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Development of Preliminary Test Estimators and Confidence Interval for the Reliability Characteristics of Generalized Inverted Scale Family of Distribution based on Records

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Abstract: Preliminary test estimators (PTEs) for power of parameter and two reliability measure $R(t) = P(X > Y)$ and $P = P(X > Y)$ of Generalized Inverted Scale family of distribution are developed based on record value.Preliminary test confidence interval (PTCI) is also developed based on uniformly minimum variance unbiased estimators (UMVUE), maximum likelihood estimator (MLE). A comparative study of different methods of estimation done through simulation establishes that PTEs perform better than ordinary UMVUE and MLE.

Keywords: Record values, Generalized Inverted Scale family of distribution , Preliminary test estimator, Preliminary test confidence interval,Coverage Probability ,Simulation studies.

1 Introduction

A scale of family of distributions plays an important role in reliability analysis with some of its most common members i.e. exponential distribution, Rayleigh distribution, half-logistic distribution etc. Gupta and Kundu [1,2,3] introduced the generalized exponential distribution. If Y is an exponential random variable (rv), then $X = 1/Y$ has an inverted exponential distribution. Lin et al. [4] and Dey [5] discussed inverted exponential distribution (IED) to analyze lifetime data. Abouammoh and Alshingiti [6] discussed generalized inverted exponential distribution (GIED) by introducing a shape parameter and addressed their statistical and reliability properties. Under Type II censoring, Krishna and Kumar [7] estimated reliability characteristics of GIED. Potdar and Shirke [8,9] discussed inference on the scale family of lifetime distributions based on progressively censored data and generalized inverted scale family of distributions respectively.
Chandler [10] introduced the concept of record values. Based on records, inferential procedures for the parameters of different distributions have been developed by Glick [11], Nagaraja [12,13], Balakrishan et al. [14], Arnold et al. [15], Habibi Rad et al. [16], Arashi and Emadi [17], Razmkhah and Ahmadi [18], Belaghi et al. [19,20] and others.
In the present paper, we derive PTES for two measures of reliability functions. The reliability function R(t) is defined as the probability of failure-free operation until time t. Thus, if the random variable (rv) X denotes the lifetime of an item or a system, then $R(t) = P(X > t)$. One may refer to Sinha [21] for further reading. Another measure of reliability under stress strength setup is the probability $P = P(X > Y)$, which represents the reliability of an item or a system of random strength X subject to random stress Y. A lot of work has been done in the literature for the point estimation and testing of R(t) and P. For a brief review, one may refer to Chaturvedi, Ghosh and Komal [22], Chaturvedi and Malhotra [23,24,25]
In the present paper, the ambit of our work dovetails with Generalized Inverted Scale family of distribution . In Sect. 2, the estimators (point as well as interval) are obtained for the powers of parameter, R(t) and P for Generalized Inverted

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Scale family of distribution. In Sect. 3, we propose the PTEs. In Sect. 4, the PTCIs are developed. In Sections 5 and 6 we provide numerical findings and also present analysis of real-life data. Section 6 is devoted to conclusion.

2 Generalized Inverted Scale Family Of Distributions

Let Y be a rv having distribution belonging to a scale family of distributions with cumulative distribution function (cdf) G, probability density function (pdf) g and scale parameter λ.Potdar and Shirke [9] generalized this family by introducing a shape parameter α to obtain a generalized scale family of distributions. Let$X = 1/Y$,then distribution of X belongs to generalized inverted scale family of distributions

$$f_x(x;\lambda,\alpha) = \frac{\alpha}{\lambda x^2} g\left(\frac{1}{\lambda x}\right)\left[G\left(\frac{1}{\lambda x}\right)\right]^{\alpha-1} \quad ;x>0,\lambda>0,\alpha>0 \tag{1}$$

Its cumulative distribution function(cdf) is represented as

$$F_x(x;\lambda,\alpha) = 1-\left[G\left(\frac{1}{\lambda x}\right)\right]^{\alpha} \quad ;x>0,\lambda>0,\alpha>0 \tag{2}$$

Let $X_1,X_2,...$ be an infinite sequence of independent and identically distributed random variables from equation (1).An observation X_j will be called an upper record value if its value exceeds that of all previous observations.Thus X_j is a record if $X_j > X_i$ for ever $i < j$.The record time sequence $\{T_n, n \geq 0\}$ is defined, as follows:

$$T_n = \begin{cases} T_0 = 1; with\ probability\ 1 \\ T_n = min\{j : X_j > X_{T_n-1}\}; n \geq 1 \end{cases}$$

and the record value sequence R_n is then defined as

$$R_n = X_{T_n} \ ;n=0,1,2,...$$

Assuming λ to be known.The likelihood function of the first n +1 upper records is $R_0,R_1,.....,R_n$ is

$$L(\alpha \mid R_0,R_1,.....,R_n) = f_x(R_n;\lambda,\alpha)\prod_{i=0}^{n-1}\frac{f_x(R_i;\lambda,\alpha)}{1-F_x(R_i;\lambda,\alpha)} \tag{3}$$

It is easy to obtain that

$$L(\alpha \mid R_0,R_1,R_2,...,R_n) = \left(\frac{\alpha}{\lambda}\right)^{n+1} exp\left(-\alpha log\left(\frac{1}{G(1/\lambda R_n)}\right)\right)\prod_{i=0}^{n}\frac{g(1/\lambda R_i)}{R_i^2 G(1/\lambda R_i)} \tag{4}$$

In the present paper we plan to use appropriately certain results on uniformly minimum variance unbiased estimators (UMVUE)and Maximum likelihood estimator (MLE) for power of parameter (α^p), R(t) and P, derived by Chaturvedi and Mahlotra [26]. These are consolidated and reproduced below for quick reference through Result 1 and Result 2.

Result 1. The UMVUE and MLE of α^q and R(t) are, as follows:

(i) For $q\varepsilon(-\infty,\infty), q \neq 0$, the UMVUE of α^q is given by

$$\tilde{\alpha}_U^q = \left\{\frac{\Gamma(n+1)}{\Gamma(n-q+1)}\right\}(U(R_n))^{-q} \ ;n>q-1 \tag{5}$$

where $U(R_n)$ is a complete and sufficient statistic for α and belongs to exponential family.

(ii) For $q \neq 0$, the MLE of α^q is given by

$$\hat{\alpha}_{ML}^q = \left(\frac{(n+1)}{U(R_n)}\right)^q \tag{6}$$

Sometimes,due to past knowledge or experience, the experimenter may be in a position to make an initial guess on some of the parameters of interest.This prior information can be expressed in the form of the following hypothesis:
For $H_0 : \alpha = \alpha_0$ against $H_0 : \alpha \neq \alpha_0$,then the critical region is obtained, as follows:

$$\{0 < U(R_n) < k_0\} \cup \{k_0^{'} < U(R_n) < \infty\}$$

where $k_0 = \frac{\chi^2_{2(n+1)}(\frac{\varepsilon}{2})}{2\alpha_0}$ and $k_0^{'} = \frac{\chi^2_{2(n+1)}(1-\frac{\varepsilon}{2})}{2\alpha_0}$ as ε is the level of significance.We reject H_0 if

$$2\alpha_0 U(R_n) < C_2 \ or \ 2\alpha_0 U(R_n) > C_1 \tag{7}$$

where $C_1 = \chi^2_{2(n+1)}(1-\frac{\varepsilon}{2})$ and $C_2 = \chi^2_{2(n+1)}(\frac{\varepsilon}{2})$.

