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Optical Solitons of The Extended Gerdjikov-Ivanov Equation in DWDM System by Extended Simplest Equation Method

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Abstract: The extended simplest equation method has been employed to obtain optical soliton solutions to the improved Gerdjikov-Ivanov equation in dense wavelength division multiplexed (DWDM) system for both kerr law and parabolic law nonlinearities. The procedure reveals new singular soliton solutions, bright soliton solutions, solutions in terms of Jacobi's elliptic function. Moreover, in the limiting case of the modulus of ellipticity, new singular and singular-periodic soliton solutions are obtained.

Keywords: DWDM system, extended simplest equation method, Gerdjikov-Ivanov model, optical solutions

1 Introduction

The Gerdjikov-Ivanov (GI) model is one of the models that investigate the dynamics of optical soliton propagation for transmission technology along with transcontinental and transoceanic distances, optical fibers, data transmission across, and telecommunications industry. This model has been studied for polarization-preserving fibers along with strategic algorithms such as modified simple equation scheme, the csch method, the extended $\tanh-\coth$ method, $\frac{G'}{G^2}$-expansion method, sine-cosine method, trial and extended trial equation methods, trial equation integration architecture, Kudryashov method, extended Kudryashov's method, $\tan(\frac{\phi(\zeta)}{2})$-expansion method, and the $\exp(-(\phi))$-expansion method [1–16]. DWDM technology is an essential feature that needs to be integrated in fiber-optic communication system [17–22]. This multiplies the information carrying capacity through these fibers. Thus, parallel transmission of data is possible across trans-continental and trans-oceanic distances in just a matter of a few femto-seconds. Only perfecting DWDM technology can achieve such an engineering marvel. The extended simplest equation method has been applied to the extended GI model in DWDM system for both kerr law and parabolic law nonlinearities which also improve the model. Strategic singular and bright soliton solutions are retrieved. Also, solutions in terms of Jacobi's elliptic function and, in the limiting case of the modulus of ellipticity, singular and singular-periodic soliton solutions have been listed with their respective existence criteria.

2 Governing model

The Gerdjikov-Ivanov equation $[1-11]$ is represented as

$$i\,\psi_t + a\,\psi_{xx} + b\,|\psi|^4\,\psi + i\,e\,\psi^2\,\psi_x^* = 0. \tag{1}$$

The first term denotes the temporal evolution of pulses when the existence of group velocity dispersion is supplied by the coefficient of a in this quite important governing model. The complex valued function $\psi(x,t)$ signifies the wave profile. The coefficient of b is named as the nonlinear term that signifies quintic nonlinearity. The existence of a form of dispersive phenomenon is ensured with the coefficient of e.

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2.1 Kerr law nonlinearity

For Kerr law nonlinearity in DWDM system Eq. (1) generalizes to

$$i\psi_t^{(l)}+a_l\psi_{xx}^{(l)}+b_l|\psi^{(l)}|^4\psi^{(l)}+ie_l(\psi^{(l)})^2(\psi_x^{(l)})^*$$
$$+\left\{c_l\psi_{xt}^{(l)}+d_l|\psi^{(l)}|^2\psi^{(l)}+\sum_{n\neq l}^{N}\alpha_{ln}|\psi^{(n)}|^2\psi^{(l)}\right\}=0. \quad (2)$$

The coefficients of a_l and c_l correspond to group velocity dispersion and the spatio-temporal dispersion respectively. Moreover, the coefficients of d_l are indicated by self-phase modulation while the coefficients of α_{ln} stand for cross-phase modulation effect. The dependent variable $\psi^{(l)}(x,t)$ represents soliton profile in every single channel for $1\leq l\leq r$ and r is the number of channels.

In this subsection we obtain singular soliton solutions, bright soliton solutions, and solutions in terms of Jacobi's elliptic function to Eq.(2) by the extended simplest equation method [4,5,17,20]. To solve Eq.(2), we use the following wave transformation.

$$\psi^{(l)}(x,t)=w_l(\zeta(x,t))e^{i\theta(x,t)}, \quad (3)$$

where ζ represents the amplitude component of the soliton and θ is the phase component of the soliton that is described as

$$\zeta(x,t)=k_1x-\nu t, \quad (4)$$

$$\theta(x,t)=-k_2x+\mu t+k_3. \quad (5)$$

Here, ν is the velocity of the soliton, k_2 is the frequency of the solitons in each of the two components while w is the solution wave number and k_3 is the phase constant. Putting (3) along with (4), (5) into (2) we get

$$-\mu w_l-i\nu w_l'-k_2^2a_lw_l-2ia_lk_1k_2w_l'+k_1^2a_lw_l''$$
$$+b_lw_l^5+k_2c_l\mu w_l+ic_lk_1w_l'+ic_lk_2\nu w_l'-k_1c_l\nu w_l''$$
$$+d_lw_l^3-k_2e_lw_l^3+ik_1e_lw_l^2w_l'+\left(\sum_{n\neq l}^{N}\alpha_{ln}w_n^2\right)w_l=0, \quad (6)$$

