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A Decomposition Algorithm for Solving Multi-Level Large-Scale Linear Programming Problems With Neutrosophic Parameters in the Constrains

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Abstract: Multi-level programming (MLP) is an important branch of operation research. The majority of optimization problems-humans currently face- have very large numbers of variables and constraints and are called large-scale programming problems. However, practical situations entail some imprecision regarding some decisions and performances. Neutrosophic sets play a vital role by considering three independent degrees, namely, the truth membership degree, indeterminacy membership degree, and falsity membership degree, of any aspect of an uncertain decision. The present study focuses on solving multi-level large-scale linear programming problems with neutrosophic parameters in the constrains by considering the problem coefficients to be trapezoidal neutrosophic numbers. The neutrosophic form of the problem is transformed into an equal crisp model in the first stage of the solution methodology to reduce the problem's complexity. In the second stage, a decomposition algorithm is used to obtain the Pareto optimal solution among conflicted decision levels. The proposed algorithm is validated by an illustrative example.

Keywords: Large scale, Linear programming, Multi-level programming, Neutrosophic set, Trapezoidal neutrosophic number.

1 Introduction

Multi-level optimization problems include decentralized planning problems with multiple decision makers (DMs) in a multi-level or hierarchical organization where decision makers interact with each other [1].

A key feature of multi-level programming problems (MLPP) is that a planner at a certain level of the hierarchy can have its objective function and decision space partially defined by other levels. In addition, each planner can use control instruments to influence the policies on other levels to achieve their own objectives [1].

In real-world applications, most optimization problems are large-scale programming problems that include numerous variables and constraints [2]. The block angular structure of constraints is a notable structure in major large-scale programming problems [3]. In this structure, a large problem is divided into smaller sub-problems that appear together and share common resources in the upper-most interconnected constraints. From this perspective, Dantzig and Wolfe [4] proposed a decomposition algorithm for large-scale programming problems with block angular structures.

Neutrosophic sets are described by three independent degrees, specifically, the truth membership degree (T), indeterminacy membership degree (I), and falsity membership degree (F), where T, I, and F are standard or non-standard subsets of $]0^{-},1^{+}[$ [5].

Considerable studies have been conducted on multi-level and multi-objective large-scale programming problems [2, 3, 6–9]. In [8], Emam et al. proposed a decomposition algorithm for solving a multi-level large-scale quadratic programming problem with stochastic parameters in the objective functions. In the first stage, the stochastic nature of the problem is transformed into an equivalent crisp problem to reduce the complexity. In the second stage, Taylor series and a decomposition algorithm are combined to produce the optimal solution.

Sultan et al. [6] considered a three-level large-scale linear programming problem in which the objective functions at all levels must be maximized. A three-level programming problem may be seen as a static version of the

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Stackelberg strategy. The authors proposed a procedure to solve a three-planner model and obtained a solution to this problem. On every level, the goal is to optimize the problem independently as a large-scale programming problem by means of the Dantzig and Wolfe decomposition method. Consequently, the optimization process is performed as a series of sub-problems that can be solved independently.

Several studies have been conducted in the field of neutrosophic linear programming problems [5, 10–13].

Hussian et al. [10] proposed linear programming problems based on a neutrosophic environment. They converted neutrosophic linear programming problems into crisp programming models using neutrosophic set parameters.

Abdel-Baset et al. [11] introduced the neutrosophic integer programming problem (NIPP) in which the parameters are trapezoidal neutrosophic numbers. The degrees of the T, F and I membership functions of the objectives are taken into account simultaneously. The NIPP has been converted into a crisp programming model using the T, F, and I membership functions and single-valued triangular neutrosophic numbers.

This rest of this paper is arranged, as follows: Section 2 contains some preliminaries. In Section 3, a three-level large-scale linear programming problem (TLLSLPP) with neutrosophic parameters in the constraints (N-TLLSLPP)is formulated. In Section 4, the neutrosophic nature of the problem is simplified into an equivalent crisp form. In Section 5, a decomposition algorithm for the TLLSLPP is presented. An algorithm for solving the (N-TLLSLPP) with neutrosophic parameters in the constraints is proposed in Section 6. Furthermore, the results and the solution algorithm are illustrated with a numerical example in Section 7. Section 8 is devoted to conclusion and suggestions for further research.

