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Optimal Allocation in Agriculture using Intuitionistic Fuzzy Assignment Problem

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Abstract: This paper deals with Intuitionistic fuzzy assignment problem in which cost $\widetilde{C}_{ij}$ has been considered as an Intuitionistic triangular fuzzy number. The proposed accuracy function is used to defuzzifying the assignment costs which are converted into crisp values. The optimum solution of the formulated problem is obtained through Licenced LINGO software using Branch and Bound method.

Keyword: Intuitionistic fuzzy assignment problem, optimal allocation, Branch and Bound method, Intuitionistic Triangular Fuzzy Numbers.

1 Introduction

Problems in the field of Agriculture, Engineering, Science and Management are analyzed from the framework of economic techniques and concepts. In order to meet growing demand of the expounding population globally, there arises a need of innovative methods of production. In the field of Agriculture, farmers face problems in relation to allocation of fertilizer & fields, manpower to fields, and optimal number of plants in the farm. Similarly, there are other fields of study where allocation problems are encountered - sales person to area, operator to machines, product to product mix. In fact every field of study lend themselves to allocation problems where resources allocation demands optimal use in order to achieve goal(s) of the organization. In Agriculture field, experiments in the field are common ways of research to understand myriad factor(s) which affect productivity and quality of produce. Planning is the most impact aspect of agriculture field driven experiments capable of study with aid of mathematical programming. A special branch of Mathematical programming popularly known as Assignment problems is highly useful to analyze problems in Agriculture. Assignment problems offer a unique arrangement whereby problems as resource allocation assignments are understood as matrix of allocation of job(s) to the number of persons. While doing so, it is made sure that the objective function is maximized (profit or sales) or minimized (cost).

A number of techniques within assignment problems framework offer solutions including Hungarian method, Neural Networks, Genetic Algorithm etc which are in vogue. The ultimate aim of assignment problem is to determine, optimum allocation of number of jobs (origins) to the equal number of persons (destinations) in such a way that the total cost or profit of allocation problem is minimum or maximum. In order to obtain the optimal allocation of assignment problem different algorithms / methods viz, Hungarian method, neural networks, genetic algorithm etc have been developed. [1] founded the first mathematical formulation of fuzziness. For real world problems a numerous attempt made by [2] to explore the ability of fuzzy set theory. [3] Introduced Intuitionistic fuzzy sets as an extension of Zadeh's notion of fuzzy set. In all divisions of decision making problems of Fuzzy methods have been developed in [4-6].The FLPP with fuzzy variables in parametric form is given in [7]. A fuzzy assignment model proposed by [8] and proved some theorems that considers all individuals to have same skills. The concepts of fuzzy set theory involving uncertainty and imprecision into the decision-making problems can be

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found in [9].
Fuzzy generalized assignment problem with credibility Constraints investigated by[10]. Various kinds of fuzzy assignment problems and Intuitionistic Fuzzy assignment problems has been discussed by several authors (see, [11-18]. On area compensation fuzzy sets and systems used Ranking and defuzzification methods and can be found in [19]. [20] presented Ranking of trapezoidal Intuitionistic fuzzy numbers. The Hungarian method for the assignment and transportation problems given by [21].

2 Preliminaries

Fuzzy set: Let A be a classical set, $\mu_{\bar{A}}(x)$ be a function from A to [0, 1].A fuzzy set $\bar{A}$ with the membership function $\mu_{\bar{A}}(x)$ is defined by $\bar{A} = \{(x, \mu_{\bar{A}}(x)); x \in A, \mu_{\bar{A}}(x) \in [0,1]\}$.

Intuitionistic Fuzzy set (IFS): Let X denote universe of discourse, then an intuitionistic fuzzy set $\bar{A}^I$ in X is given by $\bar{A} = \{(x, \mu_{\bar{A}^I}(x), v_{\bar{A}^I}(x)); x \in X,]\}$ where, $\mu_{\bar{A}^I}(x), v_{\bar{A}^I}(x))$: X□□0,1□□are functions such that 0 □□($\mu_{\bar{A}^I}(x) + v_{\bar{A}^I}(x)$) □□1 □for all x□X. For each x the membership function $\mu_{\bar{A}^I}(x)$ and $v_{\bar{A}^I}(x)$ represent the degree of membership and non-membership of the element x□X to A□□X respectively.

Intuitionistic fuzzy number (IFN). An intuitionistic fuzzy set of real line R is called an Intuitionistic fuzzy number if the following holds:

(i). There exists $x_o \in R$, $\mu_{\bar{A}^I}(x_o)$= 1 and $v_{\bar{A}^I}(x_o)$ = 0, x_o is called the mean value of $\bar{A}^I$.

