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A Mathematical Model of Risk Factors in HIV/AIDS Transmission Dynamics: Observational Study of Female Sexual Network in India

S. Saravanakumar[1], *A. Eswari*[2], *L. Rajendran*[3] *and Marwan Abukhaled*[4,*]

[1]Department of Mathematics, Sri Ramakrishna Institute of Technology, Coimbatore, India
[2]Department of Mathematics, Agricultural Engineering College and Research Institute, Tamil Nadu Agricultural University, Coimbatore, India
[3]Department of Mathematics, AMET (Deemed to be University), Tamil Nadu, India
[4]Department of Mathematics and Statistics, American University of Sharjah, Sharjah, UAE

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Abstract: In this paper, a mathematical model for the transmission dynamics of HIV/AIDS epidemic with emphasis on the role of female sex workers is considered. The model is a system of nine nonlinear differential equations that represent nine different groups of an HIV population. A modified approach of the homotopy perturbation method is used to derive an approximate analytical expression for each of the nine different groups that form HIV population. The analytical results are shown to be consistent with the numerical results obtained by the highly accurate fourth-order Runge-Kutta method. The analytical solution will simplify studying the effect of each parameter on the governing equation and identifying the dynamics of HIV prevalence. Thus, effective prevention strategies can be adopted.

Keywords: Analytical solution, HIV transmission dynamics, Mathematical modeling, Prevention strategies.

1 Introduction

Understanding the transmission dynamics of HIV and its deadly consequences (AIDS) helps propose effective treatment for the infected patients and adopt effective prevention measures. Mathematical models of HIV transmission aim to describe the dynamics of the disease prevalence using systems of differential equations where the transition rates from a state to another are defined quantitatively [1]. These models, which are complex because of biological and behavioral variables incorporation, become fairly reliable if their validity is justified [2]. In literature, the four families of mathematical models that are used to model HIV transmission are stochastic, deterministic, statistical, and state-space model (Kalman filter model) [3].

Stochastic models assume that the response variables are random indexed by time, so the HIV epidemic is a stochastic process [3]. Stochastic difference and differential equations have been used as models for primary HIV infection [4], models for spread of HIV in a mobile heterosexual population [5] and as models to study the effects of antiretroviral therapy (ART) and HIV vaccines on HIV transmission using empirical data [6]. The fact that the variables involved in modeling infectious diseases are subject to random variation makes stochastic models more realistic than deterministic models. However, adopting stochastic models involves some obstacles. For example, closed form solutions are almost impossible to be found in real world applications [7] and numerical solutions in either weak or strong sense may become unstable over large interval domains [8].

In deterministic models, the population is divided into compartments consisting of those who are susceptible in each of the infection stages or in the AIDS phase. The movement between these compartments, which ranges from infected to progressing to AIDS is expressed in terms of systems of difference, differential or integral equations. In addition, deterministic models consider the biological, epidemiological and clinical aspects of the disease. Among deterministic models that are used to

* Corresponding author e-mail: mabukhaled@aus.edu

study the HIV infection and its consequences, we mention modeling the early stages of infection, the dynamics of prevalence, and control of HIV/AIDS [9–13].

Statistical models have also contributed to understanding the infection and the spread of HIV as well as its development to AIDS. Brookmeyer and Liao [14] proposed a back calculation method that aims to reconstruct the pattern of HIV infections in the past to predict the future cases and evaluate the impact of therapeutic advances on these cases. Downs et al. [15] applied an empirical Bayesian back-calculation method to AIDS incidence data to reconstruct and predict HIV/AIDS epidemic in the European countries. For more on statistical models see [16–18]

The iterative computational algorithm (the state space method or Kalman filtering method) aims to improve noisy measurements, current state estimates and forecasts calculations. Tan and We [19] employed the Kalman filtering method for simultaneous estimation of HIV infection and incubation distributions, the numbers of susceptible and infective people and AIDS cases. Cazelles and Chau [20] used the Kalman filter and dynamic models to assess the changing HIV/AIDS epidemic.

1.1 HIV/AIDS in India

The presence of HIV in India became evident when first cases were documented in the southern city of Chennai in 1986. Prevalence of HIV is broadly divided into three groups as per states and union territories [21]:

–High prevalence states: Forty-five districts in 6 HIV prevalence states fall in this category
–Moderate prevalence states: Three states fall into this group where the HIV prevalence is more than 5% among high risk groups.
–Low prevalence states: The remaining states in the country are classified within this group

Figures 1, 2 and 3 present review of HIV spread across India as a whole and within the states in addition to AIDS related deaths from 1982 to 2017 [22].

In 2002, it was estimated that around 500,000 were diagnosed as HIV positive in the southern Indian State of Tamil Nadu. The number of reported AIDS cases was 24667 in 2003 and 1092 cases in 1998. Heterosexuality is the main cause of HIV transmission. Commercial sex workers, homosexuals, and intravenous injection drug users are classified among the high risk groups, whereas migrant population workers of unorganized sectors, street children, and youth adolescent are among the most vulnerable groups [21].