2 Preliminaries

This section presents a review of key neutrosophic set concepts and definitions.

Definition 1. (A single-valued neutrosophic set) [14]

Let Y be a universe of discourse. A single-valued neutrosophic set N over Y is an object having the form N = $\{\langle y, T_N(y), I_N(y), F_N(y)\rangle : y \in Y\}$, where $T_N(y)$: Y $\rightarrow$[0,1], $I_N(y)$: Y $\rightarrow$[0,1] and $F_N(y)$: Y $\rightarrow$[0,1] with $0 \leq T_N(y) + I_N(y) + F_N(y) \leq 3$ for all $y \in Y$. The intervals $T_N(y)$, $I_N(y)$ and $F_N(y)$ denote the T, I and the F membership functions of y to N, respectively.

Definition 2 [13] The trapezoidal neutrosophic number $\tilde{G}$ is a neutrosophic set in R with the following T, I and F membership functions:

$$T_{\tilde{G}}(X) = \begin{cases} \propto_{\tilde{G}} \left(\frac{x-g_1}{g_2-g_1}\right) & (g_1 \leq x \leq g_2) \\ \propto_{\tilde{G}} & (g_2 \leq x \leq g_3) \\ \propto_{\tilde{G}} & (g_3 \leq x \leq g_4) \\ 0 & otherwise \end{cases} \quad (1)$$

$$I_{\tilde{G}}(X) = \begin{cases} \frac{\left(g_2-x+\theta_{\tilde{G}}(X-g'_1)\right)}{(g_2-g'_1)} & (g'_1 \leq x \leq g_2) \\ \theta_{\tilde{G}} & (g_2 \leq x \leq g_3) \\ \frac{\left(x-g_3+\theta_{\tilde{G}}(g'_4-x)\right)}{(g'_4-g_3)} & (g_3 \leq x \leq g'_4) \\ 1 & otherwise \end{cases} \quad (2)$$

$$F_{\tilde{G}}(X) = \begin{cases} \frac{\left(g_2-x+\beta_{\tilde{G}}(X-g''_1)\right)}{(g_2-g'_1)} & (g''_1 \leq x \leq g_2) \\ \beta_{\tilde{G}} & (g_2 \leq x \leq g_3) \\ \frac{\left(x-g_3+\beta_{\tilde{G}}(g''_4-x)\right)}{(g''_4-g_3)} & (g_3 \leq x \leq g''_4) \\ 1 & otherwise \end{cases} \quad (3)$$

where $\propto_{\tilde{G}}$, $\theta_{\tilde{G}}$ and $\beta_{\tilde{G}}$ represent the maximum truthiness degree, minimum indeterminacy degree, minimum falsity degree, respectively, and $\propto_{\tilde{G}}$, $\theta_{\tilde{G}}$ and $\beta_{\tilde{G}} \in [0, 1]$. In addition, $g''_1 \leq g_1 \leq g'_1 \leq g_2 \leq g_3 \leq g'_4 \leq g_4 \leq g''_4$.

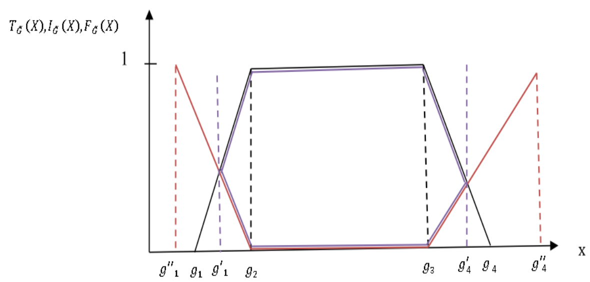

Fig. 1: The T, I, and F membership functions of the trapezoidal neutrosophic number.