(ii) $\mu_{\bar{A}^I}$ is a continuous mapping from R to the closed interval [0,1] and for all x□R, the relation $o \le \mu_{\bar{A}^I} + v_{\bar{A}^I} \le 1$holds.

Triangular intuitionistic fuzzy number (TrIFN):

A triangular intuitionistic fuzzy number $\bar{A}^I$is an intuitionistic fuzzy subset in R with the following membership function $\mu_{\bar{A}^I}(x)$and non-membership function $v_{\bar{A}^I}(x)$.

$$\mu_{\bar{A}^I}(x) = \begin{cases} 0, & x < a_1 \\ \dfrac{(x-a_1)}{a_2-a_1}, & a_1 \le x \le a_2 \\ \dfrac{(a_3-x)}{a_3-a_2}, & a_2 \le x \le a_3 \\ 0 & x > a_3 \end{cases}$$

$$v_{\bar{A}^I}(x) = \begin{cases} 1, & x < a_1' \\ \dfrac{(a_2-x)}{a_2-a_1'}, & a_1' \le x \le a_2 \\ \dfrac{(x-a_2)}{a_3'-a_2}, & a_2 \le x \le a_3' \\ 1 & x > a_3' \end{cases}$$

$a_1^{'} \le a_1 \le a_2 \le a_3 \le a_3^{'}$ and $\{\mu_{\bar{A}^I}(x), v_{A^{-I}}(x)\} \le 0.5$, $\mu_{\bar{A}^I}(x) = v_{\bar{A}^I}(x)$ for all x□R. The ***TrIFN*** is given by $\bar{A}^I = (a_1, a_2, a_3; a_1^1 a_2, a_3^1)$.

3 Intuitionistic Fuzzy Assignment Problem (IFAP):

Suppose there are M jobs to be performed and M persons are available for doing these jobs. Assume that each persons can do each job at a time, though with different. Let $\tilde{C}_{pq}$ be an Intuitionistic fuzzy cost of assigning $p^{th}$the person to the q^{th}job. Let the decision variable y_{ij} denoting he assignment of the $p^{th}$the person to the q^{th}job. The problem is to find an assignment (which job should be assigned to which person on one – one basis) so that total cost of performing all jobs is minimum. Problems of this kind are known as assignment problem. Mathematically an IFAP is given below:

$$\text{Minimize } Z = \sum_{p=1}^{M} \sum_{q=1}^{M} \tilde{C}_{pq} X_{pq}$$

Subject to

$$\sum_{i=1}^{M} X_{pq} = 1 \qquad for \ p = 1, 2, ..., M.$$

$$\sum_{j=1}^{M} X_{pq} = 1 \qquad for \ q = 1, 2, ..., M.$$

where X_{pq} =

$$\begin{cases} 1 & , if the p^{th} \text{the crop is assigned to the } q^{th} \text{paddock} \\ 0, & if the\ p^{th} \text{the crop is not assigned to the } q^{th} \text{paddock} \end{cases}$$

$$\tilde{C}_{pq}^{I} = (C_{pq}^1, C_{pq}^2, C_{pq}^3)(C_{pq}^{1'}, C_{pq}^2, C_{pq}^{3'})$$

Crops	**Paddocks**					
	1	***2***	⋯	***q***	⋯	***M***
1	$\tilde{a}_{11}$	$\tilde{a}_{12}$	⋯	$\tilde{a}_{1q}$	⋯	$\tilde{a}_{1M}$
2	$\tilde{a}_{21}$	$\tilde{a}_{21}$	⋯	$\tilde{a}_{2q}$	⋯	$\tilde{a}_{2M}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
P	$\tilde{a}_{p1}$	$\tilde{a}_{p1}$		$\tilde{a}_{pq}$		$\tilde{a}_{qM}$
⋮	⋮	⋮	⋮	⋮	⋮	
M	$\tilde{a}_{M1}$	$\tilde{a}_{M2}$	⋯	$\tilde{a}_{Mq}$	⋯	$\tilde{a}_{MM}$

4 Numerical Illustrations

Let us consider an IFAP, where a farmer intends to plant four different crops in each of four equal sized paddocks. The four different crops namely C1, C2, C3 and C4 are representing along rows and four equal sized paddocks like P1, P2, P3 & P4 are representing along Columns. The required nutrient requirements vary for different crops and the paddocks vary in soil fertility. Thus the total fertilizer cost which must be applied depends on which crop is grown in which paddock. Let the cost matrix be $[\widetilde{C}_{ij}]$ whose elements are given triangular Intuitionistic fuzzy numbers. The farmer's main objective is to determine the optimal allocation of assignment of paddocks to crops in such a way that the total fertilizer cost becomes Minimum. The parts of data has been taken from [23] and to defuzzify a triangular Intuitionistic fuzzy number an Accuracy function defined in [22].

