Progress in Fractional Differentiation and Applications
An International Journal

http://dx.doi.org/10.18576/pfda/080309

A Game Theory-Based Fractional Order Model for the Simulation of Human Responses in an Emerging Epidemic

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Received: 2 Sep. 2020, Revised: 18 Oct. 2020, Accepted: 20 Nov. 2020
Published online: 1 Jul. 2022

Abstract: A non-integer order model stemmed from an evolutionary game theory is presented in this work to study the impact of changes in human behavior during an emerging epidemic. Introducing fractional order derivatives in Caputo sense to the integer order model may give a better understanding of the numerous effects of memory and learning process of individuals on the prevalence of the infectious diseases when epidemics occur. Numerical simulations are presented to figure out the features of using the non-integer order (fractional order) models in epidemiology.

Keywords: Approximate solution, parabolic Fokker-Planck equation, local fractional derivative operators, Cantor sets.

1 Introduction

Human behavior plays an essential role in the prevalence of infectious diseases and epidemics [1]. Human behavioral changes during epidemics shape the disease spreading and figure out the features of the successful strategies used to control and eradicate epidemic outbreaks [1, 2]. People change their behaviors during an epidemic outbreak in order to prevent infection. On the other hand, human behavioral responses to emerging epidemic outbreak including taking antiviral drugs, vaccination, avoiding crowded environments, limiting travels, wearing face masks, washing hands regularly with antiseptics and respecting cough and sneezes etiquettes influence the dynamic of the epidemic outbreak. Historically, changes in human behavior during emerging epidemics have been linked with the prevalence of infectious diseases in human societies [3, 2]. In the middle ages, the Black Death plague killed about one-third of the total population in Europe. In 1665, the plague was spreading rapidly in Europe and the streets were empty in big cities even in London as people fled the big cities and traveled to the countryside [2, 4]. During the 1918 influenza epidemic waves which had struck the world and caused more than 50 million deaths worldwide, individuals were shunning infected people [5]. More recently, in 2014, Ebola epidemic that caused in more than 11000 deaths in six countries is considered as a behavioral disease as the transmission is affected by human behaviors [6, 7, 8]. Ebola outbreak reduced travelling to West Africa and resulted in applying trade restrictions to control the spread of Ebola virus [5, 7, 8]. People avoid touching wild animals and shun eating bat soup in the infected areas [7]. In 2016, Zika virus has struck about 61 countries and territories [9]. Zika is a vector borne disease that can be transmitted sexually or by Aedes mosquitoes. Also Zika can be transmitted from infected pregnant woman to her unborn baby [10]. In the infected areas, human behaviors have been changed as people have taken some relativity new actions during Zika outbreak to prevent Zika infection (e.g., mosquito insect repellent spry, removing long grass in the house garden, sleeping under bed nets, using magnetic window screen, wearing clothing that covers arms and legs, and removing standing water from their property and using condoms during sex) [10]. Various mathematical models have been investigated to study the influence of uncoordinated human behavioral changes on the spread of epidemics [1, 2, 11]. Nevertheless, most of such models are integer order models. Fractional order models of infectious diseases are considered as models with memory effect while the integer order models cannot display the memory effect in the transmission process [12, 13, 14, 15, 16, 17, 18]. The fractional order derivative ”α” is the memory index and its related learning process [19, 20, 21]. We believe that, introducing the

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fractional order α to the integer order model will improve the system performance through adding a new degree of freedom to study the effect of memory and its associate learning process on the numerical results [2, 22, 23]. The future states of the outbreak will be affected dramatically by the previous responses. If $\alpha \to 0$, then the model has a perfect memory and depends strongly on the history of the transmission process and conversely if $\alpha \to 1$ the system has a short memory [20]. Hence, any decrease in human memory and learning process will increase the epidemic spread and vice versa. For instant, during Zika epidemic, increasing human awareness through media in order to help individuals learning how to use mosquito nets and mosquito repeller, reduced mosquito biting rate and helped to control the disease as people used their previous experience and the stored information about the disease in their memory to reduce the risk of infection [21]. Consequently, we present here a fractional order model based on evolutionary game theory to predict human behavior during epidemic outbreaks. During an emerging epidemic, memory and learning process play a significant role in disease transmutation as they affect directly in human behaviors.

The rest of the paper is arranged as follows. In Section 2, basic definitions and model derivation are presented while in Section 3 we show that the proposed model has non- negative solutions. The existence of the uniformly stable solutions of the presented model is discussed in Section 4. The numerical solutions of the proposed fractional order system is presented in Section 5.

2 Model derivation

First of all, we introduce some of the basic definitions of the fractional calculus which has become the focus of several scientific fields [12, 13, 14, 15, 16, 17, 18].

