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The Weighted Power Shanker Distribution with Characterizations and Applications of Real Life Time Data

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Abstract: In this paper, we propose a new extension of power Shanker distribution known as weighted power Shanker distribution. The different mathematical and statistical properties of the newly introduced distribution are derived and discussed in detail such as moments, order statistics, likelihood Ratio test, Income distribution curves, and entropy and reliability measures. The maximum likelihood estimators of the parameters of new distribution are executed and also the Fisher's information matrix is discussed. Application of the new distribution with three real life data sets are executed to show the supremacy of weighted power shanker distribution in analyzing real life time data.

Keywords: Power Shanker distribution, weighted distribution, order statistics, maximum likelihood estimation, reliability measures.

1 Introduction

Weighted distributions introduced by Fisher [5] is important in technique of fitting models to the unknown weight functions when the samples can be taken both from the original distribution and the developed distribution and later Rao [14] modified and formulated in general terms in deals with modelling statistical data when the usual practice of using existing standard distributions was found to be inappropriate. Weighted distributions are applied in many fields such as reliability, medicine and ecology. Weighted distributions are modified with reference to the probabilities of events as observed and transcribed. The concept of weighted distributions provides an access on collectively basis to deal with problems of model specification and data interpretation. Weighted distributions arise when the observations generated from a stochastic process are not given equal chances of being recorded; instead they are recorded according to some weight function. Weighted distributions reduce to length biased distribution, when the weight function depends only on the length of units. When the sampling mechanism selects units with probability proportional to the measure of the unit size, resulting distribution is called size biased. Warren [21] was the first to apply the weighted distributions in connection with sampling wood cells. Patil and Rao [13] introduced the concept of size biased sampling and weighted distributions by identifying some situations where the underlying models retain their form. The statistical interpretation of weighted and size biased distributions was originally identified by Buckland and Cox [2] in the context of renewal theory. There are various good sources which provide detailed description of weighted distributions. Dar et al. [3] obtained weighted transmuted power distribution and its properties and applications. Rather and Subramanian [17] obtained the new generalization of Akshaya distribution with applications in engineering science. Eyob, Shanker, Shukla and Leonida [4] derived weighted quasi Akash distribution with properties and its applications. Para and Jan [12] introduced three parameter weighted Pareto Type II distribution as a new life time distribution with applications in medical sciences. Hassan et al. [9,8,7] introduced three new weighted probability models with application to handle life time data in engineering and medical sciences. Rather and Ozel [16] discusses on the weighted power lindley distribution with applications on the lifetime data. Recently, Ganaie, Rajagopalan and Rather [6] derived the weighted two parameter quasi shanker distribution with properties and its application, which indicates that the newly executed distribution is more flexible and reliable than the classical distribution.

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Power shanker distribution was suggested by Shanker and Shukla [18] is a newly introduced life time distribution. They introduced a new generalization of the shanker distribution by considering the power transformation of the random variable $X = Y^{\frac{1}{\alpha}}$. The different statistical properties of the proposed distribution are derived including shapes of density and hazard rate function, moments, skewness, kurtosis, stochastic ordering and maximum likelihood estimation. Shanker [20] also discussed another life time distribution called as Shanker distribution and its applications. Various mathematical and statistical properties of the proposed distribution are derived and defined. The proposed Shanker distribution is a particular case of power shanker distribution. Shanker and Shukla [19] obtained a new generalization of shanker distribution known as quasi shanker distribution and its applications and discussed its several mathematical and statistical properties. The goodness of fit of power shanker distribution is discussed with real lifetime data set and the fit shows quite satisfactory over two parameter power Lindley and one parameter Lindley, Shanker and exponential distributions.

In this paper we introduce a new distribution with three parameters, known as weighted Power Shanker (WPS) distribution. We introduce the new distribution with the hope that it will provide a better result and will be reliable and flexible in comparison with other distributions. On applying the weighted version, the third parameter in this distribution makes it more flexible to describe different types of real data than its sub-models. The weighted power shanker distribution, due to its flexibility in accommodating different forms of the hazard function, seems to be more suitable distribution that can be used in various problems in fitting survival data.

2 Weighted Power Shanker (WPS) Distribution

The probability density function of power Shanker (PS) distribution is given by

$$f\left(x;\theta,\alpha\right) = \frac{\alpha\,\theta^2}{\left(\theta^2+1\right)} x^{\alpha-1}\left(\theta + x^{\alpha}\right) e^{-\theta x^{\alpha}} \; ; \; x > 0, \theta > 0, \; \alpha > 0 \tag{1}$$

and the cumulative distribution function of power Shanker distribution is given by

$$F(x;\theta,\alpha) = 1 - \left(1 + \frac{\theta x^{\alpha}}{\theta^2+1}\right) e^{-\theta x^{\alpha}} \; ; \; x > 0, \theta > 0, \alpha > 0 \tag{2}$$

Suppose X is a non-negative random variable with probability density function $f(x)$ .Let $w(x)$ be the non-negative weight function, then, the probability density function of the weighted random variable X_w is given by

$$f_W(x) = \frac{w(x) f(x)}{E(w(x))}, \quad x > 0.$$

Where $w(x)$ be the non-negative weight function and $E(w(x)) = \int w(x) f(x) dx < \infty.$

In this paper, we have to introduce the weighted power Shanker distribution by considering the weight function as $w(x) = x^c$ and by using the definition of weighted distribution, the probability density function of weighted power shanker distribution is given as

$$f_W(x;\theta,\alpha,c) = \frac{x^c f(x;\theta,\alpha)}{E(x^c)} \tag{3}$$

Where $E(x^c) = \int_0^{\infty} x^c f(x;\theta,\alpha) dx$

$$E(x^{c}) = \frac{\theta^{2}\Gamma\frac{(\alpha+c)}{\alpha} + \Gamma\frac{(2\alpha+c)}{\alpha}}{\alpha\theta^{\frac{c}{\alpha}}\left(\theta^{2}+1\right)} \tag{4}$$

Substitute equations (1) and (4) in equation (3), we will obtain the probability density function of weighted power Shanker distribution.

$$f_{w}(x;\theta,\alpha,c) = \frac{\alpha\theta^{\frac{2\alpha+c}{\alpha}}}{\theta^{2}\Gamma\frac{(\alpha+c)}{\alpha} + \Gamma\frac{(2\alpha+c)}{\alpha}} x^{\alpha+c-1}\left(\theta+x^{\alpha}\right)e^{-\theta x^{\alpha}} \tag{5}$$

Now cumulative distribution function of weighted power Shanker distribution is obtained as

$$F_{w}(x;\theta,\alpha,c) = \int_{0}^{x} f_{w}(x;\theta,\alpha,c)dx$$

$$F_{w}(x;\theta,\alpha,c) = \int_{0}^{x} \frac{\alpha\theta^{\frac{2\alpha+c}{\alpha}}}{\theta^{2}\Gamma\frac{(\alpha+c)}{\alpha} + \Gamma\frac{(2\alpha+c)}{\alpha}} x^{\alpha+c-1}\left(\theta+x^{\alpha}\right)e^{-\theta x^{\alpha}}dx$$

$$F_{w}(x;\theta,\alpha,c) = \frac{1}{\theta^{2}\Gamma\frac{(\alpha+c)}{\alpha} + \Gamma\frac{(2\alpha+c)}{\alpha}}\left(\alpha\theta^{\frac{3\alpha+c}{\alpha}}\int_{0}^{x} x^{\alpha+c-1}e^{-\theta x^{\alpha}}dx + \alpha\theta^{\frac{2\alpha+c}{\alpha}}\int_{0}^{x} x^{2\alpha+c-1}e^{-\theta x^{\alpha}}dx\right) \tag{6}$$