Thus we define PTE based on UMVUE and MLE as

$$\hat{\alpha}^p_{PT-U} = \hat{\alpha}^p_U - (\hat{\alpha}^p_U - \alpha^p_0)I(A) \tag{8}$$

$$\hat{\alpha}^p_{PT-ML} = \hat{\alpha}^p_{ML} - (\hat{\alpha}^p_{ML} - \alpha^p_0)I(A) \tag{9}$$

Where $I(A)$ is a indicator function of the set

$$I(A) = \{\chi^2_{2(n+1)} : C_2 \leq \chi^2_{2(n+1)} \leq C_1\}$$

(iii) The UMVUE of R(t) is given by

$$\tilde{R}(t) = \begin{cases} \left[1 - \frac{U(t)}{U(R_n)}\right]^n ; U(t) < U(R_n) \\ 0; \qquad otherwise \end{cases} \tag{10}$$

(iv) The MLE of R(t) is given by

$$\hat{R}(t) = exp\left\{\frac{-(n+1)U(t)}{U(R_n)}\right\} \tag{11}$$

Now we define PTE based on $R(t)$ viz

$$\tilde{R}(t)_{PT-U} = \tilde{R}(t) - (\tilde{R}(t) - R_0(t))I(A)$$

$$\hat{R}(t)_{PT-ML} = \hat{R}(t) - (\hat{R}(t) - R_0(t))I(A)$$

where $R_0(t) = exp\{-\alpha U(t)\}$

Suppose X and Y belong to the same family of distributions,

$$P = \alpha_2(\alpha_1 + \alpha_2)^{-1}$$

Suppose we want to test $H_0 : P = P_0$ and $H_1 : P \neq P_0$

It follows that H_0 is equivalent to $\alpha_2 = k\alpha_1$ where $k = P_0(1-P_0)^{-1}$. Hence,

$$H_0 : \alpha_2 = k\alpha_1 \text{ and } H_1 : \alpha_2 \neq k\alpha_1$$

denoting by $F_{a,b}(.)$, the $F-$ Statistics with (a,b) degree of freedom using the fact that $\frac{(m+1)\alpha_1 U(R_n)}{(n+1)\alpha_2 U^*(R_m)} \backsim F_{2(n+1),2(m+1)}$,the critical region is given by

$$\left\{\frac{U(R_n)}{U(R^*_m)} < k_2\right\} \cup \left\{\frac{U(R_n)}{U(R^*_m)} > k_2^{'}\right\}$$

Where $k_2 = \frac{k(n+1)}{(m+1)}F_{2(n+1),2(m+1)}(\frac{\varepsilon}{2})$ and $k_2^{'} = \frac{k(n+1)}{(m+1)}F_{2(n+1),2(m+1)}(1-\frac{\varepsilon}{2})$

Result 2. The UMVUE and MLE of P is given by

(i) The UMVUE,of P is given by

$$\tilde{P} = \begin{cases} \sum_{i=0}^{m-1} 1 - m\frac{(-1)^i m! n!}{(m-i-1)!(n+i+1)!}\left\{\frac{U(R_n)}{U(R^*_m)}\right\}^{i+1} ; R_n \leq R^*_m \\ \sum_{i=0}^{n} \frac{(-1)^i n! m!}{(n-i)!(m+i)!}\left\{\frac{U(R^*_m)}{U(R_n)}\right\}^{i} \qquad ; R^*_m < R_n \end{cases} \tag{12}$$

(ii) The MLE,of P is given by

$$\hat{P} = \frac{(m+1)U(R_n)}{(m+1)U(R_n)+(n+1)U(R_m^*)} \tag{13}$$

Now we define PTE based on P viz

$$\tilde{P}_{(PT-U)} = \tilde{P} - (\tilde{P} - P_0)I(B) \tag{14}$$

$$\hat{P}_{(PT-ML)} = \hat{P} - (\hat{P} - P_0)I(B) \tag{15}$$

Where $I(B)$ is the indicator function of the set

$$B = \left\{ F_{2(n+1),2(m+1)} : C_4 < F_{2(n+1),2(m+1)} < C_3 \right\}$$

Where $C_3 = F_{2(n+1),2(m+1)}(1-\frac{\varepsilon}{2})$ and $C_4 = F_{2(n+1),2(m+1)}(\frac{\varepsilon}{2})$

3 Proposed Preliminary Test Estimators

In this section we plan to develop Bias and MSE of Preliminary Test Estimators(PTEs) based on UMVUE and MLE. We also provide the bias and MSE expressions for PTES based on UMVUE and MLE of the reliability functions R(t)and P. Suppose $\delta = \frac{\alpha}{\alpha_0}$, we have

Theorem 1 The Bias and MSE Of PTE of α^q based on UMVUE are

$$Bias(\hat{\alpha}^q_{PT_U}) = \alpha_0^q [H_{2(n+1)}(\delta C_1) - H_{2n+2}(\delta C_2) - (\delta)^q \{H_{2(n+p+1)}(\delta C_1) - H_{2(n+p+1)}(\delta C_2)\}] \tag{16}$$

and

$$\begin{aligned} MSE(\alpha_U^q) &= (\delta\alpha_0)^{2q}\left[\frac{\Gamma(n-2q+1)\Gamma(n+1)}{\Gamma^2(n-q+1)} - 1\right] \\ &- (\delta\alpha_0)^{2q}\frac{\Gamma(n-2q+1)\Gamma(n+1)}{\Gamma^2(n-q+1)}\{H_{2(n-2q+1)}(\delta C_1) - H_{2(n-2q+1)}(\delta C_2)\} \\ &+ (\alpha_0^{2q} - 2\delta^q\alpha_0^{2q})\{H_{2n+2}(\delta C_1) - H_{2n+2}(\delta C_2)\} \\ &+ 2(\delta\alpha_0)^{2q}\{H_{2(n-q+1)}(\delta C_1) - H_{2(n-q+1)}(\delta C_2)\}. \end{aligned} \tag{17}$$

Where $H_\gamma(.)$ is for the cdf of χ^2 distribution with γ degree of freedom.

Proof The bias of the PTE given at 8 is

$$\begin{aligned} Bias(\hat{\alpha}^q_{PT-U}) &= Bias[\hat{\alpha}^q_U - (\hat{\alpha}^q_U - \alpha_0^q)I(A)] \\ &= E[\hat{\alpha}^q_U - (\hat{\alpha}^q_U - \alpha_0^q)I(A) - \alpha^q] \\ &= \alpha_0^q[H_{2n+2}(\delta C_1) - H_{2n+2}(\delta C_2) \\ &- (\delta)^q\{H_{2(n-q+1)}(\delta C_1) - H_{2(n-q+1)}(\delta C_2)\}] \end{aligned} \tag{18}$$

Also

$$\begin{aligned} MSE(\hat{\alpha}^q_{PT-U}) &= E[(\hat{\alpha}^q_{PT-U} - \alpha^q)^2] \\ &= Var(\hat{\alpha}^q_{PT-U}) + [Bias(\hat{\alpha}^q_{PT-U})]^2. \end{aligned}$$