Definition 3 [15] A ranking function of neutrosophic numbers is a function $N(R) \rightarrow R$, *where* $N(R)$ is a set of neutrosophic numbers defined on the set of real numbers, which maps each neutrosophic number onto the real line. Let $\tilde{C} = \langle (c_1, c_2, c_3, c_4); \propto_{\tilde{C}}, \theta_{\tilde{C}}, \beta_{\tilde{C}}\rangle$ and $\tilde{D} = \langle (d_1, d_2, d_3, d_4); \propto_{\tilde{D}}, \theta_{\tilde{D}}, \beta_{\tilde{D}}\rangle$ be two trapezoidal neutrosophic numbers; then,

1.If $R(\tilde{C}) > R(\tilde{D})$ then $\tilde{C} > \tilde{D}$,
2.If $R(\tilde{C}) < R(\tilde{D})$ then $\tilde{C} < \tilde{D}$,
3.If $R(\tilde{C}) = R(\tilde{D})$ then$\tilde{C} = \tilde{D}$.

3 Problem Formulation and Solution Concept

The (N-TLLSLPP) be formulated with neutrosophic parameters in the constraints as follows:

[First Level]

$$\max_{x_1,x_2} F_1 = \max_{x_1,x_2} \sum_{j=1}^{m} c_{1j}x_j \,, \tag{4}$$

where $x_3,\ldots,x_m$ solves

[Second Level]

$$\max_{x_3,x_4} F_2 = \max_{x_3,x_4} \sum_{j=1}^{m} c_{2j}x_j \,, \tag{5}$$

where $x_5,\ldots,x_m$ solves

[Third Level]

$$\max_{x_5,\, x_6} F_3 = \max_{x_5,\, x_6} \sum_{j=1}^{m} c_{3j}x_j \,, \tag{6}$$

where $x_7,\ldots,x_m$ solves

Subject to

$$x \in G. \tag{7}$$

where

$$\begin{aligned} G = \{a_{01}x_1 + a_{02}x_2 \quad & + a_{0m}x_m \leq \tilde{b}_0, \\ d_1x_1 \quad & \leq \tilde{b}_1, \\ d_2x_2 \quad & \leq \tilde{b}_2, \\ & d_mx_m \leq \tilde{b}_m, \\ & x_1,\ldots,x_m \geq 0\}. \end{aligned}$$

In problems (4) - (7), $x_j \in R, (j = 1,2,...,m)$ is a real vector of variables, G is the large-scale linear constraint set where $b = (\tilde{b}_0,\ldots,\tilde{b}_m)^T$ are trapezoidal neutrosophic numbers, $b = (\tilde{b}_0,\ldots,\tilde{b}_m)^T$ is an $(m+1)$ vector, and $a_{01},...,a_{0m},d_1,...,d_m$ are constants. Therefore, $F_i : R^m \to R, (i = 1,2,3)$ represents the first-level objective function, second-level objective function, and third-level objective function, respectively. Furthermore, the first-level decision maker (FLDM) has x_1,x_2, representing the first decision level choice; as well as the second-level decision maker (SLDM) and the third-level decision maker (TLDM) have x_3,x_4 and x_5,x_6, representing the second decision level choice and the third decision level choice, respectively.

Definition 4

For any $(x_1,x_2 \in G_1 = \{x_1,x_2 | \, (x_1,...,x_m) \in G\})$ given by the FLDM and $(x_3,x_4 \in G_2 = \{x_3,x_4 \,|\, (x_1,...,x_m) \in G\})$ given by the SLDM, if the decision-making variable $(x_5,x_6 \in G_3 = \{x_5,x_6 \,|\, (x_1,...,x_m) \in G\})$ is the Pareto optimal solution of the TLDM, then $(x_1,...,x_m)$ is a feasible solution of the TLLSLPP.

Definition 5

If $x^* \in R^m$ is a feasible solution of the TLLSLPP with probability $\prod_{i=1}^{m} \alpha_i$, no other feasible solution $x \in G$ exists, such that $F_1(x^*) \leq F_1(x)$. Thus, x^* is the Pareto optimal solution of the TLLSLPP.

