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Study of Disaster Probability when Strength Follows Power Function Distribution and Stress Follows Odd Generalised Exponential Gompertz Distribution(OGE-G)

Surinder Kumar, Vaidehi Singh and Prem Lata Gautam*

Department of Statistics, School for Physical and Decision Sciences, Babasaheb Bhimrao Ambedkar University, Lucknow, India - 226025

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Abstract: In this article, the probability of disaster is studied when the strength of the items follows power function distribution and the stress of the manufactured items/devices follows OGE-G distribution. In order to study the probability of disaster, a relationship between the parameters of OGE-G and power function distribution is established through the reliability measure $P = Pr(Y > X)$. Finally, through the relationship among the parameters involved in the model is used to get the optimum cost function when the cost function is linear in terms of parameters.

Keywords: Odd Generalised Exponential Gompertz distribution, Power function distribution, Probability of disaster, Stress-Strength reliability, Tolerance level and Optimum cost.

1 Introduction

For any complex manufactured system, the reliability of its component or the whole system is always a topic of discussion for the manufacturers as well as the buyers. Overestimation and underestimation of stress and strength of the components, items or systems may engender great losses in terms of system failures as well as human injuries. There are several statistical methods/models existing in the literature to study the reliability of a system. For example, $R(t) = P(X > t)$, where t is the given time, $P = Pr(X > Y)$, where X and Y represent the strength and stress of the model respectively, $P = Pr(X > \theta)$, where θ is the maximum range of the strength distribution etc. For a brief review, one may refer to Church and Harris (1970)[1], Enis and Geisser (1971)[2], Downton (1973)[3], Tong (1974)[4], Kelly et al. (1976)[5], Sathe and Shah (1981)[6], Chao (1982)[7], Awad and Gharraf (1986)[8], Kundu and Gupta (2005)[9] Raqab and Kundu (2005)[10], Kundu and Raqab (2009)[11], Chaturvedi and Sharma (2010)[12], Rezaei et al. (2010)[13].

In this study, the OGE-G lifetime model is considered, which has many advantages over the other well known life testing models such as Exponential, Generalised Exponential, Gompertz, Generalised Gompertz and Beta Gompertz distribution [see El-Damcese et al. (2015)[14]]. The probability density function (pdf) and cumulative density function (cdf) of the OGE-G distribution, which is considered here to represent the stress of the manufactured devices is defined as

$$f(x;\Theta) = \gamma\beta\lambda e^{cx} e^{\frac{\lambda}{c}(e^{cx}-1)} e^{-\gamma\left\{e^{\frac{\lambda}{c}(e^{cx}-1)}-1\right\}} \left[1 - e^{-\gamma\left\{e^{\frac{\lambda}{c}(e^{cx}-1)}-1\right\}}\right]^{\beta-1} \tag{1}$$

* Corresponding author e-mail: singhvaidehi92@gmail.com

and

$$F(x;\Theta)=\left[1-e^{-\gamma\left\{e^{\frac{\lambda}{c}(e^{cx}-1)}-1\right\}}\right]^{\beta} \quad ;x>0, \qquad \gamma,\lambda,\beta,c>0 \tag{2}$$

where $\Theta=(c,\gamma,\lambda,\beta)$ and c, γ, λ are the scale parameters and β is the shape parameter.

Let us assume that the strength of the manufactured items/devices follows the power function distribution with (pdf)

$$g(y;\theta,\mu)=\frac{\mu}{\theta}\left(\frac{y}{\theta}\right)^{\mu-1};0<y<\theta, \qquad \mu>0 \tag{3}$$

This paper has manifolds in Section 2; the theoretical expressions for the probability of disaster is obtained. In Section 4 and Section 5, the stress-strength reliability for the model $P=Pr(Y>X)$ is obtained when strength follows power function distribution and OGE-G distribution, respectively. In Section 3 and Section 6, the numerical study is done and the findings are discussed. Finally, the whole study is illustrated with an example in Section 7.

2 Probability of disaster i.e. $\alpha=P(X>\theta)$

Theorem 2.1: If the random variable X follows OGE-G distribution given at 1 and θ is the maximum range of a random variable Y which follows Power function distribution given at 3 respectively, then α is given by

$$\alpha=P(X>\theta)$$

$$=1-\left[1-e^{-\gamma\left\{e^{\frac{\lambda}{c}(e^{p}-1)}-1\right\}}\right]^{\beta}$$

where p=cθ.