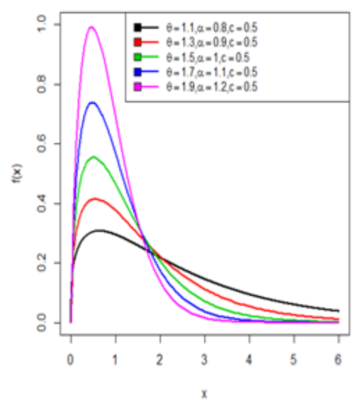
$$F_{w}(x;\theta,\alpha,c) = \frac{1}{\theta^{2}\Gamma\frac{(\alpha+c)}{\alpha} + \Gamma\frac{(2\alpha+c)}{\alpha}}\int_{0}^{x} \alpha\theta^{\frac{2\alpha+c}{\alpha}} x^{\alpha+c-1}\left(\theta+x^{\alpha}\right)e^{-\theta x^{\alpha}}dx$$

Put $\theta x^{\alpha} = t \Rightarrow x^{\alpha} = \frac{t}{\theta} \Rightarrow x = \left(\frac{t}{\theta}\right)^{\frac{1}{\alpha}}$

Also $\alpha\theta x^{\alpha-1}dx = dt \Rightarrow dx = \frac{dt}{\alpha\theta x^{\alpha-1}} \Rightarrow dx = \frac{dt}{\alpha\theta\left(\frac{t}{\theta}\right)^{\frac{\alpha-1}{\alpha}}}$

After simplifying the equation (6) we obtain the cumulative distribution function of weighted power Shanker distribution as

$$F_W(x;\theta,\alpha,c)=\frac{1}{\theta^2\Gamma\frac{(\alpha+c)}{\alpha}+\Gamma\frac{(2\alpha+c)}{\alpha}}\left(\theta^2\gamma\left(\frac{(\alpha+c)}{\alpha},\theta x^\alpha\right)+\gamma\left(\frac{(2\alpha+c)}{\alpha},\theta x^\alpha\right)\right) \tag{7}$$



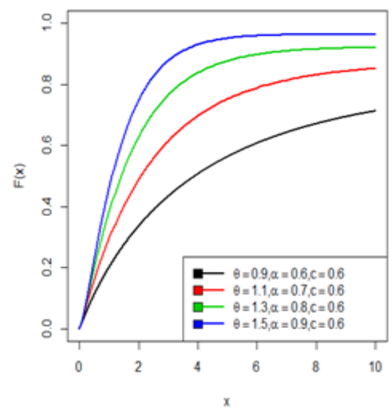


Fig.1: pdf plot of weighted power Shanker distribution. **Fig.2** cdf plot of weighted power Shanker distribution.

3 Reliability Measures

In this section, we obtain the reliability function, hazard rate and Reverse hazard rate functions of the Proposed weighted power Shanker distribution.

3.1 Reliability Function

The reliability function is also known as survival function and is also called as complement of the cdf and the reliability function of weighted power Shanker distribution is given by

$$R(x)=1-F_W(x;\theta,\alpha,c)$$

$$R(x)=1-\frac{1}{\theta^2\Gamma\frac{(\alpha+c)}{\alpha}+\Gamma\frac{(2\alpha+c)}{\alpha}}\left(\theta^2\gamma\left(\frac{(\alpha+c)}{\alpha},\theta x^\alpha\right)+\gamma\left(\frac{(2\alpha+c)}{\alpha},\theta x^\alpha\right)\right)$$

3.2 Hazard Function

The hazard function is also termed as hazard rate, instantaneous failure rate or force of mortality and is given by

$$h(x)=\frac{f_W(x;\theta,\alpha,c)}{R(x)}$$

$$h(x)=\frac{\alpha\theta^{\frac{2\alpha+c}{\alpha}}x^{\alpha+c-1}\left(\theta+x^\alpha\right)e^{-\theta x^\alpha}}{\left(\theta^2\Gamma\frac{(\alpha+c)}{\alpha}+\Gamma\frac{(2\alpha+c)}{\alpha}\right)-\left(\theta^2\gamma\left(\frac{(\alpha+c)}{\alpha},\theta x^\alpha\right)+\gamma\left(\frac{(2\alpha+c)}{\alpha},\theta x^\alpha\right)\right)}$$

3.3 Reverse Hazard Function

The reverse hazard function of weighted power Shanker distribution is given by

$$h^r(x) = \frac{f_W(x;\theta,\alpha,c)}{F_W(x;\theta,\alpha,c)}$$

$$h^r(x) = \frac{\alpha\theta^{\frac{2\alpha+c}{\alpha}} x^{\alpha+c-1}\left(\theta+x^{\alpha}\right)e^{-\theta x^{\alpha}}}{\left(\theta^2\gamma\left(\frac{(\alpha+c)}{\alpha},\theta x^{\alpha}\right)+\gamma\left(\frac{(2\alpha+c)}{\alpha},\theta x^{\alpha}\right)\right)}$$

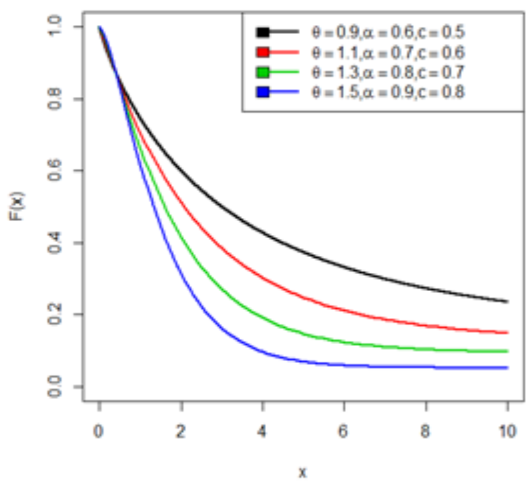


Fig.3 Reliability of weighted power Shanker distribution.

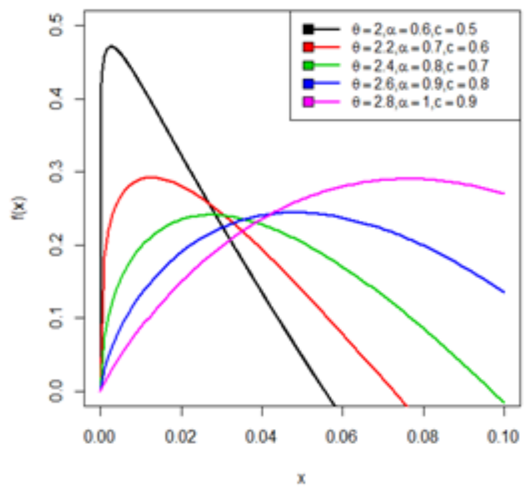


Fig.4 Hazard plot of weighted power Shanker distribution.

4 Structural Properties

In this section we will investigate some statistical properties of weighted power Shanker distribution.