The present paper aims to find an analytical solution to a mathematical model for the transmission dynamics of HIV/AIDS epidemic with emphasis on the role of female workers in the disease prevalence. It is believed that analytical solutions are more useful in the study of disease transmission dynamics than numerical solutions

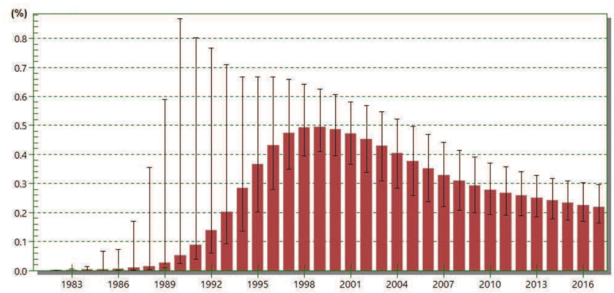

Fig. 1: Adult HIV prevalence in India from 1982 to 2017.

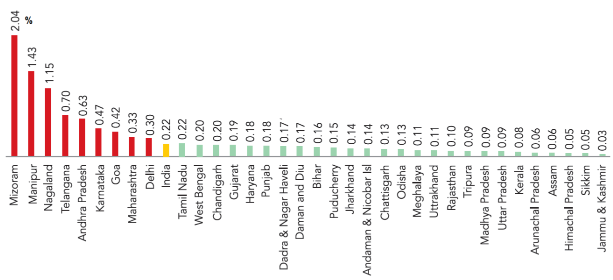

Fig. 2: State-wise adult HIV Prevalence in 2017.

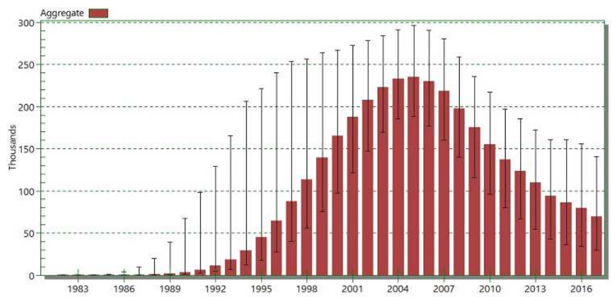

Fig. 3: AIDS related deaths over years from 1982 to 2017.

because they enable us to obtain quantitative information on the variables that cannot be measured easily. In this paper, we will employ a modified form of the powerful homotopy perturbation method [23–25] to solve the underlined nonlinear system modeling the dynamics of HIV transmission. Other methods that can be used to obtain analytical solutions include, but not limited to, the variational iteration method [26, 27], differential transformation method [28] and Green function based method [29–31].

This paper is organized as follows: Section 2 describes the mathematical model of HIV infection.

Section 3 addresses the analytical solution of the nonlinear model. In section 4, we present numerical simulation with discussion. Section 5 is devoted to conclusions.

2 Mathematical model for HIV infection

A sexually active population at time t denoted by $P(t)$ consists of the following nine mutually exclusive compartments: HIV-susceptible males, females, and female sex-workers denoted S_m, S_f, and S_{fs}, respectively, HIV-infected males, females, and female sex-workers denoted I_m, I_f an I_{fs}, respectively, AIDS-infected males, females, and female sex-workers denoted A_m, A_f and A_{fs}, respectively. The following system of deterministic nonlinear differential equations describes the model of HIV/AIDS infection [11]

$$\frac{dS_m}{dt} = \Lambda_1 - \beta_1 S_m I_f - \beta_2 S_m I_{fs} - \mu S_m, \quad (1)$$
$$\frac{dI_m}{dt} = \beta_1 S_m I_f + \beta_2 S_m I_{fs} - (b_1 + \mu) I_m, \quad (2)$$
$$\frac{dA_m}{dt} = b_1 I_m - (\mu + d) A_m, \quad (3)$$
$$\frac{dS_f}{dt} = \Lambda_2 - \beta_3 S_f I_m - \mu S_f, \quad (4)$$
$$\frac{dI_f}{dt} = \beta_3 S_f I_m - (b_2 + \mu) I_f, \quad (5)$$
$$\frac{dA_f}{dt} = b_2 I_f - (\mu + d) A_f, \quad (6)$$
$$\frac{dS_{fs}}{dt} = \Lambda_3 - \beta_4 S_{fs} I_m - \mu S_{fs}, \quad (7)$$
$$\frac{dI_{fs}}{dt} = \beta_4 S_{fs} I_m - (b_3 + \mu) I_{fs}, \quad (8)$$
$$\frac{dA_{fs}}{dt} = b_3 I_{fs} - (\mu + d) A_{fs}, \quad (9)$$

with the following initial conditions

$$S_m > 0,\ S_f > 0,\ S_{fs} > 0, \quad (10)$$
$$I_m \geq 0,\ I_f \geq 0,\ I_{fs} \geq 0, \quad (11)$$
$$A_m \geq 0,\ A_f \geq 0,\ A_{fs} \geq 0, \quad (12)$$

where Λ_1, Λ_2 and Λ_3 denote the recruitment rates of male, female and female sex-workers, respectively, $\beta_i, i = 1,2,3,4$ are the rates of transmission of infection from infective to susceptible, μ is the natural death rate constant, d is the disease induced mortality rate in AIDS classes, b_1, b_2, and b_3 are the progression rates from HIV infective male, female and female sex workers to respective AIDS class. The disease dynamic transmission is illustrated in Figure 4 [11].

