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Optimal Discrete Search for a Randomly Moving COVID19

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Abstract: In this paper, we subedit a search for a randomly moving Coronavirus (COVID-19) among a finite set of different states. We use a monitoring system to search for COVID-19 which is hidden in one of the n cells of the respiratory system in the human body in each fixed number of time intervals m. The expected rescue time of the patient and detecting COVID-19 has been obtained. Also, we extend the results and obtain the total optimal expected search time of COVID-19. The optimal search strategy is derived suing a dynamic programming algorithm. An illustrative real life example introduced to clear the applicability of this model.

Keywords: Monitoring system, COVID-19, Moving target, Optimal search policy, Imperfect sensor.

1 Introduction

Since the beginning of 2020, mankind has faced one of the biggest global disasters, which is the spread of Coronavirus (COVID-19) worldwide. The question we think about a lot now is: How does the Coronavirus attack the human body?

COVID-19 is a respiratory virus that affects the upper and lower respiratory tracts and as same as other respiratory viruses, but the difference is that it is highly contagious and does not spread through droplets from an infected person to a healthy person only, but also through surfaces and places with a spray, because it lives for hours on the places where it spread. And enter through the nose, eye and mouth.

The virus needs a cell in order for its growth to complete, it is possible that the cell is in the throat or certain receptors within the respiratory system, where it penetrates the receptors and attaches to them between the virus cell and the respiratory system cell, then growth and replication of the DNA cells occurs, and the virus begins to multiply and multiply with millions and its symptoms appear in the body including ”sore throat, difficulty breathing, and dry cough”.

Our mission in this paper is to calculate the expected total rescue time of the patient and detecting COVID-19 before multiplying with millions and its side effects appear on the human body. Also, we will obtain the total optimal excepted search time of COVID-19.

In our daily lives, we all suffer from the disappearance of some important things that require us to search for them and find them as quickly as possible, different models that have been studied previously using different search strategies and are also interesting at the same time. All this was aimed for finding the lost target quickly and with the minimum cost. To mention some important models, better to start with a linear search strategy, which has many life and mission applications, for example, searching for a damaged until in a large linear system (electrical power lines, telephone lines and petrol or gas support lines), whether this linear system is independent or intersecting (see A. B. El-Rayes, A. A. Mohamed, and Fergani [1], A. B. El-Rayes, A. A. Mohamed [2], A. A. Mohamed [3], A. A. Mohamed and H. M. Abou Gabal [4,5,6], Zaid [7]). Also, the coordinated search technique has been studied in case of linear search by many authors (see A. A. Mohamed, H. M. AbouGabal, and W. A. Afifi [8,9,10]), and discussed the coordinated search technique for a located target on two intersected lines, when the located target has symmetric and unsymmetrical distribution, A. A. Mohamed and W. A. Afifi [11], W. A. Afifi, El-Bagoury and S. N. Al-Aziz [12], presented more advanced work and use coordinated

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search technique for a moving target on one line and many independent lines, respectively. Recently, El-Hadidy, and El-Bagoury [13,14], A. A. Mohamed and El-Bagoury [15] and M. A. Kassem, El-Bagoury, Afifi and Alaziz [16], studied a more sophisticated search model in the three dimensional space to find a 3-D randomly located target by one searcher, two searchers and four searchers.

The discrete search problems are not new. In this technique a target is assumed to be in one of several cells and the searcher must distribute his effort (time, energy, etc.) among the cells to find the target. The probability of being in a certain cell at a certain time and the detection function are supposed to be known to the searcher, the searcher wishes to find the optimal distribution of the effort over the set of possible cells such that the overall probability of detecting the target is maximal [17].

In this paper, we consider the monitoring system with the main aim that the system is to detect the lost moving COVID-19 between n cells according to some random motion. For every m time period, one cell is occupied by COVID-19. We assume that the searches in different time intervals are independent. Place of COVID-19 is uncertain, so there are some previous probability distribution that can be quantified using some previous information. The n locations in each time interval i are searched sequentially by the sensor, where $i = 1,2,...,m$ (i. e., the sensor searches sequentially all the locations from cell 1 to n_i cell in each time interval i), is imperfect and therefore, the outgoing signals from the sensor are subject to errors. Verification of positive detections using sensor only from the investigation team is imperfect in order to verify whether it is true or false. This type of search takes time and the objective is to find a search policy that minimizes the expected search time until COVID-19 is found.

Here we have two cases:

Case 1: The total tie of detecting COVID-19 by the monitoring system in any time interval i is less than or equal to the time of existing COVID-19 in this time interval before it moves to the next time interval $i+1$.