Definition 1. The fractional integral of order $\alpha > 0$ of a function $f : R^+ \to R$ is given as follows:

$$J^{\alpha} f(x) = \frac{1}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} f(t)dt, \ \ 0 < \alpha \leq 1, \ 0 < x \leq T,$$
$$J^0 f(x) = f(x).$$

Definition 2. Riemann–Liouville and Caputo fractional derivatives of order α where $\alpha \in (m-1, m)$ of a continuous function $f : R^+ \to R$ is given respectively by

$$D_*^{\alpha} f(x) = D^m \left(J^{m-\alpha} f(x)\right),$$
$$D^{\alpha} f(x) = J^{m-\alpha} \left(D^m f(x)\right),$$

Where

$$m-1 < \alpha \leq m, \ m \in N.$$

Now, the fractional-order derivative in Caputo sense is introduced into the SIR epidemic model proposed in [11] to be as follows:

$$\begin{aligned}
D^{\alpha} S_n^{tr}(t) &= -\lambda S_n, \\
D^{\alpha} S_a^{tr}(t) &= -q\lambda S_a, \\
D^{\alpha} I_s^{tr}(t) &= p\left[\lambda S_n + q\lambda S_a\right] - \gamma I_s, \\
D^{\alpha} I_{A_n}^{tr}(t) &= (1-p)\lambda S_n - \gamma I_{A_n}, \\
D^{\alpha} I_{A_a}^{tr}(t) &= (1-p)q\lambda S_a - \gamma I_{A_a}, \\
D^{\alpha} R_s^{tr}(t) &= \gamma I_s, \\
D^{\alpha} R_{A_n}^{tr}(t) &= \gamma I_{A_n}, \\
D^{\alpha} R_{A_a}^{tr}(t) &= \gamma I_{A_a},
\end{aligned} \tag{1}$$

Where $\lambda = \beta_s I_s + \beta_A I_A + q\beta_A I_{A_a}$ is the infection force and $\alpha \in (0,1]$ is the fractional order. For $\alpha \in (0, 0.5]$, we believe that the rate of change cannot be approximately described by the fractional derivatives [24].

S represents susceptible people, I_S represents the infective symptomatic people, I_A represents the asymptomatic people, R_S represents people who were recovered from symptomatic infections, and R_A represents people who were recovered from asymptomatic infections. The population fractions S_n, I_{A_n}, R_{A_n} represent individuals with normal behavior while S_a, I_{A_a}, R_{A_a} represent individuals with alerted behavior. β_s and β_A are the rate of transmission for symptomatic people and asymptomatic people respectively. The potentially infectious contacts is decreased by q. The mean length of the infection period is presented by $1/\gamma$ while p represents the probability of developing symptoms of the disease.

The information about the spread and transmission of the infectious diseases is a crucial aspect as it presents the number of infected symptomatic people over time. Based on [11, 25], the information variable $M(t)$ which introduces the information about the current/past states of the disease can be presented as:

$$D^{\alpha}M(t) = p[\lambda(t)S_n(t) + q\lambda(t)S_a(t)] - \nu I(t)$$

Where $1/\nu$ is the average time of the symptomatic cases memory and $0 < \alpha \leq 1$ is the fractional order.
The outlines of imitation dynamics for two fractions x and $(1-x)$ of population performing two different strategies have been presented in [11] as an integer order ordinary differential equation. Here, the imitation dynamics can be illustrated as a fractional order differential equation as follows:

$$D^{\alpha}(x) = \widetilde{\omega}x(1-x)\varphi\Delta P \tag{2}$$

where $0 < \alpha \leq 1$, $\widetilde{\omega}$ is the rate of face to face communication between individuals, ΔP is the difference between the payoffs $(\Delta P = P_n - P_a)$ of the two different strategies and φ is a proportionality constant as $D^{\alpha}(x) \propto \Delta P$. From (2), it is clear that the equilibrium points of (2) are $x=0, x=1$. In other words, if some individuals play the same strategy then such strategy can spread through the population. The authors in [11], considered that individuals rarely change strategy to avoid the effect of the equilibria. So, equation (2) becomes:

$$D^{\alpha}(x) = \omega x(1-x)\Delta P + \widetilde{\mu}(1-x) - \widetilde{\mu}x,$$

where $\omega := \widetilde{\omega}\varphi$ and $\widetilde{\mu} \ll 1$ is the rate at which people change their strategies.
The imitation dynamics presents changes of individuals' strategy but it does not assume any movement between the population classes$(S-I-R)$.
So, for susceptible people, the imitation model as follows

$$\begin{aligned}
D^{\alpha}S_n^{im}(\tau) &= \omega[S_nS_a + S_a(I_{A_n} + R_{A_n})]\Delta P + \widetilde{\mu}S_a - \widetilde{\mu}S_n \quad &\text{if } \Delta P > 0\\
D^{\alpha}S_n^{im}(\tau) &= \omega[S_nS_a + S_n(I_{A_a} + R_{A_a})]\Delta P + \widetilde{\mu}S_a - \widetilde{\mu}S_n \quad &\text{if } \Delta P < 0\\
D^{\alpha}S_a^{im}(\tau) &= -\omega[S_nS_a + S_a(I_{A_n} + R_{A_n})]\Delta P - \widetilde{\mu}S_a + \widetilde{\mu}S_n \quad &\text{if } \Delta P > 0\\
D^{\alpha}S_a^{im}(\tau) &= -\omega[S_nS_a + S_n(I_{A_a} + R_{A_a})]\Delta P - \widetilde{\mu}S_a + \widetilde{\mu}S_n \quad &\text{if } \Delta P < 0
\end{aligned}$$

