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Newly Proposed Solutions Using Caputo, Caputo–Fabrizio and Atangana–Baleanu Fractional Derivatives: A Comparison

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Abstract: The aim of this paper is to demonstrate the extent to which the new iterative Sumudu transform method (NISTM) helps in solving three fractional KdV–Burgers equations (KdVB). In fact, new explanatory solutions are being obtained by using Caputo sense, which represents kernels power law type, Caputo–Fabrizio (CF) standing for exponentially with decaying type kernel and the Atangana–Baleanu (AB) representing the Mittag-Leffler type kernel. It is found that the model consisting of ABC fractional derivatives are affected more by the past than Caputo fractional derivative and CF fractional derivative. The accuracy and efficiency of the NISTM has been shown by studying the convergence of this technique.

Keywords: KdV-Burgers equation, Caputo sense, Caputo–Fabrizio sense, Atangana–Baleanu sense, Sumudu transform.

1 Introduction

The soliton phenomenon are important natural phenomena, which were established through extensive research [1]. The KdVB type equation [2–6] has gained considerable attention due to its diverse implementations in shallow water surfaces and plasma physics. In recent years, there has been a great deal of interest in fractional calculus [7–15] that exhibits self-organization phenomena, and which introduces a new parameter (fractional derivative) to these systems. There is a solid reason as to why we have used fractional differential equations (FDEs), viz., a physical phenomenon realistic modelling depends not only on the instant time, but also on the past that was successfully achieved with the help of fractional calculus. Some methods have been used for solving non-linear FPDE [16–28]. One of the central methods for transformation is NISTM, which was implemented by Prakash et al. [29]. It is considered a rather good approach to solving nonlinear fractio nal differential equations. The original contributions in this paper is to study three fractional KdVB equations under fractional-order operators with the Caputo [29], the CF [30] and the AB [31] in the Caputo sense to determine the equation with highest efficiency rate. We propose NISTM to solve these nonlinear fractional differential equations in the Caputo sense, which take the following forms: The KdVB equation of fractional order under the Caputo sense is given as

$$^{C}D_t^{\alpha} v(x,t) + av\frac{\partial v}{\partial x} + b\frac{\partial^2 v}{\partial x^2} + c\,\frac{\partial^3 v}{\partial x^3} = 0. \tag{1}$$

The KdVB equation of fractional order under the CF sense is given as

$$^{CF}D_t^{\alpha} v(x,t) + av\frac{\partial v}{\partial x} + b\frac{\partial^2 v}{\partial x^2} + c\,\frac{\partial^3 v}{\partial x^3} = 0. \tag{2}$$

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Finally, the KdVB equation of fractional order under the AB sense is given as

$$^{ABC}D_t^{\alpha}v(x,t)+av\frac{\partial v}{\partial x}+b\frac{\partial^2 v}{\partial x^2}+c\frac{\partial^3 v}{\partial x^3}=0. \tag{3}$$

Subject to the initial condition

$$v(x,0)=f_0(x), \tag{4}$$

where a, b, c are given constants, and α, is the parameter describing the order of fractional derivative. The sub-sections that constitute the outline of this paper are presented here. In Section 2, we present a succinct summary of the fractional derivatives and natural transform. In Section 3, we present the convergence of NISTM. Section 4 contains new approximate results for the fractional KdVB equations. The paper concludes with a statement of the findings.

2 Mathematical groundwork

In this section, we present some basic definitions of Sumudu transform, which will be used for the purposes of this paper [32–35].

Definition 1. The Sumudu transform of the function is defined over the set of functions for some positive constant A

$$U=\left\{v(t):\exists A,\tau_1,\tau_2>0,|v(t)|<Ae^{\frac{|t|}{\tau_j}},if t\in(-1)^j\times[0,\infty)\right\}$$

by

$$S_t[v(t)]=\int_0^{\infty}v(\mathrm{u\,t})e^{-t}dt,u\in(-\tau_1,\tau_1), \tag{5}$$

where $S_t[v(t)]$ is the time function natural transform and u is the Sumudu transform variable.

