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A New Extension of Two Parameter Pranav Distribution with Applications in Industrial and Medical Sciences

V. Rajagopalan [1], R. A. Ganaie [1,], and S. R. Nair[2]*

[1]Department of Statistics, Faculty of Science, Annamalai University, Tamil Nadu, India
[2]Department of Statistics, N.S.S College for women, Thiruvananthapuram, Kerala, India

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Abstract: In this study, a new distribution has been proposed called as weighted three parameter Pranav distribution. The different statistical properties of new distribution have been obtained which include moments, harmonic mean, order statistics, survival analysis, entropies, bonferroni and Lorenz curves. For estimating the model parameters of the new distribution, the technique of maximum likelihood estimation has been used. Finally, three real life data sets have been fitted in new distribution to discuss its goodness of fit.

Keywords: Two parameter Pranav distribution, Weighted distribution, Survival analysis, Order statistics, Maximum likelihood estimation.

1 Introduction

The study of weighted distributions is useful in distribution theory because it provides a new understanding of the existing standard probability distributions due to the introduction of additional parameter in the model which creates flexibility in their nature. The addition of extra parameter to the existing classical distribution makes the distribution more reliable and flexible in comparison with other distributions. The weighted distributions also provide a technique for fitting the models to the unknown weight function when the samples can be taken both from original and developed distributions. The concept of weighted distributions introduced by fisher (1934) is a traceable work in respect of his studies on how the method of ascertainment can affect the form of distribution of recorded observations. Later, Rao (1965) introduced and formalized in a unifying theory for problems where the observations fall in non-experimental, non-replicated and non-random manner. The weighted distributions arise when the observations generated from a stochastic process are not given equal chances of being recorded, instead they are recorded according to some weight function. When observations are recorded by an investigator in the nature according to certain stochastic model, the distribution of recorded observations will not have the original distribution unless each and every observation is given an equal chance of being recorded. The weighted distributions play a major role in analyzing and modeling lifetime data in many applied sciences like engineering, medicine, behavioral sciences, finance, and insurance. The weighted distributions are used as a tool in selection of appropriate models for observed data, especially when samples are drawn without a proper frame. The weighted distributions are applied in various research areas related to reliability, biomedicine, ecology, analysis of family data, meta-analysis, analysis of intervention data and other areas for the improvement of proper statistical models. The weighted distribution reduces to length biased distribution when the weight function considers only the length of units of interest. The concept of length biased sampling was introduced by Cox (1969) and Zelen (1974). The statistical interpretation of length biased distribution was originally introduced by Cox (1962) in the context of renewal theory.

There are various good sources which provide the detailed description of weighted distributions. A lot of work has been done by many researchers to introduce some new probability distributions along with their significant role in handling data sets from various practical fields. Para and Jan (2018) obtained the three-parameter weighted Pareto type II distribution with properties and applications in medical sciences. Gupta and Tripathi (1996) studied the weighted version of the bivariate three parameter logarithmic series distribution which has application in many fields such as ecology, social and behavioral sciences, and species abundance studies. Aleem et al. (2013) introduced a class of modified weighted weibull distribution (MWWD) and its properties. Afaq et al. (2016) constructed the length biased weighted Lomax distribution with properties and applications. Saghir et al. (2016) studied the length-biased weighted exponentiated inverted weibull

*Corresponding author e-mail: rashidau7745@gmail.com

distribution. Kilany (2016) constructed the weighted Lomax distribution and obtain its several structural properties. Fatima and Ahmad (2017) discussed on the weighted inverse Rayleigh distribution along with its properties and applications. Bashir and Rasul (2015) studied the weighted Lindley distribution and discuss its various statistical properties. Gupta and Kundu (2009) discussed on the weighted exponential distribution along with its different structural properties. Rao and Pandey (2020) studied the parameter estimation of area biased Rayleigh distribution. Kersey (2010) constructed the weighted inverse Weibull distribution and beta inverse Weibull distribution. Rather and Ozel (2020) studied the weighted power lindley distribution along with its applications on the lifetime data. Ganaie et al. (2021) developed the weighted power Shanker distribution with characterizations and applications of real-life data. Recently, Mohiuddin et al. (2022) studied the weighted Amarendra distribution with properties and applications to real life data.

Two parameter Pranav distribution was proposed by Edith Umeh and Amuche Ibenegbu (2019) which is a special case of one parameter Pranav and Ishita distribution. Its various structural properties which include moments, coefficient of variation, skewness, kurtosis, index of dispersion, hazard rate function, mean residual life function, stochastic ordering, mean deviation, bonferroni and Lorenz curves have been discussed. For estimating its parameters, the method of moments and the method of maximum likelihood estimation have been used. The goodness of fit of two parameter Pranav distribution with real lifetime data set from medical sciences have been illustrated and the fit has been found quite satisfactory as compared over one parameter distributions and two parameter distributions.

2 Weighted Three Parameter Pranav (WTPP) Distribution

The probability density function of two parameter Pranav distribution is given by

$$f(x;\theta,\alpha)=\frac{\theta^4}{\alpha\theta^4+6}\left(\alpha\theta+x^3\right)e^{-\theta x};\ x>0,\theta>0,\alpha\geq 0 \tag{1}$$

and the cumulative distribution function of two parameter Pranav distribution is given by

$$F(x;\theta,\alpha)=1-\left(1+\frac{\theta x\left(\theta^2x^2+3\theta x+6\right)}{\alpha\theta^4+6}\right)e^{-\theta x};x>0,\theta>0,\alpha\geq 0 \tag{2}$$

Let the random variable X follows non-negative condition with probability density function $f(x)$.Let its non-negative weight function be $w(x)$, then the probability density function of the weighted random variable X_w is given by

$$f_W(x)=\frac{w(x)f(x)}{E(w(x))},\quad x>0.$$

Where the non - negative weight function be $w(x)$ and $E(w(x))=\int w(x)f(x)dx<\infty.$