The Balancing principle leads to
$w_n=w_l$.
Breaking down into real and imaginary parts we get

$$(-\mu-k_2^2a_l+k_2c_l\mu)w_l+b_lw_l^5+(k_1^2a_l-k_1c_l\nu)w_l''$$
$$+(d_l-k_2e_l+\alpha_l)w_l^3=0, \quad (7)$$

$$(-\nu-2a_lk_1k_2+k_1c_l\mu+k_2c_l\nu+k_1e_lw_l^2)w_l^2=0, \quad (8)$$

from (8) the velocity of the soliton solution is

$$\nu=\frac{2a_lk_1k_2-k_1c_l\mu}{k_2c_l-1}, \quad (9)$$

and we obtain the conditions $k_2c_l-1\neq 0$ and $e_l=0$. Balancing w_l'' with w_l^5 in equation (7) gives $N=\frac{1}{2}$ Since N is not integer we set $w_l=\sqrt{\varphi_l}$. Substituting into (7) and multiplying by $4\varphi_l\sqrt{\varphi_l}$ we get

$$4(-\mu-k_2^2a_l+k_2c_l\mu)\varphi_l^2+4b_l\varphi_l^4 \quad (10)$$
$$+\frac{2k_1^2c_l^2\mu-2k_1^2a_l-2k_1^2k_2a_lc_l}{k_2c_l-1}\varphi_l\varphi_l''$$
$$-\frac{k_1^2c_l^2\mu-k_1^2a_l-k_1^2k_2a_lc_l}{k_2c_l-1}(\varphi_l')^2$$
$$+4(d_l-k_2e_l+\alpha_l)\varphi_l^3=0.$$

Balancing $\varphi_l\varphi_l''$ with φ^4 gives $N=1$.

2.2 The Application

The following assumption is made to retrieve singular soliton solutions, bright soliton solutions, and solutions in terms of Jacobi's elliptic function to Eq.(10) using the extended simplest equation method.

$$\varphi_l=\sum_{i=0}^{N}A_i^{(l)}u^i, \quad (11)$$

where

$$(u')^2=\Gamma(u)=\frac{\Theta(u)}{\Upsilon(u)}=\frac{\sum_{i=0}^{\tau}\lambda_iu^i}{\sum_{i=0}^{\rho}\chi_iu^i}, \quad (12)$$

with λ_i, χ_i, $A_i^{(l)}$ are constants, λ_τ, χ_ρ, $A_N^{(l)}$ are non-zero, and $\Theta(u),\Upsilon(u)$ are polynomials of u. We derive the terms $(\varphi_l')^2$ and φ_l'' from Eq. (11) and Eq. (12) as

$$(\varphi_l')^2=\frac{\Theta(u)}{\Upsilon(u)}\left(\sum_{i=0}^{N}iA_i^{(l)}u^{i-1}\right)^2, \quad (13)$$

and

$$\varphi_l''=\frac{\Theta'(u)\Upsilon(u)-\Theta(u)\Upsilon'(u)}{2\Upsilon(u)^2}\left(\sum_{i=0}^{N}iA_i^{(l)}u^{i-1}\right)$$
$$+\frac{\Theta(u)}{\Upsilon(u)}\left(\sum_{i=0}^{N}i(i-1)A_i^{(l)}u^{i-2}\right). \quad (14)$$

Equation (12) can be formulated as

$$\pm(\zeta-\zeta_0)=\int\frac{du}{\sqrt{\Gamma(u)}}=\int\sqrt{\frac{\Upsilon(u)}{\Theta(u)}}du. \quad (15)$$

The balancing principle applied to (10) implies

$$\tau=\rho+2N+2, \quad (16)$$

setting $N=1$ and $\rho=0$ we get $\tau=4$. Thus, from (11) we have

$$\varphi_l = A_0^{(l)} + A_1^{(l)} u, \tag{17}$$

$$(\varphi_l^{'})^2 = \frac{(A_1^{(l)})^2 \Sigma_{i=0}^{4} \lambda_i u^i}{\chi_0}, \tag{18}$$

$$\varphi_l^{''} = \frac{A_1^{(l)} \Sigma_{i=0}^{4} i\lambda_i u^{i-1}}{2\chi_0}, \tag{19}$$

where $\lambda_4 \neq 0$ and $\chi_0 \neq 0$. Substituting Eqs. $(17)-(19)$ into Eq. (10), we obtain a system of algebraic equations. Solving the system, we get:

$$\lambda_0=\lambda_0,\ \lambda_1=\lambda_1,\ A_0^{(l)}=A_0^{(l)},\ A_1^{(l)}=A_1^{(l)},\ \chi_0=\chi_0$$

$$\lambda_2 = \frac{R_1}{2k_1^2((A_0^{(l)})^4 b_l c_l^2 - (A_0^{(l)})^2 a_l + (4A_0^{(l)})^3 c_l^2 (d_l - k_2 e_l + \alpha_l))},$$