Thus, the cost matrix obtained by using accuracy function

		Paddocks			
		P1	**P2**	**P3**	**P4**
Crops	**C1**	(7,9,11)(5,9,13)	(5,19,27)(1,19,32)	(10,23,30)(6,23,30)	(15,28,59)(11,28,59)
	C2	(7,8,15)(1,8,21)	(6,8,12)(4,8,19)	(4,14,28)(3,14,31)	(3,14,26)(3,14,27)
	C3	(4,8,21)(1,8,24)	(2,10,24)(1,10,27)	(8,20,23)(5,20,26)	(7,9,11)(5,9,13)
	C4	(2,13,25)(1,13,26)	(7,11,24)(4,11,27)	(11,12,19)(5,12,25)	(10,23,30)(6,23,30)

from [22]

$$\begin{bmatrix} 9.00 & 17.62 & 21.00 & 32.00 \\ 9.50 & 9.12 & 15.25 & 14.37 \\ 10.50 & 11.75 & 17.75 & 09.00 \\ 13.25 & 13.25 & 13.50 & 21.00 \end{bmatrix}$$

The Proposed Accuracy function is

$$H(\bar{a}^{*I}) = \frac{((a_3-a_1)+(a_2-a_1))+((a_3'-a_1')+(a_2-a_1'))}{4}$$

The cost matrix obtained by using proposed Accuracy function =

$$\begin{bmatrix} 4.50 & 21.25 & 18.50 & 30.50 \\ 9.00 & 6.75 & 18.25 & 17.25 \\ 12.75 & 16.25 & 15.75 & 04.50 \\ 17.75 & 12.75 & 9.00 & 18.50 \end{bmatrix}$$

Mathematically, the assignment problem using proposed accuracy function can be presented as:

MinZ=4.50 x_{11}+21.25x_{12}+18.50x_{13}+30.50x_{14}+9.00x_{21}+6.75x_{22}+18.25x_{23}+17.25x_{24}+12.75x_{31} +16.25x_{32}+15.75x_{33} +4.50x_{34} +17.75x_{41}+12.75x_{42}+9.00x_{43}+18.50x_{44}

Subject to the constraints:

$x_{11} + x_{12} + x_{13} + x_{14} = 1$
$x_{21} + x_{22} + x_{23} + x_{24} = 1$
$x_{31} + x_{32} + x_{33} + x_{34} = 1$
$x_{41} + x_{42} + x_{43} + x_{44} = 1$
$x_{11} + x_{21} + x_{31} + x_{41} = 1$
$x_{12} + x_{22} + x_{32} + x_{42} = 1$
$x_{13} + x_{23} + x_{33} + x_{43} = 1$
$x_{14} + x_{24} + x_{34} + x_{44} = 1$
$x_{ij} \in \{0,1\}$

5 Solution Procedures and Discussion

In order to get the optimal allocation, we solve the above problem using Branch and Bound Method.

Step1: Initially, no paddock is assigned to any crop, the assignment (σ) at the root node is 0 of the branching tree is a null set {ϕ} &, the subsequent lower level is also 0.

Step 2: Branching: the four different sub problems under the root node and the lower bounds for all.

Compute the lower bound P_{11}^{1}

$$V_\sigma = \sum_{i,j\in A} C_{i.j} + \sum_{i\in X} \left(\sum_{j\in Y} \min C_{i.j}\right)$$

$$Where \ \in \{1,1\}, \quad A = \{1,1\}, \quad X = \{2,3,4\},\ Y = \{2,3,4\}$$

$$V_{11} = C_{11} + \sum_{i\in(2,3,4)} \left(\sum_{j\in(2,3,4)} \min C_{i.j}\right)$$

$$P_{11}^{1} = C_{11} + \{\min(C_{22}, C_{23}, C_{24}) + \min(C_{32}, C_{33}, C_{34}) + \min(C_{42}, C_{43}, C_{44})\}$$

$$P_{11}^{1} = 4.5 + 6.75 + 4.50 + 9.0) = 24.75$$

$$P_{12}^{1} = 21.25 + \{\min(9.0,\ 18.25,\ 17.25) + \min(12.75,15.75,4.50) + \min(17.75,9.0,18.50)\} = 43.75$$

$$P_{13}^{1} = 18.5 + 6.75 + 4.5 + 12.75 = 42.50$$

$$P_{14}^{1} = 59.0$$

Step 3: Further branching is done from the terminal node which has the least lower bound i,e. P_{11}^{1} and the further branching can be done from this node. Eliminate the first row and first column.