In this paper, we have to obtain the weighted three parameter Pranav distribution, so consequently we will take at $w(x)=x^c$ in order to obtain the weighted three parameter Pranav distribution. Then the probability density function of weighted three parameter Pranav distribution is given by

$$f_W(x)=\frac{x^c f(x)}{E(x^c)} \tag{3}$$

Where $E(x^c)=\int_0^\infty x^c f(x)dx$

$$E(x^c)=\frac{\alpha\,\theta^4\Gamma(c+1)+\Gamma(c+4)}{\theta^c\left(\alpha\theta^4+6\right)} \tag{4}$$

By using the equations (1) and (4) in equation (3), we will obtain the probability density function of weighted three parameter Pranav distribution as

$$f_w(x) = \frac{\theta^{c+4}}{\alpha\theta^4\Gamma(c+1) + \Gamma(c+4)} x^c \left(\alpha\theta + x^3\right) e^{-\theta x} \quad (5)$$

and the cumulative distribution function of weighted three parameter Pranav distribution can be obtained as

$$F_w(x) = \int_0^x f_w(x) dx$$

$$F_w(x) = \int_0^x \frac{\theta^{c+4}}{\alpha\theta^4\Gamma(c+1) + \Gamma(c+4)} x^c \left(\alpha\theta + x^3\right) e^{-\theta x} dx$$

$$F_w(x) = \frac{1}{\alpha\theta^4\Gamma(c+1) + \Gamma(c+4)} \int_0^x x^c \theta^{c+4} \left(\alpha\theta + x^3\right) e^{-\theta x} dx$$

$$F_w(x) = \frac{1}{\alpha\theta^4\Gamma(c+1) + \Gamma(c+4)} \left(\alpha\theta^{c+5} \int_0^x x^c e^{-\theta x} dx + \theta^{c+4} \int_0^x x^{c+3} e^{-\theta x} dx \right) \quad (6)$$

Put $\theta x = t \Rightarrow \theta dx = dt \Rightarrow dx = \frac{dt}{\theta}$, As $x \to x, t \to \theta x,\ x \to 0, t \to 0$

After the simplification of equation (6), we will obtain the cdf of weighted three parameter Pranav distribution as

$$F_w(x) = \frac{1}{\alpha\theta^4\Gamma(c+1) + \Gamma(c+4)} \left(\alpha\theta^4 \gamma(c+1, \theta x) + \gamma(c+4, \theta x) \right) \quad (7)$$

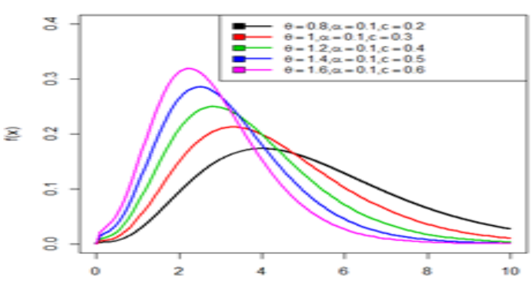

Fig.1: Pdf Plot of WTPPD

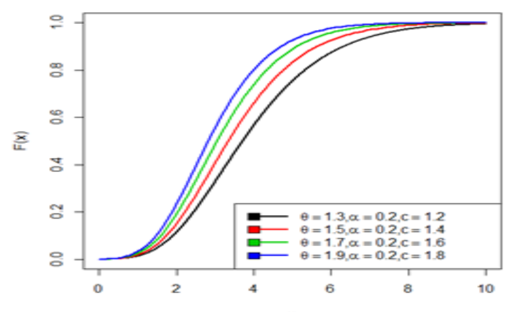

Fig.2: Cdf Plot of WTPPD

3 Survival Analysis

In this section, we will discuss some measures of the survival analysis of proposed distribution which are given as

3.1 Survival Function

The survival function of weighted three parameter Pranav distribution can be obtained as

$$S(x) = 1 - F_w(x)$$

$$S(x) = 1 - \frac{1}{\alpha\theta^4\Gamma(c+1) + \Gamma(c+4)} \left(\alpha\theta^4 \gamma(c+1, \theta x) + \gamma(c+4, \theta x) \right)$$

3.2 Hazard Function

The hazard function of proposed model is given by

$$h(x)=\frac{f_w(x)}{1-F_w(x)}$$

$$h(x)=\frac{x^c\theta^{c+4}\left(\alpha\theta+x^3\right)e^{-\theta x}}{\left(\alpha\theta^4\Gamma(c+1)+\Gamma(c+4)\right)-\left(\alpha\theta^4\gamma(c+1,\theta x)+\gamma(c+4,\theta x)\right)}$$

3.3 Reverse Hazard Function

The reverse hazard function of proposed model is given by

$$h_r(x)=\frac{f_w(x)}{F_w(x)}$$

$$h_r(x)=\frac{x^c\theta^{c+4}\left(\alpha\theta+x^3\right)e^{-\theta x}}{\left(\alpha\theta^4\gamma(c+1,\theta x)+\gamma(c+4,\theta x)\right)}$$

3.4 Mills Ratio

$$M.R=\frac{1}{h_r(x)}=\frac{\left(\alpha\theta^4\gamma(c+1,\theta x)+\gamma(c+4,\theta x)\right)}{x^c\theta^{c+4}\left(\alpha\theta+x^3\right)e^{-\theta x}}$$

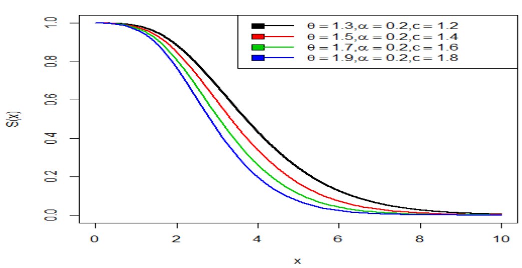

Fig.3: Survival Plot of WTPPD

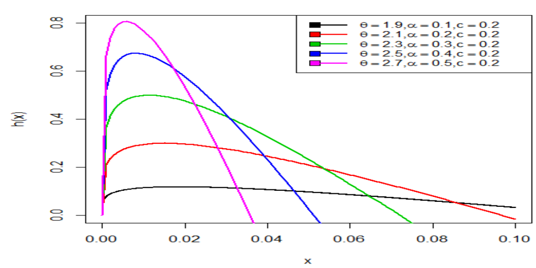

Fig.4: Hazard Plot of WTPPD

4 Structural Properties

Some of the statistical properties of proposed weighted three parameter Pranav distribution have been discussed which includes moments, harmonic mean, MGF and characteristic function.

