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A Schematic Study of Nuclear Structure for the Nd Isotopes

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Abstract: The results of Interacting Boson Model-2 (IBM-2) calculations for Neodymium isotopes have been performed to make a schematic study of nuclear structure for these isotopes. This calculation includes energy spectra, electromagnetic transition moments and mixing ratios are presented in this work. In general the IBM-2 results are in a good agreement with the experimental values.

Keywords: IBM-2, Nuclear structure, electric transition probability, magnetic transition probability, Mixing ratios.

1 Introduction

he Interacting Boson Model (IBM), was used in this present work, was proposed by F. Iachello and A. Arima [1,2,3,4], where interacting bosons are used to describe collective excitations in nuclei. From the symmetry properties of the model's boson operators, three types of idealized nuclei were found whose properties can be calculated analytically. These three limits of nuclei can be used as benchmarks with which to classify different nuclei. It was found that different regions of the nuclear chart exhibit properties that are similar to one of these idealized limits.

The interacting boson model (IBM)] has been success in describing the medium and heavy even-even nuclei collective low-lying energy states. In the IBM-2, the Hamiltonian is diagonalized in the boson space using group theory method. The collective Hamiltonian was written in terms of interaction of proton paired and neutrons paired, identified as proton bosons and neutron bosons, each pair can coupled to $L = 0$ and $L = 2$, The proton (neutron) bosons with angular momentum $L = 0$ are treated by s_π and s_ν and are called s-boson, while proton bosons and neutron bosons with angular momentum $L = 2$ are denoted by d_π and d_ν, and are called d-bosons.

In IBM-2, assumes that for the even-even nucleus with N_π number of proton bosons and N_ν is the number of neutron bosons out side the major shell. the number of bosons are counted from the nearest major shell. The total number of bosons is given $N_\pi + N_\nu = N$, the bosons number which accounted from the beginning to the middle of the shell is called particles, while the number of bosons which counted from middle to the end of the shell is called holes. In this work we employed the IBM-2 on the $^{144-154}Nd$ isotopes ($Z = 60, 82 < N < 126$), to study the nuclear structure and electromagnetic transitions in these isotopes. The Nd isotopes are the chain of nuclei members around mass number 140 and they show an ideal case for studying the shape transition influence from the spherical (vibrational shape character) nuclei to deformed (rotor deformed) nuclei. In this work, we study the energy spectra, electromagnetic transitions B(E2), B(M1) transition probabilities and mixing ratios of the $^{144-154}Nd$ isotopes. The proton-neutron interaction in the valence shell of nuclei has been attributed as being responsible for the formation of collectivity in nuclei. There have been fits made for the strength of this interaction using phenomenological IBM-2 for a number of nuclei, especially in the $A = 140$ mass region, but data are still sparse. The IBM-2 distinguishes between proton and neutron bosons. The Hamiltonian of IBM-2 can be written as [4,5,6]:

$$H = H_\pi + H_\nu + V_{\pi\nu} \quad (1)$$

$$H = \varepsilon(n_{d\pi} + n_{d\nu}) + kQ_\pi.Q_\nu + V_{\pi\pi} + V_{\nu\nu} + M_\pi \quad (2)$$

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Where the $n_{d\rho}$ are the d-boson number operators for protons and neutrons with the respective d-boson energies ε_ρ (where $\rho = \pi$ or ν). The Q_ρ denote the quadrupole operator for proton-bosons and neutron-bosons. The last term of Eq.(2) denotes the so-called Majorana interaction force, this parameter fixed state location with mixed proton bosons-neutron bosons symmetry with respect to totally symmetric states, and is defined as [5,6]:

$$M_{\pi\nu} = \frac{1}{2}\xi_2(s_\pi^+ + d_\nu^+ - d_\pi^+ s_\nu^+).(s_\pi d_\nu^\sim - d_\nu^\sim s_\pi) - \sum_{K=1,3} \xi_K([d_\pi^+ d_\nu^+]^{(K)}.[d_\pi^\sim d_\nu^\sim]^{(K)}) \quad (3)$$

The operator of quadrupole moment in the IBM-2 for proton and neutron boons is written as [7]:

$$Q_\pi^{\chi_\pi} = (d_\pi^+ d_\pi^\sim)^{(2)} + \chi_\pi(s_\pi^+ d_\pi^\sim + d_\pi^+ s_\pi)^{(2)}$$
$$Q_\nu^{\chi_\nu} = (d_\nu^+ d_\nu^\sim)^{(2)} + \chi_\nu(s_\nu^+ d_\nu^\sim + d_\nu^+ s_\nu)^{(2)} \quad (4)$$