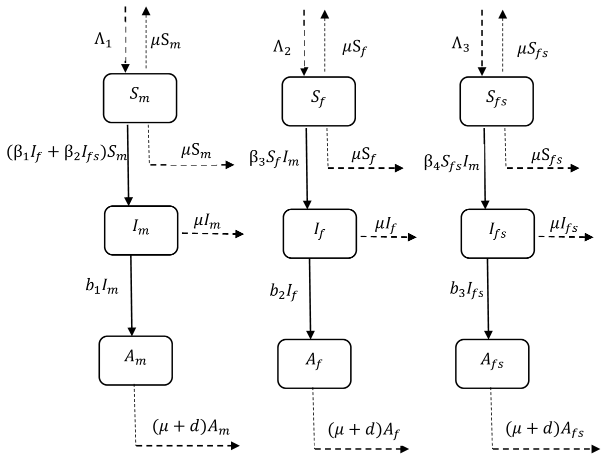

Fig. 4: Schematic illustration of the Dynamic transmission of the deterministic model.

3 Analytical solution of the nonlinear HIV/AIDS transmission dynamics

In this section, a new approach of the homotopy perturbation method (HPM) is employed to derive an analytical solution of the nonlinear system (1)-(12). The main idea of the HPM is presented in Appendix A. Constructing the homotopy (A.4) to each equation in the nonlinear system (1)-(12) gives

$$(1-p)\left[\frac{dS_m}{dt} + \mu S_m\right] + p\left[\frac{dS_m}{dt} - \Lambda_1 + (\beta_1 I_s + \beta_2 I_{fs} + \mu) S_m\right] = 0 \quad (13)$$

$$(1-p)\left[\frac{dI_m}{dt} + (b_1 + \mu) I_m\right] + p\left[\frac{dI_m}{dt} + (b_1 + \mu) I_m - (\beta_1 I_f - \beta_2 I_{fs}) S_m\right] = 0 \quad (14)$$

$$(1-p)\left[\frac{dA_m}{dt} + (\mu + d) I_m\right] + p\left[\frac{dA_m}{dt} - b_1 I_m + (\mu + d) A_m\right] = 0 \quad (15)$$

$$(1-p)\left[\frac{dS_f}{dt} + \mu S_f\right] + p\left[\frac{dS_f}{dt} - \Lambda_2 + \mu S_f - \beta_3 S_f I_m\right] = 0 \quad (16)$$

$$(1-p)\left[\frac{dI_f}{dt}+(b_2+\mu)IS_f\right] + p\left[\frac{dI_f}{dt}-\Lambda_2+\mu S_f-\beta_3 S_f I_m\right]=0 \quad (17)$$

$$(1-p)\left[\frac{dA_f}{dt}+(\mu+d)A_f\right] + p\left[\frac{dA_f}{dt}+(\mu+d)A_f-b_2 I_f\right]=0 \quad (18)$$

$$(1-p)\left[\frac{dS_{fs}}{dt}+\mu S_{fs}\right] + p\left[\frac{dS_{fs}}{dt}-\Lambda_3+\mu S_{fs}-\beta_4 S_{fs} I_m\right]=0 \quad (19)$$

$$(1-p)\left[\frac{dI_{fs}}{dt}+(b_3+\mu)I_{fS}\right] + p\left[\frac{dI_{fs}}{dt}+(b_3+\mu)I_{fS}-\beta_4 S_{fs} I_m\right]=0 \quad (20)$$

$$(1-p)\left[\frac{dA_{fs}}{dt}+(\mu+d)A_{fs}\right] + p\left[\frac{dA_{fs}}{dt}+(\mu+d)A_{fs}-b_3 I_{fs}\right]=0 \quad (21)$$

The solution to each equation in the system (13)-(21) is expressed in terms of power series, as follows:

$$S_m = S_{m0}+pS_{m1}+p^2S_{m2}+p^3S_{m3}+\cdots \quad (22)$$

$$S_f = S_{f0}+pS_{f1}+p^2S_{f2}+p^3S_{f3}+\cdots \quad (23)$$

$$S_{fs} = S_{fs0}+pS_{fs1}+p^2S_{fs2}+p^3S_{fs3}+\cdots \quad (24)$$

$$I_m = I_{m0}+pI_{m1}+p^2I_{m2}+p^3I_{m3}+\cdots \quad (25)$$

$$I_f = I_{f0}+pI_{f1}+p^2I_{f2}+p^3I_{f3}+\cdots \quad (26)$$

$$I_{fs} = I_{fs0}+pI_{fs1}+p^2I_{fs2}+p^3I_{fs3}+\cdots \quad (27)$$

$$A_m = A_{m0}+pA_{m1}+p^2A_{m2}+p^3A_{m3}+\cdots \quad (28)$$

$$A_f = A_{f0}+pA_{f1}+p^2A_{f2}+p^3A_{f3}+\cdots \quad (29)$$

$$A_{fs} = A_{fs0}+pA_{fs1}+p^2A_{fs2}+p^3A_{fs3}+\cdots \quad (30)$$

Back substitution of Eqs. (22)-(30) into Eqs. (13)-(21) and arranging the coefficients of the powers of p produce the following systems of differential equations:

$$\begin{aligned} p^0 &: \frac{dS_{m0}}{dt}+\mu S_{m0}=0 \\ p^1 &: \frac{dS_{m1}}{dt}+\mu S_{m1}+\beta_1 S_{m0}I_{f0}+\beta_2 S_{m0}I_{fs0}=0 \\ p^2 &: \frac{dS_{m2}}{dt}+\mu S_{m2}+\beta_1(S_{m0}I_{f1}+S_{m1}I_{f0}) \\ &\quad +\beta_2(S_{m0}I_{fs1}+S_{m1}I_{fs0})=0 \end{aligned} \quad (31)$$