4.1 Moments

Let the random variable X follows weighted three parameter Pranav distribution with parameters θ, α and c, then the r^{th} order moment of proposed new distribution can be obtained as

$$E(X^r)=\mu_r'=\int_0^\infty x^r f_w(x)dx$$

$$=\int_0^\infty x^r\frac{\theta^{c+4}}{\alpha\theta^4\Gamma(c+1)+\Gamma(c+4)}x^c\left(\alpha\theta+x^3\right)e^{-\theta x}dx$$

$$= \frac{\theta^{c+4}}{\alpha\theta^4\Gamma(c+1)+\Gamma(c+4)}\int_0^\infty x^{c+r}\left(\alpha\theta+x^3\right)e^{-\theta x}dx$$

$$= \frac{\theta^{c+4}}{\alpha\theta^4\Gamma(c+1)+\Gamma(c+4)}\left(\alpha\theta\int_0^\infty x^{(c+r+1)-1}e^{-\theta x}dx+\int_0^\infty x^{(c+r+4)-1}e^{-\theta x}dx\right) \tag{8}$$

After the simplification of equation (8), we obtain

$$E(X^r)=\mu_r' = \frac{\alpha\theta^4\Gamma(c+r+1)+\Gamma(c+r+4)}{\theta^r\left(\alpha\theta^4\Gamma(c+1)+\Gamma(c+4)\right)} \tag{9}$$

By putting r = 1, 2, 3 and 4 in equation (9), we will obtain the first four moments of weighted three parameter Pranav distribution.

$$E(X)=\mu_1' = \frac{\alpha\theta^4\Gamma(c+2)+\Gamma(c+5)}{\theta\left(\alpha\theta^4\Gamma(c+1)+\Gamma(c+4)\right)}$$

$$E(X^2)=\mu_2' = \frac{\alpha\theta^4\Gamma(c+3)+\Gamma(c+6)}{\theta^2\left(\alpha\theta^4\Gamma(c+1)+\Gamma(c+4)\right)}$$

$$E(X^3)=\mu_3' = \frac{\alpha\theta^4\Gamma(c+4)+\Gamma(c+7)}{\theta^3\left(\alpha\theta^4\Gamma(c+1)+\Gamma(c+4)\right)}$$

$$E(X^4)=\mu_4' = \frac{\alpha\theta^4\Gamma(c+5)+\Gamma(c+8)}{\theta^4\left(\alpha\theta^4\Gamma(c+1)+\Gamma(c+4)\right)}$$

$$\text{Variance} = \frac{\left(\alpha\theta^4\Gamma(c+3)+\Gamma(c+6)\right)\left(\alpha\theta^4\Gamma(c+1)+\Gamma(c+4)\right)-\left(\alpha\theta^4\Gamma(c+2)+\Gamma(c+5)\right)^2}{\theta^2\left(\alpha\theta^4\Gamma(c+1)+\Gamma(c+4)\right)^2}$$

$$S.D(\sigma) = \frac{\sqrt{\left(\alpha\theta^4\Gamma(c+3)+\Gamma(c+6)\right)\left(\alpha\theta^4\Gamma(c+1)+\Gamma(c+4)\right)-\left(\alpha\theta^4\Gamma(c+2)+\Gamma(c+5)\right)^2}}{\theta\left(\alpha\theta^4\Gamma(c+1)+\Gamma(c+4)\right)}$$

$$C.V = \frac{\sigma}{\mu_1'} = \frac{\sqrt{\left(\alpha\theta^4\Gamma(c+3)+\Gamma(c+6)\right)\left(\alpha\theta^4\Gamma(c+1)+\Gamma(c+4)\right)-\left(\alpha\theta^4\Gamma(c+2)+\Gamma(c+5)\right)^2}}{\alpha\theta^4\Gamma(c+2)+\Gamma(c+5)}$$

4.2 Harmonic Mean

The harmonic mean for the proposed model can be obtained as

$$H.M = E\left(\frac{1}{x}\right) = \int_0^\infty \frac{1}{x} f_w(x)dx$$

$$= \int_0^\infty \frac{\theta^{c+4}}{\alpha\,\theta^4\Gamma(c+1)+\Gamma(c+4)} x^{c-1}\left(\alpha\theta + x^3\right) e^{-\theta x} dx$$

$$= \frac{\theta^{c+4}}{\alpha\theta^4\Gamma(c+1)+\Gamma(c+4)}\left(\alpha\theta\int_0^\infty x^{c-1} e^{-\theta x} dx + \int_0^\infty x^{c+2} e^{-\theta x} dx\right)$$

$$= \frac{\theta^{c+4}}{\alpha\theta^4\Gamma(c+1)+\Gamma(c+4)}\left(\alpha\theta\int_0^\infty x^{(c+1)-2} e^{-\theta x} dx + \int_0^\infty x^{(c+3)-1} e^{-\theta x} dx\right) \tag{10}$$

After the simplification of equation (10), we obtain

$$H.M = \frac{\theta^{c+4}}{\alpha\theta^4\Gamma(c+1)+\Gamma(c+4)}\left(\alpha\theta\gamma(c+1,\,\theta x) + \gamma(c+3,\,\theta x)\right)$$