Case 2: The total time of detecting COVID-19 by the monitoring system in any time interval i is more than the time of existing COVID-19 in this time interval then COVID-19 moves to the next time interval $i+1$ before the surveillance system has finished its search in time interval i.

2 Search plan

COVID-19 is assumed to be in one of several cells not necessarily identical regions. Let the number of cells be n. Through each of m time intervals, only one state is being gilled with COVID-19. In our model let COVID-19 be hidden in one of cells of the nasopharynx and located in each one of n possible area cells AC_{j_i} where $i = 1,2,...,m$ and $j = 1,2,...,n$ (e. g., the respiratory system) during each of m time intervals. COVID-19 is randomly moving from one cell to another in each time interval i where it can change its place (cell) in each time interval i and occupies a new cell during each of m time intervals and the searches in each time intervals are independent of searcher's action. Our aim is to detect COVID-19 as quickly as possible. In each time interval, the imperfect sensor searches the n ACs sequentially.

After locating the place, where COVID-19 is in the AC_{j_i} (e. g., the affecteed cell) using the imperfect sensor. To investigate the detection, a rescue team (PCR test) is sent to that cell, the test detects the virus's DNA through "polymerase chain reaction" so that the virus's genome is determined. If PCR test is positive, then we have three kinds of detection as following:

1- Perfect Detection: The sensor identifies correctly the place (the affected cell), where COVID-19 is kept.
2- Partial Detection: The sensor correctly identifies the AC_{j_i}, where COVID-19 is held, but incorrectly identifies the specific place captivity.
3- False Detection: COVID-19 is not hidden in the AC_{j_i}, where the sensor has recorded a detection.

In the following Figure (1), the affected part in human naspharynx is divided into 10 ACs, and a search in AC8 in a certain time interval $i = 3$ yields a detection at a specific cell. The rescue team completes the search for COVID-19 in the rest of AC_{j_i} even if it was sent to the wrong address in AC_{j_i} from the beginning, and if the COVID-19 is found there by the team then it will be the case of partial detection. If COVID-19 is not detected in AC_{j_i} (the case of false detection), the AC_{j_i} will be permanently erased by the sensor until the end of the search.

Suppose that AC_{j_i}, where $i = 1,2,...,m$ and $j = 1,2,...,n$, is the cell which into stat j at time interval i, and let θ_i be the parameter description of AC_{j_i}, where COVID-19 exists in the time interval, that is, $\theta_i = j_i$ when COVID-19 is contained in AC_{j_i}.

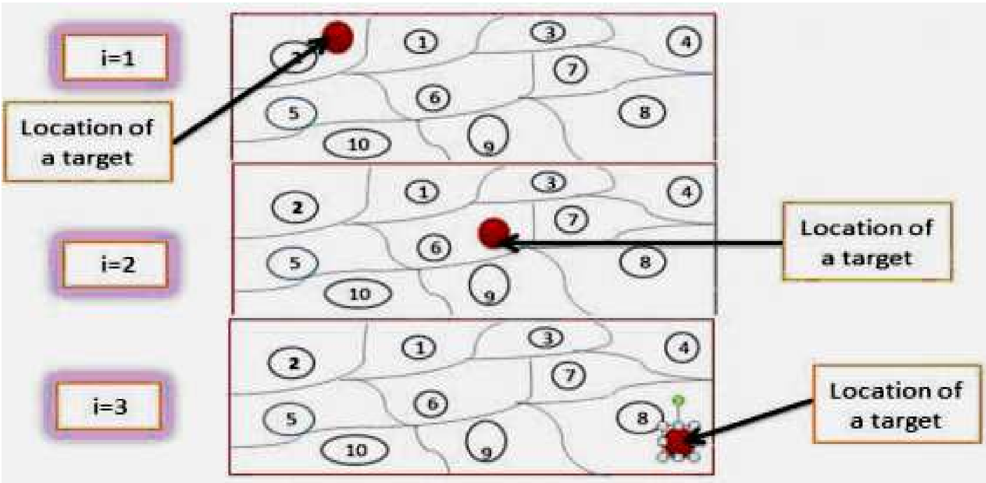

Fig. 1: The part in human nasopharynx is divided into 10 ACs, COVID-19 moves from one cell to another in every time interval until it is discovered.

Let the prior probability mass function (p. m. f.) of θ_i be

$$\pi_i : (1_i, 2_i, ..., n_i) \to [0,1].$$

We write $\pi_{j_i} = P(\theta_i = j_i)$.

The sensor is incomplete (imperfect) as we have seen before. Suppose that $p_{j_i} = P$ (sensor indicates detection in $AC_{j_i} \backslash \theta_i = j_i$): that is p_{j_i} is the probability that the sensor is correctly identified the AC_{j_i}, where COVID-19 is hidden at time interval i.