$$\begin{aligned} p^0 &: \frac{dI_{m0}}{dt}+(b_1+\mu)I_{m0}=0 \\ p^1 &: \frac{dI_{m1}}{dt}+(b_1+\mu)I_{m1}-\beta_1 S_{m0}I_{f0}-\beta_2 S_{m0}I_{fs0}=0 \end{aligned} \quad (32)$$

$$\begin{aligned} p^0 &: \frac{dA_{m0}}{dt}+(\mu+d)A_{m0}=0 \\ p^1 &: \frac{dA_{m1}}{dt}+(\mu+d)A_{m1}-b_1 I_{m0}=0 \end{aligned} \quad (33)$$

$$\begin{aligned} p^0 &: \frac{dS_{f0}}{dt}+\mu S_{f0}=0 \\ p^1 &: \frac{dS_{f1}}{dt}-\Lambda_2+\mu S_{f1}+\beta_3 S_{f0}I_{m0}=0 \end{aligned} \quad (34)$$

$$\begin{aligned} p^0 &: \frac{dI_{f0}}{dt}+(b_2+\mu)I_{f0}=0 \\ p^1 &: \frac{dI_{f1}}{dt}+(b_2+\mu)I_{f1}-\beta_3 S_{f0}I_{m0}=0 \\ p^2 &: \frac{dI_{f2}}{dt}+(b_2+\mu)I_{f2}-\beta_3(S_{f0}I_{m1}+S_{f0}I_{m1})=0 \end{aligned} \quad (35)$$

$$\begin{aligned} p^0 &: \frac{dA_{f0}}{dt}+(\mu+d)A_{f0}=0 \\ p^1 &: \frac{dA_{f1}}{dt}+(\mu+d)A_{f1}-b_2 I_{f0}=0 \\ p^2 &: \frac{dA_{f2}}{dt}+(\mu+d)A_{f2}-b_2 I_{f1}=0 \end{aligned} \quad (36)$$

$$p^0 : \frac{dS_{fs0}}{dt} - \Lambda_3 + \mu S_{fs0} = 0$$
$$p^1 : \frac{dS_{fs1}}{dt} - \Lambda_3 + \mu S_{fs1} + \beta_4 S_{fs0} I_{m0} = 0 \quad (37)$$
$$p^2 : \frac{dS_{fs2}}{dt} + \mu S_{fs2} + \beta_4 (S_{fs0} I_{m1} + S_{fs1} I_{m0}) = 0$$

$$p^0 : \frac{dI_{fs0}}{dt} + (b_3 + \mu) I_{f0} = 0$$
$$p^1 : \frac{dI_{fs1}}{dt} + (b_3 + \mu) I_{fS1} - \beta_4 S_{fs0} I_{m0} = 0 \quad (38)$$
$$- \beta_4 (S_{fs0} I_{m1} + S_{fs1} I_{m0}) = 0$$

$$p^0 : \frac{dA_{fs0}}{dt} + (\mu + d) A_{fs0} = 0$$
$$p^1 : \frac{dA_{fs1}}{dt} + (\mu + d) A_{fs1} - b_3 I_{fs0} = 0 \quad (39)$$
$$p^2 : \frac{dA_{fs2}}{dt} + (\mu + d) A_{fs2} - b_3 I_{fs1} = 0$$

The solutions of the initial value systems (31)-(39) with initial conditions (10)-(12) are given by

$$S_{m0}(t) = le^{-\mu t}$$
$$S_{m1}(t) = \left(-\frac{\beta_1 lR}{\mu + b_2} - \frac{\beta_2 ly}{\mu + b_3} - \frac{\Lambda_1}{\mu} \right) e^{-\mu t} + \frac{\Lambda_1}{\mu}$$
$$+ \frac{\beta_1 lR}{\mu + b_2} e^{-(2\mu + b_2)t} + \frac{\beta_2 ly}{\mu + b_3} e^{-(2\mu + b_3)t} \quad (40)$$
$$S_{m2}(t) = \left(\frac{\beta_2 \beta_4 lL}{(b_3 - b_1 - \mu)(-2\mu - b_1)} \right) \times \left(e^{-\mu t} - e^{-(3\mu + b_1)t} \right)$$

$$I_{m0}(t) = Le^{-(b_1 + \mu)t}$$
$$I_{m1}(t) = \frac{\beta_1 lR}{b_1 - \mu - b_2} \left(-e^{-(b_1 + \mu)t} + e^{-(2\mu + b_2)t} \right) \quad (41)$$
$$+ \frac{\beta_2 ly}{b_1 - \mu - b_3} \left(-e^{-(b_1 + \mu)t} + e^{-(2\mu + b_3)t} \right)$$

$$A_{m0}(t) = ke^{-(d + \mu)t}$$
$$A_{m1}(t) = \frac{b_1 L}{b_1 - d} \left(e^{-(d + \mu)t} - e^{-(b_1 + \mu)t} \right) \quad (42)$$

$$S_{f0}(t) = Qe^{-\mu t}$$
$$S_{f1}(t) = \frac{\beta_3 QL}{\mu + \beta_1} (e^{-(b_1 + 2\mu)t} - e^{-\mu t}) + \frac{\Lambda_2}{\mu} (1 - e^{-\mu t}) \quad (43)$$