4.3 Moment Generating Function and Characteristic Function

Let X be the random variable following weighted three parameter Pranav distribution with parameters θ, α and c, then the MGF of X can be obtained as

$$M_X(t) = E\left(e^{tx}\right) = \int_0^\infty e^{tx} f_w(x)dx$$

Using Taylor's series, we obtain

$$= \int_0^\infty \left(1 + tx + \frac{(tx)^2}{2!} + \ldots\right) f_w(x)dx$$

$$= \int_0^\infty \sum_{j=0}^\infty \frac{t^j}{j!} x^j f_w(x)dx$$

$$= \sum_{j=0}^\infty \frac{t^j}{j!}\left(\frac{\alpha\,\theta^4\Gamma(c+j+1) + \Gamma(c+j+4)}{\theta^j\left(\alpha\theta^4\Gamma(c+1)+\Gamma(c+4)\right)}\right)$$

$$M_X(t) = \frac{1}{\alpha\theta^4\Gamma(c+1)+\Gamma(c+4)} \sum_{j=0}^\infty \frac{t^j}{j!\theta^j}\left(\alpha\,\theta^4\Gamma(c+j+1) + \Gamma(c+j+4)\right)$$

Similarly, the characteristic function of proposed model can be obtained as

$$\varphi_X(t) = M_X(it)$$

$$M_X(it) = \frac{1}{\alpha\theta^4\Gamma(c+1)+\Gamma(c+4)} \sum_{j=0}^\infty \frac{it^j}{j!\theta^j}\left(\alpha\,\theta^4\Gamma(c+j+1) + \Gamma(c+j+4)\right)$$

5 Order Statistics

Order statistics have been extensively used in statistical sciences and have wide range of application in the field of reliability and life testing. Let $X_{(1)}, X_{(2)}, \ldots, X_{(n)}$ denotes the order statistics of a random sample $X_1, X_2, \ldots, X_n$ from a continuous population with probability density function $f_x(x)$ and cumulative distribution function $F_X(x)$, then the probability density function of r^{th} order statistics $X_{(r)}$ is given by

$$f_{x(r)}(x) = \frac{n!}{(r-1)!(n-r)!} f_X(x)\left(F_X(x)\right)^{r-1}\left(1-F_X(x)\right)^{n-r} \tag{11}$$

By using the equations (5) and (7) in equation (11), we will obtain the probability density function of r^{th} order statistics of weighted three parameter Pranav distribution as

$$f_{x(r)}(x) = \frac{n!}{(r-1)!(n-r)!}\left(\frac{\theta^{c+4}}{\alpha\theta^4\Gamma(c+1)+\Gamma(c+4)} x^c\left(\alpha\theta + x^3\right)e^{-\theta x}\right)$$

$$\times\left(\frac{1}{\alpha\theta^4\Gamma(c+1)+\Gamma(c+4)}\left(\alpha\theta^4\gamma(c+1,\,\theta x)+\gamma(c+4,\,\theta x)\right)\right)^{r-1}$$

$$\times\left(1-\frac{1}{\alpha\theta^4\Gamma(c+1)+\Gamma(c+4)}\left(\alpha\theta^4\gamma(c+1,\theta x)+\gamma(c+4,\theta x)\right)\right)^{n-r}$$

Therefore, the pdf of higher order statistic $X_{(n)}$ of weighted three parameter Pranav distribution can be obtained as

$$f_{x(n)}(x) = \frac{n\theta^{c+4}}{\alpha\theta^4\Gamma(c+1)+\Gamma(c+4)} x^c\left(\alpha\theta + x^3\right)e^{-\theta x}$$

$$\times\left(\frac{1}{\alpha\theta^4\Gamma(c+1)+\Gamma(c+4)}\left(\alpha\theta^4\gamma(c+1,\,\theta x)+\gamma(c+4,\,\theta x)\right)\right)^{n-1}$$

and the pdf of first order statistic $X_{(1)}$ of weighted three parameter Pranav distribution can be obtained as

$$f_{x(1)}(x) = \frac{n\theta^{c+4}}{\alpha\theta^4\Gamma(c+1)+\Gamma(c+4)} x^c\left(\alpha\theta + x^3\right)e^{-\theta x}$$

$$\times\left(1-\frac{1}{\alpha\theta^4\Gamma(c+1)+\Gamma(c+4)}\left(\alpha\theta^4\gamma(c+1,\,\theta x)+\gamma(c+4,\,\theta x)\right)\right)^{n-1}$$

6 Likelihood Ratio Test

Let the random sample $X_1, X_2, \ldots\ldots X_n$ of size n drawn from the weighted three parameter Pranav distribution. To test, we introduce the hypothesis

$$H_o : f(x) = f(x;\theta,\alpha) \quad \text{against} \quad H_1 : f(x) = f_w(x;\theta,\alpha,c)$$

In order to analyse and investigate, whether the random sample of size n comes from the two parameter Pranav distribution or weighted three parameter Pranav distribution, the following test statistic is used.

$$\Delta = \frac{L_1}{L_o} = \prod_{i=1}^{n}\frac{f_w(x;\theta,\alpha,c)}{f(x;\theta,\alpha)}$$

$$\Delta = \frac{L_1}{L_o} = \prod_{i=1}^{n} \left(\frac{x_i^{\,c} \theta^{\,c}\left(\alpha\theta^4 + 6\right)}{\alpha\theta^4\Gamma(c+1) + \Gamma(c+4)} \right)$$

$$\Delta = \frac{L_1}{L_o} = \left(\frac{\theta^c\left(\alpha\theta^4 + 6\right)}{\alpha\theta^4\Gamma(c+1) + \Gamma(c+4)} \right)^n \prod_{i=1}^{n} x_i^{\,c}$$

We should reject the null hypothesis if

$$\Delta = \left(\frac{\theta^c\left(\alpha\theta^4 + 6\right)}{\alpha\theta^4\Gamma(c+1) + \Gamma(c+4)} \right)^n \prod_{i=1}^{n} x_i^{\,c} > k$$

$$\text{Or } \Delta^* = \prod_{i=1}^{n} x_i^{\,c} > k\left(\frac{\alpha\theta^4\Gamma(c+1) + \Gamma(c+4)}{\theta^c\left(\alpha\theta^4 + 6\right)} \right)^n$$

$$\Delta^* = \prod_{i=1}^{n} x_i^{\,c} > k^*, \text{Where } k^* = k\left(\frac{\alpha\theta^4\Gamma(c+1) + \Gamma(c+4)}{\theta^c\left(\alpha\theta^4 + 6\right)} \right)^n$$

For large sample of size *n*, *2log Δ* is distributed as chi-square distribution with one degree of freedom and also *p* value is obtained from the chi-square distribution. Thus, we refused to accept the null hypothesis, when the probability value is given by

$p\left(\Delta^* > \gamma^*\right)$, Where $\gamma^* = \prod_{i=1}^{n} x_i^{\,c}$ is less than a specified level of significance and $\prod_{i=1}^{n} x_i^{\,c}$ is the. observed value of the statistic Δ^*.