Definition 2. The Sumudu transform $S_t[v(x,t)]$ of Caputo's fractional derivative D_t^{α} is defined as:

$$S_t\left[{}^{C}D_t^{\alpha}v(x,t)\right]=u^{-\alpha}\left(S_t[v(x,t)]-v(x,0)\right),\qquad 0<\alpha\leq 1 \tag{6}$$

Definition 3. The Sumudu transform $S_t[v(x,t)]$ of CF fractional derivative $^{CF}D_t^{\alpha}$ is defined as:

$$S_t\left[{}^{CF}D_t^{\alpha}v(x,t)\right]=\frac{C(\alpha)}{1-\alpha(1-u)}\left(S_t[v(x,t)]-v(x,0)\right),\qquad 0<\alpha\leq 1 \tag{7}$$

Definition 4. The Sumudu transform $S_t[v(x,t)]$ of AB fractional derivative in Caputo sense $^{ABC}D_t^{\alpha}$ is defined as:

$$S_t\left[{}^{ABC}D_t^{\alpha}v(x,t)\right]=\frac{AB(\alpha)S_tE_{\alpha}\left(-\frac{\alpha t^{\alpha}}{1-\alpha}\right)}{(1-\alpha)}\left(S_t[v(x,t)]-v(x,0)\right),\qquad 0<\alpha\leq 1 \tag{8}$$

3 Convergence of NISTM

In this section, we briefly discuss the convergence of the NISTM series of solutions.

Theorem 1. The infinite series $\sum_{n=0}^{\infty}v_n(x,t)$ converges to the analytical solution $v(x,t)$ whenever $R_n=\frac{\|v_n(x,t)\|}{\|v_{n-1}(x,t)\|}$, $R\in[0,1)$, where the norms are defined as [36]:

$$\|v_n(x,t)\|=\sqrt{\int_a^b\int_c^d|v_n(x,t)|^2dtdx},$$

4 Numerical examples

Three examples of fractional KdVB equation have been addressed in this section to illustrate NISTM performance and efficiency.

Example 1. NISTM for fractional KdVB equation by using Caputo sense

$$^{C}D_t^{\alpha} v(x,t) = -avv_x - bv_{2x} - cv_{3x}. \tag{9}$$

With a given initial condition

$$v(x,0) = \frac{12b^2}{25} - \frac{12b^2}{25} Tanh\left(\frac{bx}{5}\right) + \frac{6}{25}\left(Sech\left(\frac{bx}{5}\right)\right)^2. \tag{10}$$

According to the NISTM, we apply Sumudu transform on both sides of equation (9), and we have

$$S_t\left[^{C}D_t^{\alpha} v(x,t)\right] = -S_t\left[avv_x + bv_{2x} + cv_{3x}\right]. \tag{11}$$

Using equation (5), equation (6) and equation (10), we have

$$u^{-\alpha}\left(\widetilde{v}(x,u) - v(x,0)\right) = -S_t\left[avv_x + bv_{2x} + cv_{3x}\right]. \tag{12}$$

On simplifying,

$$\widetilde{v}(x,u) = v(x,0) - u^{\alpha} S_t\left[avv_x + bv_{2x} + cv_{3x}\right]. \tag{13}$$

We can write equation (13) in the form

$$\widetilde{v}(x,u) = v(x,0) - u^{\alpha} S_t\left[H(v)\right], where\ H(v) = avv_x + bv_{2x} + cv_{3x}. \tag{14}$$

By comparing both sides of equation (14), we have

$$\widetilde{v}_0(x,u) = v(x,0) = f_0(x), \tag{15}$$

$$\widetilde{v}_{k+1}(x,u) = -u^{\alpha} S_t\left[H\left(\sum_{r=0}^{k} v_k\right) - H\left(\sum_{r=0}^{k-1} v_{k-1}\right)\right]. \tag{16}$$

By taking inverse Sumudu transform to equation (15), and equation (16), we get

$$v_0(x,t) = v(x,0) = f_0(x), \tag{17}$$

$$v_{k+1}(x,t) = -S_t^{-1}\left(u^{\alpha} S_t\left[H\left(\sum_{r=0}^{k} v_k\right) - H\left(\sum_{r=0}^{k-1} v_{k-1}\right)\right]\right). \tag{18}$$