$$I_{f0}(t) = Re^{-(b_2 + \mu)t}$$
$$I_{f1}(t) = \left(\frac{\beta_3 QL}{b_2 - \mu - b_1} \right) (-e^{-(b_2 + \mu)t} + e^{-(b_1 + 2\mu)t})$$
$$I_{f2}(t) = \left(\frac{\beta_3 \Lambda_2 L}{\mu (b_2 - \mu - b_1)} \right) (e^{-(b_2 + \mu)t} - e^{-(b_1 + 2\mu)t})$$
$$- \left(\frac{\beta_3 \Lambda_2 L}{\mu (b_2 - b_1)} \right) (e^{-(b_2 + \mu)t} + e^{-(b_1 + \mu)t}) \quad (44)$$

$$A_{f0}(t) = Je^{-(d + \mu)t}$$
$$A_{f1}(t) = \left(\frac{b_2 R}{b_2 - d} \right) (e^{-(d + \mu)t} - e^{-(b_2 + \mu)t})$$
$$A_{f2}(t) = \frac{b_2 \beta_3 QL}{(b_2 - b_1 - \mu)(d - b_2)} (e^{-(\mu + d)t} - e^{-(b_2 + \mu)t})$$
$$- \frac{b_2 \beta_3 QL}{(b_2 - b_1 - \mu)(d - b_1 - \mu)} (e^{-(\mu + d)t} - e^{-(2\mu + b_1)t}) \quad (45)$$

$$S_{fs0}(t) = Xe^{\mu t}$$
$$S_{fs1}(t) = \frac{\Lambda_3}{\mu} (1 - e^{-\mu t}) + \frac{\beta_4 XL}{b_1 + \mu} (-e^{-\mu t} + e^{-(b1 + 2\mu)t})$$
$$S_{fs2}(t) = \frac{\beta_4 \Lambda_3 L}{\mu (\mu + b_1)} (e^{-\mu t} - e^{-(2\mu + b_1)t})$$
$$- \frac{\beta_4 \Lambda_3 L}{\mu + b_1} (e^{-(b_1 + \mu)} t - e^{-\mu t}) \quad (46)$$

$$I_{fs0}(t) = ye^{-(b_3 + \mu)t}$$
$$I_{fs1}(t) = \left(\frac{\beta_4 XL}{-\mu - b_1 + b_3} \right) (e^{-(b_1 + 2\mu)t} - e^{-(b3 + \mu)t})$$
$$I_{fs2}(t) = \frac{\beta_4 \Lambda_3 L}{\mu (b_3 - b_1 - \mu)} (e^{-(b_3 + \mu)t} - e^{-(2\mu + b_1)t})$$
$$- \frac{\beta_4 \Lambda_3 L}{\mu (b_3 - b_1)} (e^{-(b_3 + \mu)t} - e^{-(b_1 + \mu)t}) \quad (47)$$

$$A_{fs0}(t) = ze^{-(d + \mu)t}$$
$$A_{fs1}(t) = \left(\frac{b_3 \beta_4 XL}{(-\mu - b_1 + b_3)(d - b_3)} \right) (e^{-(b_1 + 2\mu)t} - e^{-(b3 + \mu)t})$$
$$S_{fs2}(t) = \frac{b_3 \beta_4 XL}{(b_3 - b_1 - \mu)(d - b_3)} (e^{-(\mu + d)} - e^{-(b_3 + \mu)})$$
$$- \frac{b_3 \beta_4 XL}{(b_3 - b_1 - \mu)(d - b_1 - \mu)} (e^{-(\mu + d)} + e^{-(2\mu + b_1)}) \quad (48)$$

Equations (40)-(48) are now used to construct the following analytical solutions

$$S_m(t) = \lim_{p\to 1} S_m(t) \approx S_{m0} + S_{m1} + S_{m2} \quad (49)$$

$$I_m(t) = \lim_{p\to 1} I_m(t) \approx I_{m0} + I_{m1} \quad (50)$$

$$A_m(t) = \lim_{p\to 1} A_m(t) \approx A_{m0} + A_{m1} \quad (51)$$

$$S_f(t) = \lim_{p\to 1} S_f(t) \approx S_{f0} + S_{f1} \quad (52)$$

$$I_f(t) = \lim_{p\to 1} I_f(t) \approx I_{f0} + I_{f1} + I_{f2} \quad (53)$$

$$A_f(t) = \lim_{p\to 1} A_f(t) \approx A_{f0} + A_{f1} + A_{f2} \quad (54)$$

$$S_{fs}(t) = \lim_{p\to 1} S_{fs}(t) \approx S_{fs0} + S_{fs1} + S_{fs2} \quad (55)$$

$$I_{fs}(t) = \lim_{p\to 1} I_{fs}(t) \approx I_{fs0} + I_{fs1} + I_{fs2} \quad (56)$$

$$A_{fs}(t) = \lim_{p\to 1} A_{fs}(t) \approx A_{fs0} + A_{fs1} + A_{fs2} \quad (57)$$

4 Numerical simulation and discussion

For numerical simulations, the following experimental values for the parameters are used [11]: $\Lambda_1 = 80, \Lambda_2 = 60, \Lambda_3 = 50, \beta_1 = 0.00005, \beta_2 = 0.0002, \beta_3 = 0.0001, \beta_4 = 0.00005, 0.0003, b_1 = 0.1, b_2 = 0.0924, b_3 = 0.25, \mu = 0.0743$, and $d = 0.123$. The analytical solution obtained by the proposed method is compared to a numerical solution obtained by the fourth order Runge-Kutta method using MATLAB's function ode45.