7 Income Distribution Curves

The income distribution curves known as Bonferroni and Lorenz curves or classical curves are applied in different fields like reliability, medicine, insurance, and demography, but mostly it is also being used to study the distribution of inequality in income or poverty. The income distribution or classical curves are defined as

$$B(p) = \frac{1}{p\mu_1'} \int_0^q x f(x)\,dx$$

$$\text{and } L(p) = pB(p) = \frac{1}{\mu_1'} \int_0^q x f(x)\,dx$$

$$\text{Where } \mu_1' = E(X) = \frac{\alpha\theta^4\Gamma(c+2) + \Gamma(c+5)}{\theta\left(\alpha\theta^4\Gamma(c+1) + \Gamma(c+4)\right)} \quad \text{and } q = F^{-1}(p)$$

$$B(p)=\frac{\theta\left(\alpha\theta^4\Gamma(c+1)+\Gamma(c+4)\right)}{p\left(\alpha\theta^4\Gamma(c+2)+\Gamma(c+5)\right)}\int_0^q \frac{\theta^{c+4}}{\alpha\theta^4\,\Gamma(c+1)+\Gamma(c+4)}x^{c+1}\left(\alpha\theta+x^3\right)e^{-\theta x}dx$$

$$B(p)=\frac{\theta^{c+5}}{p\left(\alpha\,\theta^4\Gamma(c+2)+\Gamma(c+5)\right)}\int_0^q x^{c+1}\left(\alpha\theta+x^3\right)e^{-\theta x}dx$$

$$B(p)=\frac{\theta^{c+5}}{p\left(\alpha\,\theta^4\Gamma(c+2)+\Gamma(c+5)\right)}\left(\alpha\theta\int_0^q x^{(c+2)-1}e^{-\theta x}dx+\int_0^q x^{(c+5)-1}e^{-\theta x}dx\right)$$

After simplification, we get

$$B(p)=\frac{\theta^{c+5}}{p\left(\alpha\,\theta^4\Gamma(c+2)+\Gamma(c+5)\right)}\left(\alpha\theta\gamma(c+2,\theta q)+\gamma(c+5,\theta q)\right)$$

$$L(p)=\frac{\theta^{c+5}}{\left(\alpha\,\theta^4\Gamma(c+2)+\Gamma(c+5)\right)}\left(\alpha\theta\gamma(c+2,\theta q)+\gamma(c+5,\theta q)\right)$$

8 Entropies

The concept of entropy was given by German physicist Rudolf Clausius in 1850 and the term entropy is a scientific concept as well as a measurable physical property that is commonly associated with state of disorder, randomness or uncertainty.

8.1 Renyi Entropy

The concept namely Renyi entropy was given by Alfred Renyi and in information theory the Renyi entropy generalizes the Hartley entropy, the Shannon entropy, the collision entropy and the min-entropy. The Renyi entropy of order β is defined as

$$e(\beta)=\frac{1}{1-\beta}\log\left(\int f^{\beta}(x)dx\right)$$

Where, $\beta>0$ and $\beta\neq 1$

$$e(\beta)=\frac{1}{1-\beta}\log\int_0^\infty\left(\frac{\theta^{c+4}}{\alpha\theta^4\Gamma(c+1)+\Gamma(c+4)}x^c\left(\alpha\theta+x^3\right)e^{-\theta x}\right)^\beta dx$$

$$e(\beta)=\frac{1}{1-\beta}\log\left(\left(\frac{\theta^{c+4}}{\alpha\theta^4\Gamma(c+1)+\Gamma(c+4)}\right)^\beta\int_0^\infty x^{\beta c}e^{-\theta\beta x}\left(\alpha\theta+x^3\right)^\beta dx\right) \quad (12)$$

Using binomial expansion in equation (12), we obtain

$$e(\beta)=\frac{1}{1-\beta}\log\left(\left(\frac{\theta^{c+4}}{\alpha\theta^4\Gamma(c+1)+\Gamma(c+4)}\right)^\beta\sum_{j=0}^\infty\binom{\beta}{j}(\alpha\theta)^{\beta-j}x^{3j}\int_0^\infty x^{\beta c}e^{-\theta\beta x}dx\right)$$

$$e(\beta)=\frac{1}{1-\beta}\log\left(\left(\frac{\theta^{c+4}}{\alpha\theta^4\Gamma(c+1)+\Gamma(c+4)}\right)^\beta\sum_{j=0}^\infty\binom{\beta}{j}(\alpha\theta)^{\beta-j}\int_0^\infty x^{(\beta c+3j+1)-1}e^{-\theta\beta x}dx\right)$$

$$e(\beta)=\frac{1}{1-\beta}\log\left(\left(\frac{\theta^{c+4}}{\alpha\theta^{4}\Gamma(c+1)+\Gamma(c+4)}\right)^{\beta}\sum_{j=0}^{\infty}\binom{\beta}{j}(\alpha\theta)^{\beta-j}\frac{\Gamma(\beta c+3j+1)}{(\theta\beta)^{\beta c+3j+1}}\right)$$