The terms $V_{\pi\pi}$ is the interaction of proton-proton bosons and $V_{\nu\nu}$ is the interaction of neutron-neutron bosons only and given by [7]:

$$V_{\pi\pi} = \sum_{J=0,2,4} C_{L\rho}[(d^+d^+)_\pi^{(L)}(d^\sim d^\sim)_\pi^{(L)}]^{(0)}$$

$$V_{\nu\nu} = \sum_{J=0,2,4} C_{L\rho}[(d^+d^+)_\nu^{(L)}(d^\sim d^\sim)_\nu^{(L)}]^{(0)} \quad (5)$$

2 Results and Discussion

2.1 Choice of the IBM-2 Parameters

The IBM-2 Hamiltonian parameters are listed in Table (1), one can see the parameters ε, k, χ_ν and $\xi(\xi = \xi_1 = \xi_2 = \xi_3)$ vary smoothly from isotope to another, these parameters treated as free parameters. The free parameters ε and k as a function of bosons number i.e., as a function of neutron bosons and proton bosons, $\varepsilon = \varepsilon(N_\pi, N_\nu)$, $k = k(N_\pi, N_\nu)$ and the Majorana force parameter depend on N_ν and N_π, the parameter χ_ν depend on N_ν. The other parameters depend only on N_ν or N_π, the parameter $\chi_\pi = \chi_\pi(N_\pi)$, $C_{L\pi} = C_{L\pi}(N_\pi)$ and $C_{L\nu} = C_{L\nu}(N_\nu)$. The number of proton bosons account from nearest major shell ($Z = 50$), the Nd isotopes have 60 protons (10) protons outside the major shell ($Z = 50$), therefore we have $N_\pi = 5$ proton bosons, while the number of neutron bosons accounts from the nearest neutron closed shell $N = 82$, therefore the number of neutron bosons varies from $N_\nu = 1$ to $N_\nu = 6$. The parameter χ_π is a constant for all isotopes, this is due to the number of proton bosons are constant in whole isotopes, whereas we include $C_{0\pi}$ and $C_{2\pi}$ terms in $V_{\pi\pi}$ proton-proton bosons interaction parameter and don't include $V_{\nu\nu}$, because for most $^{144-152}Nd$ isotopes $N_\pi > N_\nu$. In the Majorana force parameter we set $\xi_1 = \xi_2 = \xi_3$ for the whole isotopic chain. In genera the IBM-2 Hamiltonian parameters which are given in Table (1) are estimated by fitted with the experimental values, since vary one parameter, while keeping others constants until to get a perfect value to fit with experiment. We can use these parameters to evaluate the energy levels and electromagnetic transition rates using the computer code **NPBOS** program [8].

Table 1: The IBM-2 Hamiltonian for $^{144-154}$Nd isotopes, all parameters in MeV units except χ_π and χ_ν are dimensionless ($N_\pi = 5$).

Parameter	Nd-144	Nd-146	Nd-148	Nd-150	Nd-152	Nd-154
N_ν	1	2	3	4	5	6
N_π	5	5	5	5	5	5
ε	0.95	0.9	0.7	0.47	0.34	0.3
k	-0.18	-0.15	-0.1	-0.07	-0.089	-0.085
χ_ν	0.00	0.00	0.80	-1.00	-1.10	-1.20
χ_π	-1.20	-1.20	-1.20	-1.20	-1.20	-1.20
$C_{0\pi}$	0.400	0.400	0.400	0.400	0.400	0.400
$C_{2\pi}$	0.200	0.200	0.200	0.200	0.200	0.200
$C_{4\pi}$	0.00	0.00	0.00	0.00	0.00	0.00
$C_{0\nu}$	0.00	0.00	0.00	0.00	0.00	0.00
$C_{2\nu}$	0.00	0.00	0.00	0.00	0.00	0.00
$C_{4\nu}$	0.00	0.00	0.00	0.00	0.00	0.00
$\xi_1 = \xi_2 = \xi_3$	0.06	0.08	0.22	0.37	0.22	0.20

2.2 Energy Spectra

In the Figures (1) to (6), we show the IBM-2 calculations together with experimental values; the agreement between them is quite well. One can observe the discrepancies between our calculations and experimental data appears in high spin states, such as states in gamma band, 2β band and 2γ band, this is due to, these states are outside of the IBM-2 space. One must be careful the low-lying energy levels which calculated in IBM-2 are agree very well with the experimental values, where all these states have a collective nature.

The energy ratios for $^{144-154}Nd$ isotopes are given in Table (2), the ratios increased gradually and smoothly from the ^{144}Nd isotope to ^{154}Nd isotope. The energy ratio $R_1 = 1.890$ for ^{144}Nd isotope and 2.302 for ^{146}Nd isotope, this indicates these isotopes are shows a vibrational shape character (near spherical shape, corresponds to a anharmonic vibrator). i.e., lies in $SU(5)$ symmetry [9], because these isotopes are near to the neutron major shell ($Z = 82$). The energy ratios R_1 for $^{146-148}Nd$ isotopes equal 2.498 and 2.930 respectively, these results means the isotopes $^{148-150}Nd$ are corresponds to the transitional nuclei (corresponds to γ-soft or γ-unstable) [10]. Finally, the ^{152}Nd isotope tends to deformed nucleus ($SU(3)$ symmetry), while the ^{154}Nd isotope (94 neutron) appears deformed nucleus character (rotor), lies in $SU(3)$ symmetry. This due to, far out than the neutron major shell ($N = 82$ and $N = 126$).