$$\lambda_3 = \frac{R_2}{(6A_0^{(l)} k_1^2((A_0^{(l)})^3 b_l c_l^2 - A_0^{(l)} a_l + (A_0^{(l)})^2 c_l^2 (d_l - k_2 e_l + \alpha_l))},$$

$$\lambda_4 = \frac{R_3}{3k_1^2((A_0^{(l)})^4 b_l c_l^2 - (A_0^{(l)})^2 a_l + (A_0^{(l)})^3 c_l^2 (d_l - k_2 e_l + \alpha_l))},$$

$$\mu = \frac{R_4}{4\chi_0 (A_0^{(l)})^2 c_l^2 k_2^2 - 8\chi_0 (A_0^{(l)})^2 c_l k_2 + 4\chi_0 (A_0^{(l)})^2 + \lambda_1 A_0^{(l)} A_1^{(l)} c_l^2 k_1^2 - \lambda_0 (A_1^{(l)})^2 c_l^2 k_1^2}.$$

Where

$$\begin{aligned}
R_1 =& 8(A_0^{(l)})^4 b_l \chi_0 + 4(A_0^{(l)})^3 \chi_0 (d_l - k_2 e_l + \alpha_l) \\
&+ 2(A_1^{(l)})^2 \lambda_0 a_l k_1^2 + 8(A_0^{(l)})^4 b_l c_l \chi_0 k_2 (c_l k_2 + 2) \\
&- 2A_0^{(l)} A_1^{(l)} \lambda_1 a_l k_1^2 (1 + 2(A_0^{(l)})^2 a_l^{-1} b_l c_l^2) \\
&+ 4(A_0^{(l)})^3 c_l^2 \chi_0 k_2^2 (d_l - k_2 e_l - 2c_l^{-1} k_2^{-1} + \alpha_l (1 - 2c_l^{-1} k_2^{-1})) \\
&- 3A_0^{(l)} (A_1^{(l)})^2 \lambda_0 c_l^2 k_1^2 (d_l - k_2 e_l + \alpha_l) \\
&+ 3(A_0^{(l)})^2 A_1^{(l)} \lambda_1 c_l^2 k_1^2 (d_l - k_2 e_l + \alpha_l) - 4(A_0^{(l)})^2 (A_1^{(l)})^2 \lambda_0 b_l c_l^2 k_1^2, \\
R_2 =& (3A_1^{(l)} d_l - k_2 e_l + 3A_1^{(l)} \alpha_l + 8A_0^{(l)} A_1^{(l)} b_l)(4\chi_0 (A_0^{(l)})^2 c_l^2 k_2^2 \\
&- 8\chi_0 (A_0^{(l)})^2 c_l k_2 + 4\chi_0 (A_0^{(l)})^2 + \lambda_1 A_0^{(l)} A_1^{(l)} c_l^2 k_1^2 - \lambda_0 (A_1^{(l)})^2 c_l^2 k_1^2), \\
R_3 =& (A_1^{(l)})^2 b_l (4\chi_0 (A_0^{(l)})^2 c_l^2 k_2^2 - 8\chi_0 (A_0^{(l)})^2 c_l k_2 + 4\chi_0 (A_0^{(l)})^2 \\
&+ \lambda_1 A_0^{(l)} A_1^{(l)} c_l^2 k_1^2 - \lambda_0 (A_1^{(l)})^2 c_l^2 k_1^2), \\
R_4 =& (4(A_0^{(l)})^4 b_l \chi_0 - 4(A_0^{(l)})^2 a_l \chi_0 k_2^2)(1 - c_l k_2) \\
&+ 4(A_0^{(l)})^3 \chi_0 (d_l - k_2 e_l - c_l d_l k_2 + (1 - c_l k_2)\alpha_l) \\
&- ((A_1^{(l)})^2 \lambda_0 a_l k_1^2 - A_0^{(l)} A_1^{(l)} \lambda_1 a_l k_1^2)(1 + c_l k_2).
\end{aligned}$$

Substituting into (12) and (15), we get

$$\pm(\zeta - \zeta_0) = Q \int \frac{du}{\sqrt{\Gamma(u)}}, \tag{20}$$

where $Q = \sqrt{\frac{\chi_0}{\lambda_4}},\ \Gamma(u) = \Sigma_{i=0}^{4} \frac{\lambda_i}{\lambda_4} u^i$.

