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A New Measurement of Nuclear Radius from the study of β+–Decay Energy of Finite-Sized Nuclei

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Abstract: The nuclear radius is the fundamental parameter used to describe the structure as well as the effective interactions of atomic nucleus. Any improvement to its measurements or the techniques applied could be a milestone to the understanding of nuclear structure and its complex dynamics. There is a great challenge from both theoretical and experimental measurement of nuclear radius that the range of nuclear force is far from being constant, especially when there is a significant difference between proton and neutron numbers. Attempt to tackled with this challenge have been made especially, to improve the A – dependence formula of measuring nuclear radius or to use another approaches that could provide constant R/A1/3 value. In view of these observations, this study, proposed a new approach to measure the nuclear size, from the study of the β+–decay and coulomb energy difference of finite-size nuclei. However, the study modeled nucleus as a positively charged object of charge +Ze equals in magnitude with negatively charged (-e) orbiting leptons, from which the nuclear potential charge radii, RC are measured. This measurement takes into account the interaction of leptons and successfully produced a simple formula that can be applied to measure the size of nuclear potential radius RC. The results are in good agreement with the previously measured values of RC using nuclear finite-size model. Therefore, the present study improves the validity of previously measured RC. For the improved nuclear finite-size model, the studies could provide more information on the understanding of nuclear matter and charge radius, nuclear potential, charge distributions, coulomb energy, electron energy levels and on their future measurements.

Keywords: Nuclear radii, β+ decay, Coulomb Energy Difference, Finite-Size, Charge Distributions, and Potentials.

1 Introduction

The study of nuclear stability played an important role in the foundation and development of theoretical and experimental information about the nuclear charge radius. Many atomic nuclei are unstable and decay naturally to emit α–particles, β–particles, and γ–rays. The α decay corresponds to a very asymmetric spontaneous fission, where a nucleus ${}_{Z}^{A}X$ transforms into ${}_{Z-2}^{A-4}Y$ with the ejection of a ${}^{4}He$ nucleus (α–particle). The β decay can occur in nuclei where the neutron-to-proton ratio is not optimal. In this process the parent and the daughter nuclei have the same atomic mass: an electron (positron) plus; the nucleus transforms from ${}_{Z}^{A}X$ into ${}_{Z+1}{}^{A}Y$ for β^- or ${}_{Z-1}{}^{A}Y$ for β^+ and electron capture. These are accompanied by an electron anti-neutrino (neutrino) emission. Generally, α and β decays are also accompanied by emission of γ quanta, conversion electrons and $e^+ – e^-$ pairs and/or by emission of subsequent atomic radiation, X-rays and/or Auger and Coster-Kronig electrons. More recently, the cluster decays, the 2β decay, the spontaneous fission, the nucleon, di-nucleon, tri-nucleon emission and higher-order electromagnetic phenomena forms of radioactive decays have been considered [1].

The energy of β^+ decay has been studied to measure with good accuracy, the change in the coulomb energy of mirror nuclei. Mirror nuclei are isobars, which their stable decay products each contain just one more neutron than the number of protons and their mass number is $A = 2Z – 1$. Some states of mirror nuclei with the same isospin, spin/parity can form the isospin or isotopic multiplets and then approximately have the same wavefunctions, energy differences in the excited analogue states and the binding energies [2-4]. The change in coulomb energy or coulomb energy difference of mirror nuclei has been studied to measured value of nuclear charge radius [5].

The nuclear radius is the fundamental parameter used to describe the structure as well as the effective interactions of atomic nucleus. Therefore, any developments in its measurement techniques can improve the understanding of

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nuclear structure and its complex dynamics [6-8]. The nuclear radius is mainly characterized by *rms* radii $\langle r^2\rangle^{1/2}$ or by matter radii, *R*.

When nucleus is treated as an incompressible quantum drops with sharp density $\rho_0 \simeq 0.15\ fm^{-3}$ and nucleon number *A*, then its size can be determined from the relation: $R = r_0 A^{1/3}$, where r_0 is the range of the nuclear force [9,10]. In turn, the root-mean-square charge radius of such a uniform distribution is given by $R_{rms} = \langle r^2\rangle^{1/2} = \sqrt{3/5}R$. However, the conventional *A* – dependent formula is not globally valid for all nuclei as the ratio $R/A^{1/3}$ is far from being constant especially when there is a significant difference between proton and neutron numbers [11,12]. The variation in the ratio $R/A^{1/3}$ from both theoretical and experimental data make the measurement of nuclear charge radius a great challenge to nuclear physicist [13-8]. In view of these reasons, the $A^{1/3}$ dependence formula has been improve, for example, by including corrections such as surface effect [19], isospin [20], shell correction [21] exotic nuclei [22], halo nuclei [23], finite-size nucleus [24], volume effect [25] and/or proposing $Z^{1/3}$ dependence formula [22] to describe nuclei much better [6].