The following iteration is then deduced, as follows:

$$v_0(x,t) = f_0 = \frac{12b^2}{25} - \frac{12b^2}{25} Tanh\left(\frac{bx}{5}\right) + \frac{6}{25}\left(Sech\left(\frac{bx}{5}\right)\right)^2, \tag{19}$$

$$v_1(x,t) = -f_1 \frac{t^{\alpha}}{\Gamma(\alpha+1)}, \tag{20}$$

$$v_2(x,t) = f_2 \frac{t^{2\alpha}}{\Gamma(2\alpha+1)} - f_1 \partial_x f_1 \frac{a\Gamma(2\alpha+1) t^{3\alpha}}{\Gamma(3\alpha+1)\Gamma(\alpha+1)^2}, \tag{21}$$

where

$$f_1 = af_0\partial_x f_0 + b\partial_{x,x} f_0 + c\partial_{x,x,x} f_0,$$
$$f_2 = af_0\partial_x f_1 + af_1\partial_x f_0 + b\partial_{x,x} f_1 + c\partial_{x,x,x} f_1.$$

And so on. Therefore, we get the solution of equation (9) as

$$v(x,t) = v_0(x,t) + \sum_{m=1}^{\infty} v_m(x,t).$$

See Figures 1, 4(a) and 5.

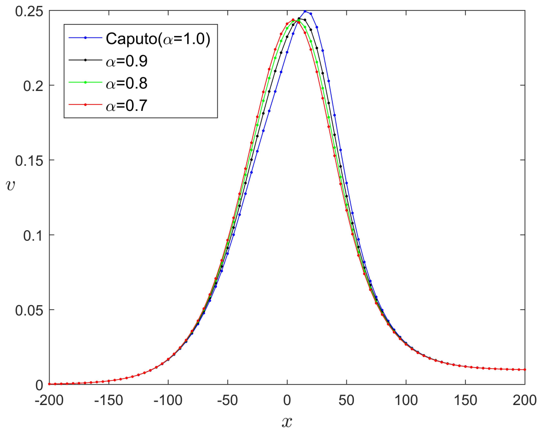


Fig. 1: Graphs of v(x,t) for example 1 at $a = 1$, $b = -0.1$, $c = 0.5$, $t = 60$ at various values α.

Table 1: Comparison of the absolute errors between NISTM and HATM [6] at $t = 0.1$, $\alpha = 1$.

x	Exact [6]	|Exact-NISTM| Caputo	|Exact-NISTM| CF	|Exact-NISTM| ABC	|Exact-HATM| [6]
0.1	0.24480	1.540E-05	1.540E-05	1.540E-05	1.921E-05
0.2	0.24480	3.508E-05	3.508E-05	3.508E-05	3.842E-05
0.3	0.24480	5.475E-05	5.475E-05	5.475E-05	5.763E-05
0.4	0.24470	7.443E-05	7.443E-05	7.443E-05	7.684E-05
0.5	0.24470	9.410E-05	9.410E-05	9.410E-05	9.604E-05
0.6	0.24470	1.138E-04	1.138E-04	1.138E-04	1.153E-04
0.7	0.24470	1.334E-04	1.334E-04	1.334E-04	1.345E-04
0.8	0.24470	1.531E-04	1.531E-04	1.531E-04	1.537E-04
0.9	0.24460	1.728E-04	1.728E-04	1.728E-04	1.729E-04
1	0.24460	1.925E-04	1.925E-04	1.925E-04	1.921E-04

Example 2. NISTM for fractional KdVB equation using CF sense

$$^{CF}D_t^{\alpha} v(x,t) = -avv_x - bv_{2x} - cv_{3x}. \tag{22}$$

With a given initial condition

$$v(x,0) = \frac{12b^2}{25} - \frac{12b^2}{25} Tanh\left(\frac{bx}{5}\right) + \frac{6}{25}\left(Sech\left(\frac{bx}{5}\right)\right)^2. \tag{23}$$