Theorem 2 The Bias and MSE Of PTE of α^q based on MLE are

$$\begin{aligned} Bias(\alpha^q{}_{PT-ML}) &= (\delta\alpha_0(n+1))^q\frac{\Gamma(n-p+1)}{\Gamma(n+1)} \\ &[1 - \{H_{2(n-p+1)}(\delta C_1) - H_{2(n-p+1)}(\delta C_2)\}] \\ &+ \alpha^p{}_0\{H_{2(n+1)}(\delta C_1) - H_{2(n+1)}(\delta C_1)\} - (\delta\alpha_0)^q. \end{aligned} \tag{19}$$

and

$$
\begin{aligned}
MSE(\alpha_{PT-ML}{}^{q}) = & \left\{(\delta\alpha_0(n+1))^{q}\frac{\Gamma(n-q+1)}{\Gamma(n+1)}\right\}^{2}\left[\frac{\Gamma(n-2q+1)\Gamma(n+1)}{\Gamma^{2}(n-q+1)}-1\right] \\
& -(\delta\alpha_0(n+1))^{2q}\frac{\Gamma(n-2q+1)}{\Gamma(n+1)}\{H_{2(n-2q+1)}(\delta C_1)-H_{2(n-2q+1)}(\delta C_2)\} \\
& -\left\{(\delta\alpha_0(n+1))^{q}\frac{\Gamma(n-p+1)}{\Gamma(n+1)}\right\}^{2}\{H_{2(n-q+1)}(\delta C_1)-H_{2(n-q+1)}(\delta C_2)\}^{2} \\
& +\alpha_0{}^{2p}\{H_{2n+2}(\delta C_1)-H_{2n+2}(\delta C_2)\}[1-\{H_{2n+2}(\delta C_1)-H_{2n+2}(\delta C_2)\}] \\
& +2\{(\delta\alpha_0(n+1))^{q}\frac{\Gamma(n-q+1)}{\Gamma(n+1)}\}^{2}\{H_{2(n-q+1)}(\delta C_1)-H_{2(n-q+1)}(\delta C_2)\} \\
& +2\{(\delta\alpha_0(n+1))^{q}\frac{\Gamma(n-q+1)}{\Gamma(n+1)}\}^{2}\{H_{2(n-q+1)}(\delta C_1)-H_{2(n-q+1)}(\delta C_2)\} \\
& \times[\{H_{2(n-q+1)}(\delta C_1)-H_{2(n-q+1)}(\delta C_2)\}-1] \\
& +[(\delta\alpha_0(n+1))^{q}\frac{\Gamma(n-q+1)}{\Gamma(n+1)}[1-\{H_{2(n-q+1)}(\delta C_1)-H_{2(n-q+1)}(\delta C_2)\}] \\
& +\alpha_0^{q}\{H_{2(n+1)}(\delta C_1)-H_{2(n+1)}(\delta C_2)\}-(\delta\alpha_0)^{q}]
\end{aligned} \tag{20}
$$

Proof The Bias and MSE can be derived using Result (i). Now, the result can be derived easily.

The following theorem provides the Bias and MSE of R(t) based on UMVUE

Theorem 3 The Bias and MSE of PTE of R(t) based on UMVUE are

$$
Bias(\tilde{R}(t)_{PT-U}) = R_0(t)\{H_{2n+2}(\delta C_1)-H_{2n+2}(\delta C_2)\}-\varphi_1 \tag{21}
$$

$$
\begin{aligned}
MSE(\tilde{R}(t)_{PT-U}) = & R-\varphi_2+(R_0(t))^{2} \\
& \times\{H_{2n+2}(\delta C_1)-H_{2n+2}(\delta C_2)\} \\
& +2\varphi_1 R(t)-2R(t)R_0(t)\{H_{2n+2}(\delta C_1)-H_{2n+2}(\delta C_2)\}
\end{aligned} \tag{22}
$$

$$
\varphi_1 = \int_{C_2}^{C_1}\left(1-\frac{2\alpha U(t)}{u}\right)^{n}\frac{u^{ne^{-u/2}}}{2^{n+1}n!}du
$$

and

$$
\varphi_2 = \int_{C_2}^{C_1}\left(1-\frac{2\alpha U(t)}{u}\right)^{2n}\frac{u^{ne^{-u/2}}}{2^{n+1}n!}du
$$

Proof The variance of UMVUE of R(t) is obtained as follows:

$$
\begin{aligned}
Var\{\tilde{R(t)}\} = & \frac{1}{n!}\left\{\alpha U(t)\right\}^{(n+1)} exp\left\{-\alpha U(t)\right\}\left[\frac{a_n}{\alpha U(t)}\right. \\
& -a_{n-1}exp\{\alpha U(t)\}E_i(-\alpha U(t)) \\
& +\Sigma_{i=0}^{n-2}a_i\left\{\Sigma_{m=1}^{n-i-1}\frac{(m-1)!}{(n-i-1)!}(-\alpha U(t))^{n-i-m-1}\right. \\
& \left.-\frac{1}{(n-i-1)!}(-\alpha U(t))^{n-i-1}exp(\alpha U(t))E_i(-\alpha U(t))\right\} \\
& \left.+\Sigma_{i=n+1}^{2n}a_i(i-n)!\left(\frac{1}{\alpha U(t)}\right)^{i-n+1}\Sigma_{r=0}^{i-n}\frac{1}{r!}(\alpha U(t))^{r}\right] \\
& -exp\{-2\alpha U(t)\},=R
\end{aligned}
$$

where $a_i = (-1)^i \binom{2n}{i}$ and $-E_i(-x) = \int_x^\infty \frac{e^{-u}}{u} du$

Now using Variance of R(t) result can easily be derived.

Theorem 4 The Bias and MSE of PTE of R(t) based on MLE are

$$Bias(\hat{R}(t)_{PT-ML}) = \frac{2}{n!}(2\alpha(n+1)U(t))^{\frac{n+1}{2}} K_{n+1}\left(2\sqrt{\alpha(n+1)U(t)}\right) - \varphi_3 + R_0(t)\{H_{2(n+1)}(\delta C_1) - H_{2(n+1)}(\delta C_2)\} - R(t)$$

$$\begin{aligned} MSE(R(t)_{PT-ML}) = & \frac{2}{n!}(2\alpha(n+1)U(t))^{\frac{n+1}{2}} K_{n+1}\left(2\sqrt{\alpha(n+1)U(t)}\right) \\ & - \left[\frac{2}{n!}(\alpha(n+1)U(t))^{\frac{n+1}{2}} K_{n+1}\left(2\sqrt{\alpha(n+1)U(t)}\right)\right]^2 - \varphi_4 - \varphi_3^2 \\ & + (R_0(t))^2\{H_{2n+2}(\delta C_1) - H_{2n+2}(\delta C_2)\}\{1 - \{H_{2n+2}(\delta C_1) - H_{2n+2}(\delta C_2)\}\} \\ & + 2\left[\frac{2}{n!}(\alpha(n+1)U(t))^{\frac{n+1}{2}} K_{n+1}\left(2\sqrt{\alpha(n+1)U(t)}\right)\right]\varphi_3 \\ & + 2R_0(t)\{H_{2n+2}(\delta C_1 - H_{2n+2}(\delta C_2)\} \\ & \times \varphi_3 - \frac{2}{n!}(\alpha(n+1)U(t))^{\frac{n+1}{2}} K_{n+1}\left(2\sqrt{\alpha(n+1)U(t)}\right) \\ & + \left[\frac{2}{n!}(\alpha(n+1)U(t))^{\frac{n+1}{2}} K_{n+1}\left(2\sqrt{\alpha(n+1)U(t)}\right) - \varphi_3 \right. \\ & \left. + R_0(t)\ \{H_{2n+2}(\delta C_1) - H_{2n+2}(\delta C_2)\} - R(t)\right]^2. \end{aligned}$$

where

$$\varphi_3 = \int_{C_2/2}^{C_1/2} \frac{z}{n!} exp\left(-\left(z + \frac{\alpha(n+1)U(t)}{z}\right)\right) dz \tag{23}$$

$$\varphi_4 = \int_{C_2/2}^{C_1/2} \frac{z}{n!} exp\left(-\left(z + \frac{2\alpha(n+1)U(t)}{z}\right)\right) dz \tag{24}$$