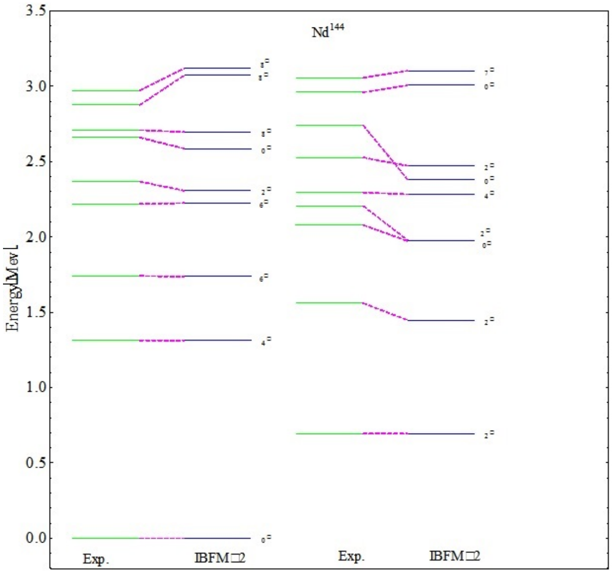

Fig. 1: Comparison between experimental data [11] and IBM-2 calculated energy levels for ^{144}Nd.

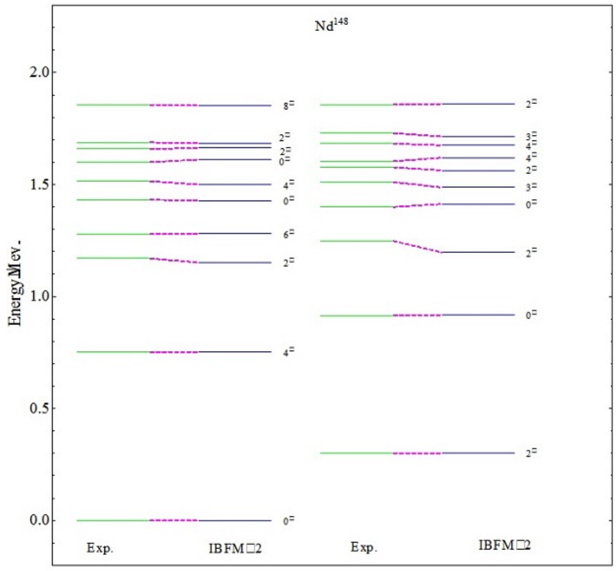

Fig. 3: Comparison between experimental data [13] and IBM-2 calculated energy levels for ^{148}Nd.

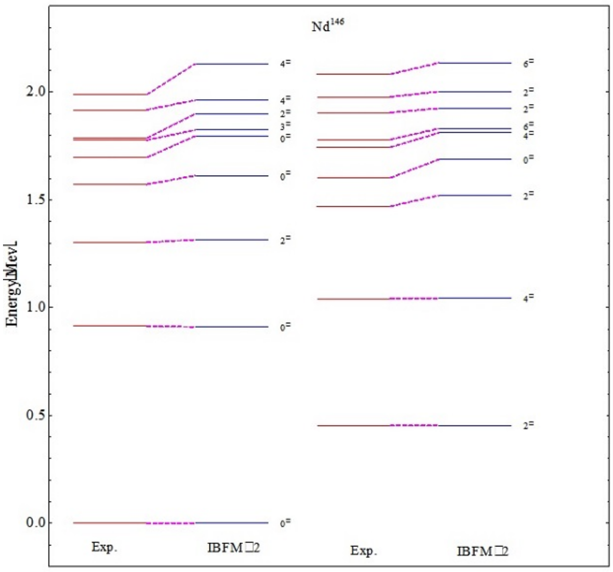

Fig. 2: Comparison between experimental data [12] and IBM-2 calculated energy levels for ^{146}Nd.