The other equations of the infected and removed classes can be defined for all other classes. In [11], τ is introduced to present the of the uncoordinated behavioral changes such that $t=\xi\tau$ is considered as the transmission dynamic time unit where $\xi \in R$. The imitation dynamics of susceptible class can be presented as follows:

$$D^{\alpha}S_n(t) = D^{\alpha}S_n^{tr} + \frac{1}{\xi}D^{\alpha}S_n^{im},$$

$$D^{\alpha}S_a(t) = D^{\alpha}S_a^{tr} + \frac{1}{\xi}D^{\alpha}S_a^{im},$$

Similarly, we can model the other population classes. $I_s^{im}(\tau)$ and $R_s^{im}(\tau)$ are assumed to be zero [11] and the full model after can be presented as follows:

$$\begin{aligned}
D^{\alpha}S_n(t) = -\lambda S_n + \frac{1}{\xi}[&\omega S_nS_a\Delta P + \omega S_a(I_{A_n} + R_{A_n})\Delta P\mathscr{H}(\Delta P)\\
&-\omega S_n(I_{A_a} + R_{A_a})\Delta P\mathscr{H}(-\Delta P) + \widetilde{\mu}S_a - \widetilde{\mu}S_n],
\end{aligned}$$

$$\begin{aligned}
D^{\alpha}S_a(t) = -q\lambda S_a + \frac{1}{\xi}[&\omega S_nS_a\Delta P - \omega S_a(I_{A_n} + R_{A_n})\Delta P\mathscr{H}(\Delta P)\\
&+\omega S_n(I_{A_a} + R_{A_a})\Delta P\mathscr{H}(-\Delta P) - \widetilde{\mu}S_a + \widetilde{\mu}S_n],
\end{aligned}$$

$$D^{\alpha}I_S(t) = p[\lambda S_n + q\lambda S_a] - \gamma I_s$$

$$\begin{aligned}
D^{\alpha}I_{A_n}(t) = (1-p)\lambda S_n - \gamma I_{A_n} + \frac{1}{\xi}[&\omega I_{A_n}I_{A_a}\Delta P + \omega I_{A_a}(S_n + R_{A_n})\Delta P\mathscr{H}(\Delta P)\\
&-\omega I_{A_n}(S_a + R_{A_a})\Delta P\mathscr{H}(-\Delta P) + \widetilde{\mu}I_{A_a} - \widetilde{\mu}I_{A_n}],
\end{aligned} \tag{3}$$

$$
\begin{aligned}
D^{\alpha}I_{A_a}(t) &= (1-p)\lambda S_a - \gamma I_{A_a} + \frac{1}{\xi}[\omega I_{A_n} I_{A_a}\Delta P - \omega I_{A_a}(S_n + R_{A_n})\Delta P \mathscr{H}(\Delta P)\\
&\quad + \omega I_{A_n}(S_a + R_{A_a})\Delta P \mathscr{H}(-\Delta P) - \widetilde{\mu} I_{A_a} + \widetilde{\mu} I_{A_n}],\\
D^{\alpha}R_S(t) &= \gamma I_s\\
D^{\alpha}R_{A_n}(t) &= \gamma I_{A_n} + \frac{1}{\xi}[\omega R_{A_n} R_{A_a}\Delta P + \omega R_{A_a}(S_n + I_{A_n})\Delta P \mathscr{H}(\Delta P)\\
&\quad - \omega R_{A_n}(S_a + I_{A_a})\Delta P \mathscr{H}(-\Delta P) + \widetilde{\mu} R_{A_a} - \widetilde{\mu} R_{A_n}],\\
D^{\alpha}R_{A_a}(t) &= \gamma I_{A_a} + \frac{1}{\xi}[\omega R_{A_n} R_{A_a}\Delta P - \omega R_{A_a}(S_n + I_{A_n})\Delta P \mathscr{H}(\Delta P)\\
&\quad + \omega R_{A_n}(S_a + I_{A_a})\Delta P \mathscr{H}(-\Delta P) - \widetilde{\mu} R_{A_a} + \widetilde{\mu} R_{A_n}],
\end{aligned}
$$

where

$$
\mathscr{H}(\Delta P) = \begin{cases} 1 & \text{if } \Delta P \geq 0 \\ 0 & \text{if } \Delta P < 0 \end{cases}
$$