Proof We can write

$$\begin{aligned} (Bias(\hat{R}(t)_{PT-ML}) &= E\left(\hat{R}(t) - (\hat{R}(t) - R_0(t))I(A) - \hat{R}(t)\right) \\ &= \frac{1}{\Gamma(n+1)}\int_0^\infty exp\left[-\left\{y + \frac{\alpha(n+1)U(t)}{y}\right\}\right] y^n dy - \varphi_3 \\ &+ R_0(t)\{H_{2n+2}(\delta C_1) - H_{2n+2}(\delta C_2)\} - R(t). \end{aligned} \tag{25}$$

Applying a result of Watson [27] given by

$$\int_0^\infty u^{-m} exp\left\{-\left(au + \frac{b}{u}\right)\right\} du = 2\left(\frac{a}{b}\right)^{\frac{m-1}{2}} K_{m-1}\left(2\sqrt{ab}\right)$$

[it is to be noted that $K_{-m}(.) = K_m(.)$ for m = 0,1,2,...]
Now, result can be derived

Theorem 5 The Bias and MSE of PTE of P based on UMVUE are

$$Bias(P_{PT-U}) = \begin{cases} P_0 P(B) - \varphi_5; \nu \leq 1 \\ P_0 P(B) - \varphi_6; \nu > 1 \end{cases} \tag{26}$$

$$MSE(\tilde{P}_{PT-U}) = \begin{cases} \varphi_7 - P^2 - \varphi_8 + 2P(\varphi_5 - P_0 P(B)) + P_0^0 P(B); \nu \leq 1 \\ \varphi_7 - P^2 - \varphi_9 + 2P(\varphi_6 - P_0 P(B)) + P_0^0 P(B); \nu > 1 \end{cases} \tag{27}$$

Where
$\varphi_5 = \Sigma_{i=0}^{m-1} \left(\frac{(-1)^i m!n!}{(m-i-1)!(n+i+1)!\beta(n+1,m+1)} \right) \left(\frac{\alpha_2}{\alpha_1} \right)^{i+1} \int_{C_4'}^{C_3'} \frac{z^{n+i+1}}{(1+z)^{n+m+2}} dz,$
$\varphi_6 = \Sigma_{i=0}^{n} \left(\frac{(-1)^i m!n!}{(m-i-1)!(n+i+1)!\beta(n+1,m+1)} \right) \left(\frac{\alpha_2}{\alpha_1} \right)^{i+1} \int_{C_4'}^{C_3'} \frac{z^{n+i+1}}{(1+z)^{n+m+2}} dz,$
$\nu = \frac{U(R_n)}{U(R_m^*)}$
$C_3' = \left(\frac{n+1}{m+1}\right) C_3$
$C_4' = \left(\frac{n+1}{m+1}\right) C_4$

$$\varphi_7 = \Sigma_{i=0}^{m-1} \Sigma_{j=0}^{m-1} \frac{a_i a_j \rho^{-(-i-j-2)}}{B(n+1,m+1)} \Sigma_{k=0}^{n+i+j+2} (-1)^k \binom{n+i+j+2}{k} \int_Q^1 r^{m+k-i-j-2} dr$$
$$+ \Sigma_{i=0}^{n} \Sigma_{j=0}^{n} \frac{b_i b_j \rho^{i+j)}}{B(n+1,m+1)} \Sigma_{k=0}^{m+i+j} (-1)^k \binom{m+i+j}{k} \int_{1-Q}^1 r^{n+k-i-j} dr$$

$\varphi_8 = \Sigma_{i=0}^{m-1} \Sigma_{j=0}^{m-1} a_i a_j \int_{C_4}^{C_3} \nu^{i+j+2} \Phi_1(F) dF$
$\varphi_9 = \Sigma_{i=0}^{n} \Sigma_{j=0}^{n} b_i b_j \left(\frac{\theta_2 m+1}{\theta_1 n+1} \right)^{i+j} \nu^{i+j} \Phi_1(F) dF$
Proof Consider

$$E(\tilde{P}^2) = E\left(\Sigma_{i=0}^{m-1} \Sigma_{j=0}^{m-1} a_i a_j (\nu)^{i+j+2} \mid \nu \le 1) P(\nu \le 1\right)$$
$$+ E\left(\Sigma_{i=0}^{n} \Sigma_{j=0}^{n} b_i b_j (\nu)^{-(i+j)} \mid \nu > 1\right) P(\nu > 1)$$

denoting,$a_i = (\frac{(-1)^i m!n!}{(m-i-1)!(n+i+1)!})$ and $b_i = (\frac{(-1)^i m!n!}{(n-i)!(m+i)!})$ Clearly expression of $Var(\tilde{P})$ depends on the evaluation $E(\nu^l \mid \nu \le 1)P(\nu \le 1)$ and $E(\nu^{-l} \mid \nu > 1)P(\nu > 1)$ for $l \ge 0$ for obtaining pdf of ν.

$$h(\nu) = \frac{\rho^{n+1}}{B(n+1,m+1)} \nu^n (1+\rho\nu)^{-n-m-2}; \nu > 0$$

For $l > 0$

$$E(\nu^l \mid \nu \le 1)P(\nu \le 1) = \int_0^1 \frac{\rho^{n-1}}{B(n+1,m+1)} \nu^{n+l} (1+\rho\nu)^{-n-m-2} d\rho$$

Substituting $r = (1+\rho\nu)^{-1}$

$$E(\nu^l \mid \nu \le 1)P(\nu \le 1) = \frac{\rho^{-l}}{B(n+1,m+1)} \Sigma_{k=0}^{n+l} (-1)^k \binom{n+l}{k} \int_Q^1 r^{m-l+k} dr,$$

where

$$\int_Q^1 r^{m-l+k} dr = \begin{cases} \frac{1-Q^{m-l+k+1}}{m-l+k+1}; k \ne l-m-1 \\ -log(Q); k = l-m-1 \end{cases}$$

where $Q = \frac{1}{1+\rho}$. Similarly, we obtain

$$E(\nu^{-l} \mid \nu > 1)P(\nu > 1) = \frac{\rho^l}{B(n+1,m+1)} \Sigma_{k=0}^{m+l} (-1)^k \binom{m+l}{k} \int_{1-Q}^1 r^{n-l+k} dr$$

where

$$\int_{1-Q}^1 r^{n-l+k} dr = \begin{cases} \frac{1-(1-Q)^{n-l+k+1}}{n-l+k+1}; k \ne l-n-`1 \\ -log(1-Q); k = l-n-`1 \end{cases}$$

Thus,$Var(\tilde{P}) = \varphi_7 - P^2$ and

$$Var(\tilde{P}I(B)) = \begin{cases} \varphi_8 - \varphi_5^2; \nu \le 1 \\ \varphi_9 - \varphi_6^2; \nu > 1 \end{cases}$$

we can obtain the result.