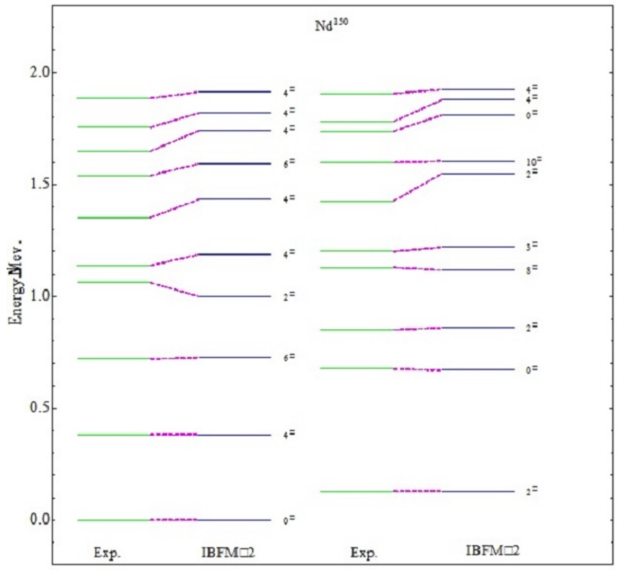

Fig. 4: Comparison between experimental data [14] and IBM-2 calculated energy levels for ^{150}Nd.

2.3 Electromagnetic Transition Probability

2.3.1 Electric Transition probability

The transition operators are sums over the proton and neutron transition operators of the IBM-2. For example,

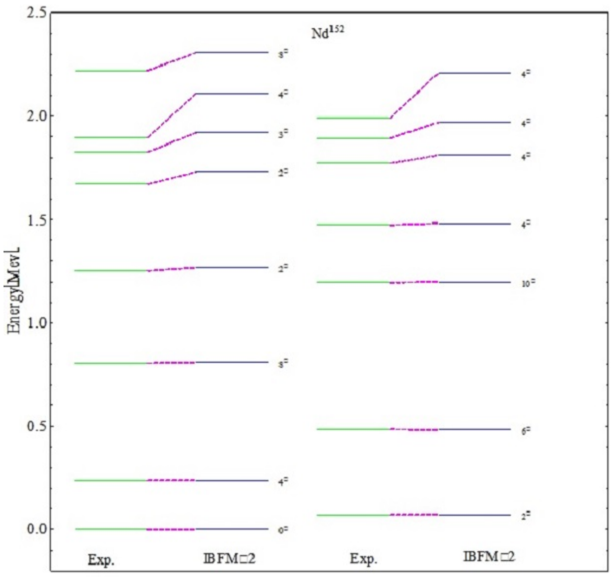

Fig. 5: Comparison between experimental data [15] and IBM-2 calculated energy levels for ^{152}Nd.

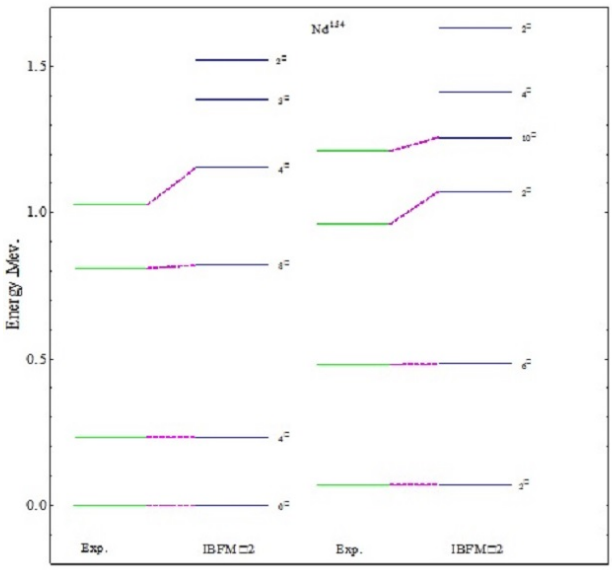

Fig. 6: Comparison between experimental data [16] and IBM-2 calculated energy levels for ^{154}Nd.

in the IBM-2, E2 operators are given by:

$$T^{(E2)} = e_\pi Q_\pi^{\chi_\pi} + e_\nu Q_\nu^{\chi_n u} \tag{6}$$

The e_π (e_ν) is the effective charges for proton (neutron) bosons respectively have ebunits, the effective charges e_π and e_ν are depending on proton bosons number and neutron bosons number. The quadruple operators $Q_\pi^{\chi_\pi}$ and $Q_\nu^{\chi_\nu}$ are defined in Eq. (4). The reduced electric quadruple transition rates between two states are given by [17]:

Table 2: The Energy Ratios for $^{144-154}$Nd isotopes.