Suppose that, $S = S_a + S_n$, $I_A = I_{A_a} + I_{A_n}$, $R_A = R_{A_a} + R_{A_n}$ and define the variable $x = \frac{S_n + I_{A_n} + R_{A_n}}{S + I_A + R_A}$. As in [24], if we a follow the assumption: $\frac{S_n}{S_n + S_a} = \frac{I_{A_n}}{I_{A_n} + I_{A_a}} = \frac{R_{A_n}}{R_{A_n} + R_{A_a}}$
then $Sx = S_n$, $I_A x = I_{A_n}$, $R_A x = R_{A_n}$ and the previous model (3) can be written as follows:

$$
\begin{aligned}
&D^{\alpha}S(t) = -\lambda\,[x + q(1-x)]S,\\
&D^{\alpha}I_A(t) = (1-p)\lambda\,[x + q(1-x)]S - \gamma I_A,\\
&D^{\alpha}I_s(t) = p\lambda\,[x + q(1-x)]S - \gamma I_s\,,\\
&D^{\alpha}R_A(t) = \gamma I_A,\\
&D^{\alpha}R_s(t) = \gamma I_s,\\
&D^{\alpha}M(t) = p\lambda\,[x + q(1-x)]S - \nu M\\
&D^{\alpha}x(t) = x(1-x)\left[\frac{pS}{1 - R_s - I_s}\right][\lambda(q-1)] +\\
&\qquad \rho\,[x(1-x)(1 - I_s - R_s)(1 - mM(t)) + \mu(1-2x)].
\end{aligned}
\tag{4}
$$

Where $m = \frac{m_n - m_a}{k}$, $\rho = \frac{k\omega}{\xi}$, $\mu = \frac{\widetilde{\mu}}{k\omega}$ and $\lambda = \beta_s I_s + \beta_A I_A x + q\beta_A I_A(1-x)$. The initial conditions are $S(0) = 1 - 10^{-3}$, $I_s(0) = 10^{-3}$, $x(0) = 1 - 10^{-6}$, $I_A(0) = R_A(0) = R_S(0) = M(0) = 0$ [24].
The above model can be modified in order to make the dimensions match [21, 24]. The new modified model can be written as follows:

$$
\begin{aligned}
&D^{\alpha}S(t) = -(\beta_s^{\alpha} I_s + \beta_A^{\alpha} I_A x + q\beta_A^{\alpha} I_A(1-x))[x + q(1-x)]S,\\
&D^{\alpha}I_A(t) = (1-p)(\beta_s^{\alpha} I_s + \beta_A^{\alpha} I_A x + q\beta_A^{\alpha} I_A(1-x))[x + q(1-x)]S - \gamma I_A,\\
&D^{\alpha}I_s(t) = p(\beta_s^{\alpha} I_s + \beta_A^{\alpha} I_A x + q\beta_A^{\alpha} I_A(1-x))[x + q(1-x)]S - \gamma I_s\\
&D^{\alpha}R_A(t) = \gamma I_A,\\
&D^{\alpha}R_s(t) = \gamma I_s,\\
&D^{\alpha}M(t) = p(\beta_s^{\alpha} I_s + \beta_A^{\alpha} I_A x + q\beta_A^{\alpha} I_A(1-x))[x + q(1-x)]S - \nu M\\
&D^{\alpha}x(t) = x(1-x)\left[\frac{pS}{1 - R_s - I_s}\right][(\beta_s^{\alpha} I_s + \beta_A^{\alpha} I_A x + q\beta_A^{\alpha} I_A(1-x))(q-1)] +\\
&\qquad \rho^{\alpha}[x(1-x)(1 - I_s - R_s)(1 - mM(t)) + \mu^{\alpha}(1-2x)]
\end{aligned}
\tag{5}
$$

3 Non-negative solutions

Denote $R_+^7 = \left\{\chi \in R^7 \mid \chi \geq 0\right\}$ and let

$$
\chi(t) = (S(t),\ I_A(t), I_s(t), R_A(t), R_s(t), M(t), x(t))^T\ .
$$

Lemma1. Consider that $f(\chi) \in C[a,b]$ and $D_a^{\alpha} f(\chi) \in C[a,b]$, $for\ 0 < \alpha \le 1$ **,** then we have [18]:

$$f(x) = f(a) + \frac{1}{\Gamma(\alpha)}(D_a^{\alpha} f)(\xi)(\chi - a)^{\alpha}$$

With $a \le \xi \le \chi$, $\forall x \in (a,b]$.
Corollary1. Consider that $f(\chi) \in C[a,b]$ and $D_a^{\alpha} f(\chi) \in C(a,b]$, $for\ 0 < \alpha \le 1$ **.**
If $D_a^{\alpha} f(\chi) \ge 0$, $\forall \chi \in (a,b)$, then $f(\chi)$ is non-decreasing for each $x \in [a,b]$.
If $D_a^{\alpha} f(\chi) \le 0$, $\forall \chi \in (a,b)$, then $f(\chi)$ is non-increasing for each $x \in [a,b]$.
Proof. This is comprehensible from **Lemma 1.**
Theorem1. There exists a unique solution

$$\chi(t) = (S(t),\ I_A(t), I_s(t), R_A(t), R_s(t), M(t), x(t))^T$$

For the system (5) over $t \ge 0$ which remains in R_+^7 .
Proof. It is understandable that the solution of system (5) on $(0, +\infty)$ is unique [27]. From (5), we have

$$\begin{aligned}
&D^{\alpha} S|_{s=0} \ \ \ge 0,\\
&D^{\alpha} I_A|_{I_A=0} \ge 0,\\
&D^{\alpha} I_s|_{I_s=0} \ \ \ge 0,\\
&D^{\alpha} R_A|_{R_A=0} \ge 0,\\
&D^{\alpha} R_s|_{R_s=0} \ \ge 0,\\
&D^{\alpha}\ M|_{M=0} \ \ge 0,\\
&D^{\alpha}\ \ x|_{x=0} \ \ \ \ge 0.
\end{aligned}$$