The basic idea in treating the (N-TLLSLPP) is to use the ranking function to transform each trapezoidal number into an equivalent crisp number.

If an NTLLSLPP is in the maximization state, the ranking function for this trapezoidal neutrosophic number can be defined as follows [15]:

$$R(\tilde{g}) = \left| \left(\frac{-\frac{1}{3}(3g^l - 9g^u) + 2(g^{m1} - g^{m2})}{2} \right) * (T_{\tilde{g}} - I_{\tilde{g}} - F_{\tilde{g}}) \right| \tag{8}$$

If the NTLLSLPP is in the minimization state, the ranking function for the trapezoidal neutrosophic number can be defined as follows [15]:

$$R(\tilde{g}) = \left(\frac{(g^l + g^u) - 3(g^{m1} + g^{m2})}{-4} \right) * (T_{\tilde{g}} - I_{\tilde{g}} - F_{\tilde{g}}) \tag{9}$$

where $(\tilde{g} = g^l, g^{m1}, g^{m2}, g^u;\ T_{\tilde{g}}, I_{\tilde{g}}, F_{\tilde{g}})$ is a trapezoidal neutrosophic number and $g^l, g^{m1}, g^{m2},$ *and* g^u are the lower bound, the first and second median values as well as upper bound for the trapezoidal neutrosophic number, respectively. Moreover, $T_{\tilde{g}}$, $I_{\tilde{g}}$ *and* $F_{\tilde{g}}$ are the truth, indeterminacy and falsity degree of the trapezoidal number.

If the reader deals with a symmetric trapezoidal neutrosophic number, which has the following form: $\tilde{g} = \langle (g^{m1}, g^{m2}); \alpha, \beta \rangle$, where $\alpha = \beta$ and $\alpha, \beta > 0$, the ranking function for the neutrosophic number will be defined as follows.

$$R(\tilde{a}) = \left(\frac{(a^{m1} + a^{m2}) + 2(\alpha + \beta)}{2} \right) * T_{\tilde{a}} - I_{\tilde{a}} - F_{\tilde{a}} \tag{10}$$

4 Deterministic Three-Level Large-Scale Linear Programming Problem

The (N-TLLSLPP) can be simplified into the following deterministic TLLSLPP after implementing the maximization ranking function in Eq. (8):

[First Level]

$$\max_{x_1,x_2} F_1 = \max_{x_1,x_2} \sum_{j=1}^{m} c_{1j}x_j \,, \tag{11}$$

where $x_3,\ldots,x_m$ solves

[Second Level]

$$\max_{x_3,x_4} F_2 = \max_{x_3,x_4} \sum_{j=1}^{m} c_{2j}x_j \,, \quad (12)$$

where $x_5,\ldots,x_m$ solves
[Third Level]

$$\max_{x_5,\,x_6} F_3 = \max_{x_5,\,x_6} \sum_{j=1}^{m} c_{3j}x_j \,, \quad (13)$$

where $x_7,\ldots,x_m$ solves
Subject to

$$x \in G. \quad (14)$$

where

$$\begin{aligned} G = \{a_{01}x_1 + a_{02}x_2 \quad & + a_{0m}x_m \le b_0, \\ d_1x_1 \quad & \le b_1, \\ d_2x_2 \quad & \le b_2, \\ & d_mx_m \le b_m, \\ & x_1,\ldots,x_m \ge 0\}. \end{aligned}$$