Theorem 6 The Bias and MSE of PTE of P based on MLE are

$$Bias(\hat{P}_{PT-ML}) = \begin{cases} -\varphi_{10} + P_0 P(B); \omega = 1 \\ \varphi_{11} - \varphi_{10} + P_0 P(B)\} - P; \omega \neq 1 \end{cases}$$

$$MSE(\hat{P}_{PT-ML}) = \begin{cases} \left(\frac{n+1}{n+m+2}\right)^2 \left[\left(\frac{n+2}{n+1}\right)\left(\frac{n+m+2}{n+m+3}\right) - 1\right] - \varphi_{12} + 2\left(\frac{n+1}{n+m+2}\right) \\ \times(\varphi_{10} - P_0 P(B) + P_0^2 P(B); \omega = 1 \\ \varphi_{13} - \varphi_{12} + \left(\frac{n+1}{n+m+3}\right)^2 + 2\left(\frac{n+1}{n+m+3}\right)(\varphi_{10} - \varphi_{11}) \\ +P_0^2 P(B) - 2\left(\frac{n+1}{n+m+2}\right) P_0 P(B); \omega \neq 1 \end{cases}$$

Where

$\varphi_{10} = \int_{C_3}^{C_4} \left(\frac{1}{1+\frac{\alpha_1}{\alpha_2 F}}\right) \Phi_1(F) dF$

$$\varphi_{11} = \frac{1}{B(n+1,m+1)} \left(\rho\left(\frac{n+1}{m+1}\right)\right)^{n+1} \frac{1}{\varepsilon^{n+m+l+1}} \times \int_1^{\omega} (t-1)^{n+l}(c-t)^m t^{-(n+m+2)} dt, \tag{28}$$

$\varphi_{12} = \int_{C_3}^{C_4} \left(\frac{1}{1+\frac{\alpha_1}{\alpha_2 F}}\right)^2 \Phi_1(F) dF$

$$\varphi_{13} = \frac{1}{B(n+1,m+1)} \left(\rho\left(\frac{n+1}{m+1}\right)\right)^{n+1} \frac{1}{\varepsilon^{n+m+l+2}} \times \int_1^{\omega} (t-1)^{n+l}(c-t)^m t^{-(n+m+2)} dt, \tag{29}$$

and

$P(B) = \{F_{2(n+1),2(m+1)}(C_3) - F_{2(n+1),2(m+1)}(C_4)\}$

Proof Consider

$$Bias(\hat{P}_{PT-ML}) = E(\hat{P}) - E(\hat{P}I(B)) + P_0 E(I(B)) - P$$

where $E(\hat{P}) = E\left(\frac{\widehat{\alpha}_2}{\widehat{\alpha}_1+\widehat{\alpha}_2}\right) = E(\widehat{Q})$,(say).Applying the approach of Constantine et al. [28], we derive the pdf of $\hat{Q}$ by transformation into two new independent rvs $r > 0$ and $\lambda\ \varepsilon\ \left(0, \frac{\Pi}{2}\right)$ where $\widehat{\alpha}_1 = \alpha_1 r(n+1)cos^2\lambda$ and $\widehat{\alpha}_2 = \alpha_2 r(m+1)sin^2\lambda$. Putting $\varphi = cos^2\lambda$, the pdf of $\hat{Q} = \left[1 + \rho\left(\frac{n+1}{m+1}\right)\left(\frac{\varphi}{1-\varphi}\right)\right]^{-1}$ is

$$g(q) = \frac{1}{B(n+1,m+1)} \left(\rho\left(\frac{m+1}{n+1}\right)\right)^{n+1} \frac{q^{n(1-q)^m}}{(1+\varepsilon q)^{n+m+2}}; 0 < q < 1, \quad \varepsilon = \rho\left(\frac{m+1}{n+1}\right) - 1, \tag{30}$$

Where $B(a,b)$ is the beta function with parameter a and b.When $\varepsilon = 0$,

$$E(\hat{Q}^l) = \frac{B(n+l+1,m+1)}{B(n+1,m+1)} \tag{31}$$

Where $\varepsilon \neq 0$, equation (30) becomes on putting $1 + \varepsilon q = t$,

$$E(\hat{Q}^l) = \frac{1}{B(n+1,m+1)} \left(\rho\left(\frac{m+1}{n+1}\right)\right)^{n+1} \frac{1}{\varepsilon^{n+m+l+1}} \times \int_1^{\omega} (t-1)^{n+l}(c-t)^m t^{-(n+m+2)} dt, \tag{32}$$

Then, the bias and MSE of PTE of P based on MLE can be derived

4 Proposed Preliminary Test Confidence Interval

In this section we construct PTCI of the scale parameter α. Suppose for known value of the shape parameter λ, we are interested in testing the hypothesis

$$H_0 : \alpha = \alpha_0$$
$$H_1 : \alpha \neq \alpha_0$$

Since $U(R_n)$ follows gamma distribution with parameter $(n+1,\alpha)$, it is easy to obtain the 100 percentage equal tail CI of α as

$$I_{ETCI} = \left[\frac{\chi^2_{2(n+1)}(\frac{\varepsilon}{2})}{2U(R_n)}, \frac{\chi^2_{2(n+1)}(1-\frac{\varepsilon}{2})}{2U(R_n)}\right]$$

From Result 1, p=1 we have obtained the same expression for the MLE and UMVUE of α as $\hat{\alpha} = \frac{(n+1)}{U(R_n)}$.Now I_{ETCI} can be written as

$$I_{ETCI} = [C_5\hat{\alpha}, C_6\hat{\alpha}]$$

denoting $C_5 = \frac{\chi^2_{2(n+1)}(\frac{\varepsilon}{2})}{2(n+1)}$
$C_6 = \frac{\chi^2_{2(n+1)}(1-\frac{\varepsilon}{2})}{2(n+1)}$
Hence, we can define PTCI of α as

$$I_{ETCI} = [C_5\hat{\alpha}_{PT}, C_6\hat{\alpha}_{PT}],$$

As $\delta = \frac{\alpha}{\alpha_0}$ and $T = 2\alpha U(R_n)$, coverage probability of Preliminary test confidence interval of α is

$$\begin{aligned} P(\alpha\ \varepsilon\ I_{PTCI}) = &P\left(\alpha\ \varepsilon\ (c_5\alpha_0, c_6\alpha_0) : \chi^2_{2(n+1)}(\frac{\varepsilon}{2}) < 2\alpha U(R_n) < \chi^2_{2(n+1)}\left(1-\frac{\varepsilon}{2}\right)\right) \\ &+P\left(\alpha\ \varepsilon\ (c_5\hat{\alpha}_U, c_6\hat{\alpha}_U) : 2\alpha U(R_n) < \chi^2_{2(n+1)}\left(\frac{\varepsilon}{2}\right)\right) \\ &+P\left(\alpha\ \varepsilon\ (c_5\hat{\alpha}_U, c_6\hat{\alpha}_U) : 2\alpha U(R_n) > \chi^2_{2(n+1)}\left(1-\frac{\varepsilon}{2}\right)\right) \end{aligned}$$

or

$$\begin{aligned} P(\alpha\ \varepsilon\ I_{PTCI}) = &P\left((c_5 < \delta < c_6) : \delta\chi^2_{2(n+1)}(\frac{\varepsilon}{2}) < T < \delta\chi^2_{2(n+1)}(1-\frac{\varepsilon}{2})\right) \\ &+P\left(\chi^2_{2(n+1)}\left(\frac{\varepsilon}{2}\right) < T < \chi^2_{2(n+1)}\left(1-\frac{\varepsilon}{2}\right), T < \delta\chi^2_{2(n+1)}\left(\frac{\varepsilon}{2}\right)\right) \\ &+P\left(\chi^2_{2(n+1)}\left(\frac{\varepsilon}{2}\right) < T < \chi^2_{2(n+1)}\left(1-\frac{\varepsilon}{2}\right), T > \delta\chi^2_{2(n+1)}\left(\frac{\varepsilon}{2}\right)\right) \end{aligned}$$