Proof: We know that

$$\alpha=P(X>\theta)$$

$$=\int_{\theta}^{\infty}\gamma\beta\lambda e^{cx}e^{\frac{\lambda}{c}(e^{cx}-1)}e^{-\gamma\left\{e^{\frac{\lambda}{c}(e^{cx}-1)}-1\right\}}\left[1-e^{-\gamma\left\{e^{\frac{\lambda}{c}(e^{cx}-1)}-1\right\}}\right]^{\beta-1}dx \tag{4}$$

On taking $1-e^{-\gamma\left\{e^{\frac{\lambda}{c}(e^{cx}-1)}-1\right\}}$=z, in equation4, we get

$$\alpha=\int_{1-e^{-\gamma\left\{e^{\frac{\lambda}{c}(e^{c\theta}-1)}-1\right\}}}^{1} z^{\beta-1}dz$$

or

$$\alpha = 1 - \left[1 - e^{-\gamma \left\{ e^{\frac{\lambda}{c}(e^{p}-1)} - 1 \right\}} \right]^{\beta} \tag{5}$$

where, p=cθ.
Hence, the theorem follows.

3 Numerical study for the Probability of Disaster α for different values of c and λ

From the expression 5, which is established for measuring the probability of disaster $\alpha = P(X > \theta)$, the numerical values are obtained for different combinations of p, c and λ and are presented in Table 3.1. It can be easily interpreted from Table 3.1 that the probability of disaster decreases with an increase in the value of p. The probability of disaster means the stress increases over the strength, i.e. disaster will happen when $X > \theta$ [Alam and Roohi (2003)[15]]. Here, it is suggested that in order to overcome the problem of disaster (i.e. to attain the smallest value of $\alpha = P(X > \theta)$), the values of p = cθ, where c is the scale parameter of OGE-G distribution and θ is the scale parameter of the power function distribution, should be considered in such a manner that the value of α tends to zero.

Table 3.1
Numerical Values for Probability of Disaster $\alpha = P(X > \theta)$ and p for different values of c and λ at $\gamma = 0.05$ and $\beta = 0.05$

p	**$c = 2.5, \lambda = 0.05$**	$c = 0.5, \lambda = 0.05$	$c = 2.5, \lambda = 0.5$	$c = 1.5, \lambda = 0.5$	$c = 2.5, \lambda = 1.5$
0.25	**0.335146**	0.279042	0.253111	0.233093	0.208776
0.50	**0.306994**	0.247991	0.220257	0.198406	0.170982
0.75	**0.287748**	0.226410	0.196959	0.173166	0.142125
1.00	**0.272036**	0.208422	0.177056	0.150930	0.115321
1.25	**0.258129**	0.192081	0.158420	0.129358	0.087933
1.50	**0.245237**	0.176427	0.139912	0.107088	0.587346
1.75	**0.232917**	0.160847	0.120702	0.083123	0.028786
2.00	**0.220888**	0.144852	0.100061	0.056925	0.005489
2.25	**0.208942**	0.127973	0.077341	0.029524	0.000015
2.75	**0.184632**	0.089554	0.026261	0.000072	0
3.25	**0.158660**	0.042331	0.000040	0	0
4.00	**0.112657**	0.000001	0	0	0
4.25	**0.094092**	0	0	0	0
5.50	**0.000100**	0	0	0	0

The values of p at different tolerance level for α are presented in Table 3.2. These values have an interpretation that as the tolerance level α decreases, the corresponding values of p increases. Further, these values are utilised to have an idea to obtain the minimum cost, which is shown in Section 7.

Table 3.2
Values of p at different tolerance level α, when $\gamma = 0.05$ $\beta = 0.05$, c = 2.5 and $\lambda = 0.05$

α	0.05	0.02	**0.01**	0.001	0.0001	0.00001
$p = c\theta$	4.74943	5.06156	**5.18660**	5.39246	5.49117	5.55367

4 Stress-Strength Reliability when the random stress and strength follows OGE-G and Power function distribution

Theorem 4.1: Let $X \sim f(x;\Theta)$ and $Y \sim g(y;\theta,\mu)$, where X represents the stress and Y represents the strength, respectively, then $P = Pr(Y > X)$ is given by

$$P = \left[1 - e^{-\gamma\left\{e^{\frac{\lambda}{c}(e^{p}-1)}-1\right\}}\right]^{\beta} - \frac{1}{p^{\mu}}\int_0^{1-e^{-\gamma\left\{e^{\frac{\lambda}{c}(e^{p}-1)}-1\right\}}} \beta log\left[1+\frac{ct}{\lambda\gamma}\right] t^{\beta-1}dt$$

where p=cθ.