5 Decomposition Algorithm for the Three-Level Large-Scale Linear Programming Problem

The TLLSLPP is solved by adopting the leader-follower Stackelberg strategy in combination with the Dantzig and Wolf decomposition method ([2, 4]).To solve the FLDM we will use decomposition technique to split the large scale into smaller n sub problems making it handled more efficiently.
The decomposition principle is based on demonstrating TLLSLPP in terms of the extreme points of the sets$d_jx_j \le b_j$, $x_j \ge 0$, $j = 1,2,..,m$.Accordingly, the solution space defined by each $d_jx_j \le b_j$,$x_j \ge 0$,$j = 1,2,..,m$ should be bounded and closed.
Afterwards, the FLDM decision variable is introduced to the SLDM to search for the optimal solution using the Dantzig and Wolf decomposition method [4]. The decomposition method breaks down the large-scale problem into n sub-problems that can be solved directly.
Last, the exact steps are followed by TLDM till the optimal solution to his problem , which is the optimal for the TLLSLPP.
Theorem 1
The decomposition algorithm terminates within a finite number of iterations, yielding a solution of the large-scale problem.
For the proof of theorem 1, the reader is referred to [4].

6 An Algorithm for Solving the (N-TLLSLPP) with Neutrosophic Parameters in the Constraints

An algorithm for solving the (N-TLLSLPP) is outlined in the following sequence of steps.
Step 1:
DMs enter their (N-TLLSLPP) with neutrosophic parameters in the constraints.
Step 2:
If the (N-TLLSLPP) is in the maximization state, every neutrosophic parameter in the constraints is converted into its equivalent crisp value by means of Eq. (8); otherwise, Eq. (9) is used. If trapezoidal neutrosophic number is a symmetric, use Eq. (10)
Step 3:
The (N-TLLSLPP) with neutrosophic parameters in the constraints is simplified into the equivalent deterministic TLLSLPP.
Step 4:
Begin with the FLDM problem and proceed to Step 5.
Step 5:
Transform the master problem in terms of the extreme points of the sets$d_jx_j \le b_j$,$x_j \ge 0$,$j = 1,2,3$.
Step 6:
Define the extreme points $x_j = \sum_{k=1}^{k_j} \beta_{jk} \hat{x}_{jk.}$, $j = 1,2,3$ by means of Balinski$'$s algorithm.
Step 7:
Set $k = 1$.
Step 8:
Calculate $z_{jk} - c_{jk} = C_B B^{-1} P_{jk} - c_{jk}$ and proceed to Step 9.
Step 9:
If $\overset{*}{z}_{jk} - \overset{*}{c}_{jk} \le 0$, proceed to Step 10; otherwise, the optimal solution is obtained, proceed to Step 15.
Step 10:
Specify $\hat{X}_{jk}$ related to min $\{\overset{*}{z}_{jk} - \overset{*}{c}_{jk}\}$, then proceed to Step 11.
Step 11:
B_{jk} related to extreme point $\hat{X}_{jk}$ must enter the solution, then proceed to Step 12.
Step 12:
Define the leaving variable, then proceed to Step 13.
Step 13:
By substituting, the vector related to the leaving variable with vector B_{jk}
The new basis is settled.
Step 14:
Set $k = k+1$, proceed to Step 8.
Step 15:
If the SLDM obtains the optimal solution, proceed to Step 19; otherwise, proceed to Step 16.

Step 16:

Set $(x_1,x_2) = (x_1^F,x_2^F)$ as the SLDM constraints, then proceed to Step 17.
Step 17:
The SLDM formulates his problem; proceed to Step 7.
Step 18:
If the TLDM obtains the optimal solution, proceed to Step 21; otherwise, proceed to Step 19.
Step 19:
Set $(x_1,x_2,x_3,x_4) = (x_1^F,x_2^F,x_3^S,x_4^S)$ as the TLDM constraints, then proceed to Step 20.
Step 20:
The TLDM formulates his problem,then proceed to Step 7.
Step 21:
$\left(x_1^F,x_2^F,x_3^S,x_4^S,x_5^T,x_{6,}^T...,x_m^T\right)$ is the optimal solution for the three-level large-scale linear programming problem, then stop.