Figure 5 shows the analytical curves representing HIV-susceptible males (S_m), HIV-susceptible females (S_f), HIV-susceptible female sex-workers (S_{fs}), HIV-infected males (I_m), and AIDS-infected males (A_m). The analytical curves representing HIV-infected females (I_f), HIV-infected female sex-workers (I_{fs}), AIDS-infected females (A_f,) and AIDS-infected female sex-workers (A_{fs}) are depicted in Figure 6.

A comparison between the analytical and numerical curves for HIV infected male (I_m), AIDS infected male (A_m), HIV susceptible female (S_f, and HIV susceptible female sex worker (S_{fs}) is shown in Figures 7-10.

Figures 11 and 12 show the effects of the rate of transmission of infection, β_2, on the number of susceptible and infected males. Figure 13 indicates that a larger rate of transmission of infection leads to an increase in AIDS infected females as time increases. Figure 14 also reveals that the number of AIDS infected female sex workers increase with time when the rate of transmission increases.

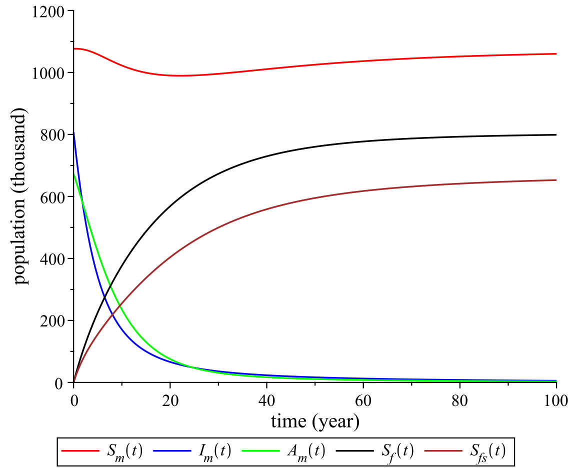

Fig. 5: Analytical curves for S_m, I_m, A_m, S_f and S_{fs}.

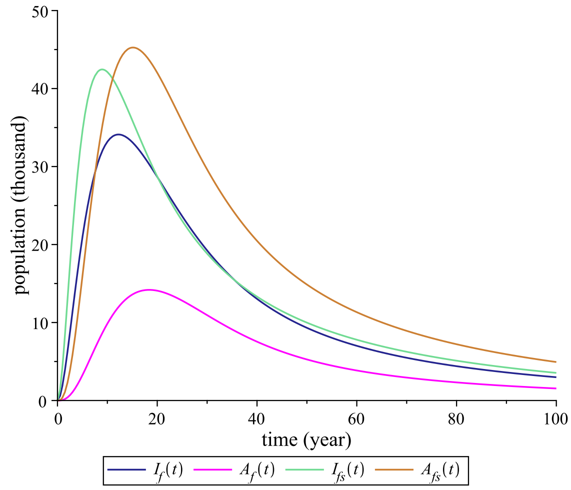

Fig. 6: Analytical curves for I_f, A_f, I_{fs} and A_{fs}.

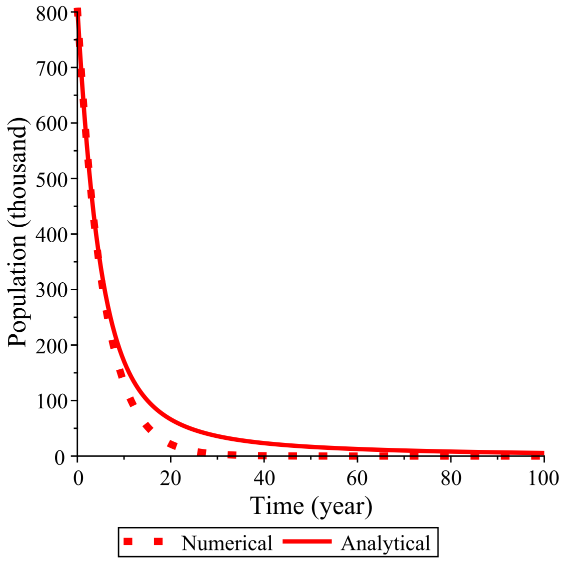

Fig. 7: Comparison between analytical and numerical solutions for HIV infected male (I_m).

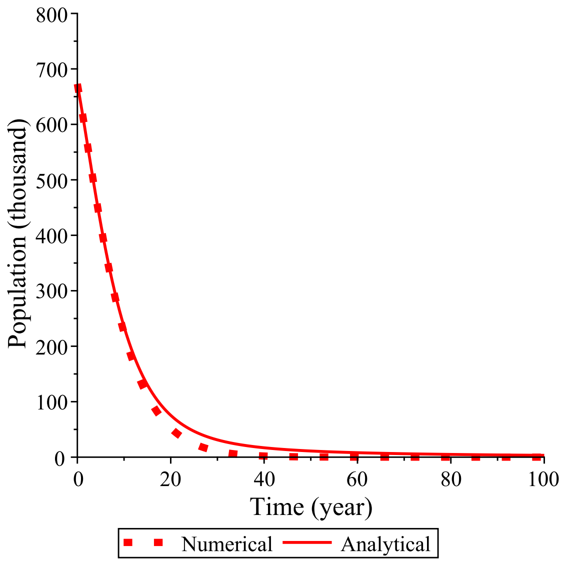

Fig. 8: Comparison between analytical and numerical solutions for AIDS infected male (A_m).