4.1 Moments

Let X represents the random variable of weighted power Shanker distribution with parameters θ, α and c, then the rth order moment $E(X^r)$ about origin is

$$E(X^r) = \mu_r' = \int_0^{\infty} x^r f_W(x;\theta,\alpha,c)dx$$

$$E(X^r) = \int_0^{\infty} \frac{\alpha\theta^{\frac{2\alpha+c}{\alpha}}}{\theta^2\Gamma\frac{(\alpha+c)}{\alpha}+\Gamma\frac{(2\alpha+c)}{\alpha}} x^{\alpha+c+r-1}\left(\theta+x^{\alpha}\right)e^{-\theta x^{\alpha}}dx$$

$$= \frac{\alpha\,\theta^{\frac{2\alpha+c}{\alpha}}}{\theta^2\Gamma\frac{(\alpha+c)}{\alpha}+\Gamma\frac{(2\alpha+c)}{\alpha}}\int_0^\infty x^{\alpha+c+r-1}\left(\theta+x^\alpha\right)e^{-\theta x^\alpha}dx$$

$$= \frac{\alpha\,\theta^{\frac{2\alpha+c}{\alpha}}}{\theta^2\Gamma\frac{(\alpha+c)}{\alpha}+\Gamma\frac{(2\alpha+c)}{\alpha}}\left(\theta\int_0^\infty x^{\alpha+c+r-1}e^{-\theta x^\alpha}dx+\int_0^\infty x^{2\alpha+c+r-1}e^{-\theta x^\alpha}dx\right) \tag{8}$$

Put $x^\alpha = t \;\Rightarrow x = t^{\frac{1}{\alpha}}$

Also $\alpha\, x^{\alpha-1}dx = dt \;\Rightarrow dx = \frac{dt}{\alpha x^{\alpha-1}} = \frac{dt}{\alpha\, t^{\frac{\alpha-1}{\alpha}}}$

After simplifying, the equation (8) becomes

$$E(X^r)=\mu_r' = \frac{\theta^2\Gamma\frac{(\alpha+c+r)}{\alpha}+\Gamma\frac{(2\alpha+c+r)}{\alpha}}{\alpha\theta^{\frac{r}{\alpha}}\left(\theta^2\Gamma\frac{(\alpha+c)}{\alpha}+\Gamma\frac{(2\alpha+c)}{\alpha}\right)} \tag{9}$$

Putting $r = 1$ in equation (9), we get the expected value of weighted power Shanker distribution.

$$E(X)=\mu_1' = \frac{\theta^2\Gamma\frac{(\alpha+c+1)}{\alpha}+\Gamma\frac{(2\alpha+c+1)}{\alpha}}{\alpha\theta^{\frac{1}{\alpha}}\left(\theta^2\Gamma\frac{(\alpha+c)}{\alpha}+\Gamma\frac{(2\alpha+c)}{\alpha}\right)}$$

and putting $r = 2$, we get the second moment as

$$E(X^2)=\mu_2' = \frac{\theta^2\Gamma\frac{(\alpha+c+2)}{\alpha}+\Gamma\frac{(2\alpha+c+2)}{\alpha}}{\alpha\theta^{\frac{2}{\alpha}}\left(\theta^2\Gamma\frac{(\alpha+c)}{\alpha}+\Gamma\frac{(2\alpha+c)}{\alpha}\right)}$$

and by putting $r = 3$ and 4, we get the the third and fourth moment as

$$E(X^3)=\mu_3'=\frac{\theta^2\Gamma\frac{(\alpha+c+3)}{\alpha}+\Gamma\frac{(2\alpha+c+3)}{\alpha}}{\alpha\theta^{\frac{3}{\alpha}}\left(\theta^2\Gamma\frac{(\alpha+c)}{\alpha}+\Gamma\frac{(2\alpha+c)}{\alpha}\right)}$$

$$E(X^4)=\mu_4'=\frac{\theta^2\Gamma\frac{(\alpha+c+4)}{\alpha}+\Gamma\frac{(2\alpha+c+4)}{\alpha}}{\alpha\theta^{\frac{4}{\alpha}}\left(\theta^2\Gamma\frac{(\alpha+c)}{\alpha}+\Gamma\frac{(2\alpha+c)}{\alpha}\right)}$$

$$\text{Variance}=\frac{\theta^2\Gamma\frac{(\alpha+c+2)}{\alpha}+\Gamma\frac{(2\alpha+c+2)}{\alpha}}{\alpha\theta^{\frac{2}{\alpha}}\left(\theta^2\Gamma\frac{(\alpha+c)}{\alpha}+\Gamma\frac{(2\alpha+c)}{\alpha}\right)}-\left(\frac{\theta^2\Gamma\frac{(\alpha+c+1)}{\alpha}+\Gamma\frac{(2\alpha+c+1)}{\alpha}}{\alpha\theta^{\frac{1}{\alpha}}\left(\theta^2\Gamma\frac{(\alpha+c)}{\alpha}+\Gamma\frac{(2\alpha+c)}{\alpha}\right)}\right)^2$$

$$S.D(\sigma)=\sqrt{\left(\frac{\theta^2\Gamma\frac{(\alpha+c+2)}{\alpha}+\Gamma\frac{(2\alpha+c+2)}{\alpha}}{\alpha\theta^{\frac{2}{\alpha}}\left(\theta^2\Gamma\frac{(\alpha+c)}{\alpha}+\Gamma\frac{(2\alpha+c)}{\alpha}\right)}-\left(\frac{\theta^2\Gamma\frac{(\alpha+c+1)}{\alpha}+\Gamma\frac{(2\alpha+c+1)}{\alpha}}{\alpha\theta^{\frac{1}{\alpha}}\left(\theta^2\Gamma\frac{(\alpha+c)}{\alpha}+\Gamma\frac{(2\alpha+c)}{\alpha}\right)}\right)^2\right)}$$

4.2 Harmonic mean

The harmonic mean of the weighted power Shanker distribution can be obtained as

$$H.M=E\left(\frac{1}{x}\right)=\int_0^\infty \frac{1}{x} f_w(x;\theta,\alpha,c)dx$$

$$=\int_0^\infty \frac{\alpha\theta^{\frac{2\alpha+c}{\alpha}}}{\theta^2\Gamma\frac{(\alpha+c)}{\alpha}+\Gamma\frac{(2\alpha+c)}{\alpha}} x^{\alpha+c-2}\left(\theta+x^\alpha\right)e^{-\theta x^\alpha}dx$$

$$=\frac{\alpha\theta^{\frac{2\alpha+c}{\alpha}}}{\theta^2\Gamma\frac{(\alpha+c)}{\alpha}+\Gamma\frac{(2\alpha+c)}{\alpha}}\int_0^\infty x^{\alpha+c-2}\left(\theta+x^\alpha\right)e^{-\theta x^\alpha}dx$$

$$= \frac{\alpha\theta^{\frac{2\alpha+c}{\alpha}}}{\theta^2\Gamma\frac{(\alpha+c)}{\alpha}+\Gamma\frac{(2\alpha+c)}{\alpha}}\left(\theta\int_0^\infty x^{\alpha+c-2}e^{-\theta x^\alpha}dx+\int_0^\infty x^{2\alpha+c-2}e^{-\theta x^\alpha}dx\right) \tag{10}$$

Put $x^\alpha = t \Rightarrow x = t^{\frac{1}{\alpha}}$

Also $\alpha x^{\alpha-1}dx = dt \Rightarrow dx = \frac{dt}{\alpha x^{\alpha-1}} \Rightarrow dx = \frac{dt}{\alpha t^{\frac{\alpha-1}{\alpha}}}$

After the simplification of equation (10), we obtain the harmonic mean of weighted power Shanker distribution as

$$H.M = \frac{\theta^{\frac{2\alpha+c}{\alpha}}}{\left(\theta^2\Gamma\frac{(\alpha+c)}{\alpha}+\Gamma\frac{(2\alpha+c)}{\alpha}\right)}\left(\theta\gamma\left(\frac{(\alpha+c-1)}{\alpha},\theta x^\alpha\right)+\gamma\left(\frac{(2\alpha+c-1)}{\alpha},\theta x^\alpha\right)\right)$$

4.3 Moment Generating Function

Let X be the random variable of weighted power Shanker distribution, then the moment generating function of X is obtained as

$$M_X(t) = E\left(e^{tx}\right) = \int_0^\infty e^{tx} f_W(x;\theta,\alpha,c)dx$$

