that the negative signs indicate the nucleus takes a prolate shape in this states, while the positive signs means the nucleus takes an oblate shape.

The intrinsic magnitude values of quadrupole moments Q_0 , increased in negative smoothly from ^{154}Gd to ^{156}Gd isotopes with increasing neutron number; this means the deformation is increased. The values of Q_0 increased gradually with spin increased in three bands.

The magnetic dipole moments evaluated from the following Equation [15]:

$$\mu_I = gI \quad (10)$$

Where g - is the g -factor or gyromagnetic ratio, which evaluated, depending on the experimental value of $\mu(2_1^+) = 0.77(4)$ [13] and the experimental value of mixing ratio for the transition $\delta(2_2^+ \rightarrow 2_1^+) = -(17.5_{-1.4}^{+1.6})$ [16], we obtain the value $g = 0.447\mu_N$, this value is used for two isotopes. Form the Table (3), the DDM values of

$\mu(I_i)$ depend on the nuclear spin (I_i^+), these values variation with nuclear spin (I_i^+) about 1-1.5%. The agreement between DDM results and experimental values are good.

3.3 Electric Transition Probability

The reduced electric transition probability is given by the following equation [15]:

$$B(E2; I_i^+ \rightarrow I_f^+) = \frac{|\langle I_f^+ || T^{(E2)} || I_i^+ \rangle|^2}{2I_i^+ + 1} \quad (11)$$

The DDM results for electric transition probability are given in the Table (4), together with experimental values; the agreement between them is well. The intraband $B(E2)$ values (transitions in the same band, i.e., ground state band) such as $B(E2; 0_1^+ \rightarrow 2_1^+)$, have highest values more than other the interband transitions, i.e., transitions from beta band to ground band or transitions from gamma band to ground band, this is due to the selection rules of these transitions.

The branching ratios calculated in DDM depending to Alga rule [15], the results of DDM branching ratios and experimental data are given in Table (5). From these results, we can see the branching ratios transition from beta band to ground state band and gamma band to ground band are reasonable agreement with experimental values. Some results of branching ratios higher and other results are small or few this due to the selection rules for transition between the bands.

3.4 Magnetic Dipole Matrix Elements

The reduced magnetic dipole matrix element which is calculated by DDM is listed in Table (6), there is no

Table 3: Quadrupol moments for $Q(I_i^+)$ in (eb) Units and magnetic dipole moments $\mu(I_i^+)$ in (μ_N^2) for $^{152-154}\text{Gd}$ isotopes.

| Band | J_i^+ | $^{154}\text{Gd } Q(I_i^+)$ | | $^{156}\text{Gd } Q(I_i^+)$ | | $^{154}\text{Gd } \mu(I_i^+)$ | | $^{156}\text{Gd } \mu(I_i^+)$ | |
|----------------|----------------|-----------------------------|-------|-----------------------------|-------|-------------------------------|------------------|-------------------------------|------------------|
| | | Exp.[13] | DDM | Exp.[14] | DDM | β (rms) | γ^0 (rms) | β (rms) | γ^0 (rms) |
| gsb | 0 ₁ | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| | 2 ₁ | -1.82(4) | -1.84 | -1.93(4) | -1.98 | 0.91(4) | 0.99 | 0.77(4) | 0.85 |
| | 4 ₁ | - | -1.95 | - | -2.55 | - | 1.24 | 1.24(8) | 1.69 |
| | 6 ₁ | - | -2.22 | - | -2.86 | - | 1.89 | 1.5(13) | 2.70 |
| | 8 ₁ | - | -2.8 | - | -2.90 | - | 2.34 | - | 2.87 |
| β -band | 0 ₂ | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| | 2 ₂ | - | -1.67 | - | -1.89 | - | 0.78 | - | 0.81 |
| | 4 ₂ | - | -2.11 | - | -2.22 | - | 1.75 | - | 1.86 |
| | 6 ₂ | - | -2.11 | - | -2.50 | - | 1.85 | - | 1.88 |
| γ -band | 2 ₃ | - | 1.54 | - | 1.88 | 0.83 $^{+7}_{-9}$ | 0.73 | - | 0.83 |
| | 0 ₃ | - | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| | 3 ₁ | - | -1.17 | - | -1.24 | - | -1.21 | - | 1.32 |
| | 4 ₃ | - | -1.19 | - | -1.44 | - | 1.54 | - | 1.91 |
| | 6 ₃ | - | -1.44 | - | -1.72 | - | 1.97 | - | 2.03 |

Table 4: Electric Transition probability B(E2) in e^2b^2 Units for $^{154-156}\text{Gd}$ isotopes.