The measurement of nuclear size is a challenge particularly considering the fact that there are many theoretical and experimental data source yielded different results. This is because the nucleus is an object whose properties are much more difficult to characterize [26]. Nuclei are composed of nucleons which themselves are built from fundamental particles called quarks. This study built a picture of spherical object with charge density $\rho(r) = 3Ze/4\pi R^3$, possessing a positive charge $+Ze$, equals the magnitude of charge ($-e$) of orbiting leptons. From this nuclear model, a new quantity is proposed, based on the study of the β^+–decay and Coulomb energy difference, to measure the nuclear size.

2 Description of the Calculations

The common *Z*/*r* potential which has been used to describe the interaction between an electron (or muon) with nucleus did not adequately provide information on the details of nuclear interactions as it does not account for the charge density inside the nucleus. This makes it necessary to choose a suitable potential [27,28]. For a nucleus with spherically symmetric charge distribution $\rho(r) = 3Ze/4\pi R^3$, the effective interaction can best be described by the lepton-nuclear potential energy $U(r)$ [16]:

$$U(r) = -ke\left[\frac{4\pi}{r}\int_0^R \rho(r')r'^2\,dr' + 4\pi\int_R^\infty \frac{1}{r'}\rho(r')r'^2\,dr'\right]$$

where within a nuclear radius $r \le R$, the expression is described by

$$U(\vec{r}, R) = -\frac{Zke^2}{2r}\left[\frac{3r}{R} - \left(\frac{r}{R}\right)^3\right] \qquad (1)$$

and outside the nucleus, $r > R$, this expression reduces to *Z*/*r* potential

$$U(r) = -\frac{Zke^2}{r} \qquad (2)$$

The potential (1) has a constant value of $U(\vec{r}) = -Zke^2/2R$ inside the nucleus. [29]. The modification to nuclear potential (2) reflects on the nuclear charge radius [24], coulomb energy [30], energy levels of an electron [27,28] and other related calculations such as isotope shift [31] and quantum electrodynamics' calculations [32,33]. The coulomb energy derived previously from the study of potential model (1) and electrodynamics theory is given by

$$E_{FN} = \beta_C \frac{Z^2}{A^{1/3}} \qquad (3)$$

where

$$\beta_C = \frac{ke^2}{r_C} \qquad (4)$$

and r_C is the nuclear charge parameter which determine the range of nuclear potential [30]. For a pair of finite-sized mirror nuclei of charges $+Ze$ and $+(Z-1)e$, the corresponding coulomb energy (3) are respectively

$${}^{A}_{Z}X \rightarrow E_{FN} = \beta_C \frac{Z^2}{A^{1/3}} \qquad (5)$$

and

$${}_{Z-1}^{\ \ A}Y \rightarrow E_{FN'} = \beta_C \frac{(Z-1)^2}{A^{1/3}} \qquad (6)$$

Hence, the difference ΔE_{FN} of the coulomb energy (6) and (5) will be

$$\Delta E_{FN} = \beta_C \frac{(2Z-1)}{A^{1/3}} = \beta_C A^{2/3} \qquad (7)$$

The first member of the pair of mirror nuclei is usually β^+ active and undergoes β^+ transformation into the second as

$${}^{A}_{Z}X \rightarrow {}_{Z-1}^{\ \ A}Y + \beta^+ + v$$

and the Q-value for the β^+–decay is:

$$E(\beta^+) = [\Delta m({}^{A}_{Z}X) - \Delta m({}_{Z-1}^{\ \ A}Y) - 2m_e]c^2 \qquad (8)$$

Where $\Delta m({}^{A}_{Z}X) = Zm_p + (A-Z)m_n - m_{nucleus}$, and the binding energy

$$\Delta E_B = \Delta m({}^{A}_{Z}X) - \Delta m({}_{Z-1}^{\ \ A}Y) \qquad (9)$$

The expression (8) gives the value of β^+ transition energy between the mirror nuclei $^{A}X_{Z} - {}^{A}Y_{Z-1}$ in terms of binding energy of nucleus (9).