Therefore the traveling wave solutions to Eq.(2) are:
When $\Gamma(u) = (u-\vartheta_1)^4$

$$\psi^{(l)}(x,t) = \sqrt{A_0^{(l)} + A_1^{(l)} \vartheta_1 \pm \frac{A_1^{(l)} Q}{k_1 x - (\frac{2a_l k_1 k_2 - k_1 c_l \mu}{k_2 c_l - 1})t - \zeta_0}} \times e^{i(-k_2 x + \mu t + k_3)}, \tag{21}$$

When $\Gamma(u) = (u-\vartheta_1)^3 (u-\vartheta_2)$, and $\vartheta_2 > \vartheta_1$

$$\psi^{(l)}(x,t) = \sqrt{A_0^{(l)} + A_1^{(l)} \vartheta_1 + \frac{4A_1^{(l)} Q^2 (\vartheta_2 - \vartheta_1)}{4Q^2 - M^2}} \times e^{i(-k_2 x + \mu t + k_3)}, \tag{22}$$

When $(u-\vartheta_1)^2 (u-\vartheta_2)^2$

$$\psi^{(l)}(x,t) = \sqrt{A_0^{(l)} + A_1^{(l)} \vartheta_j + \frac{(-1)^{j+1} A_1^{(l)} (\vartheta_1 - \vartheta_2)}{\exp(\frac{M}{Q}) - 1}} \times e^{i(-k_2 x + \mu t + k_3)}, \tag{23}$$

where $M = (\vartheta_1 - \vartheta_2)(k_1 x - (\frac{2a_l k_1 k_2 - k_1 c_l \mu}{k_2 c_l - 1})t - \zeta_0)$, and $j = 1, 2$. When $\Gamma = (u-\vartheta_1)^2 (u-\vartheta_2)(u-\vartheta_3)$, and $\vartheta_1 > \vartheta_2 > \vartheta_3$

$$\psi^{(l)}(x,t) = \sqrt{A_0^{(l)} + A_1^{(l)} \vartheta_1 - \frac{2A_1^{(l)} (\vartheta_1 - \vartheta_2)(\vartheta_1 - \vartheta_3)}{2\vartheta_1 - \vartheta_2 - \vartheta_3 + (\vartheta_3 - \vartheta_2)\cosh(Y_1)}} \times e^{i(-k_2 x + \mu t + k_3)}, \tag{24}$$

where $Y_1 = \frac{k_1 \sqrt{(\vartheta_1 - \vartheta_2)(\vartheta_1 - \vartheta_3)}}{Q} (k_1 x - (\frac{2a_l k_1 k_2 - k_1 c_l \mu}{k_2 c_l - 1})t)$.
When $\Gamma = (u-\vartheta_1)(u-\vartheta_2)(u-\vartheta_3)(u-\vartheta_4)$, and $\vartheta_1 > \vartheta_2 > \vartheta_3 > \vartheta_4$

$$\psi^{(l)}(x,t) = \sqrt{A_0^{(l)} + A_1^{(l)} \vartheta_2 + \frac{A_1^{(l)} (\vartheta_1 - \vartheta_2)(\vartheta_4 - \vartheta_2)}{\vartheta_4 - \vartheta_2 + (\vartheta_1 - \vartheta_4) sn^2(Y_2)}} \times e^{i(-k_2 x + \mu t + k_3)}, \tag{25}$$

where
$Y_2 = \left[\pm \frac{\sqrt{(\vartheta_1 - \vartheta_3)(\vartheta_2 - \vartheta_4)}}{2Q} (k_1 x - (\frac{2a_l k_1 k_2 - k_1 c_l \mu}{k_2 c_l - 1})t - \zeta_0), m\right]$
and $m^2 = \frac{(\vartheta_2 - \vartheta_3)(\vartheta_1 - \vartheta_4)}{(\vartheta_1 - \vartheta_3)(\vartheta_2 - \vartheta_4)}$.

Note that $\vartheta_i,\ i = 1, ..., 4$ are the roots of $\Gamma(u) = 0$.

When $A_0^{(l)} = -A_1^{(l)} \vartheta_1$ and $\zeta_0 = 0$, the solutions $(21)-(25)$ are reduced to the following plane wave solutions

$$\psi^{(l)}(x,t) = \sqrt{\pm \frac{A_1^{(l)} Q}{k_1 x - (\frac{2a_l k_1 k_2 - k_1 c_l \mu}{k_2 c_l - 1})t}} \times e^{i(-k_2 x + \mu t + k_3)}, \tag{26}$$

$$\psi^{(l)}(x,t) = \sqrt{\frac{4A_1^{(l)} Q^2 (\vartheta_2 - \vartheta_1)}{4Q^2 - M_0^2}} \times e^{i(-k_2 x + \mu t + k_3)}, \tag{27}$$

singular soliton solutions

$$\psi^{(l)}(x,t) = \sqrt{\frac{A_1^{(l)} (\vartheta_2 - \vartheta_1)}{2} (1 \mp \coth(\frac{M_0}{2Q}))} \times e^{i(-k_2 x + \mu t + k_3)}, \tag{28}$$

and bright soliton solutions

$$\psi^{(l)}(x,t) = \sqrt{\left(\frac{D}{C + \cosh(B(k_1 x - 2a_1 k_1 k_2 t))}\right)} \times e^{i(-k_2 x + \mu t + k_3)}, \tag{29}$$

where $M_0 = (\vartheta_1 - \vartheta_2)(k_1 x - (\frac{2a_l k_1 k_2 - k_1 c_l \mu}{k_2 c_l - 1})t)$, $D = \frac{2A_1^{(l)} (\vartheta_1 - \vartheta_2)(\vartheta_1 - \vartheta_3)}{(\vartheta_3 - \vartheta_2)}$,