Proof: We know that

$$P = \int_0^{\theta}\int_x^{\theta} f(x;\Theta)g(y;\theta,\mu)dydx \quad (6)$$

$$= \int_0^{\theta}\int_x^{\theta} \gamma\beta\lambda e^{cx} e^{\frac{\lambda}{c}(e^{cx}-1)} e^{-\gamma\left\{e^{\frac{\lambda}{c}(e^{cx}-1)}-1\right\}}\left[1 - e^{-\gamma\left\{e^{\frac{\lambda}{c}(e^{cx}-1)}-1\right\}}\right]^{\beta-1} \frac{\mu}{\theta}\left(\frac{y}{\theta}\right)^{\mu-1} dydx \quad (7)$$

On substituting $y = vx$ in 7, we get

$$= \int_0^{\theta}\int_1^{\frac{\theta}{x}} \gamma\beta\lambda e^{cx} e^{\frac{\lambda}{c}(e^{cx}-1)} e^{-\gamma\left\{e^{\frac{\lambda}{c}(e^{cx}-1)}-1\right\}}\left[1 - e^{-\gamma\left\{e^{\frac{\lambda}{c}(e^{cx}-1)}-1\right\}}\right]^{\beta-1} \frac{\mu}{\theta}\left(\frac{vx}{\theta}\right)^{\mu-1} dvdx$$

$$= \int_0^{\frac{p}{c}}\int_1^{\frac{p}{cx}} \gamma\beta\lambda e^{cx} e^{\frac{\lambda}{c}(e^{cx}-1)} e^{-\gamma\left\{e^{\frac{\lambda}{c}(e^{cx}-1)}-1\right\}}\left[1 - e^{-\gamma\left\{e^{\frac{\lambda}{c}(e^{cx}-1)}-1\right\}}\right]^{\beta-1} \mu\left(\frac{x}{\theta}\right)^{\mu} v^{\mu-1} dvdx$$

$$= \int_0^{\frac{p}{c}} \gamma\beta\lambda e^{cx} e^{\frac{\lambda}{c}(e^{cx}-1)} e^{-\gamma\left\{e^{\frac{\lambda}{c}(e^{cx}-1)}-1\right\}}\left[1 - e^{-\gamma\left\{e^{\frac{\lambda}{c}(e^{cx}-1)}-1\right\}}\right]^{\beta-1} \left(\frac{x}{\theta}\right)^{\mu}\left[\left(\frac{p}{cx}\right)^{\mu}-1\right] dx \quad (8)$$

On substituting $1-e^{-\gamma\left\{e^{\frac{\lambda}{c}\left(e^{cx}-1\right)}-1\right\}}$=t in 8 and solving the above integrals, we get

$$P=\left[1-e^{-\gamma\left\{e^{\frac{\lambda}{c}\left(e^{p}-1\right)}-1\right\}}\right]^{\beta}-\frac{1}{p^{\mu}}\int_{0}^{1-e^{-\gamma\left\{e^{\frac{\lambda}{c}\left(e^{p}-1\right)}-1\right\}}}\beta log\left[1+\frac{ct}{\lambda\gamma}\right]t^{\beta-1}dt \tag{9}$$

Hence, the theorem follows.

Stress-strength reliability for the item is obtained for the fixed values of λ, c, γ, β and for different values of p and μ, from 9 and shown in Table 4.1. A brief interpretation of Table 4.1 is given in the Section 6.