Using Taylor series, we obtain

$$= \int_0^\infty\left(1+tx+\frac{(tx)^2}{2!}+....\right)f_W(x;\theta,\alpha,c)dx$$

$$= \int_0^\infty\sum_{j=0}^\infty\frac{t^j}{j!}x^j f_W(x;\theta,\alpha,c)dx$$

$$= \sum_{j=0}^\infty\frac{t^j}{j!}\mu_j'$$

$$= \sum_{j=0}^\infty\frac{t^j}{j!}\left(\frac{\theta^2\Gamma\frac{(\alpha+c+j)}{\alpha}+\Gamma\frac{(2\alpha+c+j)}{\alpha}}{\alpha\theta^{\frac{j}{\alpha}}\left(\theta^2\Gamma\frac{(\alpha+c)}{\alpha}+\Gamma\frac{(2\alpha+c)}{\alpha}\right)}\right)$$

$$M_X(t)=\frac{1}{\alpha\left(\theta^2\Gamma\frac{(\alpha+c)}{\alpha}+\Gamma\frac{(2\alpha+c)}{\alpha}\right)}\sum_{j=0}^{\infty}\frac{t^j}{j!\theta^{\frac{j}{\alpha}}}\left(\theta^2\Gamma\frac{(\alpha+c+j)}{\alpha}+\Gamma\frac{(2\alpha+c+j)}{\alpha}\right)$$

Similarly,the characteristic function of weighted power Shanker distribution is given by

$$\varphi_x(t)=M_X(it)$$

$$M_X(it)=\frac{1}{\alpha\left(\theta^2\Gamma\frac{(\alpha+c)}{\alpha}+\Gamma\frac{(2\alpha+c)}{\alpha}\right)}\sum_{j=0}^{\infty}\frac{it^j}{j!\theta^{\frac{j}{\alpha}}}\left(\theta^2\Gamma\frac{(\alpha+c+j)}{\alpha}+\Gamma\frac{(2\alpha+c+j)}{\alpha}\right)$$

5 Order Statistics

Let $X_{(1)}$, $X_{(2)}$,…, $X_{(n)}$ be the order statistics of a random sample $X_1, X_2,\ldots, X_n$ from a Continuous population with probability density function $f_x(x)$ andcumulative distribution function $F_X(x)$ then the probability density function of r^{th} order statistics $X_{(r)}$ is given by

$$f_{x(r)}(x)=\frac{n!}{(r-1)!\,(n-r)!}\,f_X(x)\left[F_X(x)\right]^{r-1}\left[1-F_X(x)\right]^{n-r} \tag{11}$$

Using equations (5) and (7) in equation (11), the probability density function of r^{th}order statistics of weighted power Shanker distribution is given by

$$f_{x(r)}(x)=\frac{n!}{(r-1)!(n-r)!}\left(\frac{\alpha\theta^{\frac{2\alpha+c}{\alpha}}}{\theta^2\Gamma\frac{(\alpha+c)}{\alpha}+\Gamma\frac{(2\alpha+c)}{\alpha}}x^{\alpha+c-1}\left(\theta+x^\alpha\right)e^{-\theta x^\alpha}\right)$$
$$\times\left(\frac{1}{\theta^2\Gamma\frac{(\alpha+c)}{\alpha}+\Gamma\frac{(2\alpha+c)}{\alpha}}\left(\theta^2\gamma\left(\frac{(\alpha+c)}{\alpha},\theta x^\alpha\right)+\gamma\left(\frac{(2\alpha+c)}{\alpha},\theta x^\alpha\right)\right)\right)^{r-1}$$
$$\times\left(1-\frac{1}{\theta^2\Gamma\frac{(\alpha+c)}{\alpha}+\Gamma\frac{(2\alpha+c)}{\alpha}}\left(\theta^2\gamma\left(\frac{(\alpha+c)}{\alpha},\theta x^\alpha\right)+\gamma\left(\frac{(2\alpha+c)}{\alpha},\theta x^\alpha\right)\right)\right)^{n-r}$$

Therefore, the probability density function of higher order statistics $X_{(n)}$ of weighted power Shanker distribution can be obtained as

$$f_{x(n)}(x) = \frac{n\alpha\theta^{\frac{2\alpha+c}{\alpha}}}{\theta^2\Gamma\frac{(\alpha+c)}{\alpha} + \Gamma\frac{(2\alpha+c)}{\alpha}} x^{\alpha+c-1}\left(\theta + x^{\alpha}\right) e^{-\theta x^{\alpha}}$$

$$\times\left(\frac{1}{\theta^2\Gamma\frac{(\alpha+c)}{\alpha} + \Gamma\frac{(2\alpha+c)}{\alpha}}\left(\theta^2\gamma\left(\frac{(\alpha+c)}{\alpha}, \theta x^{\alpha}\right) + \gamma\left(\frac{(2\alpha+c)}{\alpha}, \theta x^{\alpha}\right)\right)\right)^{n-1}$$

and the probability density function of first order statistics $X_{(1)}$ of weighted power Shanker distribution can be obtained as

$$f_{x(1)}(x) = \frac{n\alpha\theta^{\frac{2\alpha+c}{\alpha}}}{\theta^2\Gamma\frac{(\alpha+c)}{\alpha} + \Gamma\frac{(2\alpha+c)}{\alpha}} x^{\alpha+c-1}\left(\theta + x^{\alpha}\right) e^{-\theta x^{\alpha}}$$

$$\times\left(1 - \frac{1}{\theta^2\Gamma\frac{(\alpha+c)}{\alpha} + \Gamma\frac{(2\alpha+c)}{\alpha}}\left(\theta^2\gamma\left(\frac{(\alpha+c)}{\alpha}, \theta x^{\alpha}\right) + \gamma\left(\frac{(2\alpha+c)}{\alpha}, \theta x^{\alpha}\right)\right)\right)^{n-1}$$

6 Likelihood Ratio Test

Let $X_1, X_2, \ldots. X_n$ be a random sample of size *n* from the weighted power Shanker distribution. We use the hypothesis

$$H_o : f(x) = f(x;\theta,\alpha) \quad \text{against} \quad \mathrm{H}_1 : f(x) = f_w(x;\theta,\alpha,c)$$

In order to test whether the random sample of size *n* comes from the power Shanker distribution or weighted power Shanker distribution, the following test statistic is used

$$\Delta = \frac{L_1}{L_o} = \prod_{i=1}^{n}\frac{f_w(x;\theta,\alpha,c)}{f(x;\theta,\alpha)}$$

$$\Delta = \frac{L_1}{L_o} = \prod_{i=1}^{n}\left(\frac{\alpha\theta^{\frac{c}{\alpha}} x_i^{c}\left(\theta^2+1\right)}{\theta^2\Gamma\frac{(\alpha+c)}{\alpha} + \Gamma\frac{(2\alpha+c)}{\alpha}}\right)$$

$$\Delta = \frac{L_1}{L_o} = \left(\frac{\alpha\theta^{\frac{c}{\alpha}}\left(\theta^2+1\right)}{\theta^2\Gamma\frac{(\alpha+c)}{\alpha} + \Gamma\frac{(2\alpha+c)}{\alpha}}\right)^{n}\prod_{i=1}^{n} x_i^{c}$$

We should reject the null hypothesis if

$$\Delta=\left(\frac{\alpha\theta^{\frac{c}{\alpha}}\left(\theta^{2}+1\right)}{\theta^{2}\Gamma\frac{(\alpha+c)}{\alpha}+\Gamma\frac{(2\alpha+c)}{\alpha}}\right)^{n}\prod_{i=1}^{n}x_i^{\,c}>k$$

$$\text{or }\Delta^{*}=\prod_{i=1}^{n}x_i^{\,c}>k\left(\frac{\theta^{2}\Gamma\frac{(\alpha+c)}{\alpha}+\Gamma\frac{(2\alpha+c)}{\alpha}}{\alpha\theta^{\frac{c}{\alpha}}\left(\theta^{2}+1\right)}\right)^{n}$$

$$\Delta^{*}=\prod_{i=1}^{n}x_i^{\,c}>k^{*},\text{ Where }k^{*}=k\left(\frac{\theta^{2}\Gamma\frac{(\alpha+c)}{\alpha}+\Gamma\frac{(2\alpha+c)}{\alpha}}{\alpha\theta^{\frac{c}{\alpha}}\left(\theta^{2}+1\right)}\right)^{n}$$

For large sample size *n*, *2log Δ* is distributed as chi-square distribution with one degree of freedom and also the chi-square distribution is used for getting p value. Thus, we reject the null hypothesis, when the probability value is given by

$p\left(\Delta^{*}>\lambda^{*}\right)$, Where $\lambda^{*}=\prod_{i=1}^{n}x_i^{\,c}$ is less than a specified level of significance and $\prod_{i=1}^{n}x_i^{\,c}$ is the observed value of the statistic Δ^{*}.