Isotopes	$R_1 = E(4_1^+)/E(2_1^+)$		$R_1 = E(6_1^+)/E(2_1^+)$		$R_1 = E(8_1^+)/E(2_1^+)$	
	exP.	IBM-2	Exp.	IBM-2	Exp.	IBM-2
^{144}Nd	3.880	3.897	2.499	2.576	1.887	1.890
^{146}Nd	5.684	5.4610	4.028	3.929	2.302	2.302
^{148}Nd	6.168	6.166	4.250	4.249	2.497	2.498
^{150}Nd	8.750	8.684	5.541	5.538	2.930	2.930
^{152}Nd	11.21	11.195	6.722	6.722	3.277	3.277
^{154}Nd	11.223	11.571	6.871	6.871	3.290	3.290
$SU(5)$	2		3		4	
$O(6)$	2.5		4.5		7	
$SU(3)$	3.3		7		12	

$$B(E2; i \to f) = \frac{|< I_i || T^{(E2)} || I_f >|^2}{2I_i + 1} \tag{7}$$

The quadrupole moment definition for state characterized by angular momentum I of a nucleus [17]:

$$Q_I = \sqrt{\frac{16\pi}{5}} \begin{bmatrix} I2I \\ -I0I \end{bmatrix} < I || T^{(E2)} || I > \tag{8}$$

In order to calculate the electric transition probability, from Eq.(6) we note than an B(E2) depending mainly on the identifying effective charges for proton bosons and neutron bosons. The values of effective charges for proton and neutron bosons were determined from the experimental $B(E2; 2_1^+ \to 0_1^+)$ value. However, the $^{144-146}Nd$ isotopes lies in $U(5)$ symmetry, $^{148-150}Nd$ isotopes corresponds in $O(6)$ symmetry and $^{152-154}Nd$ isotopes lies in $SU(3)$ limit, therefore, the relationships between the electric transition probability B(E2) for the three symmetries and bosons effective charges e_π (e_ν) are given as [18]:

$SU(5)$ symmetry

$$B(E2; i \to 0_1^+) = \frac{5}{N}(e_\pi N_\pi + e_\nu N_\nu)^2 \tag{9}$$

$O(6)$ symmetry

$$B(E2; i \to 0_1^+) = \frac{(N+4)}{N}(e_\pi N_\pi + e_\nu N_\nu)^2 \tag{10}$$

$SU(3)$ symmetry

$$B(E2; i \to 0_1^+) = \frac{(2N+3)}{N}(e_\pi N_\pi + e_\nu N_\nu)^2 \tag{11}$$

Where N is bosons total number, N_π (N_ν) is the proton (neutron) bosons number respectively and $B(E2; 2_1^+ \to 0_1^+)$ is the experimental E2 transition probability rate. When analyzed the experimental values, we are interesting in the ratios which given in Eqs. (1), (2) and (3). The proton bosons effective charge is constant for whole $^{144-154}Nd$ isotopes ($e_{\pi} = 0.353eb$), because the number of proton bosons is constant(N_{π} =5), the neutron bosons effective charges are given in Table (3), we observe is a suitable values and varies smoothly and gradually from isotope to another.

Table 3: The neutron bosons effective charges in *eb* units.

$e_{\nu}(eb)$	^{144}Nd	^{146}Nd	^{148}Nd	^{150}Nd	^{152}Nd	^{154}Nd
	0.0848	0.0851	0.0863	0.0872	0.0881	0.0912

The IBM-2 results for E2 transition probability rates and experimental values have been listed in Table (4). From the IBM-2 results for B(E2) it is found that the $B(E2;2_1^+ \rightarrow 0_1^+)$ values where there are much experimental data for this transition, we can see this transition increased gradually with increasing neutron number toward of the shell middle. The $B(E2;4_1^+ \rightarrow 2_1^+)$ transition probability values have the same behivour of $B(E2;2_1^+ \rightarrow 0_1^+)$ transition and in same magnitude order, increased smoothly with increasing neutron number. The agreement between IBM-2 results and experimental values is quite good.

From the Table (4), the $B(E2;6_1^+ \rightarrow 4_1^+)$ and $B(E2;8_1^+ \rightarrow 6_1^+)$ values are of the same magnitude order and typical display increased to the end of the shell, and this nicely produced by IBM-2, there is no enough experimental data to compare with IBM-2 calculations.

The values of $B(E2;2_2^+ \rightarrow 2_1^+)$, are small or weak in sometimes and fluctuation in this values of this transition because this transition is include admixture of M1 and this quantity is difficult of measured as consequence to this case.

Other transition probabilities especially inter-band transition values are small or weak, this is due to the selection rules of these transitions from β-band to ground state band or transitions from γ-band to ground state band (cross over transition).

The results of quadrupole moment for first excited state $Q(2_1^+)$ values are tabulated in Table (5). These values of $Q(2_1^+)$ are increased in negative with increasing neutron number, this is means these isotopes in the first excited states are taken a prolate shape character.

2.3.2 Magnetic Transition probability

In this work, we also studied magnetic transition probability and the magnetic dipole moment $\mu(2_1^+)$. The M1 boson operator is given as [7]:

$$T^{(M1)} = g_{\pi} L_{\pi}^{(1)} + g_{\nu}^{(1)} \tag{12}$$

Table 4: B(E2) values for $^{144-154}Nd$ isotopes in e^2b^2 Units.