Let $r_{j_i} = P$ (The location and detection are correctly determined by the sensor in $AC_{j_i} \backslash \theta_i = j_i$).

It is clear; $r_{j_i} \leq p_{j_i}$ and $p_{j_i} - r_{j_i}$ is the probability that the wrong place of COVID-19 was located in the AC_{j_i} by the sensor, where COVID-19 is hidden.

Also, let $q_{j_i} = P$ (detection indicated by the sensor in $AC_{j_i} \backslash \theta_i \neq j_i$); $1 - q_{j_i}$ is the sensor specificity in AC_{j_i}. Also, while keeping the generality, we assume that $p_{j_i} \geq q_{j_i}$.

Given a prior p. m. f. π_i, we choose an action $a_i(\pi_i) \in \{1, 2, ..., n_i\}$ that select that AC, begin searching in the next time interval i. An action $a_i(\pi_i) \neq \theta_i$ leads us to one of two following results: a false detection or no detection. Following either of these outcomes, posterior probabilities are obtained and the prior p. m. f. of θ_i are updated. In the case of a no detection, the posterior p. m. f. $\Pi^-_{a_i,j_i}(\pi_i) = (\Pi^-_{a_i,1}, \cdots, \Pi^-_{a_i,n_i})$ is given by

$$\Pi^-_{a_i,j_i}(\pi_i) = \begin{cases} \frac{(1-p_{a_i})\pi_{a_i}}{1-p_{a_i}\pi_{a_i}-q_{a_i}(1-\pi_{a_i})}, & \text{if} \quad j_i = a_i, \\ \frac{(1-q_{a_i})\pi_{a_i}}{1-p_{a_i}\pi_{a_i}-q_{a_i}(1-\pi_{a_i})}, & \text{if} \quad j_i \neq a_i. \end{cases} \tag{1}$$

The posterior p. m. f. is represented by $\Pi^+_{a_i,j_i}(\pi_i) = (\Pi_{a_i,1}, \cdots, \Pi^+_{a_i,n_i})(\pi_i)$ when COVID-19 detection is false.

$$\Pi^+_{a_i,j_i}(\pi_i) = \begin{cases} 0 & \text{if} \quad j_i = a_i \\ \frac{\pi_{j_i}}{1-\pi_{a_i}} & \text{if} \quad j_i \neq a_i \end{cases} \tag{2}$$

On the other case, no detection or false detection, we update the prior π_i by $\Pi^-_{a_i}(\pi_i)$ and $\Pi^+_{a_i}(\pi_i)$, respectively. We must obtain a sequence of priors to get a true detection.

Let c_{j_i} be the time which the sensor needs to search AC_{j_i}. In perfect detection case, the rescue mission can be completed by rescue team in $C^{(1)}_{j_i}$ time units. In partial detection case, the rescue mission length is $C^{(2)}_{j_i} > C^{(1)}_{j_i}$. The total verification time by the resuce team in AC_{j_i}, before it is declared to be clear, is $C^{(3)}_{j_i}$ in false detection.

The goal of the monitoring system is to minimize the expected total time which it takes to rescue the patient and detect COVID-19. Before we introduce the optimal search plan, we need to clear that the total time it takes to rescue the patient consists of two parts:

1- **Search Time:** The time taken to detect the AC_{j_i} in which COVID-19 exists in time interval i, including all verification time $C^{(3)}_{j_i}$ spent in a wrong AC_{j_i} following a false detection.

2- Rescue Time: The time used for finding and rescuing the patient after locating the AC_{j_i} in which COVID-19 is hidden, either by partial or perfect detection.

We suppose that the time taken to search COVID-19, by the monitoring system, in any time interval i is less than or equal the time of existence of COVID-19 in this time interval before it moves to the next time interval $i+1$. Since θ_i denotes the AC_{j_i} that contains COVID-19 in time interval I, conditional on $\theta_i = j_i$, the rescue time takes on values $C_{j_i}^{(1)}$ or $C_{j_i}^{(2)}$ depending on whether the detection in AC_{j_i} is perfect or partial. Therefore, the conditional expected rescue time is equal to

$$\frac{r_{j_i}}{p_{j_i}} C_{j_i}^{(1)} + \frac{p_{j_i} - r_{j_i}}{p_{j_i}} C_{j_i}^{(2)}. \tag{3}$$

Because at the beginning of the search, there is probability π_{j_i} that COVID-19 is hidden in AC_{j_i}, the expected total rescue time is

$$\sum_{i=1}^{m} \sum_{j=1}^{n} \pi_{j_i} \left(\frac{r_{j_i}}{p_{j_i}} C_{j_i}^{(1)} + \frac{p_{j_i} - r_{j_i}}{p_{j_i}} C_{j_i}^{(2)} \right), \tag{4}$$

which is a constant in the search plane. We can introduce other important objective function, which minimize the expected search time until correctly detecting the AC_{j_i} in which COVID-19 is hidden, either by a partial or a perfect detection. To simplify the notation, let $C_{j_i} = C_{j_i}^{(3)}$.