So, the solution of (5) remains in R_+^7

4 Existence of uniformly stable solution

In order to figure out the outlines of the existence and uniqueness of the solutions of the model (5), the following lemma have to be discussed:
Lemma 2 (Theorem 8.11 [25]) Let $\alpha_j \in\ [0,1]$ for $j = 1,2,\dots,k$ and consider the fractional order system:

$$D_*^{\alpha_j} y_j = f_j(x, y_1(x), \dots, y_k(x))),\ j = 1,2,\dots,k \tag{6}$$

where $y_j(0) = c_j, \qquad j = 1,2,\dots,k.$
Suppose that, the continuous functions $f_j = [0,x] \times R^k \to R,\ j = 1,2,\dots,k$ satisfy Lipschitz conditions corresponding to their arguments except for the first argument. So, the above system (6) has a unique continuous solution.
For system (5), each $f_i = [0,T_1] \times R_+^7 \to R_+$; $i = 1,2,\dots 7$ is continuous. Now we will prove that systems (5) have a unique continuous solution as follows.
Let

$$x_1(t) = S(t), x_2(t) = I_A(t), x_3(t) = I_s(t),\ x_4(t) = R_A, x_5(t) = R_s(t),\ \ x_6(t) = M$$

$$x_7(t) = x$$

$$D^{\alpha} x_1(t) = f_1(x_1(t), x_2(t), \dots x_7(t)), \quad t > 0 \text{ and } x_1(0) = x_{01},$$

$$D^{\alpha} x_2(t) = f_2(x_1(t), x_2(t), \dots x_7(t)), \quad t > 0 \text{ and } x_2(0) = x_{02},$$

$$D^{\alpha} x_3(t) = f_3(x_1(t), x_2(t), \dots x_7(t)), \quad t > 0 \ \text{ and }\ x_3(0) = x_{03},$$

$$D^{\alpha} x_4(t) = f_4(x_1(t), x_2(t), \dots x_7(t)), \quad t > 0 \ \text{ and }\ x_4(0) = x_{04},$$

$$D^{\alpha} x_5(t) = f_5(x_1(t), x_2(t), \dots x_7(t)), \quad t > 0 \ \text{ and }\ x_5(0) = x_{05},$$

$$D^{\alpha} x_6(t) = f_6(x_1(t), x_2(t), \dots x_7(t)), \quad t > 0 \ \text{ and }\ x_6(0) = x_{06},$$

$$D^{\alpha} x_7(t) = f_7(x_1(t), x_2(t), \dots x_7(t)), \quad t > 0 \ \text{ and }\ x_7(0) = x_{07},$$

Let $D = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7 \in R : |x_i(t) \leq a, t \in [0,T], i = 1,2,\ldots,7|\}$. The functions f_i satisfy the Lipschitz condition on R_+^3 if

$$|f_i(s_1,x_1,y_1) - f_i(s_2,x_2,y_2)| \leq K_2 \|X_1(t) - X_2(t)\|_2, \tag{7}$$

Where,

$$\|X_1(t) - X_2(t)\|_2 = |S_1 - S_2| + |I_{A1} - I_{A2}| + |I_{s1} - I_{s2}| + |R_{A1} - R_{A2}| + |R_{s1} - R_{s2}| + |M_{s1} - M_{s2}| + |x_{s1} - x_{s2}|$$

and K_2 is the Lipschitz constant.
Condition (7) is equivalent to show that: each $\frac{\partial f_i}{\partial x_j}(x_1(t), x_2(t), \ldots x_7(t))$ exists and satisfies the following relation:

$$\frac{\partial f_i}{\partial x_j}(x_1(t), x_2(t), \ldots x_7(t)) \leq K_2, \quad \forall\, i,j = 1,2,\ldots 7 \;\; and\; n = 1,2,3,\ldots,9[20].$$

Then each of f_i satisfies the Lipschitz condition and is absolutely continuous with respect to $x_i\ \forall\, i = 1,2,\ldots 7$.
Theorem 5.1. The proposed model (5) has a unique solution which is uniformly Lyapunov stable solution.
Proof. The model (5) can be written in a matrix form as follows:

$$D^{\alpha} X(t) = F(X(t)), t > 0 \text{ and } X(0) = X_0$$

Where

$$X(t) = (x_1(t) \;\; x_2(t) \ldots\; x_7(t))^{\tau}, F(X(t)) =$$
$$(f_1(x_1(t),\, x_2(t), \ldots x_7(t))\;\; f_2(x_1(t),\, x_2(t), \ldots x_7(t)) \ldots\; f_7(x_1(t),\, x_2(t), \ldots x_7(t)))^{\tau}$$