$B = \frac{k_1\sqrt{(\vartheta_1-\vartheta_2)(\vartheta_1-\vartheta_3)}}{Q}$, and $C = \frac{2\vartheta_1-\vartheta_2-\vartheta_3}{\vartheta_3-\vartheta_2}$.
The amplitude of the soliton is given by D where the inverse width of the soliton is given by B. The solitons will exist for $A_1^{(l)} < 0$. Furthermore, when $A_0^{(l)} = -A_1^{(l)}$ and $\zeta_0 = 0$, Jacobi's elliptic function solution (25) is written as

$$\psi^{(l)}(x,t) = \sqrt{\left(\frac{D_1}{C_1+sn^2(B_jS)}\right)} \times e^{i(-k_2x+\mu t+k_3)}, \quad (30)$$

where

$$D_1 = \frac{A_1^{(l)}(\vartheta_1-\vartheta_2)(\vartheta_4-\vartheta_2)}{(\vartheta_1-\vartheta_4)},$$

$$B_j = \frac{(-1)^j k_1\sqrt{(\vartheta_1-\vartheta_3)(\vartheta_2-\vartheta_4)}}{2Q},$$

$$C_1 = \frac{2\vartheta_4-\vartheta_2}{\vartheta_1-\vartheta_4},$$

$$S = (k_1x-(\frac{2a_lk_1k_2-k_1c_l\mu}{k_2c_l-1})t), \frac{(\vartheta_2-\vartheta_3)(\vartheta_1-\vartheta_4)}{(\vartheta_1-\vartheta_3)(\vartheta_2-\vartheta_4)}, j = 1,2.$$

Remark-1: When the modulus $m \to 1$, singular optical soliton solutions are obtained as

$$\psi^{(l)}(x,t) = \sqrt{\left(\frac{D_1}{C_1+\tanh^2(B_j(k_1x-(\frac{2a_lk_1k_2-k_1c_l\mu}{k_2c_l-1})t))}\right)} \times e^{i(-k_2x+\mu t+k_3)}, \quad (31)$$

where $\vartheta_3 = \vartheta_4$.

Remark-2: When the modulus $m \to 0$, singular-periodic solutions are obtained as

$$\psi^{(l)}(x,t) = \sqrt{\left(\frac{D_1}{C_1+\sin^2(B_j(k_1x-(\frac{2a_lk_1k_2-k_1c_l\mu}{k_2c_l-1})t))}\right)} \times e^{i(-k_2x+\mu t+k_3)}, \quad (32)$$

where $\vartheta_2 = \vartheta_3$.

2.3 Parabolic law nonlinearity

This law, known also as the cubic-quintic nonlinearity, arises in the nonlinear interaction between Langmuir waves and electrons. It describes the nonlinear interaction between the high frequency Langmuir waves and the ion acoustic waves by pondermotive forces. For parabolic law nonlinearity DWDM system, Eq. (1) generalizes to

$$i\psi_t^{(l)} + a_l\psi_{xx}^{(l)} + b_l|\psi^{(l)}|^4\psi^{(l)} + ie_l(\psi^{(l)})^2(\psi_x^{(l)})^* + \Big\{c_l\psi_{xt}^{(l)} + \delta_l|\psi^{(l)}|^4\psi^{(l)} + (d_l + \sum_{n\neq l}^{N}\gamma_{ln}|\psi^{(n)}|^2)|\psi^{(l)}|^2\psi^{(l)} + \sum_{n\neq l}^{N}(\alpha_{ln}+\beta_{ln}|\psi^{(n)}|^2)|\psi^{(n)}|^2\psi^{(l)}\Big\} = 0. \quad (33)$$

The coefficients of a_l and c_l correspond to group velocity dispersion and the spatio-temporal dispersion respectively. Moreover, the coefficients of d_l and δ_{ln} are indicated by self-phase modulation while the coefficients of γ_{ln}, α_{ln}, and β_{ln} stand for cross-phase modulation effect. The dependent variable $\psi^{(l)}(x,t)$ represents soliton profile in every single channel for $1 \le l \le r$, $r \in [1,\infty)$.
Substituting, (3) along with (4),(5) into (33), gives the same imaginary part as given by (7) and so the speed will be same as (9). However, the real part of equation (33) is

$$(-\mu - k_2^2a_l + k_2c_l\mu)w_l + (b_l+\delta_l+(\beta_l+\gamma_l))w_l^5 + (k_1^2a_l - k_1c_l\nu)w_l'' + 4(d_l - k_2e_l + \alpha_l)w_l^3 = 0, \quad (34)$$