7 Numerical Example

[First Level]

$$\underset{x_1,x_2}{Max}\ F_1(x_1,x_2) = \underset{x_1,x_2}{Max} 8x_1 + 20x_2 + x_5 + x_6 - 24$$

where x_3,x_4,x_5,x_6 solves
[Second Level]

$$\underset{x_3,x_4}{Max} F_2(x_3,x_4) = \underset{x_3,x_4}{Max} x_1 + 18x_3 + 18x_4 + 2x_5 - 2620\backslash$$

where x_5,x_6 solves$_{,6,}$
[Third Level]

$$\underset{x_5,x_6}{Max} F_3(x_5,x_6) = \underset{x_5,x_6}{Max} x_1 + 2x_2 + 18x_5 + 14x_6 - 16$$

Subject to
$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \leq (8,22,25,38)$,
$3x_1 + x_2 \leq (18,24,30,40)$,
$4x_3 + 2x_4 \leq (6,10,17,20)$,
$x_5 + 4x_6 \leq (2,8,12,20)$,
$x_1,x_2,x_3,x_4,x_5,x_6 \geq 0.$
In this example, we solve a (N-TLLSLPP) with trapezoidal neutrosophic numbers in the constraints. The order of elements for trapezoidal neutrosophic numbers is as follows: lower bound, first median value, second median value and upper bound. The decision makers' confirmation degree for each trapezoidal neutrosophic number is (0.9, 0.05, 0.05).
First step:
Because this NLP problem is a maximization problem, each trapezoidal number will be transformed into its equivalent crisp number using Eq. (8).
Then, the crisp model of the previous problem is defined as follows:

[First Level]

$$\underset{x_1,x_2}{Max}\ F_1(x_1,x_2) = \underset{x_1,x_2}{Max} 8x_1 + 20x_2 + x_5 + x_6 - 24$$

where x_3,x_4,x_5,x_6 solves
[Second Level]

$$\underset{x_3,x_4}{Max} F_2(x_3,x_4) = \underset{x_3,x_4}{Max} x_1 + 18x_3 + 18x_4 + 2x_5 - 2620\backslash$$

where x_5,x_6 solves$_{,\ 1,086,\ 0}$
[Third Level]

$$\underset{x_5,x_6}{Max} F_3(x_5,x_6) = \underset{x_5,x_6}{Max} x_1 + 2x_2 + 18x_5 + 14x_6 - 16$$

Subject to
$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \leq 40,$
$3x_1 + x_2 \leq 36,$
$4x_3 + 2x_4 \leq 16,$
$x_5 + 4x_6 \leq 20,$
$x_1,x_2,x_3,x_4,x_5,x_6 \geq 0$
First, the FLDM solves his/her problem as follows:
Identify the solution space of each sub-problem

Table 1: The problem divided into sub-problems

Sub-problem 1	Sub-problem 2	Sub-problem 3
$3x_1 + x_2 \leq 36$ $x_1,x_2 \geq 0$	$4x_3 + 2x_4 \leq 16$ $x_3,x_4 \geq 0$	$x_5 + 4x_6 \leq 20$ $x_5,x_6 \geq 0$
$C_1 = (8,20)$ $A_1 = (1,1)$ $\overset{*}{X}_2 = (0,10.5)^T$	$C_2 = (0,0)$ $A_2 = (1,1)$ $X_2 = (x_3,x_4)$	$C_3 = (1,1)$ $A_3 = (1,1)$ $X_3 = (x_5,x_6)$

Second, slack variable x_7 is used to convert the common constraints into the equation, and x_8,x_9,x_{10} are artificial variables

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 = 40$$

Iteration 0
$X_B = (x_7,x_8,x_9,x_{10})^T$
$X_B = (40,1,1,1)^T$
$C_B = (0,-M,-M,-M)$
$B = 1 \quad B^{-1} = 1$
Iteration 1
For sub-problem 1, where j=1

$$Z_1 - C_1 = C_B B^{-1} \begin{bmatrix} A_1 X_1 \\ 1 \\ 0 \\ 0 \end{bmatrix} - C_1 X_1 = -8x_1 - 20x_2 - M$$