8.2 Tsallis Entropy

The Tsallis entropy is a generalization of the Shannon or Boltzmann-Gibbs entropy to the case where entropy is non extensive. The term Tsallis entropy was proposed by constantino Tsallis in 1988 as a basis for generalizing the standard statistical mechanics. The Tsallis entropy of order λ is defined as

$$S_{\lambda}=\frac{1}{\lambda-1}\left(1-\int_{0}^{\infty}f^{\lambda}(x)dx\right)$$

$$S_{\lambda}=\frac{1}{\lambda-1}\left(1-\int_{0}^{\infty}\left(\frac{\theta^{c+4}}{\alpha\theta^{4}\Gamma(c+1)+\Gamma(c+4)}x^{c}\left(\alpha\theta+x^{3}\right)e^{-\theta x}\right)^{\lambda}dx\right)$$

$$S_{\lambda}=\frac{1}{\lambda-1}\left(1-\left(\frac{\theta^{c+4}}{\alpha\theta^{4}\Gamma(c+1)+\Gamma(c+4)}\right)^{\lambda}\int_{0}^{\infty}x^{\lambda c}e^{-\lambda\theta x}\left(\alpha\theta+x^{3}\right)^{\lambda}dx\right) \tag{13}$$

Using binomial expansion in equation (13), we get

$$S_{\lambda}=\frac{1}{\lambda-1}\left(1-\left(\frac{\theta^{c+4}}{\alpha\theta^{4}\Gamma(c+1)+\Gamma(c+4)}\right)^{\lambda}\sum_{j=0}^{\infty}\binom{\lambda}{j}(\alpha\theta)^{\lambda-j}x^{3j}\int_{0}^{\infty}x^{\lambda c}e^{-\lambda\theta x}dx\right)$$

$$S_{\lambda}=\frac{1}{\lambda-1}\left(1-\left(\frac{\theta^{c+4}}{\alpha\theta^{4}\Gamma(c+1)+\Gamma(c+4)}\right)^{\lambda}\sum_{j=0}^{\infty}\binom{\lambda}{j}(\alpha\theta)^{\lambda-j}\int_{0}^{\infty}x^{(\lambda c+3j+1)-1}e^{-\lambda\theta x}dx\right)$$

$$S_{\lambda}=\frac{1}{\lambda-1}\left(1-\left(\frac{\theta^{c+4}}{\alpha\theta^{4}\Gamma(c+1)+\Gamma(c+4)}\right)^{\lambda}\sum_{j=0}^{\infty}\binom{\lambda}{j}(\alpha\theta)^{\lambda-j}\frac{\Gamma(\lambda c+3j+1)}{(\lambda\theta)^{\lambda c+3j+1}}\right)$$

9 Maximum Likelihood Estimation and Fisher's Information Matrix

In this section, we will estimate the parameters of weighted three parameter Pranav distribution by using the technique of maximum likelihood estimation. Suppose the random sample $X_1, X_2,\dots,X_n$ of size n from the weighted three parameter Pranav distribution, then the likelihood function can be expressed as

$$L(x)=\prod_{i=1}^{n}f_{w}(x)$$

$$L(x)=\prod_{i=1}^{n}\left(\frac{\theta^{c+4}}{\alpha\theta^{4}\Gamma(c+1)+\Gamma(c+4)}x_{i}^{c}\left(\alpha\theta+x_{i}^{3}\right)e^{-\theta x_{i}}\right)$$

$$L(x)=\frac{\theta^{n(c+4)}}{\left(\alpha\theta^{4}\Gamma(c+1)+\Gamma(c+4)\right)^{n}}\prod_{i=1}^{n}\left(x_{i}^{c}\left(\alpha\theta+x_{i}^{3}\right)e^{-\theta x_{i}}\right)$$

The log likelihood function is given by

$$\log L = n(c+4)\log\theta - n\log\left(\alpha\theta^4\Gamma(c+1) + \Gamma(c+4)\right) + c\sum_{i=1}^{n}\log x_i$$

$$+\sum_{i=1}^{n}\log\left(\alpha\theta + x_i^{\,3}\right) - \theta\sum_{i=1}^{n}x_i \qquad (14)$$

Now differentiating the log likelihood equation (14) with respect to parameters *θ,α* and *c*. we must satisfy the normal equations as

$$\frac{\partial \log L}{\partial\theta} = \frac{n(c+4)}{\theta} - n\left(\frac{4\alpha\theta^3\Gamma(c+1)}{\alpha\theta^4\Gamma(c+1) + \Gamma(c+4)}\right) + n\left(\frac{\alpha}{\left(\alpha\theta + x_i^{\,3}\right)}\right) - \sum_{i=1}^{n}x_i = 0$$

$$\frac{\partial \log L}{\partial\alpha} = -n\left(\frac{\theta^4\Gamma(c+1)}{\alpha\theta^4\Gamma(c+1) + \Gamma(c+4)}\right) + n\left(\frac{\theta}{\left(\alpha\theta + x_i^{\,3}\right)}\right) = 0$$

$$\frac{\partial \log L}{\partial c} = n\log\theta - n\psi\left(\alpha\theta^4\Gamma(c+1) + \Gamma(c+4)\right) + \sum_{i=1}^{n}\log x_i = 0$$

Where ψ(.) is the digamma function

The above system of non-linear equations is too complicated to solve it algebraically. Therefore, we use R and wolfram mathematics for estimating the required parameters of the proposed distribution.

We use the asymptotic normality results in order to obtain the confidence interval. We have that if $\hat{\gamma} = (\hat{\theta}, \hat{\alpha}, \hat{c})$ denotes the MLE of $\gamma = (\theta, \alpha, c)$. We can state the results as follows

$$\sqrt{n}(\hat{\gamma} - \gamma) \rightarrow N_3(0, I^{-1}(\gamma))$$

Where $I^{-1}(\gamma)$ is limiting variance - covariance - Matrix of γ.