3 The optimal search plan

A search plan σ i a sequence of actions adapted to the sequence of priors, in which each action depends only on the latest prior p. m. f. π_i. Let $T_\sigma(\pi_i)$ be the expected search time until detecting the correct AC_{j_i}, if π_i and the searcher follows search policy σ. Given two policies σ_1 and σ_2, we write $\sigma_1 \geq \pi_i \sigma_2$, when $T_{\sigma_1}(\pi_i) \leq T_2(\pi_i)$.

Following Lemma is useful in proving Theorem (3).

Lemma 3.1.

Consider two policies in each time interval

$$\delta_{1i} = \begin{pmatrix} j_i, & k_i, & a_{3_i}, & a_{4_i}, & \cdots \\ k_i, & G_i, & G_i, & G_i, & \cdots \end{pmatrix}$$

and

$$\delta_{2i} = \begin{pmatrix} k_i, & j_i, & a_{3_i}, & a_{4_i}, & \cdots \\ j_i, & G_i, & G_i, & G_i, & \cdots \end{pmatrix}$$

For any π_i, $\delta_{1i} \geq \pi_i \delta_{2i}$, that is the expected search time with policy δ_{1i}, is shorter than with policy δ_{2i} in time interval i, if and only if

$$\frac{p_{j_i} \pi_{j_i}}{c_{j_i} + q_{j_i} C_{j_i}} \geq \frac{p_{k_i} \pi_{k_i}}{c_{k_i} + q_{k_i} C_{k_i}}.$$

Lemma (3.1) has been proved in case of COVID-19 in one out of n possible locations and one time interval $i = 1$, (see Kress, M., Kyle Y. and Szechtman, R. [18]. By a similar way we can calculate the expected search time until detecting the correct AC_{j_i} with policy δ_{1i} and policy δ_{2i} at time interval i, and it is given by

$$\begin{aligned} T_{\delta_{1i}}(\pi_i) = \; & c_{j_i} + \pi_{j_i}[(1-p_{j_i})(c_{k_i} + q_{k_i}(T_{G_i}(\Pi_{ki}^{+}\Pi_{ji}^{-}(\pi_i))) + (1-q_{k_i})T_{\hat{\delta}}(\Pi_{ki}^{-}\Pi_{ji}^{-}(\pi_i)))] + \pi_{k_i}[q_{j_i}(C_{j_i} + c_{k_i} + (1-p_{k_i})TG_i \\ & (\Pi_{ki}^{-}\Pi_{ji}^{-}(\pi_i))) + (1-q_{ji}(c_{k_i} + (1-p_{k_i})T_{\hat{\delta}}(\Pi_{ki}^{-}\Pi_{ji}^{-}(\pi_i)))] + (1-\pi_{j_i}-\pi_{k_i})[q_{ji}(C_{j_i} + c_{k_i} + q_{k_i}(C_{k_i} + T_{Gi}(\Pi_{ki}^{+}\Pi_{ji}^{+} \\ & (\pi_i))) + (1-q_{k_i})T_{Gi}(\Pi_{ki}^{-}\Pi_{ji}^{+}(\pi_i)) + (1-a_{ji})(c_{k_i} + a_{k_i}(C_{k_i} + T_{G_i}(\Pi_{ki}^{+}\Pi_{ji}^{-}(\pi_i))) + (1-q_{k_i})T_{\hat{\delta}}(\Pi_{ki}^{-}\Pi_{ji}^{-}(\pi_i)))]. \end{aligned}$$

Where $T_{G_i}(\cdot)$ denotes the expected search time with the greed rule in each time interval i. Interchanging the in-dices j and k, we get an expression for $T_{\delta_{2i}}(\pi_i)$.