$$P_{22}^{2} = 4.5 + 6.75 + \{\min(15.75,\ 4.5) + \min(9.0,18.50)\}$$
$$= 4.5 + 6.75 + 4.50 + 9.0) = 24.75$$
$$P_{23}^{2} = 4.5 + 18.25 + \{\min(16.25,\ 4.5) + \min(12.75,18.50)\}$$
$$= 4.5 + 15.75 + 4.50 + 12.75) = 40$$
$$P_{24}^{2} = 46.5$$

Step 4: Further branching can be done from P_{22}^{2} node. Eliminate the second row and second column

Similarly

$$P_{33}^{3} = 45.5, \quad P_{34}^{3} = 24.75, \quad P_{43}^{4} = 24.75$$

The above procedure can represented in tree form as

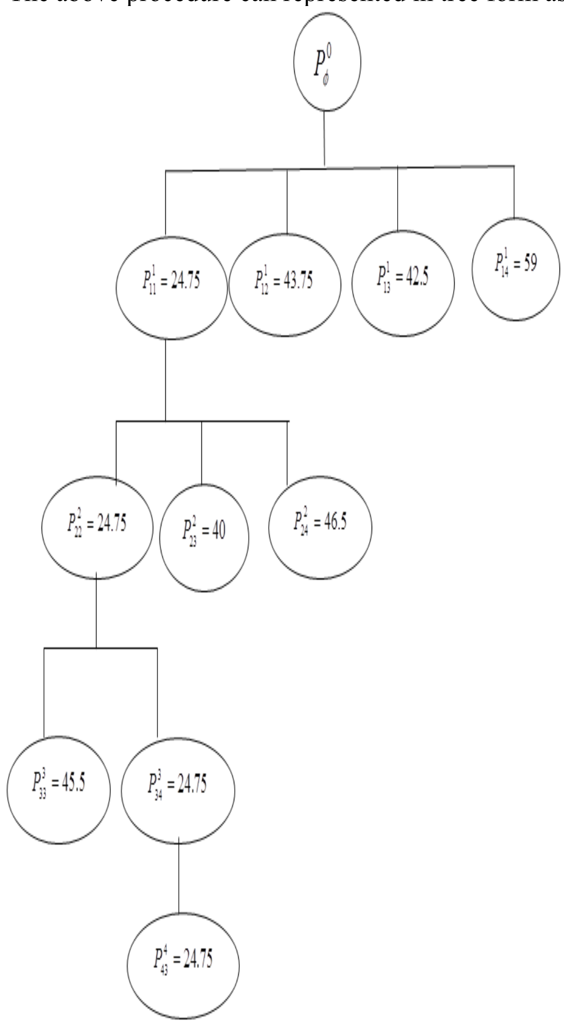

Fig1: Branch and Bound Tree.

The conventional assignment problem in the Linear programming problem form can be obtained by replacing above values for their corresponding values. After solving, we get the optimal solution 24.75and optimal allocation is(1,1), (2,2), (3,3), (4,4).

6 Discussions

We have study one of the realistic problem which is Intuitionistic fuzzy assignment. The optimal allocation of assignment problem which provides minimum fertilizer cost to the farmer when he/she faces problem in utilizing limited available resources are as: the paddock P1 is assigned to Crop1, the paddock P2 is assigned to Crop2, the paddock P3 is assigned to Crop3 and the paddock P4 is assigned to Crop4. Thus the optimal value of the said assignment problem obtained by using Branch and Bound Method is 24.75. Thus the proposed accuracy function provides better solution than the existing defined in [22]. However, the optimal allocation provides by both accuracy functions are same. Moreover, in real life situation, the sometimes non- integer optimal allocation provides infeasible solution. To avoid use of traditional method for rounding the non- integer optimal allocation to the nearest integer value, in that case Branch and Bound Method plays its important role. The solution of above Mathematical Assignment problem can also be obtained through TORA software.

7 Conclusions

In this paper, the assignment cost has been considered as an Intuitionistic fuzzy numbers. By defuzzifying, the assignment costs are converted into crisp values and the optimum solution is obtained by using Branch and Bound method. The optimal value obtained from proposed accuracy function is better than obtained from existing accuracy function.

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