The Fisher's Information 3×3 matrix is given below as

$$I(\gamma) = -\frac{1}{n}\begin{pmatrix} E\left(\frac{\partial^2 \log L}{\partial\theta^2}\right) & E\left(\frac{\partial^2 \log L}{\partial\theta\,\partial\alpha}\right) & E\left(\frac{\partial^2 \log L}{\partial\theta\,\partial c}\right) \\ E\left(\frac{\partial^2 \log L}{\partial\alpha\,\partial\theta}\right) & E\left(\frac{\partial^2 \log L}{\partial\alpha^2}\right) & E\left(\frac{\partial^2 \log L}{\partial\alpha\,\partial c}\right) \\ E\left(\frac{\partial^2 \log L}{\partial c\,\partial\theta}\right) & E\left(\frac{\partial^2 \log L}{\partial c\,\partial\alpha}\right) & E\left(\frac{\partial^2 \log L}{\partial c^2}\right) \end{pmatrix}$$

Here we show

$$E\left(\frac{\partial^2 \log L}{\partial \theta^2}\right) = -\frac{n(c+4)}{\theta^2} - n\left(\frac{\left(\alpha\theta^4\Gamma(c+1)+\Gamma(c+4)\right)\left(12\alpha\theta^2\Gamma(c+1)\right) - \left(4\alpha\theta^3\Gamma(c+1)\right)^2}{\left(\alpha\theta^4\Gamma(c+1)+\Gamma(c+4)\right)^2}\right)$$

$$-n\left(\frac{\alpha^2}{\left(\alpha\theta + x_i^3\right)^2}\right)$$

$$E\left(\frac{\partial^2 \log L}{\partial \alpha^2}\right) = n\left(\frac{\left(\theta^4\Gamma(c+1)\right)\left(\theta^4\Gamma(c+1)\right)}{\left(\alpha\theta^4\Gamma(c+1)+\Gamma(c+4)\right)^2}\right) - n\left(\frac{\theta^2}{\left(\alpha\theta + x_i^3\right)^2}\right)$$

$$E\left(\frac{\partial^2 \log L}{\partial c^2}\right) = -n\psi'\left(\alpha\theta^4\Gamma(c+1)+\Gamma(c+4)\right)$$

$$E\left(\frac{\partial^2 \log L}{\partial \theta\,\partial \alpha}\right) = -n\left(\frac{\left(\alpha\theta^4\Gamma(c+1)+\Gamma(c+4)\right)\left(4\theta^3\Gamma(c+1)\right) - \left(4\alpha\theta^3\Gamma(c+1)\right)\left(\theta^4\Gamma(c+1)\right)}{\left(\alpha\theta^4\Gamma(c+1)+\Gamma(c+4)\right)^2}\right)$$

$$-n\left(\frac{\alpha\theta}{\left(\alpha\theta + x_i^3\right)^2}\right)$$

$$E\left(\frac{\partial^2 \log L}{\partial \theta\,\partial c}\right) = \frac{n}{\theta} - n\psi\left(\frac{4\alpha\theta^3\Gamma(c+1)}{\alpha\theta^4\Gamma(c+1)+\Gamma(c+4)}\right)$$

$$E\left(\frac{\partial^2 \log L}{\partial \alpha\,\partial c}\right) = -n\psi\left(\frac{\theta^4\Gamma(c+1)}{\alpha\theta^4\Gamma(c+1)+\Gamma(c+4)}\right)$$

Since γ being unknown, we estimate $I^{-1}(\gamma)$ by $I^{-1}(\hat{\gamma})$ and this can be used to obtain asymptotic confidence intervals for θ, α and c.

10 Data Investigation

In this section, we have analyzed and investigated three real life data sets to discuss the goodness of fit of weighted three parameter Pranav distribution and the fit has been compared over two parameter Pranav, one parameter Pranav, exponential and Lindley distributions. The following data sets are given below as

The following data set in table 1 reported by Lawless (2003) represents the accelerated life test of 59 conductors and the failure times are in hours and there are no censored observations.

Table 1: Data regarding accelerated life test of conductors by Lawless (2003)

2.997	4.137	4.288	4.531	4.700	4.706	5.009	5.381	5.434	5.459
5.589	5.640	5.807	5.923	6.033	6.071	6.087	6.129	6.352	6.369
6.476	6.492	6.515	6.522	6.538	6.545	6.573	6.725	6.869	6.923
6.948	6.956	6.958	7.024	7.224	7.365	7.398	7.459	7.489	7.495
7.496	7.543	7.683	7.937	7.945	7.974	8.120	8.336	8.532	8.591
8.687	8.799	9.218	9.254	9.289	9.663	10.092	10.491	11.038	

The second real life data set reported by Smith and Naylor (1987) represents the strength of 1.5cm glass fibres measured at the National physical laboratory England and the data set is given below in table 2

Table 2: Data regarding the strength of 1.5cm glass fibres by Smith and Naylor (1987)

0.55	2	1.82	1.76	1.7	1.7	0.93	0.74	2.01	1.84.	1.77	1.78
1.25	1.04	0.77	2.24	1.84	1.89	1.36	1.27	1.11	0.81	0.84	1.49
1.39	1.28	1.13	1.24	1.52	1.49	1.42	1.29	1.3	1.61	1.59	1.54
1.5	1.51	1.64	1.61	1.6	1.55	1.55	1.68	1.66	1.62	1.61	1.61
1.73	1.68	1.66	1.62	1.63	1.81	1.76	1.69	1.66	1.67	1.58	1.53
1.5	1.48	1.48									

The third real life data set reported by Feigl and Zelen (1965) represents the survival times in weeks of 33 patients suffering from Acute Myelogeneous Leukaemia and the data set is given below as.

Table 3: Data regarding the patients suffering from Acute Myelogeneous Leukaemia by Feigl and Zelen (1965)

65	156	100	134	16	108	121	4	39
143	56	26	22	1	1	5	65	56
65	17	7	16	22	3	4	2	3
8	4	3	30	4	43			

In order to estimate the model comparison criterion values, the unknown parameters are also estimated through R Software. In order to compare the weighted three parameter Pranav distribution with two parameter Pranav, one parameter Pranav, exponential and Lindley distributions we use the criterion values *AIC* (Akaike Information Criterion), *BIC* (Bayesian Information Criterion), *AICC* (Akaike Information Criterion Corrected), *CAIC* (Consistent Akaike Information Criterion), Shannon's entropy *H(X)* and *-2logL*. The better distribution is which corresponds to the lesser values of *AIC*, *BIC, AICC, CAIC, H(X)* and *-2logL*. For calculating the criterion values *AIC, BIC, AICC, CAIC, H*(*X*) and *-2logL* can be evaluated by using the formulas as follows.

$$AIC = 2k - 2\log L, \qquad BIC = k\log n - 2\log L, \qquad AICC = AIC + \frac{2k(k+1)}{n-k-1}$$

$$CAIC = -2\log L + \frac{2kn}{n-k-1} \qquad \text{and} \qquad H(X) = \frac{-2\log L}{n}$$

Where k is the number of parameters in the statistical model, n is the sample size and $-2\log L$ is the maximized value of log-likelihood function under the considered model.