Now since $\Pi_{ki}^{-}\Pi_{ji}^{-}(\pi_i) = \Pi_{ji}^{-}\Pi_{ki}^{-}(\pi_i)$, $\Pi_{ki}^{+}\Pi_{ji}^{-}(\pi_i) = \Pi_{ji}^{-}\Pi_{ki}^{+}(\pi_i)$ and $\Pi_{ki}^{-}\Pi_{ji}^{+} = \Pi_{ji}^{+}\Pi_{ki}^{-}(\pi_i)$, and taking the difference, we have

$$T_{\delta_{i1}}(\pi_i) - T_{\delta_{2i}}(\pi_i) = -\pi_{ji} p_{j_i}(c_{k_i} + q_{k_i} C_{k_i}) + \pi_{k_i} p_{k_i}(c_{j_i} + q_{j_i} C_{j_i}).$$

Theorem 3.1. Given a prior p. m. f. π_i for θ_i, the optimal search policy follows a greedy rule, where the AC_{j_i} to search next in time interval i is one having the maximal value of

$$\frac{p_{j_i}\pi_{j_i}}{c_{j_i} + q_{j_i} C_{j_i}},$$

where $C_{j_i}^{(3)} = C_{j_i}$, $i = 1, 2, \cdots, m$ and $j = 1, 2, \cdots, n$.

To prove Theorem (3.1), we introduce two alternatives to express a feasible policy in any time interval i. First, let

$$\begin{pmatrix} a_{1_i}, & a_{2_i}, & a_{3_i}, & a_{4_i}, & \cdots \\ G_i, & G_i, & G_i, & G_i, & \cdots \end{pmatrix}$$

denote a feasible policy such that the searcher first follows the order $a_{1_i}, a_{2_i}, a_{3_i}, a_{4_i}, \cdots$ until the first detection takes place, If the first detection correctly locates COVID-19 (either perfect detection or partial detection), then the problem ends. If the first detection is a false detection, then the searcher switches to the greedy rule thereafter. Second, let

$$\begin{pmatrix} a_{1_i}, & a_{2_i}, & a_{3_i}, & a_{4_i}, & \cdots \\ a_{s_i}, & G_i, & G_i, & G_i, & \cdots \end{pmatrix}$$

denote a policy similar to the previous one, with the exception that if the first search in Aca_{1_i} results in a false detection, then the searcher is required to search in ACa_{s_i}, immediately before switching to the greedy rule.

Proof of Theorem 3.1. The proof of this theorem on the number of Acs. The theorem is trivially true for $n = 1$.

Suppose that the greedy rule is optimal if there are $n-1$ of fewer Acs. Next we show that it is also optimal when there are $nAcs$. Let

$$\frac{p_{1i}\pi_{1i}}{c_{1i} + q_{1i} C_{1i}} = \max_{\substack{i=1,2,\cdots,m \\ j=1,2,\cdots,n}} \frac{p_{j_i}\pi_{j_i}}{c_{k_{ji}} + q_{j_i} C_{j_i}}. \tag{5}$$

We consider a class of search policies in which $AC1$ is searched only in each time interval i following $[T_i - 1]$ no0detection searches elsewhere T_i denotes the detection of the target in each time interval i; that is, $a_{j_i} \neq 1$ for $i = 1, 2, \cdots, m$, $j = 1, 2, \cdots, T_i - 1$ and $a_{T_i} = 1_i$.

Let ΔT_i denote the set of these policies. In the first, we will discuss the case when $T_i < \infty$. Let

$$\delta_{1i} = \begin{pmatrix} 1, & a_{1_i}, & a_{2_i}, & \cdots \\ G, & G_i, & G_i, & \cdots \end{pmatrix},$$

$$\delta_{2i} = \begin{pmatrix} a_{1_i}, & 1, & a_{2_i}, & \cdots \\ G_i, & G_i, & G_i, & \cdots \end{pmatrix},$$

and

$$\delta_{T_i} = \begin{pmatrix} a_{1i}, & a_{2i}, & \cdots, & a_{r_i-1}, & 1_i, & a_{r_i+1}, & \cdots \\ G, & G_i, & \cdots, & G_i, & G_i, & G_i, & \cdots \end{pmatrix}.$$

From Equation (5) and Lemma 3.1, we have

$$\delta_{1i} \geq_{\pi_i} \begin{pmatrix} 1_i, & a_{1i}, & a_{2i}, & \cdots \\ G, & G_i, & G_i, & \cdots \end{pmatrix} \geq_{\pi_i} \delta_{2i}.$$

Hence, $\delta_{1i} \geq \delta_{2i}$. By repeating this process, we can see that $\delta_{1i} \geq_{\pi_i} \delta_{2i} \geq_{\pi_i} \delta_{3i} \geq_{\pi_i} \cdots \geq_{\pi_i} \delta_{Ti}$ which leads to $\delta_{1i} \geq \delta_{Ti}$. In other words, we show that for any policy in ΔT_i, with $T_i < \infty$, we can find a better policy that starts with $AC1$ in each time interval i.

In previous section, we show that $(T_{\delta_{r_i}})$ is a non-decreasing real sequence, so that $T_{\delta_{1i}} \leq T_{\delta_{\infty i}}$. Hence, for any policy in Δ_{∞_i}, the expected search time does increasee by starting the search on AC_{1i}.