or

$$\begin{aligned} P(\alpha\varepsilon I_{PTCI}) = &P\left(\delta\chi^2_{2(n+1)}\left(\frac{\varepsilon}{2}\right) < T < \delta\chi^2_{2(n+1)}\left(1-\frac{\varepsilon}{2}\right)\right) I_{c_5,c_6}(\delta) \\ &+P\left(\chi^2_{2(n+1)}\left(\frac{\varepsilon}{2}\right) < T < min\left\{\chi^2_{2(n+1)}\left(1-\frac{\varepsilon}{2}\right), \delta\chi^2_{2(n+1)}\left(\frac{\varepsilon}{2}\right)\right\}\right) \\ &+P\left(max\left\{\chi^2_{2(n+1)}\left(\frac{\varepsilon}{2}\right), \delta\chi^2_{2(n+1)}\left(1-\frac{\varepsilon}{2}\right)\right\} < T < \chi^2_{2(n+1)}\left(1-\frac{\varepsilon}{2}\right)\right) \end{aligned}$$

$$P(\alpha\ \varepsilon\ I_{PTCI}) = \begin{cases} C+1-\varepsilon\ ;\ 0 < \delta \leq \frac{\chi^2_{2(n+1)}(\frac{\varepsilon}{2})}{\chi^2_{2(n+1)}(1-\frac{\varepsilon}{2})} \\ C+P\ \left(\chi^2_{2(n+1)}\left(\frac{\varepsilon}{2}\right) < T < \delta\chi^2_{2(n+1)}\left(\frac{\varepsilon}{2}\right)\right);\frac{\chi^2_{2(n+1)}(\frac{\varepsilon}{2})}{\chi^2_{2(n+1)}(1-\frac{\varepsilon}{2})} < \delta \leq 1 \\ C+P\left(\delta\chi^2_{2(n+1)}\left(\frac{1-\varepsilon}{2}\right) < T < \chi^2_{2(n+1)}\left(\frac{1-\varepsilon}{2}\right)\right); 1 < \delta \leq \frac{\chi^2_{2(n+1)}(1-\frac{\varepsilon}{2})}{\chi^2_{2(n+1)}(\frac{\varepsilon}{2})} \end{cases}$$

Now,we will obtain length of PTCI, so we can obtain expected length of PTCI of α by following random variables:

$$L_{PT} = \begin{cases} \alpha_0(c_6 - c_5); \chi^2_{2(n+1)}\left(\frac{\varepsilon}{2}\right) < 2\alpha_0 U(R_n) < \chi^2_{2(n+1)}\left(1-\frac{\varepsilon}{2}\right) \\ \hat{\alpha}_U(c_6 - c_5); 2\alpha_0 U(R_n) < \chi^2_{2(n+1)}\left(\frac{\varepsilon}{2}\right) \end{cases}$$

Thus, expected length (EL) of the PTCI of α is:-

$$E(L_{PT}) = \alpha_0(c_6 - c_5)\Big[H_{2n+2}\left(\delta\chi^2_{2(n+1)}\left(1-\frac{\varepsilon}{2}\right)\right) - H_{2n+2}\left(\delta\chi^2_{2(n+1)}\left(\frac{\varepsilon}{2}\right)\right)$$
$$- H_{2n-2}\left(\delta\chi^2_{2(n+1)}\left(\frac{\varepsilon}{2}\right)\right) + \frac{\delta(n+1)}{n-1}$$
$$\times\left\{H_{2n-2}\left(\delta\chi^2_{2(n+1)}\left(\frac{\varepsilon}{2}\right)\right) + 1 - H_{2n-2}\left(\delta\chi^2_{2(n+1)}\left(1-\frac{\varepsilon}{2}\right)\right)\right\}\Big]$$

5 Numerical Findings

In this section, we attempt to judge the performance of preliminary test estimators based on records value.

Hence, we consider Rayleigh distribution as a particular case of generalized inverted scale family of distributions.A rv X is said to follow the Generalized inverted Rayleigh distribution if its pdf and cdf are given by

$$F_x(x;\lambda,\alpha) = 1 - [1 - e^{-(1/\lambda x)^2}]^{\alpha} \tag{33}$$

$$f_x(x;\lambda,\alpha) = \frac{2\alpha}{\lambda^2 x^2} e^{-(1/\lambda x)^2 \alpha - 1} \tag{34}$$

Since the relative efficiency $\left(\hat{\alpha}^p_{PT-ML} \mid \hat{\alpha}^p_{ML}\right)$ and relative efficiency$\left(\hat{\alpha}^p_{PT-U} \mid \hat{\alpha}^p_{U}\right)$ depend on sample size $(n+1)$ and level of significance ε. Table 1 shows the relative efficiency of $\hat{\lambda}^p_{PT-ML}$ over $\hat{\lambda}^p_{ML}$ and we observe that there exists an interval δ for which efficiency is greater than 1. Similarly, Table 2 indicates the relative efficiency of α^p_U over α^p_U and we observe that there exists an interval δ for which efficiency is greater than 1.

Table 1: Relative efficiency of PTE of α^p based on MLE over MLE of α^p for various sample size of n and level of significance ε when $\alpha = \alpha_0 = 25$ and $p = 2$

ε / n	0.01	0.05	0.10	0.20
5	25.9953	12.1000	3.2186	4.0087
10	52.5211	16.0625	7.3351	4.7223
15	61.0999	22.0500	12.1232	5.0216
20	85.4854	37.0416	18.0216	7.5291
40	123.9876	53.0284	27.0416	11.3884
60	302.1615	67.0361	35.2528	17.2044
90	343.7361	82.5112	43.6787	21.3142

Tables 1 and 2 show that irrespective of the sample size and level of significance,the PTE of α^p based on MLE and UMVUE are always more efficient.

Figure 1 shows the relative efficiency of $\hat{R}(t)_{PT-ML}$ over $\hat{R}(t)$ and relative efficiency of $\tilde{R}(t)_{PT-U}$ over $\tilde{R}(t)$ for different point and level of significance ε with respect to $\theta = \frac{R(t)}{R_0(t)}$ and we observe that preliminary test estimators of

Table 2: Relative efficiency of PTE of α^p based on UMVUE over UMVUE of α^p for various sample size of n and level of significance ε when $\alpha = \alpha_0 = 25$ and $p = 2$

ε n	0.01	0.05	0.10	0.20
5	16.4385	4.4748	1.1010	1.4670
10	17.0024	4.5434	1.1169	1.7306
15	21.6156	4.6460	1.1302	1.8549
20	26.4693	4.7821	1.1414	1.9272
40	28.1309	4.7900	1.1788	1.0079
60	31.0413	4.8412	1.1997	1.0983
90	34.1423	4.8545	2.0010	1.1227