7 Income Distribution Curves

The bonferroni and Lorenz curves also known as income distribution curves are used in economics to measure the distribution of income or wealth or poverty, but today it is used in various areas such as reliability, medicine, insurance and demography. The bonferroni and Lorenz curves are given by

$$B(p)=\frac{1}{p\mu_1{}'}\int_0^{q}x\,f(x)dx$$

$$\text{and }\;L(p)=pB(p)=\frac{1}{\mu_1{}'}\int_0^{q}x\,f(x)dx$$

$$\text{Where }\mu_1{}'=\frac{\theta^{2}\Gamma\frac{(\alpha+c+1)}{\alpha}+\Gamma\frac{(2\alpha+c+1)}{\alpha}}{\alpha\theta^{\frac{1}{\alpha}}\left(\theta^{2}\Gamma\frac{(\alpha+c)}{\alpha}+\Gamma\frac{(2\alpha+c)}{\alpha}\right)}\quad\text{and }\;q=F^{-1}(p)$$

$$B(p)=\frac{\alpha\theta^{\frac{1}{\alpha}}\left(\theta^{2}\Gamma\frac{(\alpha+c)}{\alpha}+\Gamma\frac{(2\alpha+c)}{\alpha}\right)}{p\left(\theta^{2}\Gamma\frac{(\alpha+c+1)}{\alpha}+\Gamma\frac{(2\alpha+c+1)}{\alpha}\right)}\int_0^{q}\frac{\alpha\theta^{\frac{2\alpha+c}{\alpha}}}{\theta^{2}\Gamma\frac{(\alpha+c)}{\alpha}+\Gamma\frac{(2\alpha+c)}{\alpha}}x^{\alpha+c}\left(\theta+x^{\alpha}\right)e^{-\theta x^{\alpha}}dx$$

$$B(p)=\frac{\alpha\theta^{\frac{2\alpha+c+1}{\alpha}}}{p\left(\theta^2\Gamma\frac{(\alpha+c+1)}{\alpha}+\Gamma\frac{(2\alpha+c+1)}{\alpha}\right)}\int_0^q x^{\alpha+c}\left(\theta+x^{\alpha}\right)e^{-\theta x^{\alpha}}dx$$

$$B(p)=\frac{\alpha\theta^{\frac{2\alpha+c+1}{\alpha}}}{p\left(\theta^2\Gamma\frac{(\alpha+c+1)}{\alpha}+\Gamma\frac{(2\alpha+c+1)}{\alpha}\right)}\left(\theta\int_0^q x^{\alpha+c}e^{-\theta x^{\alpha}}dx+\int_0^q x^{2\alpha+c}e^{-\theta x^{\alpha}}dx\right) \qquad (12)$$

Put $x^{\alpha}=t \Rightarrow x=t^{\frac{1}{\alpha}}$

Also $\alpha x^{\alpha-1}dx=dt \Rightarrow dx=\frac{dt}{\alpha x^{\alpha-1}} \Rightarrow dx=\frac{dt}{\alpha t^{\frac{\alpha-1}{\alpha}}}$

After the simplification of equation (12), we obtain the income distribution curves as

$$B(p)=\frac{\theta^{\frac{2\alpha+c+1}{\alpha}}}{p\left(\theta^2\Gamma\frac{(\alpha+c+1)}{\alpha}+\Gamma\frac{(2\alpha+c+1)}{\alpha}\right)}\left(\theta\gamma\left(\frac{(\alpha+c+1)}{\alpha},\theta q\right)+\gamma\left(\frac{(2\alpha+c+1)}{\alpha},\theta q\right)\right)$$

$$L(p)=\frac{\theta^{\frac{2\alpha+c+1}{\alpha}}}{\left(\theta^2\Gamma\frac{(\alpha+c+1)}{\alpha}+\Gamma\frac{(2\alpha+c+1)}{\alpha}\right)}\left(\theta\gamma\left(\frac{(\alpha+c+1)}{\alpha},\theta q\right)+\gamma\left(\frac{(2\alpha+c+1)}{\alpha},\theta q\right)\right)$$

8 Maximum Likelihood Estimation and Fisher's Information Matrix

In this section, we will discuss the parameter estimation of weighted power Shanker distribution by using the method of maximum likelihood estimation and also the Fisher's Information matrix have been derived. Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from the weighted power Shanker distribution, then the likelihood function can be written as.

$$L(x)=\prod_{i=1}^{n} f_W(x;\theta,\alpha,c)$$

$$L(x)=\prod_{i=1}^{n}\left(\frac{\alpha\theta^{\frac{2\alpha+c}{\alpha}}}{\theta^{2}\Gamma\frac{(\alpha+c)}{\alpha}+\Gamma\frac{(2\alpha+c)}{\alpha}}x_i^{\alpha+c-1}\left(\theta+x_i^{\alpha}\right)e^{-\theta x_i^{\alpha}}\right)$$

$$L(x)=\left(\frac{\alpha\theta^{\frac{2\alpha+c}{\alpha}}}{\theta^{2}\Gamma\frac{(\alpha+c)}{\alpha}+\Gamma\frac{(2\alpha+c)}{\alpha}}\right)^{n}\prod_{i=1}^{n}\left(x_i^{\alpha+c-1}\left(\theta+x_i^{\alpha}\right)e^{-\theta x_i^{\alpha}}\right)$$

$$L(x)=\frac{\alpha\theta^{n\left(\frac{2\alpha+c}{\alpha}\right)}}{\left(\theta^{2}\Gamma\frac{(\alpha+c)}{\alpha}+\Gamma\frac{(2\alpha+c)}{\alpha}\right)^{n}}\prod_{i=1}^{n}\left(x_i^{\alpha+c-1}\left(\theta+x_i^{\alpha}\right)e^{-\theta x_i^{\alpha}}\right)$$

The log likelihood function is given by

$$\log L=n\left(\frac{2\alpha+c}{\alpha}\right)\log\alpha\theta-n\log\left(\theta^{2}\Gamma\frac{(\alpha+c)}{\alpha}+\Gamma\frac{(2\alpha+c)}{\alpha}\right)+(\alpha+c-1)\sum_{i=1}^{n}\log x_i+\sum_{i=1}^{n}\log\left(\theta+x_i^{\alpha}\right)-\theta\sum_{i=1}^{n}x_i^{\alpha}\tag{13}$$