4 Algorithm

To find the optimal policy in all time intervals, we use the following algorithm to generate the search order if all the searches resulted in no detection.

1- enter n;
2- enter m;
3- for $i=1$ to m;
4- set $w_i=1$;
5- choose a_i such that $\frac{p_{a_i}\pi_{a_i}}{c_{a_i}+q_{a_i}C_{a_i}}=\max_j \frac{p_{j_i}\pi_{j_i}}{c_{j_i}+q_{j_i}C_{j_i}}$, and let $e_{w_i}=a_i$;
6- update π_i as follows:
$\pi_{a_i} \leftarrow \frac{(1-p_{a_i})\pi_{a_i}}{1-p_{aa_i}\pi_{a_i}-q_{a_i}(1-\pi_{a_i})}, j_i=a_i$ and
$\pi_{j_i} \leftarrow \frac{(1-q_{a_i})\pi_{j_i}}{1-p_{aa_i}\pi_{a_i}-q_{a_i}(1-\pi_{a_i})}, j_i \neq a_i$; and
7- let $w_i+1 \rightarrow w_i$, and go to 4.

Where $e=\{e_{w_i}\}_{w_i=1}^{\infty}$ denotes the search order generated by this algorithm.

Remark 3.1. We call the search policy in Theorem 3.1, the greedy rule, because each time we search in the AC_{j_i}, it has the maximal ratio between the probability of finding COVID-19 and the Expected (wasted) cost due to a false detection.

Theorem 3.2. If $\delta_{1i}=\begin{pmatrix} j_i, & k_i, & a_{3_i}, & a_{4_i}, & \cdots \\ k_i, & G_i, & G_i, & G_i, & \cdots \end{pmatrix}$ is the optimal search policy in eachtime interval i, then the total optimal expected search time $T_{\delta_i}(\pi_i)$ with the greedy rule until detecting the correct AC_{j_i} with policy δ_{1i} at all-time intervals i, $i=1,2,\cdots,m$; $j=1,2,\cdots,n$ is

$$T_{\delta_1}(\pi_i)=\sum_{i=1}^{m}[c_{j_i}+\pi_{j_i}[(1-p_{j_i})(c_{k_i}+q_{k_i}(T_{Gi}(\Pi_{ki}^{+}\Pi_{ji}^{-}(\pi_i)))+(1-q_{k_i})T_{\hat{\delta}}(\Pi_{ki}^{-}\Pi_{ji}^{-}(\pi_i)))]+\pi_{k_i}[q_{j_i}(C_{j_i}+c_{k_i}+(1-p_{k_i}) T_{Gi}(\Pi_{ki}^{-}\Pi_{ji}^{+}(\pi_i)))+(1-q_{j_i})(c_{k_i}+(1-p_{k_i})T_{\hat{\delta}}(\Pi_{ki}^{-}\Pi_{ji}^{-}(\pi_i)))]+(1-\pi_{j_i}-\pi k_i)[q_{ji}(C_{j_i}+c_{k_i}+q_{k_i}(C_{k_i}+T_{Gi} (\Pi_{ki}^{+}\Pi_{ji}^{+}(\pi_i)))+(1-q_{k_i})T_{Gi}(\Pi_{ki}^{-}\Pi_{ji}^{+}(\pi_i))+(1-q_{ji})(c_{k_i}+q_{k_i}(C_{k_i}+T_{G_i}(\Pi_{ki}^{+}\Pi_{ji}^{-}(\pi_i)))+(1-q_{k_i})T_{\hat{\delta}}(\Pi_{ki}^{-} \Pi_{ji}^{-})))]].$$

Proof Let $T_{\delta_{1i}}(\pi_i)=\min E(t_i)$, where $E(t_i)$ is the expected search time until COVID-19 is found in each time interval i, $i=1,2,\cdots,m$ and t_i is a positive value. But now we need to prove that:

$$\min E(t_1+t_2+t_3+...+t_m)=\min E(t_1)+\min E(t_2)+\min E(t_3)+\cdots+\min E(t_m).$$