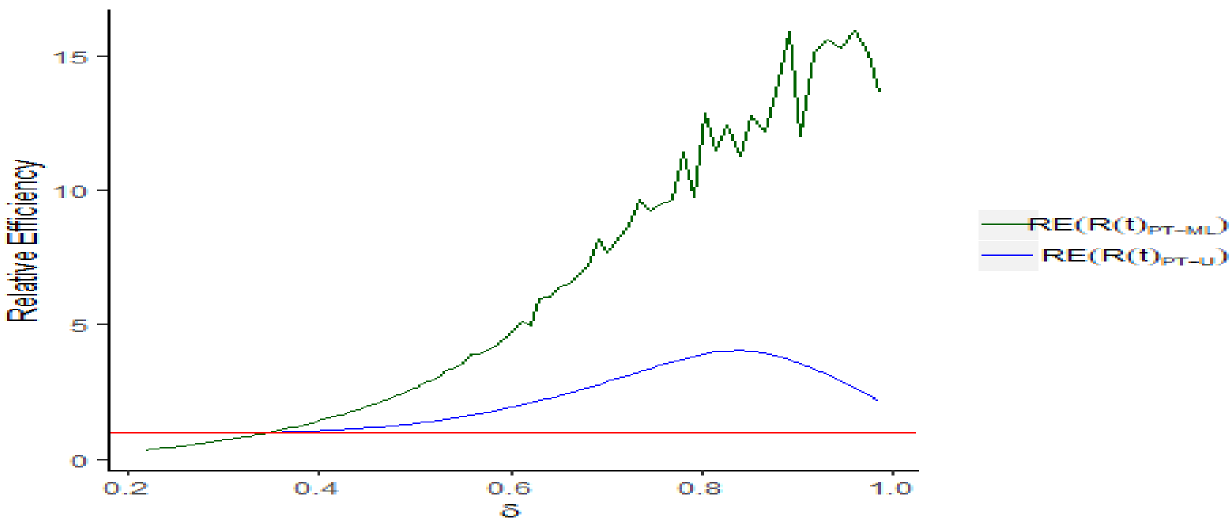

Fig. 1: Relative efficiency of $\hat{R}(t)_{PT-ML}$ over $\hat{R}(t)$ and $\tilde{R}(t)_{PT-U}$ over $\tilde{R}(t)$ with respect to $\delta = \frac{\lambda_0}{\lambda}$

$R(t)$ based on MLE and UMVUE perform better than $R(t)$ in particular interval of λ.

In Figures 2 and 3,we have shown Coverage probability of PTCI of λ with respect to $\delta = \frac{\alpha}{\alpha_0}$ for $\alpha = 0.05$ and $n = 2$ and $n = 11$,we observe that value of δ tends to 0 to ∞.The CP of PTCI is always greater than $1-\varepsilon$. This domination interval is larger for smaller sample sizes. Thus, we can conclude that the CP of PTCI of α is greater than the CP of ETCI for some values of δ in a specific interval around 1.

In Figures 4 and 5,we have shown Expected length of ETCI with PTCI of λ with respect to $\delta = \frac{\alpha}{\alpha_0}$ for $\alpha = 0.05$ and $n = 10$ and $n = 30$.The scaled EL of PTCI with the ETCI with respect to δ. We observe that there exists an interval of δ for which the EL of PTCI is lower than that of ETCI.This interval of δ for which EL of PTCI is lower decreases with the increase in sample size. We also note that as δ tends to 0 or ∞,the EL of PTCI tend to be close to the EL of ETCI.

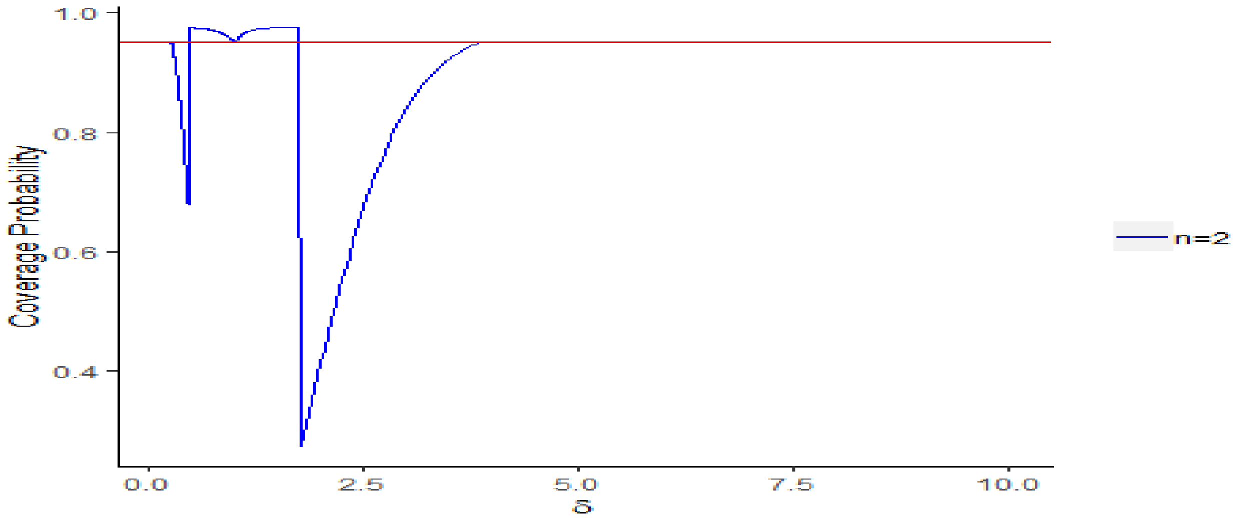

Fig. 2: Coverage probability of PTCI of α with respect to $\delta = \frac{\alpha}{\alpha_0}$ for $\varepsilon = 0.05$ and $n = 2$

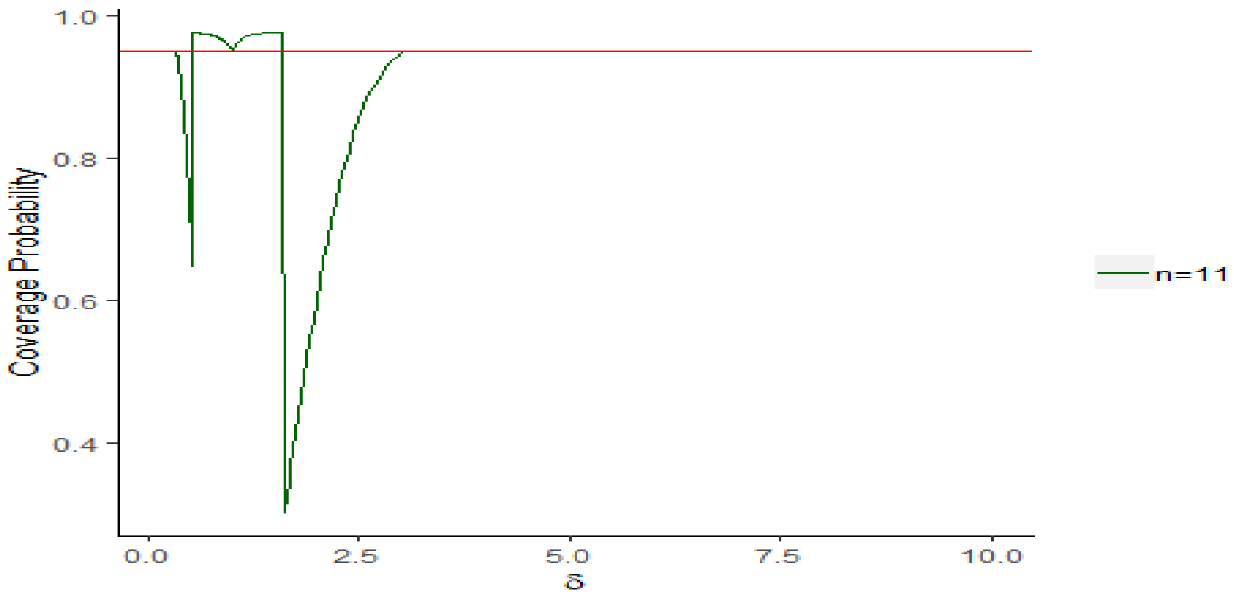

Fig. 3: Coverage probability of PTCI of α with respect to $\delta = \frac{\alpha}{\alpha_0}$ for $\varepsilon = 0.05$ and $n = 11$