The maximum likelihood estimate of θ, α and c can be obtained by differentiating equation (13) with respect to θ, α and c and must satisfy the normal equations

$$\frac{\partial\log L}{\partial\theta}=\frac{n\alpha}{\alpha\theta}\left(\frac{2\alpha+c}{\alpha}\right)-n\left(\frac{2\theta\Gamma\frac{(\alpha+c)}{\alpha}}{\theta^{2}\Gamma\frac{(\alpha+c)}{\alpha}+\Gamma\frac{(2\alpha+c)}{\alpha}}\right)+\sum_{i=1}^{n}\left(\frac{1}{\left(\theta+x_i^{\alpha}\right)}\right)-\sum_{i=1}^{n}x_i^{\alpha}=0$$

$$\frac{\partial\log L}{\partial\alpha}=-\frac{n\theta c}{\alpha^{3}\theta}-n\psi\left(\theta^{2}\Gamma\frac{(\alpha+c)}{\alpha}+\Gamma\frac{(2\alpha+c)}{\alpha}\right)+\sum_{i=1}^{n}\log x_i+\sum_{i=1}^{n}\frac{x_i^{\alpha}\log x_i}{\left(\theta+x_i^{\alpha}\right)}-\theta\sum_{i=1}^{n}x_i^{\alpha}\log x_i=0$$

$$\frac{\partial\log L}{\partial c}=\frac{n}{\alpha}\log\alpha\theta-n\psi\left(\theta^{2}\Gamma\frac{(\alpha+c)}{\alpha}+\Gamma\frac{(2\alpha+c)}{\alpha}\right)+\sum_{i=1}^{n}\log x_i=0$$

Where ψ (.) is the digamma function.

Because of the complicated form of above likelihood equations, algebraically it is very difficult to solve the system of nonlinear equations. Therefore we use the numerical technique like Newton- Raphson method for estimating the required parameters of the proposed distribution.

To obtain the confidence interval we use the asymptotic normality results. We have that if

$\hat{\lambda} = \left(\hat{\theta}, \hat{\alpha}, \hat{c}\right)$ denotes the MLE of $\lambda = \left(\theta, \alpha, c\right)$, we can state the result as

$$\sqrt{n}(\hat{\lambda} - \lambda) \rightarrow N_3\left(0, I^{-1}(\lambda)\right)$$

Where $I(\lambda)$ is the Fisher's Information matrix

$$I(\lambda) = -\frac{1}{n}\begin{pmatrix} E\left(\frac{\partial^2 \log L}{\partial\theta^2}\right) & E\left(\frac{\partial^2 \log L}{\partial\theta\,\partial\alpha}\right) & E\left(\frac{\partial^2 \log L}{\partial\theta\,\partial c}\right) \\ E\left(\frac{\partial^2 \log L}{\partial\alpha\,\partial\theta}\right) & E\left(\frac{\partial^2 \log L}{\partial\alpha^2}\right) & E\left(\frac{\partial^2 \log L}{\partial\alpha\,\partial c}\right) \\ E\left(\frac{\partial^2 \log L}{\partial c\,\partial\theta}\right) & E\left(\frac{\partial^2 \log L}{\partial c\,\partial\alpha}\right) & E\left(\frac{\partial^2 \log L}{\partial c^2}\right) \end{pmatrix}$$

Here, we define

$$E\left(\frac{\partial^2 \log L}{\partial\theta^2}\right) = -\frac{n\alpha^2}{(\alpha\theta)^2}\left(\frac{2\alpha+c}{\alpha}\right) - n\left(\frac{\left(\theta^2\Gamma\frac{(\alpha+c)}{\alpha} + \Gamma\frac{(2\alpha+c)}{\alpha}\right)\left(2\,\Gamma\frac{(\alpha+c)}{\alpha}\right) - \left(2\theta\Gamma\frac{(\alpha+c)}{\alpha}\right)^2}{\left(\theta^2\Gamma\frac{(\alpha+c)}{\alpha} + \Gamma\frac{(2\alpha+c)}{\alpha}\right)^2}\right) - \sum_{i=1}^{n}\left(\frac{1}{\left(\theta + x_i^{\alpha}\right)^2}\right)$$

$$E\left(\frac{\partial^2 \log L}{\partial\alpha^2}\right) = \frac{3n\alpha^2\theta^2 c}{\left(\alpha^3\theta\right)^2} - n\psi'\left(\theta^2\Gamma\frac{(\alpha+c)}{\alpha} + \Gamma\frac{(2\alpha+c)}{\alpha}\right) + \sum_{i=1}^{n}\frac{\left(\theta + x_i^{\alpha}\right)x_i^{\alpha}\left(\log x_i\right)^2 - \left(x_i^{\alpha}\log x_i\right)^2}{\left(\theta + x_i^{\alpha}\right)^2} - \theta\sum_{i=1}^{n}\left(x_i^{\alpha}\log x_i\right)^2$$

$$E\left(\frac{\partial^2 \log L}{\partial c^2}\right) = -n\psi'\left(\theta^2\Gamma\frac{(\alpha+c)}{\alpha} + \Gamma\frac{(2\alpha+c)}{\alpha}\right)$$

$$E\left(\frac{\partial^2 \log L}{\partial\theta\,\partial\alpha}\right) = E\left(\frac{\partial^2 \log L}{\partial\alpha\,\partial\theta}\right) = -n\psi\left(\frac{2\theta\Gamma\frac{(\alpha+c)}{\alpha}}{\theta^2\Gamma\frac{(\alpha+c)}{\alpha} + \Gamma\frac{(2\alpha+c)}{\alpha}}\right) - \sum_{i=1}^{n}\frac{x_i^{\alpha}\log x_i}{\left(\theta + x_i^{\alpha}\right)^2} - \sum_{i=1}^{n}x_i^{\alpha}\log x_i$$

$$E\left(\frac{\partial^2 \log L}{\partial\theta\,\partial c}\right) = E\left(\frac{\partial^2 \log L}{\partial c\,\partial\theta}\right) = \frac{n\alpha}{\alpha^2\theta} - n\psi\left(\frac{2\theta\Gamma\frac{(\alpha+c)}{\alpha}}{\theta^2\Gamma\frac{(\alpha+c)}{\alpha}+\Gamma\frac{(2\alpha+c)}{\alpha}}\right)$$

$$E\left(\frac{\partial^2 \log L}{\partial\alpha\,\partial c}\right) = E\left(\frac{\partial^2 \log L}{\partial c\,\partial\alpha}\right) = -\frac{n\theta}{\alpha^3\theta} - n\psi'\left(\theta^2\Gamma\frac{(\alpha+c)}{\alpha}+\Gamma\frac{(2\alpha+c)}{\alpha}\right)$$

where $\psi(.)'$ is the first order derivative of digamma function. Since λ being unknown, we estimate $I^{-1}(\lambda)$ by $I^{-1}(\hat{\lambda})$ and this can be used to obtain asymptotic confidence intervals for θ, α and c.

9 Entropy Measures

The concept of entropy is important in different areas such as probability and statistics, physics, communication theory and economics. The term entropy was given by German physicist Rudolf Clausius (1865).Entropy measures quantify the diversity, uncertainty or randomness of a system. Entropy is also termed as state of disorder or decline into disorder. Entropy of a random variable *X* is a measure of variation of the uncertainty.