| Transitions | ^{154}Gd | | ^{156}Gd | |
|----------------------------|-------------------|-------------------------------|-----------------------------------|--------------------------------|
| | Exp.[11, 14] | DDM | Exp. | DDM |
| 0 $_1^+ \rightarrow 2_1^+$ | 3.77 \pm 0.05 | 4.2 | 4.56 \times 0.05 ^a | 4.80 |
| 0 $_1^+ \rightarrow 2_2^+$ | 0.018 \pm 0.005 | 0.013 | 0.013 \times 0.004 ^a | 0.015 |
| 0 $_1^+ \rightarrow 2_3^+$ | - | 0.17 | 0.120 \times 0.004 ^a | 0.22 |
| 0 $_1^+ \rightarrow 2_4^+$ | - | 0.007 | < 0.008 ^a | 0.0098 |
| 2 $_1^+ \rightarrow 4_1^+$ | - | 0.9 \times 10 ⁻⁵ | 1.6 \times 10 ^{-5b} | 1.02 \times 10 ⁻⁵ |
| 4 $_1^+ \rightarrow 6_1^+$ | - | 3 \times 10 ⁻⁵ | 4.8 \times 10 ^{-5b} | 5 \times 10 ⁻⁵ |
| 0 $_2^+ \rightarrow 2_1^+$ | 0.0021(3) | 0.23 | 0.029(4) ^c | 0.031 |
| 2 $_1^+ \rightarrow 0_1^+$ | 0.0048(4) | 0.0051 | 0.0020(4) ^c | 0.00316 |
| 2 $_2^+ \rightarrow 2_1^+$ | 0.04(2) | 0.048 | 0.0029(4) ^d | 0.0031 |
| 2 $_3^+ \rightarrow 4_1^+$ | 0.119(8) | 0.20 | 0.0020(4) ^d | 0.0025 |
| 4 $_2^+ \rightarrow 2_1^+$ | 0.0035(8) | 0.004 | 0.0115(13) ^d | 0.01134 |
| 4 $_3^+ \rightarrow 4_1^+$ | 0.003(6) | 0.0039 | 0.0031(10) ^d | 0.0043 |
| 4 $_4^+ \rightarrow 6_1^+$ | 0.119(25) | 0.21 | 0.009(3) ^d | 0.00123 |

a-[18] b-[19] c-[20] d-[17]

experimental data to compare these values. From the results of DDM we can see the transitions from gamma band to ground band are weaker than the transitions from beta band to ground band somewhat. The magnetic transitions matrix element from gamma to gamma band is in the same order of the matrix element transition from beta band to ground band and gamma band to ground band.

3.5 Mixing Ratios

The mixing ratios for $^{154-156}\text{Gd}$ isotopes within DDM are presented using the following Equation [17]:

$$\delta(E2/M1) = 0.835E_\gamma(\text{in MeV}) \times \frac{|\langle I_f^+ || T^{(E2)} || I_i^+ \rangle|}{|\langle I_f^+ || T^{(M1)} || I_i^+ \rangle|} \quad (12)$$

Where $|\langle I_f^+ || T^{(E2)} || I_i^+ \rangle|$ is the reduced electric matrix element in eb units, and $|\langle I_f^+ || T^{(M1)} || I_i^+ \rangle|$ in μ_N^2 .

The values of mixing ratios which calculated by DDM and experimental values are taken in Table (6). These results shows, agree with the experimental values in magnitude and sign for most transitions, while two mixing ratios for transitions disagree in sign and magnitude. The discrepancy in some values occurs for high spin states transitions.

Table 5: Branching Ratios for $^{154-156}\text{Gd}$ isotopes.