Where the column vector$(x_1(t) \;\; x_2(t) \ldots\; x_7(t))^{\tau}$, is the solution vector of the system (5) and $x_1, x_2, \ldots x_7 \in C[0,T]$.
So, we can deduce that the system (5) has a unique solution which is uniformly Lyapunov stable solution [21, 28].
The two basic reproductive numbers of the system (5) have been implemented in [11] for $\alpha = 1$ as follows:

$$R_0^n = (1-p)\frac{\beta_A}{\gamma} + p\frac{\beta_S}{\gamma} \;\text{ and }\; R_0^a = q^2(1-p)\frac{\beta_A}{\gamma} + qp\frac{\beta_S}{\gamma} \tag{8}$$

Where R_0^n is for the individuals who adopt normal behavior while R_0^a is for the individuals who adopt altered behavior. Similarly, we can deduce the two basic reproductive numbers of the system (5) as follows:

$$\overline{R}_0^n = (1-p)\frac{\beta_A^{\alpha}}{\gamma} + p\frac{\beta_S^{\alpha}}{\gamma} \;\text{ and }\; \overline{R}_0^a = q^2(1-p)\frac{\beta_A^{\alpha}}{\gamma} + qp\frac{\beta_S^{\alpha}}{\gamma} \tag{9}$$

The effect of the fractional order α on the numerical values of $\overline{R}_0^n$ and $\overline{R}_0^a$ is clear from (9). In other words, the memory parameter α can affect the values of the basic reproductive numbers of the fractional order model (5) and for $\alpha \to 0$, the system (5) has a perfect memory.

5 Numerical results and discussions

Most of the fractional order models have no analytic solutions, so numerical methods can be used to obtain approximate numerical solutions for such models. In this paper, generalized Euler method is used to solve the fractional order models (4) and (5). The parameters values are given in [11] as follows:

$$\frac{1}{\gamma} = \frac{1}{\nu} = 2.8,\; \beta_S = \beta_A = 0.5, \frac{1}{m} = 0.01,\; \rho = 10, \mu = 10^{-8}.$$

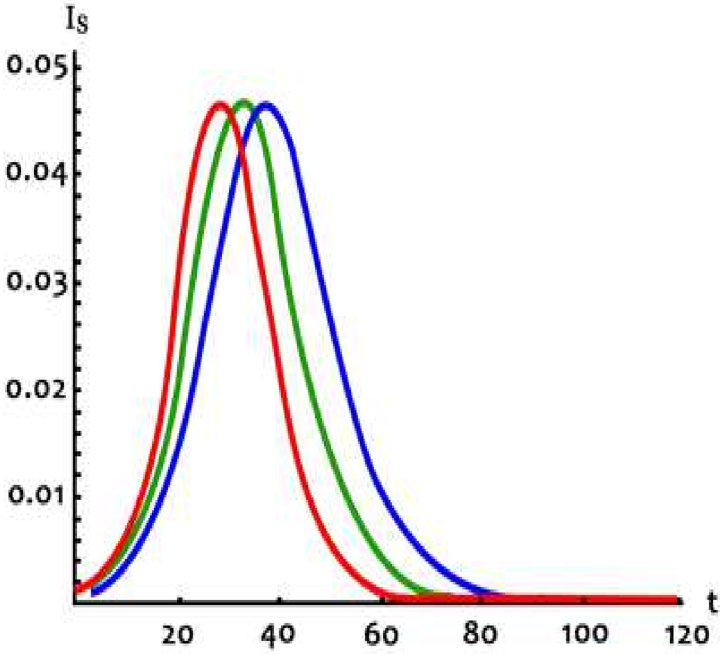

Fig. 1: The number of infective symptomatic individuals I_S of system (4) for $p = q = 1$, $\alpha = 1$(the blue line), $\alpha = 0.95$ (the green line), and $\alpha = 0.9$, (the red line).

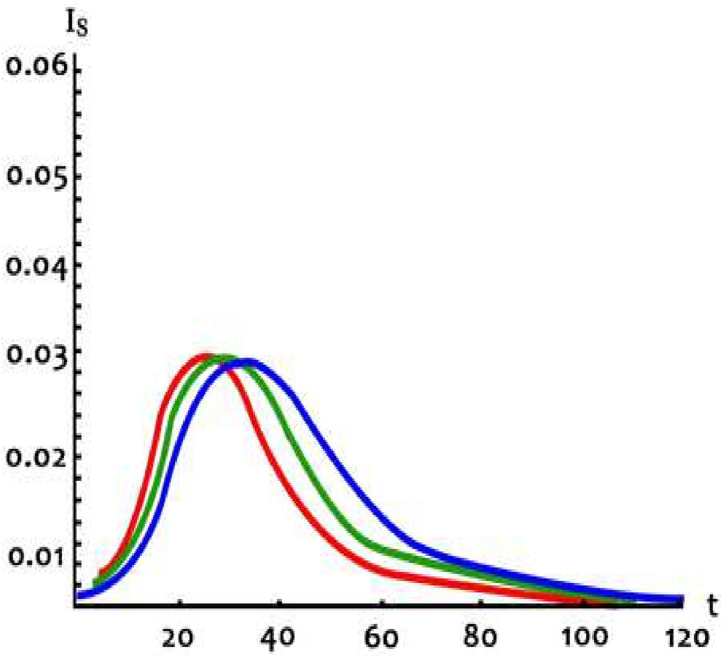

Fig. 2: The number of infective symptomatic individuals I_S of system (4) for $p = 1, q = 0.85$, $\alpha = 1$(the blue line), $\alpha = 0.95$ (the green line), and $\alpha = 0.9$, (the red line).