Table 4.1
The Stress-Strength reliability of an item for $\lambda=0.05, c=2.5$, $\gamma=0.05, \beta=0.05$ and varying the values of p and μ

p	$\mu=2$	$\mu=4$	$\mu=6$	$\mu=8$	$\mu=10$
0.75	0.657647	0.615177	0.539674	0.405447	0.166822
1.00	0.683428	0.683428	0.683428	0.683428	0.683428
1.25	0.703253	0.717155	0.726053	0.731747	0.735392
1.50	0.719952	0.739292	0.747887	0.751707	0.753405
1.75	0.734846	0.756557	0.763646	0.765960	0.766716
2.00	0.748658	0.771499	0.777209	0.778636	0.778993
2.25	0.761833	0.785285	0.789918	0.790833	0.791014
2.75	0.787439	0.811675	0.814880	0.828017	0.815359
3.00	0.800316	0.824973	0.827713	0.841317	0.828051
3.25	0.813515	0.838706	0.841091	0.855434	0.841338
3.50	0.827258	0.853148	0.855261	0.855434	0.855448
3.75	0.841800	0.868613	0.855261	0.870655	0.855448
4.00	0.857448	0.885475	0.887226	0.887336	0.887343
4.25	0.874561	0.904172	0.905811	0.905902	0.905907
4.50	0.893495	0.925137	0.926699	0.926776	0.926780
4.75	0.914364	0.948474	0.949986	0.950053	0.950056
5.00	0.936185	0.972840	0.974306	0.974364	0.974367
5.25	0.954621	**0.992503**	**0.993877**	**0.993927**	**0.993929**
5.50	0.962751	**0.998695**	**0.999883**	**0.999923**	**0.999924**
5.75	0.965973	**0.998971**	**0.999969**	**0.999984**	**0.999998**
6.00	0.968750	**0.999132**	**0.999976**	**0.999996**	**0.999999**

5 Stress-Strength Reliabilty when both the random stress and strength follow OGE-G distribution

Theorem 5.1: Let X and Y be two independent random variables from OGE-G distribution, where X and Y are the stress and the strength, respectively, which the item/component faces, then $P = Pr(Y > X)$ is given by

$$P(Y>X) = 1 - \int_0^{\infty} \gamma_1 \beta_1 \lambda_1 e^{c_1 x} e^{\frac{\lambda_1}{c_1}(e^{c_1 x}-1)} e^{-\gamma_1\left\{e^{\frac{\lambda_1}{c_1}(e^{c_1 x}-1)}-1\right\}} \left[1 - e^{-\gamma_1\left\{e^{\frac{\lambda_1}{c_1}(e^{c_1 x}-1)}-1\right\}}\right]^{\beta_1 - 1} \left[1 - e^{-\gamma_2\left\{e^{\frac{\lambda_2}{c_2}(e^{c_2 x}-1)}-1\right\}}\right]^{\beta_2} dx$$

Proof: The pdfs of the random stress X and random strength Y are as follows:

$$f(x;\Theta) = \gamma_1 \beta_1 \lambda_1 e^{c_1 x} e^{\frac{\lambda_1}{c_1}(e^{c_1 x}-1)} e^{-\gamma_1\left\{e^{\frac{\lambda_1}{c_1}(e^{c_1 x}-1)}-1\right\}} \left[1 - e^{-\gamma_1\left\{e^{\frac{\lambda_1}{c_1}(e^{c_1 x}-1)}-1\right\}}\right]^{\beta_1 - 1} \quad (10)$$

and

$$f(y;\Theta) = \gamma_2 \beta_2 \lambda_2 e^{c_2 y} e^{\frac{\lambda_2}{c_2}(e^{c_2 y}-1)} e^{-\gamma_2\left\{e^{\frac{\lambda_2}{c_2}(e^{c_2 y}-1)}-1\right\}} \left[1 - e^{-\gamma_2\left\{e^{\frac{\lambda_2}{c_2}(e^{c_2 y}-1)}-1\right\}}\right]^{\beta_2 - 1} \quad (11)$$

The probability $P = Pr(Y > X)$ is obtained on solving the following integrals:

$$P(Y>X) = \int_0^{\infty} \int_x^{\infty} f(x;\Theta) f(y;\Theta)\, dy\, dx$$

$$= \int_0^\infty \gamma_1\beta_1\lambda_1 e^{c_1x} e^{\frac{\lambda_1}{c_1}(e^{c_1x}-1)} e^{-\gamma_1\left\{e^{\frac{\lambda_1}{c_1}(e^{c_1x}-1)}-1\right\}} \left[1-e^{-\gamma_1\left\{e^{\frac{\lambda_1}{c_1}(e^{c_1x}-1)}-1\right\}}\right]^{\beta_1-1}$$