Balancing w_l'' with w_l^5 in equation (34) gives $N = \frac{1}{2}$. Since N is unreal, we set $w_l = \sqrt{\varphi_l}$. Substituting into (34) and multiplying by $4\varphi_l\sqrt{\varphi_l}$ we get

$$4(-\mu - k_2^2a_l + k_2c_l\mu)\varphi_l^2 + 4(b_l+\delta_l+(\beta_l+\gamma_l))\varphi_l^4 + \frac{2k_1^2c_l^2\mu - 2k_1^2a_l - 2k_1^2k_2a_lc_l}{k_2c_l-1}\varphi_l\varphi_l'' - \frac{k_1^2c_l^2\mu - k_1^2a_l - k_1^2k_2a_lc_l}{k_2c_l-1}(\varphi_l')^2 + 4(d_l - k_2e_l + \alpha_l)\varphi_l^3 = 0, \quad (35)$$

Balancing $\varphi_l\varphi_l''$ with φ^4 gives $N = 1$.

2.4 The Application

We will apply the extended simplest equation method to Eq.(34) to retrieve singular soliton solutions, bright soliton solutions, and solutions in terms of Jacobi's elliptic function. The balancing principle applied to (34) implies

$$\tau = \rho + 2N + 2, \quad (36)$$

setting $N = 1$ and $\rho = 0$ we get $\tau = 4$. Hence, from (11) we have

$$\varphi_l = A_0^{(l)} + A_1^{(l)}u, \quad (37)$$

where $A_0^{(l)}$ and $A_1^{(l)}$ are constants to be defined later such that $A_1^{(l)} \neq 0$ and u satisfies Eq.(12) Substituting Eq. (37) into Eq. (35), we obtain a system of algebraic equations.

Solving the system, we get:

$$\lambda_0 = \lambda_0, \lambda_1 = \lambda_1, A_0^{(l)} = A_0^{(l)}, A_1^{(l)} = A_1^{(l)}, \chi_0 = \chi_0$$

$$\lambda_2 = \frac{R_1}{2k_1^2((A_0^{(l)})^4(b_l+\delta_l)c_l^2 - (A_0^{(l)})^2 a_l + (A_0^{(l)})^3 c_l^2(d_l - k_2 e_l + \alpha_l))},$$

$$\lambda_3 = \frac{R_2}{(6A_0^{(l)}k_1^2((A_0^{(l)})^3(b_l+\delta_l+(\beta_l+\gamma_l))c_l^2 - A_0^{(l)}a_l + (A_0^{(l)})^2 c_l^2(d_l - k_2 e_l + \alpha_l))},$$

$$\lambda_4 = \frac{R_3}{3k_1^2((A_0^{(l)})^4(b_l+\delta_l+(\beta_l+\gamma_l))c_l^2 - (A_0^{(l)})^2 a_l + (A_0^{(l)})^3 c_l^2(d_l - k_2 e_l + \alpha_l))},$$

$$\mu = \frac{R_4}{4\chi_0(A_0^{(l)})^2 c_l^2 k_2^2 - 8\chi_0(A_0^{(l)})^2 c_l k_2 + 4\chi_0(A_0^{(l)})^2 + \lambda_1 A_0^{(l)} A_1^{(l)} c_l^2 k_1^2 - \lambda_0 (A_1^{(l)})^2 c_l^2 k_1^2}.$$

Where

$$\begin{aligned}
R_1 =& 8(A_0^{(l)})^4(b_l+\delta_l+(\beta_l+\gamma_l))\chi_0 + 4(A_0^{(l)})^3\chi_0(d_l - k_2 e_l + \alpha_l)\\
&+2(A_1^{(l)})^2\lambda_0 a_l k_1^2 + 8(A_0^{(l)})^4(b_l+\delta_l+(\beta_l+\gamma_l))c_l\chi_0 k_2(c_l k_2+2)\\
&-2A_0^{(l)}A_1^{(l)}\lambda_1 a_l k_1^2(1+2(A_0^{(l)})^2 a_l^{-1}(b_l+\delta_l+(\beta_l+\gamma_{ln}))c_l^2)\\
&+4(A_0^{(l)})^3 c_l^2\chi_0 k_2^2(d_l - k_2 e_l - 2c_l^{-1}k_2^{-1} + \alpha_l(1-2c_l^{-1}k_2^{-1}))\\
&-3A_0^{(l)}(A_1^{(l)})^2\lambda_0 c_l^2 k_1^2(d_l + -k_2 e_l \alpha_l) + 3(A_0^{(l)})^2 A_1^{(l)}\lambda_1 c_l^2 k_1^2(d_l - k_2 e_l + \alpha_l)\\
&-4(A_0^{(l)})^2(A_1^{(l)})^2\lambda_0(b_l+\delta_l+(\beta_l+\gamma_l))c_l^2 k_1^2,\\
R_2 =& 3A_1^{(l)}(b_l+\delta_l+(\beta_l+\gamma_l)) + 3A_1^{(l)}\alpha_l + 8A_0^{(l)}A_1^{(l)}(b_l+\delta_l+(\beta_l+\gamma_l))\\
&(4\chi_0(A_0^{(l)})^2 c_l^2 k_2^2 - 8\chi_0(A_0^{(l)})^2 c_l k_2 + 4\chi_0(A_0^{(l)})^2 + \lambda_1 A_0^{(l)}A_1^{(l)}c_l^2 k_1^2\\
&-\lambda_0(A_1^{(l)})^2 c_l^2 k_1^2,\\
R_3 =& (A_1^{(l)})^2(b_l+\delta_l+(\beta_l+\gamma_l))(4\chi_0(A_0^{(l)})^2 c_l^2 k_2^2 - 8\chi_0(A_0^{(l)})^2 c_l k_2\\
&+4\chi_0(A_0^{(l)})^2 + \lambda_1 A_0^{(l)}A_1^{(l)}c_l^2 k_1^2 - \lambda_0(A_1^{(l)})^2 c_l^2 k_1^2),\\
R_4 =& (4(A_0^{(l)})^4(b_l+\delta_l+(\beta_{ln}+\gamma_l))\chi_0 - 4(A_0^{(l)})^2 a_l\chi_0 k_2^2)(1-c_l k_2)\\
&+4(A_0^{(l)})^3\chi_0(d_l - c_l d_l k_2 - k_2^2 e_l c_l + (1-c_l k_2)\alpha_l)\\
&-((A_1^{(l)})^2\lambda_0 a_l k_1^2 - A_0^{(l)}A_1^{(l)}\lambda_1 a_l k_1^2)(1+c_l k_2).
\end{aligned}$$