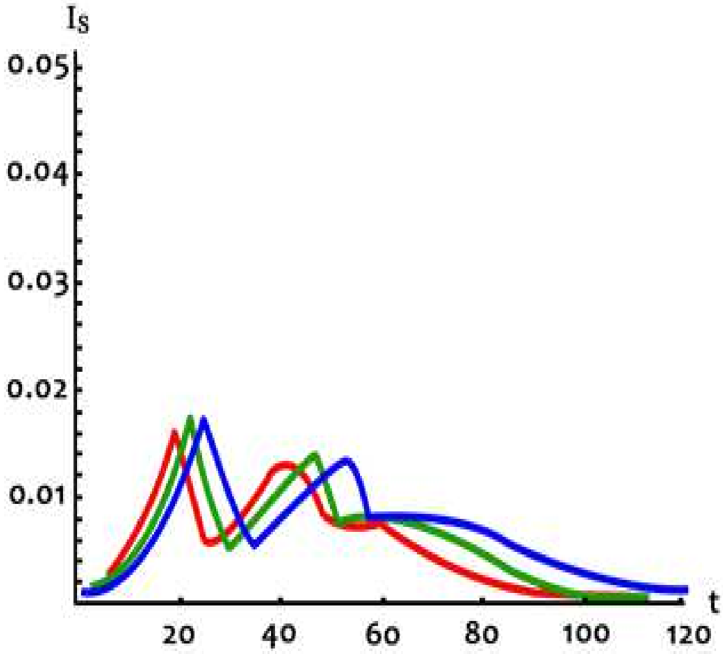

Fig. 3: The number of infective symptomatic individuals I_S of system (4) for $p = 0.7, q = 0.6$, $\alpha = 1$(the blue line), $\alpha = 0.95$ (the green line), and $\alpha = 0.9$, (the red line).

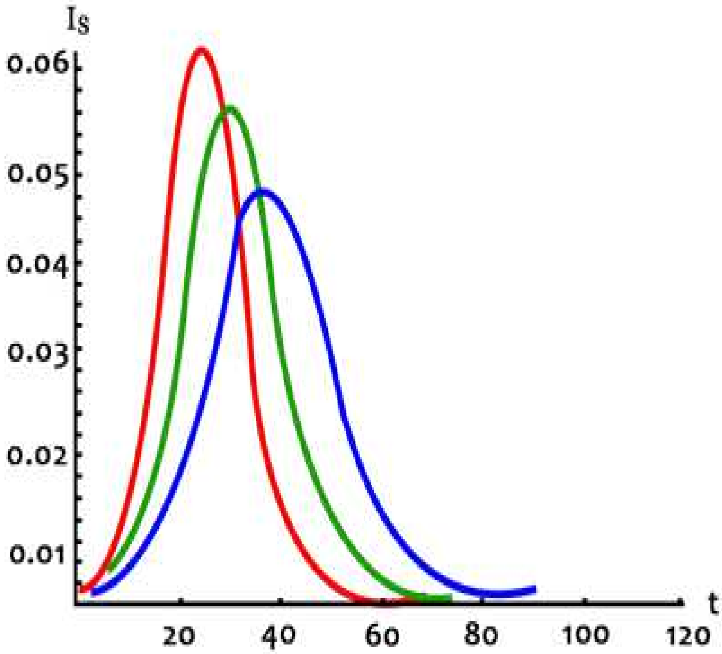

Fig. 4: The number of infective symptomatic individuals I_S of system (5) for $p = q = 1$, $\alpha = 1$(the blue line), $\alpha = 0.95$ (the green line), and $\alpha = 0.9$, (the red line).