9.1 Renyi entropy

The concept of entropy namely Renyi entropy was given by Alfred Renyi (1957). The Renyi entropy is important in ecology and Statistics as index of diversity. The Renyi entropy of order β for a random variable X is given by.

$$e(\beta) = \frac{1}{1-\beta}\log\left(\int f^{\beta}(x)dx\right)$$

Where $\beta > 0$ *and* $\beta \neq 1$

$$e(\beta) = \frac{1}{1-\beta}\log\int_0^\infty\left(\frac{\alpha\theta^{\frac{2\alpha+c}{\alpha}}}{\theta^2\Gamma\frac{(\alpha+c)}{\alpha}+\Gamma\frac{(2\alpha+c)}{\alpha}}x^{\alpha+c-1}\left(\theta+x^{\alpha}\right)e^{-\theta x^{\alpha}}\right)^{\beta}dx$$

$$e(\beta) = \frac{1}{1-\beta}\log\left(\left(\frac{\alpha\theta^{\frac{2\alpha+c}{\alpha}}}{\theta^2\Gamma\frac{(\alpha+c)}{\alpha}+\Gamma\frac{(2\alpha+c)}{\alpha}}\right)^{\beta}\int_0^\infty x^{\beta(\alpha+c-1)}e^{-\theta\beta x^{\alpha}}\left(\theta+x^{\alpha}\right)^{\beta}dx\right) \quad (14)$$

Using Binomial expansion in equation (14), we obtain

$$e(\beta)=\frac{1}{1-\beta}\log\left(\left(\frac{\alpha\theta^{\frac{2\alpha+c}{\alpha}}}{\theta^2\Gamma\frac{(\alpha+c)}{\alpha}+\Gamma\frac{(2\alpha+c)}{\alpha}}\right)^{\beta}\sum_{i=0}^{\infty}\binom{\beta}{i}\theta^{\beta-i}x^{\alpha i}\int_0^{\infty}x^{\beta(\alpha+c-1)}e^{-\theta\beta x^{\alpha}}dx\right)$$

$$e(\beta)=\frac{1}{1-\beta}\log\left(\left(\frac{\alpha\theta^{\frac{2\alpha+c}{\alpha}}}{\theta^2\Gamma\frac{(\alpha+c)}{\alpha}+\Gamma\frac{(2\alpha+c)}{\alpha}}\right)^{\beta}\sum_{i=0}^{\infty}\binom{\beta}{i}\theta^{\beta-i}\int_0^{\infty}x^{\beta(\alpha+c-1)+\alpha i}e^{-\theta\beta x^{\alpha}}dx\right) \quad (15)$$

Put $x^{\alpha}=t \Rightarrow x=t^{\frac{1}{\alpha}}$

Also $\alpha x^{\alpha-1}dx=dt \Rightarrow dx=\frac{dt}{\alpha x^{\alpha-1}}=\frac{dt}{\alpha t^{\frac{\alpha-1}{\alpha}}}$

After simplifying the equation (15), we obtain

$$e(\beta)=\frac{1}{1-\beta}\log\left(\frac{\theta^{\beta-i}}{\alpha}\left(\frac{\alpha\theta^{\frac{2\alpha+c}{\alpha}}}{\theta^2\Gamma\frac{(\alpha+c)}{\alpha}+\Gamma\frac{(2\alpha+c)}{\alpha}}\right)^{\beta}\sum_{i=0}^{\infty}\binom{\beta}{i}\frac{\Gamma\frac{\beta(\alpha+c-1)+\alpha i+1}{\alpha}}{(\theta\beta)^{\frac{\beta(\alpha+c-1)+\alpha i+1}{\alpha}}}\right)$$

9.2 Tsallis Entropy

Tsallis in 1988 introduced an entropic expression characterized by an index q which leads to a non-extensive statistics. Tsallis entropy is the basis of the so called non-extensive statistical mechanics. This generalization of B-G statistics was proposed firstly by introducing the mathematical expression of Tsallis entropy (Tsallis, 1988) for a continuous random variable is defined as follows.

$$S_{\lambda}=\frac{1}{\lambda-1}\left(1-\int_0^{\infty}f^{\lambda}(x)dx\right)$$

$$S_{\lambda}=\frac{1}{\lambda-1}\left(1-\int_0^{\infty}\left(\frac{\alpha\theta^{\frac{2\alpha+c}{\alpha}}}{\theta^2\Gamma\frac{(\alpha+c)}{\alpha}+\Gamma\frac{(2\alpha+c)}{\alpha}}x^{\alpha+c-1}\left(\theta+x^{\alpha}\right)e^{-\theta x^{\alpha}}\right)^{\lambda}dx\right)$$

$$S_{\lambda}=\frac{1}{\lambda-1}\left(1-\left(\frac{\alpha\theta^{\frac{2\alpha+c}{\alpha}}}{\theta^2\Gamma\frac{(\alpha+c)}{\alpha}+\Gamma\frac{(2\alpha+c)}{\alpha}}\right)^{\lambda}\int_0^{\infty}x^{\lambda(\alpha+c-1)}e^{-\lambda\theta x^{\alpha}}\left(\theta+x^{\alpha}\right)^{\lambda}dx\right) \quad (16)$$

Using Binomial expansion in equation (16), we get

$$S_{\lambda}=\frac{1}{\lambda-1}\left(1-\left(\frac{\alpha\theta^{\frac{2\alpha+c}{\alpha}}}{\theta^{2}\Gamma\frac{(\alpha+c)}{\alpha}+\Gamma\frac{(2\alpha+c)}{\alpha}}\right)^{\lambda}\sum_{i=0}^{\infty}\binom{\lambda}{i}\theta^{\lambda-i}x^{\alpha i}\int_{0}^{\infty}x^{\lambda(\alpha+c-1)}e^{-\lambda\theta x^{\alpha}}dx\right)$$

$$S_{\lambda}=\frac{1}{\lambda-1}\left(1-\left(\frac{\alpha\theta^{\frac{2\alpha+c}{\alpha}}}{\theta^{2}\Gamma\frac{(\alpha+c)}{\alpha}+\Gamma\frac{(2\alpha+c)}{\alpha}}\right)^{\lambda}\sum_{i=0}^{\infty}\binom{\lambda}{i}\theta^{\lambda-i}\int_{0}^{\infty}x^{\lambda(\alpha+c-1)+\alpha i}e^{-\lambda\theta x^{\alpha}}dx\right) \tag{17}$$

Put $x^{\alpha}=t \quad \Rightarrow x=t^{\frac{1}{\alpha}}$

and $\alpha x^{\alpha-1}dx=dt \quad \Rightarrow dx=\frac{dt}{\alpha x^{\alpha-1}} \quad \Rightarrow dx=\frac{dt}{\alpha t^{\frac{\alpha-1}{\alpha}}}$

After simplifying the equation (17), we obtain

$$S_{\lambda}=\frac{1}{\lambda-1}\left(1-\frac{\theta^{\lambda-i}}{\alpha}\left(\frac{\alpha\theta^{\frac{2\alpha+c}{\alpha}}}{\theta^{2}\Gamma\frac{(\alpha+c)}{\alpha}+\Gamma\frac{(2\alpha+c)}{\alpha}}\right)^{\lambda}\sum_{i=0}^{\infty}\binom{\lambda}{i}\frac{\Gamma\frac{\lambda(\alpha+c-1)+\alpha i+1}{\alpha}}{(\lambda\theta)^{\frac{\lambda(\alpha+c-1)+\alpha i+1}{\alpha}}}\right)$$

10 Data Evaluation

In this section, we will discuss the goodness of fit by inserting the three real-life data sets for fitting weighted Power Shanker distribution for comparing with Power Shanker, Shanker, Exponential and Lindley distribution.

Data set I: The first real life data set considered by Nassar and Nada (2011) represents the monthly actual tax revenue in Egypt from January 2006 to November 2010 and the actual taxes revenue data (in 1000 million Egyptian pounds) are executed below in table 1

Table 1: Data regarding taxes revenue (in 1000 million Egyptian pounds)

5.9	20.4	14.9	16.2	17.2	7.8	6.1	9.2	10.2	9.6
13.3	8.5	21.6	18.5	5.1	6.7	17	8.6	9.7	39.2
35.7	15.7	9.7	10	4.1	36	8.5	8	9.2	26.2
21.9	16.7	21.3	35.4	14.3	8.5	10.6	19.1	20.5	7.1
7.7	18.1	16.5	11.9	7	8.6	12.5	10.3	11.2	6.1

8.4	11	11.6	11.9	5.2	6.8	8.9	7.1	10.8	

Data set II: The second real life data set represents the survival times of 121 patients suffering from breast cancer reported by Lee (1992) and is executed below in table 2 .