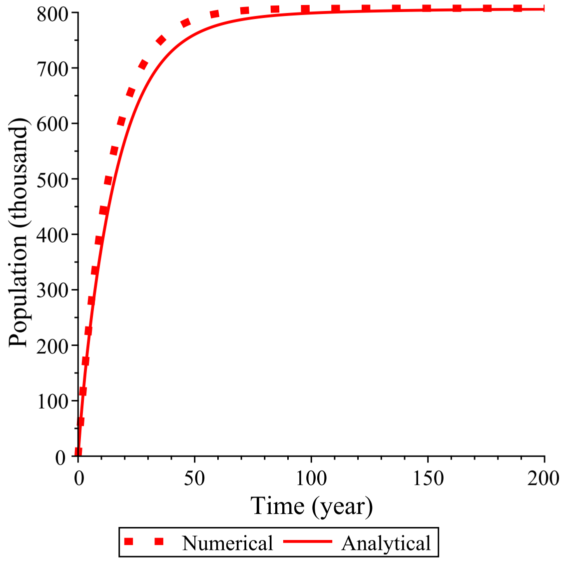

Fig. 9: Comparison between analytical and numerical solutions for HIV susceptible female (S_f).

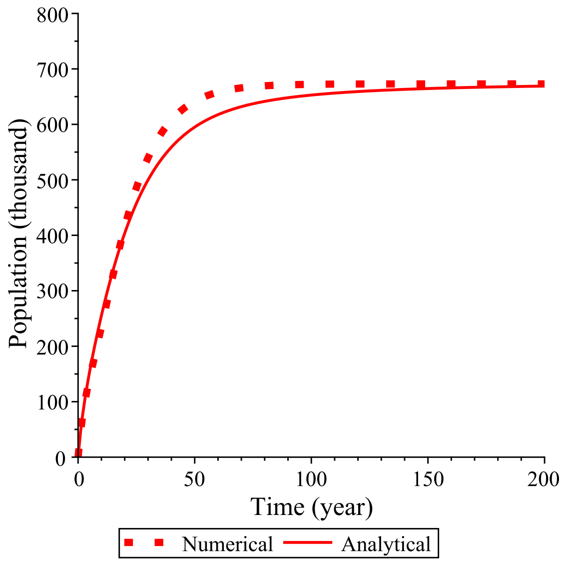

Fig. 10: Comparison between analytical and numerical solutions for HIV susceptible female sex worker (S_{fs}).

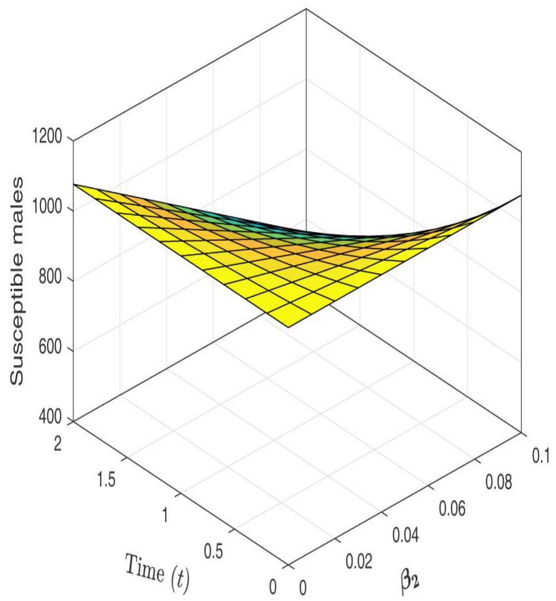

Fig. 11: Normalized analytical curve of 3-dim concentration of susceptible males as a function of time for different values of β_2.

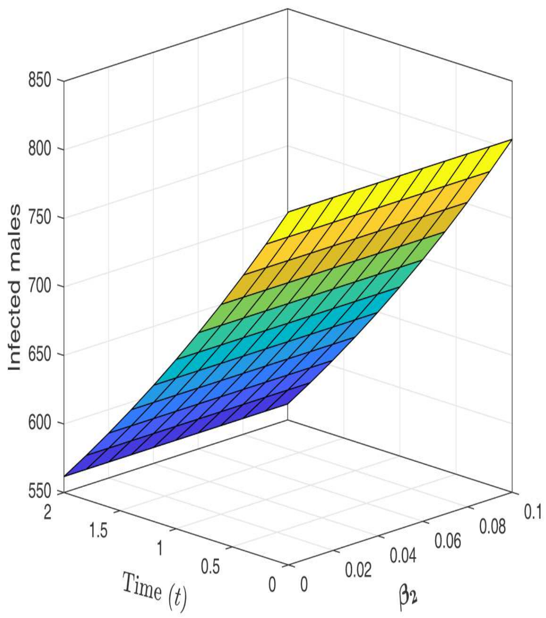

Fig. 12: Normalized analytical curve of 3-dim concentration of infected males as a function of time for different values of β_2.