Isotope	$2_1^+ \rightarrow 0_1^+$		$4_1^+ \rightarrow 2_1^+$		$6_1^+ \rightarrow 4_1^+$		$8_1^+ \rightarrow 6_1^+$	
	Exp.	IBm-2	Exp.	IBM-2	Exp.	IBM-2	Exp.	IBM-2
^{144}Nd	0.188(4)[a]	0.180	0.143(6)[a]	0.137	-	0.256	-	0.247
^{146}Nd	0.233(3)[b]	0.231	0.348[b]	0.337	-	0.412	-	0.4077
^{148}Nd	0.480[c]	0.411	0.765[c]	0.731	-	0.821	-	0.839
^{150}Nd	0.9789	0.998	1.486[a]	1.272	0.29(9)[e]	1.072	-	1.577
^{152}Nd	-	0.872	-	1.42	1.039(213)	1.131		2.383
^{154}Nd	0.47(13)[f]	0.471	-	0.621	-	1.430		1.332
Isotope	$0_2^+ \rightarrow 2_1^+$		$4_2^+ \rightarrow 2_2^+$		$2_2^+ \rightarrow 0_1^+$		$2_2^+ \rightarrow 0_2^+$	
	Exp.	IBm-2	Exp.	IBM-2	Exp.	IBM-2	Exp.	IBM-2
^{144}Nd	-	0.184	-	0.130	0.0051[a]	0.005	-	0.0134
^{146}Nd	-	0.261	0	0.210	0.128[b]	0.130	-	0.0504
^{148}Nd	-	0.410	-	0.410	0.0345[e]	0.367	-	0.0606
^{150}Nd	-	251	-	0.669	0.0218[d]	0.022	-	0.218
^{152}Nd	-	0.149		0.918	-	0.125		0.372
^{154}Nd	-	0.177		1.357	-	0.0452		0.560
Isotope	$2_2^+ \rightarrow 2_1^+$		$3_1^+ \rightarrow 2_1^+$		$3_1^+ \rightarrow 2_2^+$		$3_1^+ \rightarrow 4_1^+$	
	Exp.	IBm-2	Exp.	IBM-2	Exp.	IBM-2	Exp.	IBM-2
^{144}Nd	0.1619[a]	0.162	-	0.343	-	0.32	-	0.032
^{146}Nd	0.1557[b]	0.163	-	0.291	-	0.423	-	0.0345
^{148}Nd	0.214[c]	0.221	-	0.285	-	0.450	-	0.0412
^{150}Nd	0.0665[d]	0.077	-	0.200	-	0.140	-	0.0431
^{152}Nd	-	0.923	-	0.199	-	0.265	-	0.0451
^{154}Nd	-	1.313	-	0.144	-	0.251	-	0.0113

where a- [11] b- [12] c- [13] d- [14] e- [19] f- [20].

Table 5: Quadrupole moments for first excited states $Q(2_1^+)$ in *eb* units.

$Q(2_1^+)eb$	^{144}Nd	^{146}Nd	^{148}Nd	^{150}Nd	^{152}Nd	^{154}Nd
Exp.	-	-0.78(9) [12]	-1.46(24) [13]	-2.0(5) [14]	-	-
IBM-2	-0.723	-0.78	-1.33	-2.10	-2.246	-2.31

where $L_{\pi}^{(1)}$ and $L_{\nu}^{(1)}$ are the proton and neutron bosons angular momentum operators which are given as:

$$L_{\pi}^{(1)} = (10)^{1/2}(d_{\pi}^{+}\tilde{d_{\pi}})^{(1)}$$

$$L_{\nu}^{(1)} = (10)^{1/2}(d_{\nu}^{+}\tilde{d_{\nu}})^{(1)} \tag{13}$$

$$T^{(M1)} = \sqrt{\frac{3}{4\pi}}\left(g_{\pi}L_{\pi}^{(1)} + g_{\nu}L_{\pi}^{(1)}\right) \tag{14}$$

The g_{π} and g_{ν} are the boson g-factors which is measured in nuclear magnetons (μ_n) units. The $T^{(M1)}$ operator can be written as [7]:

$$T^{(M1)} = 0.77\left[(d^{+}d^{\sim})_{\pi} - (d^{+}d^{\sim})_{\nu}\right]^{(1)}(g_{\pi} - g_{\nu}) \tag{15}$$

The magnetic transition probability is given by [17]:

$$B(M1, i \rightarrow f) = \frac{|< I_i||T^{(M1)}||I_f >|^2}{2I_i + 1} \tag{16}$$

In order to evaluate the magnetic transition probabilities, we have to estimate the bosons g-factor for proton bosons and neutron bosons in Eq. (16), we used the relation [21]:

$$g = g_{\pi}N_{\pi}\frac{1}{N_{\pi} + N_{\nu}} + g_{\nu}N_{\nu}\frac{1}{N_{\pi} + N_{\nu}} \tag{17}$$

The equation is used to estimate the g-factor for first excited 2_1^+ state. The magnetic dipole moment value for ^{144}Nd isotope, $\mu = 0.35(3)\mu_N$ [11], and the mixing ratio for ^{144}Nd isotope to the transition $\delta(E2/M1;2_2^+ \rightarrow 2_1^+) = -1.6(5)eb/\mu_N$ [11,22], were used to produces the bosons g-factor. The predicted value of proton boson g-factor is constant for all $^{144-154}Nd$ isotopes ($g_\pi = 0.418\mu_N$), while the values of the neutron boson g-factor are shown in Table (6).