$$\left[\int_x^\infty \gamma_2\beta_2\lambda_2 e^{c_2y} e^{\frac{\lambda_2}{c_2}(e^{c_2y}-1)} e^{-\gamma_2\left\{e^{\frac{\lambda_2}{c_2}(e^{c_2y}-1)}-1\right\}} \left[1-e^{-\gamma_2\left\{e^{\frac{\lambda_2}{c_2}(e^{c_2y}-1)}-1\right\}}\right]^{\beta_2-1} dy\right] dx$$

$$= \int_0^\infty \gamma_1\beta_1\lambda_1 e^{c_1x} e^{\frac{\lambda_1}{c_1}(e^{c_1x}-1)} e^{-\gamma_1\left\{e^{\frac{\lambda_1}{c_1}(e^{c_1x}-1)}-1\right\}} \left[1-e^{-\gamma_1\left\{e^{\frac{\lambda_1}{c_1}(e^{c_1x}-1)}-1\right\}}\right]^{\beta_1-1}$$

$$\left[1-\left[1-e^{-\gamma_2\left\{e^{\frac{\lambda_2}{c_2}(e^{c_2x}-1)}-1\right\}}\right]^{\beta_2}\right] dx$$

$$P = 1-\int_0^\infty \gamma_1\beta_1\lambda_1 e^{c_1x} e^{\frac{\lambda_1}{c_1}(e^{c_1x}-1)} e^{-\gamma_1\left\{e^{\frac{\lambda_1}{c_1}(e^{c_1x}-1)}-1\right\}} \left[1-e^{-\gamma_1\left\{e^{\frac{\lambda_1}{c_1}(e^{c_1x}-1)}-1\right\}}\right]^{\beta_1-1}$$

$$\left[1-e^{-\gamma_2\left\{e^{\frac{\lambda_2}{c_2}(e^{c_2x}-1)}-1\right\}}\right]^{\beta_2} dx \quad (12)$$

Hence, the theorem follows.

The equation 12 cannot be evaluated further. Thus, in order to study the probability of the stress-strength model, the expression 12 is evaluated with the help of Mathematica software. The behaviour of the model is evaluated through assigning the different values to the underlying parameters and the results are presented in Table 5.1

Table 5.1
Numerical values of the Stress-Strength model
$P(Y > X)$ at different values of underlying parameters

β_2	$\beta_1 = 1.50$	$\beta_1 = 1.25$	$\beta_1 = 0.75$	$\beta_1 = 0.50$	$\beta_1 = 0.25$
17.5	0.921053	0.933333	0.958904	0.972222	0.985915
20.0	0.930233	0.941176	0.963855	0.975612	0.987654
22.5	0.937500	0.947368	0.967742	0.978261	0.989011
25.0	0.943396	0.952381	0.970874	0.980392	**0.990099**
27.5	0.948276	0.956522	0.973451	0.982143	**0.990991**
30.0	0.952381	0.960000	0.975613	0.983607	**0.991736**
32.5	0.955882	0.962963	0.977444	0.984848	**0.992366**
35.0	0.958904	0.965517	0.979021	0.985915	**0.992908**
37.5	0.961538	0.967742	0.980392	0.986842	**0.993377**
40.0	0.963855	0.969697	0.981595	0.987654	**0.993789**
42.5	0.965909	0.971429	0.982659	0.988372	**0.994152**
45.0	0.967742	0.972973	0.983607	0.989011	**0.994475**
47.5	0.969388	0.974359	0.984456	0.989583	**0.994764**
50.0	0.970874	0.975615	0.985222	**0.990099**	**0.995025**
52.5	0.972222	0.976744	0.985915	**0.990566**	**0.995261**
55.0	0.973451	0.977778	0.986547	**0.990991**	**0.995475**
57.5	0.974576	0.978723	0.987124	**0.991379**	**0.995671**
60.0	0.975610	0.979592	0.987654	**0.991736**	**0.995851**
62.5	0.976563	0.980392	0.988142	**0.992063**	**0.996016**
65.0	0.977444	0.981132	0.988593	**0.992366**	**0.996169**
67.5	0.978261	0.981818	0.989011	**0.992647**	**0.996310**
70.0	0.979021	0.982456	0.989399	**0.992908**	**0.996441**
72.5	0.979730	0.983051	0.989761	**0.993151**	**0.996564**
75.0	0.980392	0.983607	**0.990099**	**0.993377**	**0.996678**
77.5	0.981013	0.984127	**0.990415**	**0.993597**	**0.996785**
80.0	0.981595	0.984615	**0.990712**	**0.993789**	**0.996885**