Substituting into (12) and (15), we get

$$\pm(\zeta - \zeta_0) = Q\int \frac{du}{\sqrt{\Gamma(u)}}, \tag{38}$$

where $Q = \sqrt{\frac{\chi_0}{\sum_{i=0}^4 \lambda_i}}$, $\Gamma(u) = \sum_{i=0}^4 u^i$.
Therefore the traveling wave solutions to Eq.(33) are:
When $\Gamma(u) = (u - \vartheta_1)^4$

$$\psi^{(l)}(x,t) = \sqrt{A_0^{(l)} + A_1^{(l)}\vartheta_1 \pm \frac{A_1^{(l)}Q}{k_1 x - (\frac{2a_l k_1 k_2 - k_1 c_l \mu}{k_2 c_l - 1})t - \zeta_0}} \times e^{i(-k_2 x + \mu t + k_3)}, \tag{39}$$

When $\Gamma(u) = (u - \vartheta_1)^3(u - \vartheta_2)$, and $\vartheta_2 > \vartheta_1$

$$\psi^{(l)}(x,t) = \sqrt{A_0^{(l)} + A_1^{(l)}\vartheta_1 + \frac{4A_1^{(l)}Q^2(\vartheta_2 - \vartheta_1)}{4Q^2 - M^2}} \times e^{i(-k_2 x + \mu t + k_3)}, \tag{40}$$

When $(u - \vartheta_1)^2(u - \vartheta_2)^2$

$$\psi^{(l)}(x,t) = \sqrt{A_0^{(l)} + A_1^{(l)}\vartheta_j + \frac{(-1)^{j+1}A_1^{(l)}(\vartheta_1 - \vartheta_2)}{\exp(\frac{M}{Q}) - 1}} \times e^{i(-k_2 x + \mu t + k_3)}, \tag{41}$$

where $j = 1, 2$.
When $\Gamma = (u - \vartheta_1)^2(u - \vartheta_2)(u - \vartheta_3)$, and $\vartheta_1 > \vartheta_2 > \vartheta_3$

$$\psi^{(l)}(x,t) = \sqrt{A_0^{(l)} + A_1^{(l)}\vartheta_1 - \frac{2A_1^{(l)}(\vartheta_1 - \vartheta_2)(\vartheta_1 - \vartheta_3)}{2\vartheta_1 - \vartheta_2 - \vartheta_3 + (\vartheta_3 - \vartheta_2)\cosh(Y_1)}} \times e^{i(-k_2 x + \mu t + k_3)}, \tag{42}$$

where $Y_1 = \frac{k_1\sqrt{(\vartheta_1 - \vartheta_2)(\vartheta_1 - \vartheta_3)}}{Q}(k_1 x - (\frac{2a_l k_1 k_2 - k_1 c_l \mu}{k_2 c_l - 1})t$

When $\Gamma = (u - \vartheta_1)(u - \vartheta_2)(u - \vartheta_3)(u - \vartheta_4)$, and $\vartheta_1 > \vartheta_2 > \vartheta_3 > \vartheta_4$

$$\psi^{(l)}(x,t) = \sqrt{A_0^{(l)} + A_1^{(l)}\vartheta_2 + \frac{A_1^{(l)}(\vartheta_1 - \vartheta_2)(\vartheta_4 - \vartheta_2)}{\vartheta_4 - \vartheta_2 + (\vartheta_1 - \vartheta_4)sn^2(Y_2)}} \times e^{i(-k_2 x + \mu t + k_3)}, \tag{43}$$

where $Y_2 = \left[\pm\frac{\sqrt{(\vartheta_1 - \vartheta_3)(\vartheta_2 - \vartheta_4)}}{2Q}(k_1 x - (\frac{2a_l k_1 k_2 - k_1 c_l \mu}{k_2 c_l - 1})t - \zeta_0), m\right]$, and $m^2 = \frac{(\vartheta_2 - \vartheta_3)(\vartheta_1 - \vartheta_4)}{(\vartheta_1 - \vartheta_3)(\vartheta_2 - \vartheta_4)}$.