Table 6: Neutron bosons g-factor in (μ_N) Units.

Isotopes	^{144}Nd	^{146}Nd	^{148}Nd	^{150}Nd	^{152}Nd	^{154}Nd
$g_\nu(\mu_N)$	-0.182	-0.170	-0.156	-0.148	-0.141	-0.137

The calculations for B(M1) are listed in Table (7), from this results, we observe the value of transitions from antisymmetric states to symmetric states are quit high, this is due to the anti-symmetric component in the wave functions introduced by F-spin breaking in the Hamiltonian is increased. The transitions between low-lying collective states are weak and small, where these transitions included E2. The magnitude of M1 values are increases with spin increased for the intraband transitions, such as the transitions from β-band and γ-band to ground state band and in the interband transitions, such as $\gamma - \gamma$ band. From these results, we observe the M1 matrix elements size for $\gamma \rightarrow g$ is decrease with increasing the neutron number, specially, for the transition $\gamma \rightarrow g$ the change in M1 strengths occurs when the gamma band crosses the beta band. The magnetic transition probability $B(M1;1^+ \rightarrow 0_1^+)$ depends weakly on the strength of Majarona force parameter and proportional to the g_ν^2 factor. The value of this transition is large, this due to the transition from mixed symmetry 1^+state to the ground state (symmetric state). The IBM-2 results of $\mu(2_1^+)$ are given in Table (7), depend on the spin these values are provides the sensitive test of effective boson number within IBM-2, in the $^{144-154}Nd$ isotopes with $N = 84 - 94$, support the validity of assuming the proton boson number strong change when the neutron boson number is increased from 88 to 94. The agreement between experimental data and IBM-2 results is quite well.

2.3.3 Mixing Ratios

The mixing ratios for $^{144-154}Nd$ isotopes within IBM-2 are presented using the following Equation [17]:

$$\delta(E2/M1) = 0.835E_\gamma(inMeV) \times \frac{|<I_f^+||T^{(E2)}||I_i^+>|}{|<I_f^+||T^{(M1)}||I_i^+>|} \quad (18)$$

Where $|<I_f^+||T^{(E2)}||I_i^+>|$ is the reduced electric matrix element in eb units, and $|<I_f^+||T^{(M1)}||I_i^+>|$ in μ_N^2, E_γ is the γ-ray energy.

Table 7: Magnetic Transition Probability for $^{144-154}Nd$ isotopes in μ_N^2 Units.

Isotopes	$2_2^+ \rightarrow 2_1^+$		$2_3^+ \rightarrow 2_1^+$		$2_3^+ \rightarrow 2_2^+$	$3_1^+ \rightarrow 2_1^+$
Nd-144	0.00006		0.00012		0.00044	0.0022
Nd-146	0.00075		0.00018		0.00058	0.0043
Nd-148	0.000876		0.00023		0.00064	0.0054
Nd-150	0.00055		0.00028		0.00066	0.0044
Nd-152	0.000552		0.00041		0.00089	0.0052
Nd-154	0.00066		0.00054		0.00090	0.0068
Isotopes	$3_1^+ \rightarrow 2_2^+$		$3_2^+ \rightarrow 3_1^+$		$3_1^+ \rightarrow 4_1^+$	$1_1^+ \rightarrow 0_1^+$
Nd-144	0.0048		0.00002		0.0056	0.732
Nd-146	0.0033		0.00063		0.0057	0.747
Nd-148	0.0085		0.0010		0.0059	0.824
Nd-150	0.0091		0.0055		0.0061	0.902
Nd-152	0.019		0.0059		0.0068	0.086
Nd-154	0.022		0.0073		0.077	1.30
Isotopes	**Nd-144**	**Nd-146**	**Nd-148**	**Nd-150**	**Nd-152**	**Nd-154**
$\mu(2_1^+)$ Exp.	0.35(3) [11]	0.58(2) [12]	0.64(8) [13]	0.644(18) [14]	-	-
$\mu(2_1^+)$ IBM-2	0.33	0.57	0.69	0.62	0.53	0.50

In Table (8), we compare the IBM-2 and experimental calculated results for mixing ratios for $^{144-154}Nd$ isotopes, the agreement between IBM-2 results and experimental is quit well in sign and magnitude.