| I_i | I_f/I_i | ^{154}Gd | | ^{156}Gd | |
|----------------|--------------------------------|-------------------|-------|--------------------------------|-------|
| | | Exp. [21] | DDM | Exp. | DDM |
| 2 ₂ | 0 ₁ /2 ₁ | 0.121 \pm 0.04 | 0.22 | 0.186 \pm 0.019 ^a | 0.180 |
| 2 ₂ | 4 ₁ /2 ₁ | 2.75 \pm 0.08 | 3.0 | 1.36 \pm 0.10 ^a | 1.77 |
| 2 ₂ | 0 ₁ /4 ₁ | 0.044 \pm 0.004 | 0.171 | 0.137 \pm 0.016 ^a | 0.160 |
| 4 ₂ | 2 ₁ /4 ₁ | 0.086 \pm 0.003 | 0.07 | 0.19 \pm 0.07 ^b | 0.22 |
| 4 ₂ | 6 ₁ /4 ₁ | 2.38 \pm 0.08 | 5.6 | 3.4 ^b | 4.5 |
| 4 ₂ | 2 ₁ /6 ₁ | - | 0.44 | 0.05 ^c | 1.8 |
| 2 ₃ | 0 ₁ /2 ₁ | 0.456 \pm 0.011 | 0.89 | 0.055 \pm 0.22 ^c | 1.99 |
| 2 ₃ | 4 ₁ /2 ₁ | 0.144 \pm 0.005 | 2.3 | 0.086 \pm 0.017 | 0.006 |
| 2 ₃ | 0 ₁ /4 ₁ | 3.29 \pm 0.4 | 5.3 | 7.9 \pm 1.6 | 1.5 |
| 3 ₁ | 2 ₁ /4 ₁ | 0.984 \pm 0.026 | 0.77 | 1.52 \pm 0.08 | 5.1 |
| 4 ₃ | 2 ₁ /4 ₁ | 0.139 \pm 0.007 | 0.204 | 0.152 \pm 0.013 ^b | 1.22 |
| 4 ₃ | 6 ₁ /4 ₁ | 0.27 \pm 0.4 | 0.56 | 0.07 ^b | 0.09 |
| 4 ₃ | 6 ₁ /2 ₁ | - | 0.221 | 0.43 ^b | 0.54 |
| 5 ₁ | 4 ₁ /6 ₁ | - | 6.9 | 0.71 \pm 0.09 ^d | 1.009 |
| 2 ₄ | 2 ₂ /2 ₁ | 1.0 \pm 0.02 | 7.9 | 1.35 \pm 0.15 ^d | 2.90 |

a-[22] b-[23] c-[24] d-[26]

Table 6: Mixing ratios for $^{154-156}\text{Gd}$ isotopes.

| Transitions | ^{154}Gd | | ^{156}Gd | |
|---------------------------|----------------------|-------|------------------------|-------|
| | Exp. [16] | DDM | Exp. [16, 18] | DDM |
| $2_2^+ \rightarrow 2_1^+$ | -9.7(5) | -8.77 | $-5.9^{+1.4}_{-2.8}$ | -4.80 |
| $4_2^+ \rightarrow 4_1^+$ | -4.1(4) | -4.36 | - | 0.66 |
| $2_3^+ \rightarrow 2_1^+$ | $8.3^{+1.5}_{-1.1}$ | 11 | -1.8 ± 3 | -22 |
| $3_1^+ \rightarrow 2_1^+$ | -26^{+4}_{-6} | -33 | -10.0 ± 0.6 | 0.009 |
| $3_1^+ \rightarrow 4_1^+$ | -5.6(2) | -6.7 | - | 34.2 |
| $4_3^+ \rightarrow 4_1^+$ | $1.7 < \delta < 4.3$ | 5.3 | - | 12.8 |
| $2_4^+ \rightarrow 2_1^+$ | - | 3.2 | $0.38^{+0.06}_{-0.05}$ | 0.311 |
| $4_4^+ \rightarrow 4_1^+$ | - | 11 | - | 0.76 |

4 Conclusions

The dynamic deformation model is employed to study the nuclear structure and electromagnetic properties for Gadolinium isotopes. In this work we give a general feature of nuclear deformations, energy levels, and electromagnetic moments for these isotopes. Now we summarize the conclusions of the main results in this work:

1. The energy levels in ground, beta and gamma bands reproduced very well. The calculated beta and gamma band-heads are shifted by 0.1 and 0.3 MeV respectively.
2. The electric transition probability values produced very well for intra-band more than inter-band transitions.
3. The quadrupole moments for available states variation in signs and magnitudes for calculated and experimental values, this implies that the negative signs indicate the nucleus takes a prolate shape in this states, while the positive signs means the nucleus takes an oblate shape.
4. The energy ratios are presented in Table (2), we observe from these results the energy ratios are increased with increasing neutron number far out the closed shell ($N = 82$), therefore, these nuclei appears a deformed character (rotor character).
5. The DDM values of mixing ratios agree with the experimental data in sign and magnitude in almost transitions except the two transitions disagree in sign.

Results show once again that the DDM is applicable to spherical (vibrational) as well as deformed (rotational) nuclei. This conclusion was already drawn previously on the basis of this type of calculation for the transitional (prolate-oblate shape transition) osmium region nuclei, and the transitional (spherical-deformed shape transition) samarium nuclei. What is new here in this regard is that the theory has been extended to more deformed nuclei ($^{158-160}\text{Gd}$ isotopes are among the best examples of such nuclei, $E(4_1^+)/E(2_1^+)$ being 3.3)

Our main conclusion is that the dynamic, microscopic method of dynamic deformation model of Kumar can be used for well-deformed nuclei. We know of no other

available method which provides such a wealth of predictions with so few parameters. Finally this model success to describe the nuclear structure of well deformed (rotor) nuclei.

Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this article.

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