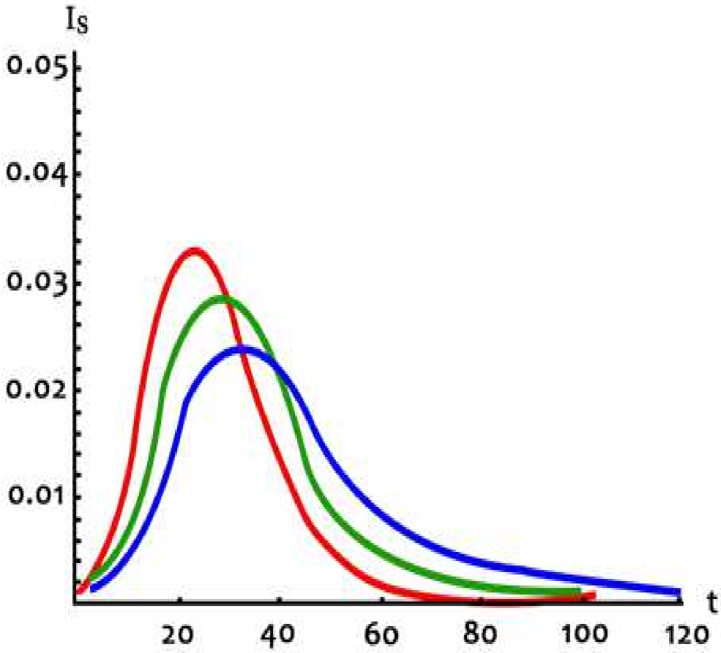

Fig. 5: The number of infective symptomatic individuals I_S of system (5) for $p = 1, q = 0.85$, $\alpha = 1$(the blue line), $\alpha = 0.95$ (the green line), and $\alpha = 0.9$, (the red line).

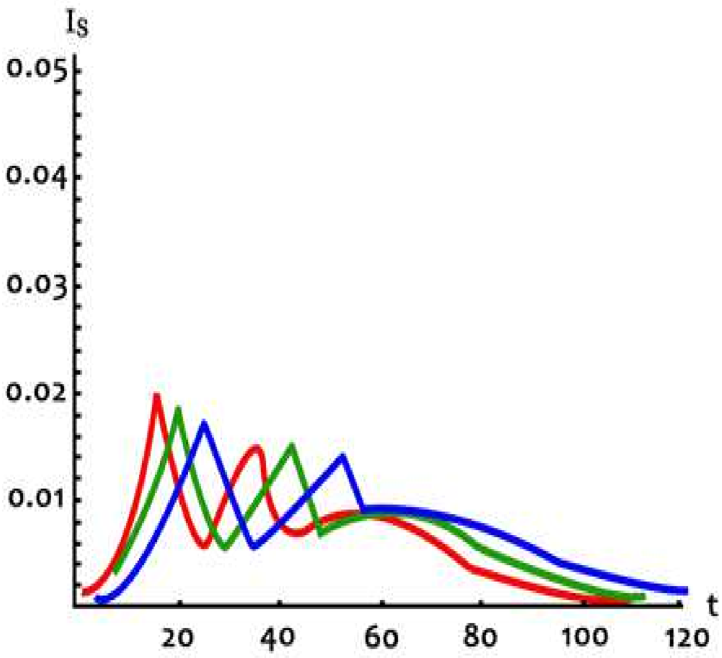

Fig. 6: The number of infective symptomatic individuals I_S of system (5) for $p = 0.7, q = 0.6$, $\alpha = 1$(the blue line), $\alpha = 0.95$ (the green line), and $\alpha = 0.9$, (the red line).

Numerical results indicate that, decreasing the values of p and q implies to a reduction in the number of infective symptomatic individuals I_S (see figures 1-6). For $x \neq 0$, people take some time to amend their behavior as shown in figures 1 & 4. Increasing the value of q implies to effective early strategy in human population as shown in figures 2, 3, 5, 6. On the other hand, when q decreases, the number of infected people embracing amended behavior decreases (see figures 3 & 6). Nevertheless, small values of q implies to enormous epidemic waves as shown in figures 3 & 6. Only infected symptomatic people can transmit the disease For $p = 1$ (see figures 1,2,4,5). But if $p \neq 1$, then asymptomatic and symptomatic people can transmit diseases with the same rate. Also when $\alpha \to 0$, the numerical results strongly depend of the previous states not only on the current state. In other words, means that when the memory parameter $\alpha \to 0$ the altered learning behavior of the population increase and the epidemic risk will vanish faster as shown in figures (1-6).

6 Conclusion

In this paper, a non-integer order model is proposed to study the impact of people behavioral changes in the spread of infectious diseases during emerging epidemics. The model is circumscribed by an evolutionary game theory. We studied the impact of memory and altered learning behavior in human population which plays a significant role in epidemic dynamics. Numerical simulations show how small changes in the values of the non-integer order α (the memory index) can make dramatic changes in the number of infective symptomatic individuals. The impact of the reduction factor of the infectious contacts number q and the developing symptoms probability q are discussed in this work. The epidemic size can be decreased by decreasing the value of q as shown in Figures (1,2,4,5). For $p \neq 1$, both asymptomatic and symptomatic people become infectious. However, for small values of q several epidemic waves can take place as shown in Figures (3,6). We improve the performance of the integer system presented in [11] by introducing the fractional order α to this system. In other words, we add a new degree of freedom in order to study the impact of population memory and its associated learning process on the epidemic dynamics. The memory effect on the basic reproductive number of the presented fractional order model. For $\alpha \to 0$, the fractional order system has a perfect memory. Studying human behavioral changes during outbreaks is essential to help public health organizations and decision makers to make strategic action plans for epidemic control.

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