Since

$$E(t_1+t_2+t_3+\cdots+t_m)=E(t_1)+E(t_2)+E(t_3)+\cdots+E(t_m)=T_0.$$

Hence,

$$T_0 \geq \min E(t_1)+E(t_2)+E(t_3)+\cdots+E(t_m)=T_1.$$

Also,

$$T_1 \geq \min E(t_1)+\min E(t_2)+E(t_3)+\cdots+E(t_m)=T_2.$$

Also, we find

$$T_2 \geq \min E(t_1)+\min E(t_2)+\min E(t_3)+\cdots+E(t_m)=T_3.$$

By the same logic, we can write

$$T_{n-1} \geq \min E(t_1)+\min E(t_2)+\min E(t_3)+\cdots+\min E(t_m)=T_n.$$

Hence, the minimum of the sequence $\{T_0,T_1,T_2,\cdots,T_n\}$ is

$$T_n = \min T_1 + \min T_2 + \min T_3 + \cdots + \min T_m,$$

i. e.,

$$T_0 = \min T_1 + \min T_2 + \min T_3 + \cdots + \min T_m.$$

Hence,

$$T_{\delta_1}(\pi_i) = \sum_{i=1}^{m} T_{\delta_{i1}}(\pi_i).$$

And according to out search plane
$\delta_{1i} = \begin{pmatrix} j_i, & k_i, & a_{3_i}, & a_{4_i}, & \cdots \\ k_i, & G_i, & G_i, & G_i, & \cdots \end{pmatrix}$ is the optimal search plan in each time interval i. Therefore,

$$T_{\delta_1}(\pi_i) = \sum_{i=1}^{m}[c_{ji} + \pi_{j_i}[(1-p_{j_i})(c_{k_i} + q_{k_i}(T_{Gi}(\Pi^+_{ki}\Pi^-_{ji}(\pi_i))) + (1-q_{k_i})T_{\hat{\delta}}(\Pi^-_{ki}\Pi^-_{ji}(\pi_i)))] + \pi_{k_i}[q_{j_i}(C_{j_i}+c_{k_i}+(1-p_{k_i}) T_{G_i}(\Pi^-_{ki}\Pi^+_{ji}(\pi_i))) + (1-q_{j_i})(c_{k_i}+(1-p_{k_i})T_{\hat{\delta}}(\Pi^-_{ki}\Pi^-_{ji}(\pi_i)))] + (1-\pi_{j_i}-\pi k_i)[q_{ji}(C_{j_i}+c_{k_i}+q_{k_i}(C_{k_i}+T_{Gi} (\Pi^+_{ki}\Pi^+_{ji}(\pi_i))) + (1-q_{k_i})T_{Gi}(\Pi^-_{ki}\Pi^+_{ji}(\pi_i)) + (1-q_{ji})(c_{k_i}+q_{k_i}(C_{k_i}+T_{G_i}(\Pi^+_{ki}\Pi^-_{ji}(\pi_i))) + (1-q_{k_i})T_{\hat{\delta}}(\Pi^-_{ki} \Pi^-_{ji})))]].$$

5 Application

Suppose COVID-19 is a randomly moving according to a three states Markov chain with a transition matrix

$$W = \begin{pmatrix} 0.5 & 0.2 & 0.3 \\ 0.0 & 0.7 & 0.3 \\ 0.4 & 0.0 & 0.6 \end{pmatrix}.$$

The initial probabilities are given by: $\pi_{10=0.3}$, $\pi_{20=0.2}$, $\pi_{30=0.5}$, and the time it takes to search COVID-19 by a monitoring system in each time interval $i = 1,2,3,4,5$ between 3 cells, j, $j = 1,2,3$ is less than or equal 5 seconds before COVID-19 moves to another cell, next time interval. We get π_{ji} and suppose the values of p_{ji}, r_{ji} and q_{ji} , where the sum of them in each cell, which the sensor indicates detection in it, equals 1 as the following:

Table 1: The values of the three kinds of detections in each time interval.

Time Interval 1	π_{1i}	π_{2i}	π_{3i}	p_{1i}	p_{2i}	p_{3i}	r_{1i}	r_{2i}	r_{3i}	q_{1i}	q_{2i}	q_{3i}
1	0.350	0.20000	0.4500	0.6400	0.5000	0.210	0.2000	0.3000	0.1200	0.1600	0.2000	0.6700
2	0.355	0.21000	0.4350	0.6560	0.2370	0.550	0.1440	0.1140	0.2500	0.2000	0.6490	0.2000
3	0.351	0.21800	0.4305	0.4624	0.4350	0.600	0.2000	0.2320	0.2000	0.3376	0.3330	0.2000
4	0.347	0.22290	0.4291	0.6432	0.2289	0.500	0.3568	0.1158	0.2500	0.0000	0.6553	0.2500
5	0.345	0.13918	0.4287	0.2313	0.6000	0.425	0.1153	0.1000	0.2369	0.6530	0.3000	0.3379

Also, we suppose $C_{ji}^{(}1)$ and $C_{ji}^{(}2)$ according to the previous constraints as the following:

Table 2: The values of rescue team time in each time interval.