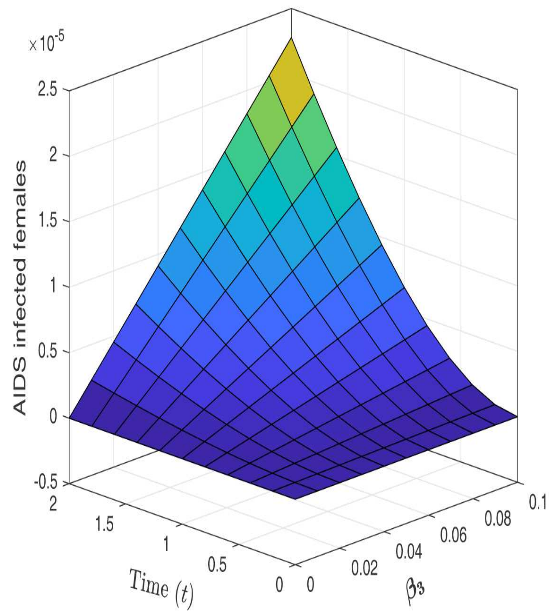

Fig. 13: Normalized analytical curve of 3-dim concentration of AIDS infected females as a function of time for different values of β_3.

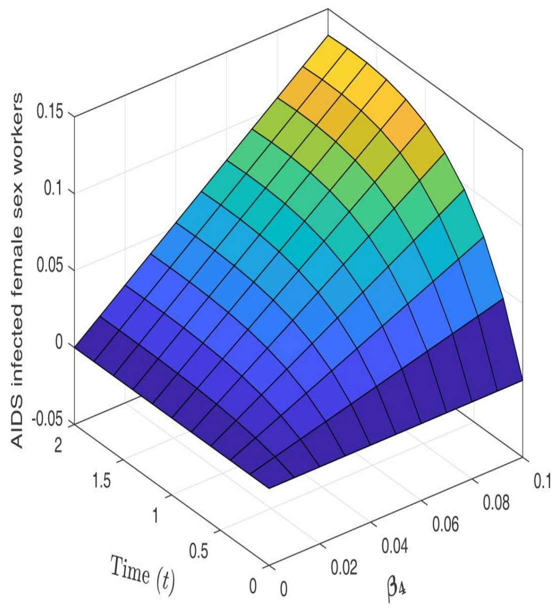

Fig. 14: Normalized analytical curve of 3-dim concentration of AIDS infected female sex workers as a function of time for different values of β_4.

5 Conclusion

A deterministic mathematical model of the HIV virus with nonlinear incidence function in a population was presented and analyzed. A modified version of the homotopy perturbation method was employed to obtain analytical expressions for the concentrations of HIV-susceptible male (S_m), female (S_f) and female sex-workers (S_{fs}), HIV-infected male (I_m) female (I_f) and female sex workers (I_{fs}), AIDS-infected males (A_M), female (A_f) and female sex workers (A_{fs}). The obtained analytical solution are consistent with the numerical results obtained by MATLAB ode45 routine. The obtained accurate analytical results can be employed to run sensitivity analysis of the parameters of the governing system to better understand the spread mechanism of the disease and suggest effective prevention measures.

Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this article.

Appendix A: The basic idea of HPM

Consider the nonlinear differential equation

$$A(u) - f(r) = 0, \quad r \in \Omega, \tag{A.1}$$

with the boundary condition

$$B\left(u, \frac{du}{dr}\right) = 0, \quad r \in \Gamma, \tag{A.2}$$

where $A, B, f(r)$ and Γ are a general differential operator, a boundary operator, a known analytical function and the boundary of the domain Ω , respectively. Expressing $A(u)$ as the sum of linear (L) and nonlinear (N) parts, Eq. (A.1) becomes

$$L(u) + N(u) - f(r) = 0. \tag{A.3}$$

The homotopy technique begins by defining $v(r, p) : \Omega \times [0,1] \to R$, such that

$$H(v, p) = (1-p)[L(v) - L(u_0)] + p[A(v) - f(r)] = 0, \tag{A.4}$$

where $p \in [0,1]$ is an embedding parameter and u_0 is an initial approximation of Eq. (A.1) that satisfies boundary conditions (A.2). Evidently, Eq.(A.4) implies that

$$H(v, 0) = L(v) - L(u_0) = 0, \tag{A.5}$$

$$H(v, 1) = A(v) - f(r) = 0. \tag{A.6}$$

As p changes from 0 to 1, $v(r, p)$ changes from u_0 to u_r, a process known as a homotopy. The solution of Eq. (A.4) may be expressed in terms of power series in the form:

$$v = v_0 + pv_1 + p^2 v_2 + \cdots. \tag{A.7}$$

With $p = 1$, an approximate solution to Eq. (A.4) is given by:

$$u = \lim_{p \to 1} v = v_0 + v_1 + v_2 + \cdots. \tag{A.8}$$

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S. Saravanakumar is an assistant professor of mathematics at Sri Ramakrishna Institue of Technology, Coimbatore, TN, India. He received his Ph.D in mathematics from Anna University, Chennai in 2020. His research interests include fuzzy sets and systems, numerical methods and intuitionistic fuzzy differential equations.

A. Eswari is an assistant professor of mathematics at Agricultural Engineering College and Research Institute, Tamil Nadu Agricultural University, Coimbatore, TN, India. She received her Ph.D. in mathematics from Manonmaniam Sundaranar University, Tirunelveli in 2013. Her research interests include mathematical modeling, stability analysis and fuzzy differential equations.

L. Rajendran is a senior Professor at the Department of Mathematics, Academy of Maritime Education and Training (Deemed to be University), Chennai, India. He has published over 200 articles in international and national journals and supervised more than 35 PhD students. He was a research fellow in Germany, Poland and UAE.

Marwan Abukhaled is a professor of mathematics at the American University of Sharjah. He received his PhD in mathematics from Texas Tech University, USA in 1995. His research interest include mathematical modeling, numerical analysis, optimal control and stochastic differential equations.