6 An Example on Real Data

Let us now consider the real data set used by Lawless [29] data is from Nelson [30]. Concerning the time breakdown of an insulating fluid between electrodes at a voltage of 34 kv(minutes), the 19 times breakdown are

$0.96, 4.15, 0.19, 0.78, 8.01, 31.75, 7.35, 6.50, 8.27, 33.91, 32.52, 3.16, 4.85,$
$2.78, 4.67, 1.31, 12.06, 36.71, 72.89$

We apply Kolmogorov Smirnov (K-S)test to check weather for a fixed voltage level, time to break down has generalized

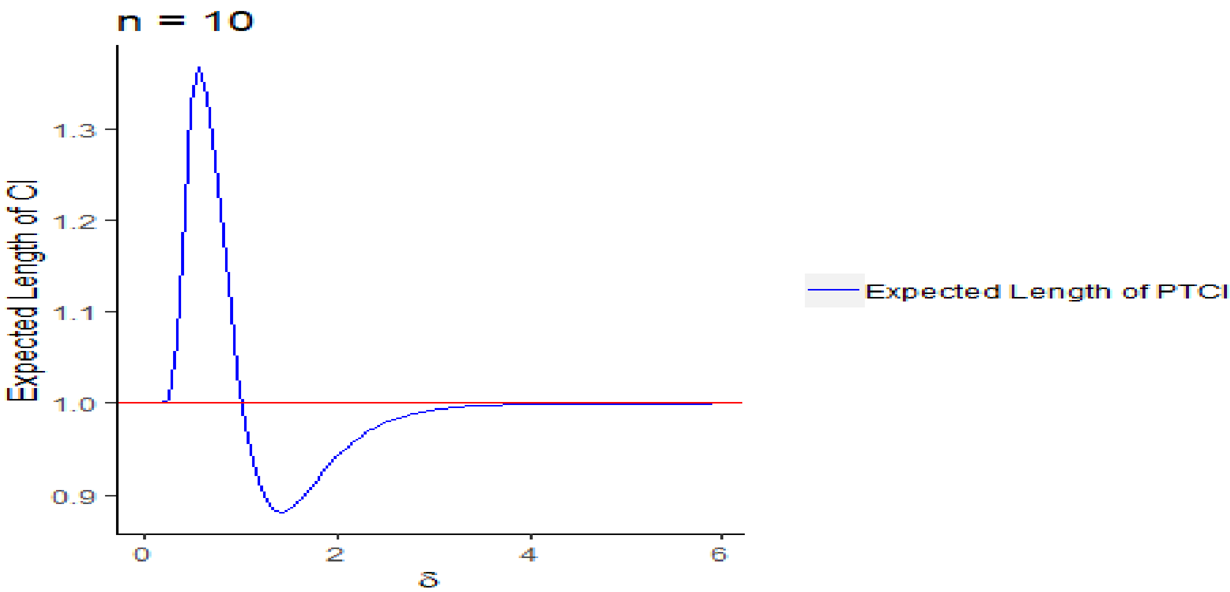

Fig. 4: Expected length of ETCI with PTCI of α with respect to $\delta = \frac{\alpha}{\alpha_0}$ for $\varepsilon = 0.05$ and $n = 10$

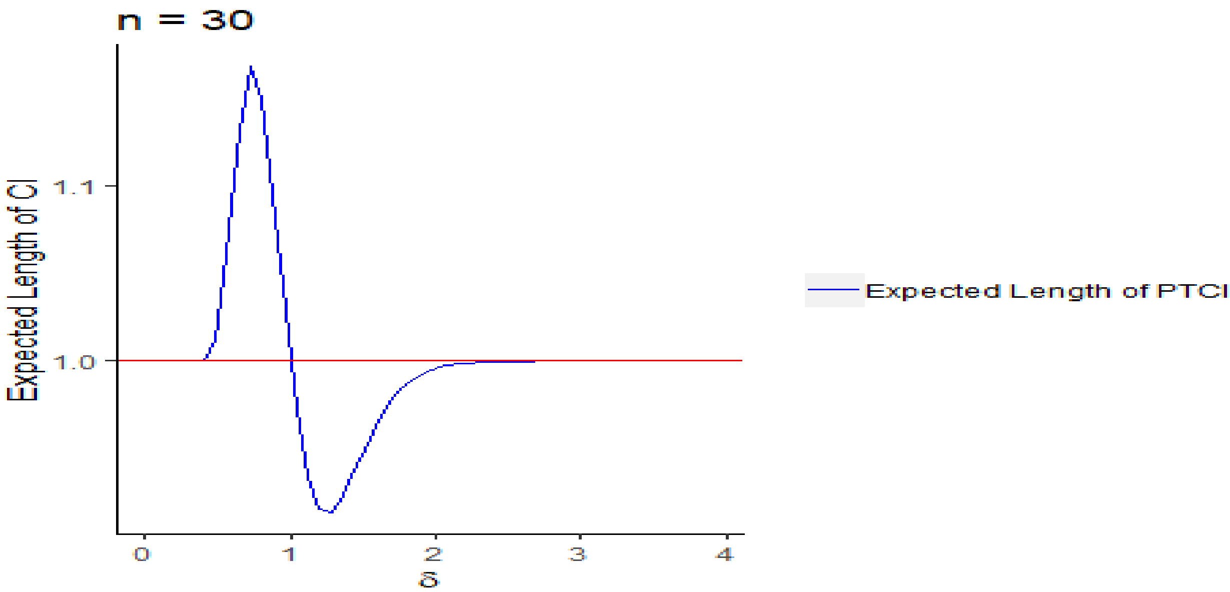

Fig. 5: Expected length of ETCI with PTCI of α with respect to $\delta = \frac{\alpha}{\alpha_0}$ for $\varepsilon = 0.05$ and $n = 30$

inverted half logistic distribution. The computed K-S statistics is 0.1493 with p-value 0.6538 . It indicated that generalized inverted half logistic distribution is suitable for data. Using an iterative algorithm, we obtained the maximum Likelihood estimator $\hat{\alpha}_{ML} = 1.9982$ for p=1

$$H_0 : \lambda = 0.15$$

$$H_1 : \lambda \neq 0.15$$

Computed test-statistic $2\alpha_0 U(R_n) = 15.5352$ which lies in the confidence interval $[7.741328.0073]$.Thus,we do not reject the null hypothesis at 5% level of significance which indicates that $\hat{\alpha}_{PT-ML} = 0.15$. Furthermore, the estimated value of $\delta = \frac{\alpha}{\alpha_0} = 1.2472$ which lies in the range of $(0.7328, 2.486)$.

7 Conclusion

The bias and MSE of all the PTEs were obtained. The numerical analysis illustrated that the proposed PTEs perform better than the classical estimators whenever the true value of parameter is close to the prior guessed value. According to the analysis of real-life data set, the PTCI of the parameter has greater coverage probability than that of the equal tail confidence interval in the neighborhood of null hypothesis. Thus, one can construct the PTEs and PTCIs which are superior to their classical counterparts whenever some prior information is available.

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Conflict of Interest

The authors declare that they have no conflict of interest.

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