Time Interval 1	C^1_{1i}	C^1_{2i}	C^1_{3i}	C^2_{1i}	C^2_{2i}	C^2_{3i}
1	2.0	1.3	1.0	2.6	3.2	3.5
2	1.7	0.5	1.5	3.2	2.5	3.5
3	2.0	1.0	1.0	3.5	4.0	2.8
4	1.6	2.0	1.0	3.4	3.0	2.0
5	1.0	1.5	2.2	2.0	3.0	2.8

From relation (4), we can get the expected total rescue time as the following:

$$E(\text{total rescue time}) = \sum_{i=1}^{5}\sum_{j=1}^{3}\pi_{j_i}\left(\frac{r_{j_i}}{p_{j_i}}C^{(1)}_{j_i} + \frac{p_{j_i}-r_{j_i}}{p_{j_i}}C^{(2)}_{j_i}\right).$$

Hence,

$$E(\text{total rescue time}) = 11.12428\,\text{seconds}.$$

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Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this article

References

[1] A. B. El-Rayes, A. A. Mohamed, H. Fergani, On The Generalized Linear Search Problem, *Delta Journal*, **6**, Second Part, 1-10, 1993.
[2] A. B. El-Rayes, A. A. Mohamed, Optimal Search for a Randomly Moving Target, *Mathematical Method of Operations Research*, 321-326, 1986.
[3] A. A. Mohamed, The Generalized Search for One Dimensional Random Walker, *International Journal of Pure and Applied Mathematics*, **19 (3)**, 375-387, 2005.
[4] A. A. Mohamed, H. M. Abou Gabal, *Generalized Optimal Search Paths for a Randomly Located Target*, In: Annual Conference (Cairo) ISSR, **35**, Math. Statistics, 17-29, 2000.
[5] A. A. Mohamed, H. M. Abou Gabal, *Linear Search with Multiplem Searchers for a Randomly Moving Target*, International Conference for Statistics, Computer Science and Its Application, Egypt, 115-124, 2003.
[6] A. A. Mohamed, H. M. Abou Gabal, Multiplicative Linear Search Problem, *Egyptian Statistical Journal*, Cairo University, **48 (1)**, 34-45, 2004.
[7] Z. T. Balkhi, The Generalized Optimal Search Paths for Continuous Univariate Random Variable, *Journal of the Operations Research*, **23 (1)**, 67-96, 1989.
[8] A. A. Mohamed, H. M. Abou Gabal, W. A. Afifi, On the Coordinated Search Problem, *International Journal of Applied Mathematics*, **25 (5)**, 627-636, 2007.
[9] A. A. Mohamed, H. M. Abou Gabal, W. A. Afifi, Coordinated Search for a Randomly Located Target, *International Journal of Contemporary Mathematical Sciences*, **8**, 5-8, 2013.
[10] A. A. Mohamed, H. M. Abou Gabal, W. A. Afifi, Generalized Coordinated Search for a Randomly Located Target, *Delta Journal. Sci.*, **38**, 2016.
[11] A. A. Mohamed, W. A. Afifi, Quasi-Coordinate Search for a Randomly Moving Target, *Journal of Mathematics and physics*, **7**, 1814-1825, 2019.
[12] W. A. Afifi,A. H. EL-Bagoury, S. N. AL-Aziz, A Novel Search Algorithm for a Multi Searchers Random Walk, *Journal of the Applied Mathematics and Information Science*, **14 (1)**, 115-122, 2020.

[13] M. El-Hadidy, A. H. EL-Bagoury, Optimal search strategy for a three dimensional randomly located target, *International Journal of Operational Research*, **29 (1)**, 115–126, 2017.

[14] A. A. Mohamed, M. El-Hadidy, A. H. EL-Bagoury, 3-Dimensional Coordinated search technique for a Randomly Located target, *International Journal of Computing Science and Mathematics*, 2018.

[15] A. A. Mohamed, A. H. EL-Bagoury, Minimizing the expected time to detect a randomly located lost target using 3-dimensional search technique, *Journal of Communications in Statistics-theory and methods*, DOI:10.1080/03610926.2019.1588323, 2019.

[16] M. A. Kassem, A. H. EL-Bagoury, W. A. Afifi, S. N. AL-Aziz, On Minimum Expected Search Time Algorithm for 3-Dimensional Randomly Located Target, *Journal of Statistics Applications & Probability*, 2020, (In Press).

[17] S. S. Brown, Optimal Search for a Moving Target in Discrete Time and Spaces, *Journal of Operations Research*, **31**, 739-751, 1980.

[18] Moshe Kress, Kyle Y. Lin, and Roberto Szechtman, Optimal Discrete Search with Imperfect Specificity. Operations Research, *Naval Postgraduate School*, Monterey, California 93943, 2007.

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