Table 2: Data regarding 121 patients suffering from breast cancer reported by Lee (1992)

0.3	0.3	4.0	5.0	5.6	6.2	6.3	6.6	6.8	7.4
7.5	8.4	8.4	10.3	11.0	11.8	12.2	12.3	13.5	14.4
14.4	14.8	15.5	15.7	16.2	16.3	16.5	16.8	17.2	17.3
17.5	17.9	19.8	20.4	20.9	21.0	21.0	21.1	23.0	23.4
23.6	24.0	24.0	27.9	28.2	29.1	30.0	31.0	31.0	32.0
35.0	35.0	37.0	37.0	37.0	38.0	38.0	38.0	39.0	39.0
40.0	40.0	40.0	41.0	41.0	41.0	42.0	43.0	43.0	43.0
44.0	45.0	45.0	46.0	46.0	47.0	48.0	49.0	51.0	51.0
51.0	52.0	54.0	55.0	56.0	57.0	58.0	59.0	60.0	60.0
60.0	61.0	62.0	65.0	65.0	67.0	67.0	68.0	69.0	78.0
80.0	83.0	88.0	89.0	90.0	93.0	96.0	103.0	105.0	109.0
109.0	111.0	115.0	117.0	125.0	126.0	127.0	129.0	129.0	139.0
154.0									

Data set III: The third data set reported by Bader and priest (1982) represents the strength measured in GPA for single carbon fibres and impregnated 1000-carbon fibre tows. Single fibres were tested under tension at guage length of 10mm with sample size (n = 63) and is given below in table 3.

Table 3: Data regarding the strength of carbon fibres measured in GPA reported by Bader & Priest (1982).

1.901	2.132	2.203	2.228	2.257	2.350	2.361	2.396	2.397	2.445	2.454	2.474	2.518
2.522	2.525	2.532	2.575	2.614	2.616	2.618	2.624	2.659	2.675	2.738	2.740	2.856
2.917	2.928	2.937	2.937	2.977	2.996	3.030	3.125	3.139	3.145	3.220	3.223	3.235
3.243	3.264	3.272	3.294	3.332	3.346	3.377	3.408	3.435	3.493	3.501	3.537	3.554
3.562	3.628	3.852	3.871	3.886	3.971	4.024	4.027	4.225	4.395	5.020		

The estimation of model comparison criterion values along with the estimation of unknown parameters are determined through the R software. In order to compare the performance of weighted Power Shanker distribution with Power Shanker, Shanker, exponential and Lindley distribution, we consider the criterion values like Bayesian Information criterion (BIC), Akaike Information Criterion (AIC) and Akaike Information Criterion Corrected (AICC). The better distribution is which corresponds to lesser values of *AIC, BIC, AICC* and *-2logL*. For calculating the criterion values AIC, BIC, AICC and -2logL can be evaluated by using the formulas as follows

$$\text{AIC} = 2k - 2\log L \qquad \text{AICC} = AIC + \frac{2k(k+1)}{n-k-1} \qquad \text{and} \qquad \text{BIC} = k\log n - 2\log L$$

Where k is the number of parameters, n is the sample size and $-2\log L$ is the maximized value of the log-likelihood function under the considered model are shown in Table 4.

Table 4 Shows Parameter estimations, corresponding standard errors, Criterion values & goodness of fit test

Data sets	Distribution	MLE	S.E	- 2logL	AIC	BIC	AICC
1	Weighted Power Shanker	$\hat{\alpha} = 0.68706348$ $\hat{\theta} = 0.36026352$ $\hat{c} = 0.55869402$	$\hat{\alpha} = 0.04394397$ $\hat{\theta} = 0.12066469$ $\hat{c} = 0.55352084$	319.0235	325.0235	331.2561	325.4598
	Power Shanker	$\hat{\alpha} = 1.30760594$ $\hat{\theta} = 0.06255676$	$\hat{\alpha} = 0.11648546$ $\hat{\theta} = 0.02140566$	389.2111	393.2111	397.3661	393.4253
	Shanker	$\hat{\theta} = 0.14620621$	$\hat{\theta} = 0.01332504$	397.0605	399.0605	401.138	399.1306
	Exponential	$\hat{\theta} = 0.074143727$	$\hat{\theta} = 0.009650935$	425.0136	427.0136	429.0912	427.0837
	Lindley	$\hat{\theta} = 0.13922068$	$\hat{\theta} = 0.01286375$	401.2586	403.2586	405.3361	403.3287
2	Weighted Power Shanker	$\hat{\alpha} = 0.5800188$ $\hat{\theta} = 0.1332690$ $\hat{c} = 0.0010000$	$\hat{\alpha} = 0.1124560$ $\hat{\theta} = 0.2345766$ $\hat{c} = 0.0015670$	811.9158	817.9158	826.3032	818.1209
	Power Shanker	$\hat{\alpha} = 0.86137770$ $\hat{\theta} = 0.07585994$	$\hat{\alpha} = 0.05841748$ $\hat{\theta} = 0.01847130$	1160.657	1164.657	1170.248	1164.7586

	Shanker	$\hat{\theta}=0.043180645$	$\hat{\theta}=0.002771516$	1165.784	1167.784	1170.58	1167.8176
	Exponential	$\hat{\theta}=0.021597929$	$\hat{\theta}=0.001959228$	1170.256	1172.256	1175.051	1172.2896
	Lindley	$\hat{\theta}=0.042301604$	$\hat{\theta}=0.002718848$	1160.863	1162.863	1165.659	1162.8966
3	Weighted Power Shanker	$\hat{\alpha}=1.28865471$ $\hat{\theta}=8.47772195$ $\hat{c}=43.40726675$	$\hat{\alpha}=0.03438267$ $\hat{\theta}=1.07388782$ $\hat{c}=5.72444567$	46.95586	52.95586	59.38526	53.3626
	Power Shanker	$\hat{\alpha}=3.55017161$ $\hat{\theta}=0.03152676$	$\hat{\alpha}=0.31041753$ $\hat{\theta}=0.01241348$	119.0899	123.0899	127.3762	123.2899
	Shanker	$\hat{\theta}=0.55232815$	$\hat{\theta}=0.04614215$	236.0158	238.0158	240.1589	238.0813
	Exponential	$\hat{\theta}=0.32687301$	$\hat{\theta}=0.04118174$	266.8915	268.8915	271.0347	268.9570
	Lindley	$\hat{\theta}=0.53923226$	$\hat{\theta}=0.04958387$	242.7153	244.7153	246.8584	244.7808

From results given above in table 4 clearly indicated that the weighted Power Shanker distribution have the lower AIC,BIC , AICC and -2logL values as compared to the Power Shanker, Shanker, exponential and Lindley distributions, which concludes that the weighted power Shanker distribution leads to a better fit than the Power Shanker, Shanker, exponential and Lindley distributions.

11 Conclusion

In the present paper, a new model of power Shanker distribution is executed called as weighted power Shanker distribution with three parameters and its different statistical and mathematical properties are investigated and derived. The subject distribution is generated by using the weighted technique and the parameters have been obtained by using the maximum likelihood estimator. The main purpose behind this manuscript completion is to aware one so that he realize how important are the new extensions in expressing some random processes even though when we have already a number of existing standard distributions. It is also observed that the considered data sets executed in newly introduced model proves a better fit rather than the baseline distribution i.e. Power shanker distribution. Finally, the significance of newly introduced distribution is established by applying the three real life data sets and the result of the data sets proved that the weighted power Shanker distribution fits better than the power Shanker, Shanker, exponential and Lindley distribution.

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