3 Results

The value of nuclear binding energy, ΔE_B is computed using the standard values of masses of nuclei from Ref. [34]. The β^+-decay energy, $E(\beta^+)$ is computed from (8) in terms of nuclear binding energy, ΔE_B. The results are presented in Table 1.

Table 1: The computed values of binding energy and β^+-decay energy of mirror nuclei.

$^{A}X_{Z} - {}^{A}X_{Z-1}$	$A^{2/3}$	ΔE_B (MeV)	$E(\beta^+)$ (MeV)
$^{13}N_7 - {}^{13}C_6$	5.5288	3.0031	2.9009
$^{15}O_8 - {}^{15}N_7$	6.0822	3.5360	3.4338
$^{23}Mg_{12} - {}^{23}Na_{11}$	8.0876	4.8391	4.7369
$^{31}S_{16} - {}^{31}P_{15}$	9.8683	6.1758	6.0736
$^{39}Ca_{20} - {}^{39}K_{19}$	11.5003	7.3131	7.2109
$^{51}Fe_{26} - {}^{51}Mn_{25}$	13.7525	8.8063	8.7041

The information represented in Table 1 is used to plot a graph of the β^+–decay energy as a function of $A^{2/3}$.

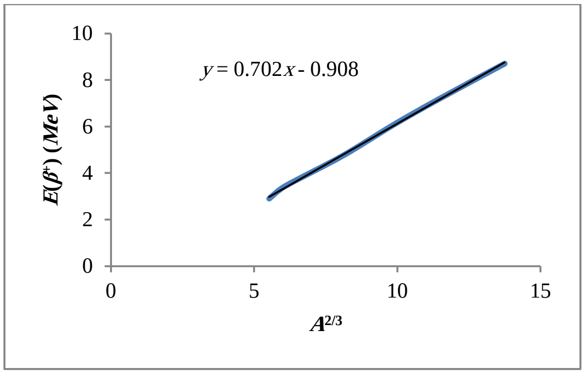

Fig. 1: The plot of β^+ transition energy against $A^{2/3}$

Figure 1 showed a plot of β^+–decay energy as a function of $A^{2/3}$, which gives a straight line equation:

$$E(\beta^+) = \beta_C A^{2/3} - 0.908\, MeV \qquad (10)$$

where the slope of the graph is

$$\beta_C = \frac{1.44\times10^{-15}\, MeV}{r_C} = 0.702\, MeV$$

This implies

$$r_C = \frac{1.44}{0.702} \times 10^{-15}\, m$$

The value of nuclear radius parameter, $r_C = 2.0513\ fm$, is determined from the slope of the graph (Figure 1). Hence the size of nuclear potential can take the form

$$R_C = 2.0513 A^{1/3}\, fm \qquad (11)$$

The quantity R_C can simply relate to nuclear matter radius, R or root mean square radius, R_{rms} as

$$R_C = \sqrt{3} R \qquad (12)$$

$$R_C = \sqrt{5} R_{rms} \qquad (13)$$

where

$$r_C = \sqrt{3} r_0 \qquad (14)$$

for $r_0 = 1.184\ fm$. The result is in agreement with the value $r_C = \sqrt{3} r_0$ obtained from Ref. [24] when studying the effect of nuclear finite-size on potential interaction. Thus, , the nuclear radius parameters r_C obtained from the β^+ transformation energy could be applied to determine the matter and charge radii for various nuclei. It can be observed that the nuclear potential radius, R_C is higher than nuclear matter radius, R and the root mean square charge radius, R_{rms}. This is because the quantity, R_C is the measure of the range of the nuclear electrostatic field radius, which is independent of the internal structure or interactions of quarks and nucleons.

4 Conclusions

Despite the fact that atomic nuclei are complex finite many-body systems governed by the laws of quantum mechanics, the present study proposed a simple formulas, equation (11), (12) and (13), that can be applied to measure the size of atomic nuclei. These formulas provide another dimension for nuclear size measurement. However, more information on nuclear potential distributions, coulomb energy as well as the electron energy levels could be provided from these results.

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