Thus, the equivalent linear programming problem is
$\text{Min} w_1 = -8x_1 - 20x_2 - M$
Subject to
$3x_1 + x_2 \leq 36$
$x_1,x_2 >= 0$
$\hat{x}_{11} = (0,36) \quad \overset{*}{w}_1 = -720 - M$

For sub-problem 2, where j=2

$$Z_2 - C_2 = C_B B^{-1} \begin{bmatrix} A_2X_2 \\ 0 \\ 1 \\ 0 \end{bmatrix} - C_2X_2 = -M$$

Thus, the equivalent linear programming problem is
Min$w_2 = -M$
Subject to
$4x_3 + 2x_4 \leq 16$
$x_3, x_4 \geq 0$
$\hat{x}_{21} = (0,0) \quad \overset{*}{w}_2 = -M$

For sub-problem 3, where j=3

$$Z_3 - C_3 = C_B B^{-1} \begin{bmatrix} A_3X_3 \\ 0 \\ 0 \\ 1 \end{bmatrix} - C_3X_3 = -x_5 - x_6 - M$$

Thus, the equivalent linear programming problem is
Min$w_3 = -x_5 - x_6 - M$
Subject to
$x_5 + 4x_6 \leq 20$
$x_5, x_6 \geq 0$
$\hat{x}_{31} = (20,0) w^*{}_3 = -20 - M$ Define new basic variables
$w^*{}_1 < w^*{}_2 \& w^*{}_1 < w^*{}_3 \& w^*{}_1 < 0$, so β_{11}related to $\hat{x}_{11}$must enter the basic solution.
After 5 iterations, the FLDM obtains his optimal solution.

$$\left(x_1{}^F, x_2{}^F, x_3{}^F, x_4{}^F, x_5{}^F, x_6{}^F\right) = (0,36,0,0,4,0).$$

Now, set $(x_1,x_2) = (0,36)$ to the SLDM constraints.
Second, the SLDM solves his/her problem as follows:

$$Max\,F_2 = \underset{x_3,x_4}{Max} 18x_3 + 18x_4 + 2x_5 - 26$$

Subject to
$x_3 + x_4 + x_5 + x_6 \leq 4,$
$4x_3 + 2x_4 \leq 16,$
$x_5 + 4x_6 \leq 20,$
$x_3, x_4, x_5, x_6 \geq 0$
The SLDM takes the same action as the FLDM until the optimal solution is obtained $\left(x_3{}^S, x_4{}^S, x_5{}^S, x_6{}^S\right) = (0,4,0,0)$, . Now,set $(x_3,x_4) = (0,4)$ to the TLDM constraints.
Finally, the TLDM solves his/her problem as follows:

$$Max\,F_3 = \underset{x_5,x_6}{Max}\, 18x_5 + 14x_6 + 56$$

Subject to

$$x_5 + x_6 \leq 0,$$

$$x_5 + 4x_6 \leq 20,$$

The TLDM takes the same action as the FLDM and SLDM until he obtains the optimal solution $\left(x_5{}^T, x_6{}^T\right) = (0,0)$.
Thus, $\left(x_1{}^F, x_2{}^F, x_3{}^S, x_4{}^S, x_5{}^T, x_6{}^T\right) = (0,36,0,4,0,0)$ is the optimal solution for the TLLSLPP, where$F_1 = 696, F_2 = 46$ and $F_3 = 58$.

8 Conclusion

This paper presented a solution algorithm for solving the (N-TLLSLPP) . The neutrosophic nature of the problem was transformed into an equivalent crisp model in the first stage of the solution algorithm to reduce the difficulty. In the second stage, the decomposition algorithm was used to reach the optimal solution among conflicted decision levels. Finally, a numerical example was presented to validate the accuracy of the suggested solution algorithm.
However, a number of points open to future debate should be examined and investigated in neutrosophic multi-level linear optimization:

1. Multi-level large-scale linear decision-making problems with neutrosophic parameters in both the objective functions and constraints.
2. Multi-level large-scale linear multi-objective decision-making problems with neutrosophic parameters in both the objective functions and constraints and with integrality conditions.

Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this article

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