It is concluded from the above table that as the value of β_2 increases, the probability $P = Pr(Y > X)$ converges to one for decreasing values of β_1

6 Discussion

Any manufactured items or components has maximum limit of its strength. For example, in case of an electric bulb, its maximum voltage capacity is 220V, on the other hand, the accelerating capacity of an engine should not increase its maximum possible speed. Thus, it is desirable that the value of θ must have the maximum limit say, θ_0. For a fixed tolerance level α, suppose one wishes that θ_α is the required value of θ. In this particular case $\theta_\alpha < \theta_0$, one may obtain the desired value of μ say, μ_α, by using Table 4.1, so that the items or components are manufactured with the strength distribution parameters having parameters (μ_α, θ_α) and subsequently the required strength reliability may be achieved. In case, $\theta_\alpha > \theta_0$ then one will have to either adjust α or need some alterations in the manufactured items or components.

7 An illustrative example

Without loss of generality, let us suppose that the maximum possible value of p is 6.0. For $\alpha \leq 0.01$, we must have $p \geq 5.25$. Since p cannot exceed 6.0, we have the option of fixing the item in such a way that $5.25 \leq p \leq 6.00$ i.e. $2.0 \leq \theta \leq 2.2$ and corresponding value of μ leads to a maximum of $P(Y > X)$.
Let C_1 be the cost of adjusting one unit of θ and C_2 be the cost of adjusting one unit of μ.
Minimize $C = C_1\theta + C_2\mu$ subject to $2.1 \leq \theta \leq 2.4$ and $P(Y > X) \geq 0.99$. The problem may be solved analytically as follows:

Using Table 4.1, for p = 5.25, 5.50, 5.75, 6.00, i.e. θ = 2.1, 2.2, 2.3, 2.4 and finding those values of μ for which $P(Y > X) \geq 0.99$. Evaluating the cost function for each pair of (θ, μ):

Table 6.1
Table for obtaining cost of manufacturing item

θ	μ	$C_1\theta + C_2\mu$	θ	μ	$C_1\theta + C_2\mu$
2.1	**4**	$\mathbf{2.1C_1 + 4C_2}$	2.3	4	$2.3C_1 + 4C_2$
2.1	6	$2.1C_1 + 6C_2$	2.3	6	$2.3C_1 + 6C_2$
2.1	8	$2.1C_1 + 8C_2$	2.3	8	$2.3C_1 + 8_C2$
2.1	10	$2.1C_1 + 10C_2$	2.3	10	$2.3C_1 + 10C_2$
2.2	4	$2.2C_1 + 4C_2$	2.4	4	$2.3C_1 + 4C_2$
2.2	6	$2.2C_1 + 6C_2$	2.4	6	$2.4C_1 + 6C_2$
2.2	8	$2.2C_1 + 8C_2$	2.4	8	$2.4C_1 + 8C_2$
2.2	10	$2.2C_1 + 10C_2$	2.4	10	$2.4C_1 + 10C_2$

Clearly, the minimum of the cost lies at $2.1C_1 + 4C_2$ depending upon the numerical values of C_1 and C_2.

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Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this article.

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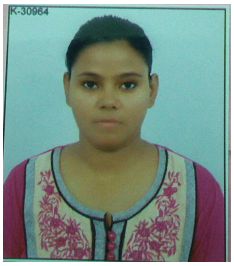
Surinder Kumar, Professor, Department of Statistics, BBAU (A Central University) Lucknow, India. He has 23 years research experience in various research fields of Statistics such as Sequential Analysis, Reliability Theory, Business Statistics and Bayesian Inference. Prof. Kumar has published more than 45 research papers in various journals of national and international repute.

Vaidehi Singh, Dept. of Statistics, BBAU (A Central University), Lucknow - India. She has research experience of 3 years, her research areas are Sequential Analysis, Reliability Theory and Bayesian Inference. Presently, she is working as an Assisant Professor in the Department of Mathematics and Statistics, in Dr. Shakuntala Misra National Rehabilitation University, Lucknow.



Prem Lata Gautam, is the research scholar of the Dept. of Statistics, BBAU (A Central University), Lucknow, India. She has research experiences of 3 years in the field of Sequential analysis, Bayesian estimation and Reliability theory and wholesome knowledge of many softwares and languages like R Software, Mathematica and Fortron.