Note that ϑ_i, $i = 1, ..., 4$ are the roots of $\Gamma(u) = 0$.
When $A_0^{(l)} = -A_1^{(l)}\vartheta_1$ and $\zeta_0 = 0$, the solutions (39) – (42) are reduced to the following plane wave solutions

$$\psi^{(l)}(x,t) = \sqrt{\pm\frac{A_1^{(l)}Q}{k_1 x - (\frac{2a_l k_1 k_2 - k_1 c_l \mu}{k_2 c_l - 1})t}} \times e^{i(-k_2 x + \mu t + k_3)}, \tag{44}$$

$$\psi^{(l)}(x,t) = \sqrt{\frac{4A_1^{(l)}Q^2(\vartheta_2 - \vartheta_1)}{4Q^2 - M_0^2}} \times e^{i(-k_2 x + \mu t + k_3)}, \tag{45}$$

singular soliton solutions

$$\psi^{(l)}(x,t) = \sqrt{\frac{A_1^{(l)}(\vartheta_2 - \vartheta_1)}{2}(1 \mp \coth(\frac{M_0}{2Q}))} \times e^{i(-k_2 x + \mu t + k_3)}, \tag{46}$$

and bright soliton solutions

$$\psi^{(l)}(x,t) = \sqrt{\left(\frac{D}{C + \cosh(B(k_1 x - 2a_1 k_1 k_2 t))}\right)} \times e^{i(-k_2 x + \mu t + k_3)}, \tag{47}$$

where $D = \frac{2A_1^{(l)}(\vartheta_1 - \vartheta_2)(\vartheta_1 - \vartheta_3)}{(\vartheta_3 - \vartheta_2)}$, $B = \frac{k_1\sqrt{(\vartheta_1 - \vartheta_2)(\vartheta_1 - \vartheta_3)}}{Q}$, and $C = \frac{2\vartheta_1 - \vartheta_2 - \vartheta_3}{\vartheta_3 - \vartheta_2}$.

The amplitude of the soliton is given by D where the inverse width of the soliton is given by B. The solitons will exist for $A_1^{(l)} < 0$. Furthermore, when $A_0^{(l)} = -A_1^{(l)}$ and $\zeta_0 = 0$, Jacobi's elliptic function solution (43) is written as

$$\psi^{(l)}(x,t) = \sqrt{\left(\frac{D_1}{C_1 + sn^2(B_j S)}\right)} \times e^{i(-k_2 x + \mu t + k_3)}, \quad (48)$$

where

$$D_1 = \frac{A_1^{(l)}(\vartheta_1 - \vartheta_2)(\vartheta_4 - \vartheta_2)}{(\vartheta_1 - \vartheta_4)},$$

$$B_j = \frac{(-1)^j k_1 \sqrt{(\vartheta_1 - \vartheta_3)(\vartheta_2 - \vartheta_4)}}{2Q},$$

$$C_1 = \frac{2\vartheta_4 - \vartheta_2}{\vartheta_1 - \vartheta_4}, and\ j = 1, 2.$$

Remark-1: When the modulus $m \to 1$, singular optical soliton solutions are obtained as

$$\psi^{(l)}(x,t) = \sqrt{\left(\frac{D_1}{C_1 + \tanh^2(B_j(k_1 x - (\frac{2a_l k_1 k_2 - k_1 c_l \mu}{k_2 c_l - 1})t))}\right)} \times e^{i(-k_2 x + \mu t + k_3)}, \quad (49)$$

where $\vartheta_3 = \vartheta_4$.

Remark-2: When the modulus $m \to 0$, singular-periodic solutions are obtained as

$$\psi^{(l)}(x,t) = \sqrt{\left(\frac{D_1}{C_1 + \sin^2(B_j(k_1 x - (\frac{2a_l k_1 k_2 - k_1 c_l \mu}{k_2 c_l - 1})t))}\right)} \times e^{i(-k_2 x + \mu t + k_3)}, \quad (50)$$

where $\vartheta_2 = \vartheta_3$.

3 Results and Discussion

The Gerdjikov-Ivanov equation has been improved in DWDM for kerr law and parabolic law nonlinearities and considered on account of acquiring optical soliton solutions. New singular soliton and bright soliton solutions were presented by applying the extended simplest equation method. New singular and singular-periodic soliton solutions were emerged using the limiting of the modulus of ellipticity of the Jacobi's elliptic function.

Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this article

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