From these results one can be observe that a change of sign appears in two transition mixing ratios, $\delta(2_2^+ \rightarrow 2_1^+)$ in ^{150}Nd isotope and $\delta(2_3^+ \rightarrow 2_1^+)$ in 148Nd isotope, this due to magnitude for E2 and M1 matrix elements.

The large values for some mixing ratios in some isotopes, due to, the very small component effect of M1 in the transition and a dominant E2 transition. The sign of the mixing ratio must be chosen according to the reduced matrix elements sign.

The experimental data are taken from refs. [11,12,13, 14,22]

Table 8: Mixing ratios for $^{144-154}Nd$ Isotopes in eb/μ_N eb/μ_N units.

Isotope	$2_2^+ \rightarrow 2_1^+$		$2_3^+ \rightarrow 2_1^+$		$3_1^+ \rightarrow 2_1^+$		$3_2^+ \rightarrow 2_1^+$	
	Exp.	**IBM-2**	**Exp.**	**IBM-2**	**Exp.**	**IBM-2**	**Exp.**	**IBM-2**
^{144}Nd	-1.6(5)	-1.041	$-1.81^{+0.24}_{-0.21}$	-2.2	-	0.0634	-	-7.11
^{146}Nd	13^{+19}_{-8}	14.2	$-0.68^{+0.56}_{-0.42}$	-1.1	-5^{+60}_{-3}	-3.99	$-1.4^{+10}_{-0.8}$	-3.22
^{148}Nd	8^{+12}_{-2}	9.55	-33.33 or $\delta > 100$	65	-66.66	-51.9	0.37	0.54
^{150}Nd	-1.5	2.5	$\delta > 1.5$ or 71.4	4.6	0.4	0.33	-	0.66
^{152}Nd	-	2.65	-	10.4	-	0.763	-	0.71
^{154}Nd	-	3.45	-	12	-	0.76	-	1.23
Isotope	$3_1^+ \rightarrow 4_1^+$		$4_2^+ \rightarrow 4_1^+$		$4_3^+ \rightarrow 4_1^+$		$4_4^+ \rightarrow 4_1^+$	
	Exp.	**IBM-2**	**Exp.**	**IBM-2**	**Exp.**	**IBM-2**	**Exp.**	**IBM-2**
^{144}Nd	-	-2.945	-	-1.919	-	-2.760	-	19
^{146}Nd	-	12.7	-	0.004	-	1.455	-	5.1
^{148}Nd	$5^{=15}_{-22}$	8	-0.53	-0.84	3 or 0.5^{+8}_{-10}	4.5	3 or 0.53^{+8}_{-10}	6.4
^{150}Nd	0.08	1.22	-1.25	-2.0	100	94	-2	-3.5
^{152}Nd	-	1.32	-	-1.33	-	11	-	1.22
^{154}Nd	-	2.76	-	0.009	-	45	-	0.098

3 Conclusions

In work we have presented calculation results of the nuclear properties of the $^{144-154}Nd$ isotopes within IBM-2 framework. In general we found a good agreement with the experimental values and our results.

The energy ratios are given in Table (2), the ratio R_1 is increased smoothly from ^{144}Nd isotope to ^{154}Nd isotope, because far out than the major shell. The value of this ratio is equal $R_1 = 1.890$ in ^{144}Nd isotope and increased gradually with increasing neutron number, for ^{154}Nd isotope which equal $R_1 = 3.290$. From the values of energy ratios, the ^{144}Nd isotopes shows intermediate a nuclear structure in the shape transition from the spherical shape ($SU(5)$ symmetry). The energy level ratios in $^{144-146}Nd$ isotopes correspond to a spherical anharmonic vibrator, and those in $^{148-150}Nd$ isotopes being a transitional nuclei lie in $O(6)$ symmetry or γ-unstable. While the isotopes $^{152-154}Nd$ characterizes a strong deformation tendency ling in $SU(3)$ symmetry (rotor shape).

Concerning the electromagnetic transition rates properties in IBM-2, we find that all calculations trends is reproduced well reasonably. The effective charges for neutron bosons and proton bosons calculated within IBM-2 are depending on the IBM-2 symmetries, we get suitable values for e_π which is a constant for all $^{144-154}Nd$ isotopes because the number of proton bosons is constant. The effective charge for neutron bosons varies from isotope to another.

The reasons of discrepancies between the IBM-2 results and the experimental energies of the beta and gamma band state may be the energy of these states belongs to basic characteristics and the usual use of the IBM-2 parameters that have to be fixed especially high spin states.

The IBM-2 predictions of M1 transition probability rates are small, this is due to the band crossing gamma transitions symmetry and selection rules for transition. The M1 matrix elements size for $\gamma \to g$ decrease with increasing neutron number, specially, for the transition $\gamma \to g$ the change in M1 strengths occurs when the gamma band crosses the beta band. Mixing ratios are studied in this work for $^{144-154}Nd$ isotopes; we get good agreement with experimental data in magnitude and sign.

Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this article.

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