Table 4: Shows values of ML Estimates and S. E estimates for data set 1, data set 2 and data set 3

Data sets	**Distributions**	**MLE**	**S. E**
	Weighted Three Parameter Pranav	$\hat{\alpha} = 20.8171738$ $\hat{\theta} = 2.7879442$ $\hat{c} = 15.9940081$	$\hat{\alpha} = 65.0872482$ $\hat{\theta} = 0.5815895$ $\hat{c} = 4.8835451$

1	**Two Parameter Pranav**	$\hat{\alpha} = 0.00100000$ $\hat{\theta} = 0.57305079$	$\hat{\alpha} = 0.11245682$ $\hat{\theta} = 0.01684117$
	Pranav	$\hat{\theta} = 0.56391498$	$\hat{\theta} = 0.03584639$
	Exponential	$\hat{\theta} = 0.14326745$	$\hat{\theta} = 0.01865093$
	Lindley	$\hat{\theta} = 0.25722036$	$\hat{\theta} = 0.02393044$
2	**Weighted Three Parameter Pranav**	$\hat{\alpha} = 0.05170527$ $\hat{\theta} = 12.53196375$ $\hat{c} = 15.46451356$	$\hat{\alpha} = 0.07198393$ $\hat{\theta} = 2.17275180$ $\hat{c} = 3.45801296$
	Two Parameter Pranav	$\hat{\alpha} = 0.00100000$ $\hat{\theta} = 2.63433132$	$\hat{\alpha} = 0.111231$ $\hat{\theta} = 0.07380349$
	Pranav	$\hat{\theta} = 1.56071577$	$\hat{\theta} = 0.07926122$
	Exponential	$\hat{\theta} = 0.66364740$	$\hat{\theta} = 0.08361152$
	Lindley	$\hat{\theta} = 0.99611639$	$\hat{\theta} = 0.09484179$
3	**Weighted Three Parameter Pranav**	$\hat{\alpha} = 20.8171738$ $\hat{\theta} = 2.7879442$ $\hat{c} = 15.9940081$	$\hat{\alpha} = 65.0872482$ $\hat{\theta} = 0.5815895$ $\hat{c} = 4.8835451$
	Two Parameter Pranav	$\hat{\alpha} = 4.920930$ $\hat{\theta} = 6.072253$	$\hat{\alpha} = 1.187860$ $\hat{\theta} = 4.182441$
	Pranav	$\hat{\theta} = 0.098007079$	$\hat{\theta} = 0.008528815$
	Exponential	$\hat{\theta} = 0.024476191$	$\hat{\theta} = 0.004253632$
	Lindley	$\hat{\theta} = 0.047815942$	$\hat{\theta} = 0.005886223$

Table 5: Comparison and Performance of fitted distributions

Data sets	Distribution	-2logL	AIC	BIC	AICC	CAIC	H(X)
1	Weighted Three Parameter Pranav	223.332	229.332	235.5647	229.7683	229.7683	3.7852
	Two Parameter Pranav	268.2498	272.2498	276.4049	272.4640	272.4640	4.5466
	Pranav	269.9899	271.9899	274.0674	272.0600	272.0600	4.5761
	Exponential	347.2809	349.2809	351.3585	349.3510	349.3510	5.8861
	Lindley	316.7054	318.7054	320.783	318.7755	318.7755	5.3678
2	Weighted Three Parameter Pranav	46.26569	52.26569	58.6951	52.6724	52.6724	0.7343
	Two Parameter Pranav	94.52527	98.52527	102.8115	98.72527	98.7252	1.5004
	Pranav	180.9627	182.9627	185.1059	183.0282	183.0282	2.8724
	Exponential	177.6606	179.6606	181.8038	179.7261	179.7261	2.8200
	Lindley	162.5569	164.5569	166.7	164.6224	164.6224	2.5802
3	Weighted Three Parameter Pranav	223.332	229.332	235.5647	230.1595	230.1595	6.7676
	Two Parameter Pranav	309.7115	313.7115	316.7045	314.1115	314.1115	9.3851
	Pranav	435.1562	437.1562	438.6527	437.2852	437.2852	13.1865

	Exponential	310.9004	312.9004	314.3969	313.0294	313.0294	9.4212
	Lindley	337.6674	339.6674	341.1639	339.7964	339.7964	10.2323

From table 5 given above, it can be easily seen from the results that the weighted three parameter Pranav distribution have the lesser *AIC*, *BIC*, *AICC*, *CAIC*, *H(X)* and *-2logL* values as compared to the two parameter Pranav, one parameter Pranav, exponential and Lindley distributions. Hence, it can be concluded that the weighted three parameter Pranav distribution leads to a better fit than the two parameter Pranav, one parameter Pranav, exponential and Lindley distributions.

11 Conclusion

In this article, a new distribution namely weighted three parameter Pranav distribution has been proposed. The proposed new distribution is generated by using the weighted technique to the classical distribution. Its various statistical properties along with some survival measures have been discussed. The parameters of new distribution have also been obtained by using the method of maximum likelihood estimation. Finally, the goodness of fit of weighted three parameter Pranav distribution has been proposed with three real life data sets and it is found from the results that the weighted three parameter Pranav distribution provides a better fit over two parameter Pranav, one parameter Pranav, exponential and Lindley distributions.

Conflict of interest

The authors declare that there is no conflict regarding the publication of this paper.

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