According to the NISTM, we apply Sumudu transform on both sides of equation (22), and we have

$$S_t\left[{}^{CF}D_t^{\alpha}v(x,t)\right] = -S_t\left[avv_x + bv_{2x} + cv_{3x}\right]. \tag{24}$$

Using equation (6) and equation (7), we have

$$\frac{C(\alpha)}{1-\alpha(1-u)}\left(\widetilde{v}(x,u) - v(x,0)\right) = -S_t\left[avv_x + bv_{2x} + cv_{3x}\right]. \tag{25}$$

On simplifying,

$$\widetilde{v}(x,u) = v(x,0) + \frac{\alpha(1-u)-1}{C(\alpha)}S_t\left[avv_x + bv_{2x} + cv_{3x}\right]. \tag{26}$$

We can write equation (26) in the form

$$\widetilde{v}(x,u) = v(x,0) + \frac{\alpha(1-u)-1}{C(\alpha)}S_t\left[H(v)\right], whereH(v) = avv_x + bv_{2x} + cv_{3x}. \tag{27}$$

By comparing both sides of equation(27), we have

$$\widetilde{v}_0(x,u) = v(x,0) = f_0(x) \tag{28}$$

$$\widetilde{v}_{k+1}(x,u) = \frac{\alpha(1-u)-1}{C(\alpha)}S_t\left[H\left(\sum_{r=0}^{k} v_k\right) - H\left(\sum_{r=0}^{k-1} v_{k-1}\right)\right]. \tag{29}$$

By taking inverse Sumudu transform to equation(28), and equation(29), we get

$$v_0(x,t) = v(x,0) = f_0(x), \tag{30}$$

$$v_{k+1}(x,t) = S_t^{-1}\left(\frac{\alpha(1-u)-1}{C(\alpha)}S_t\left[H\left(\sum_{r=0}^{k} v_k\right) - H\left(\sum_{r=0}^{k-1} v_{k-1}\right)\right]\right). \tag{31}$$

The following iteration is then deduced, as follows:

$$v_0(x,t) = f_0 = \frac{12b^2}{25} - \frac{12b^2}{25}Tanh\left(\frac{bx}{5}\right) + \frac{6}{25}\left(Sech\left(\frac{bx}{5}\right)\right)^2, \tag{32}$$

$$v_1(x,t) = \frac{f_1}{C(\alpha)}(\alpha - 1 - \alpha t), \tag{33}$$

$$v_2(x,t) = \frac{\alpha-1}{C(\alpha)}f_2 + \left(\frac{\alpha-1}{C(\alpha)}f_3 - \frac{\alpha}{C(\alpha)}f_2\right)t + \left(a\frac{\alpha^2(1-\alpha)}{C(\alpha)^3}f_1\partial_x f_1 - \frac{\alpha}{2C(\alpha)}f_3\right)t^2 - \frac{a\alpha^3\Gamma(2\alpha+1)}{3\,C(\alpha)^3\,\Gamma(\alpha+1)^2}f_1\partial_x f_1 t^3, \tag{34}$$

where

$$f_1 = af_0\partial_x f_0 + b\partial_{x,x}f_0 + c\partial_{x,x,x}f_0,$$

$$f_2 = \frac{\alpha-1}{C(\alpha)}\left(af_0\partial_x f_1 + af_1\partial_x f_0 + b\partial_{x,x}f_1 + c\partial_{x,x,x}f_1 + \frac{a(\alpha-1)}{C(\alpha)}f_1\partial_x f_1\right),$$

$$f_3 = \frac{-\alpha}{C(\alpha)}\left(af_0\partial_x f_1 + af_1\partial_x f_0 + b\partial_{x,x}f_1 + c\partial_{x,x,x}f_1 + \frac{2a(\alpha-1)}{C(\alpha)}f_1\partial_x f_1\right).$$

And so on. Therefore, we get the solution of equation(22) as

$$v(x,t) = v_0(x,t) + \sum_{m=1}^{\infty} v_m(x,t).$$

See Figures 2, 4(b) and 5.

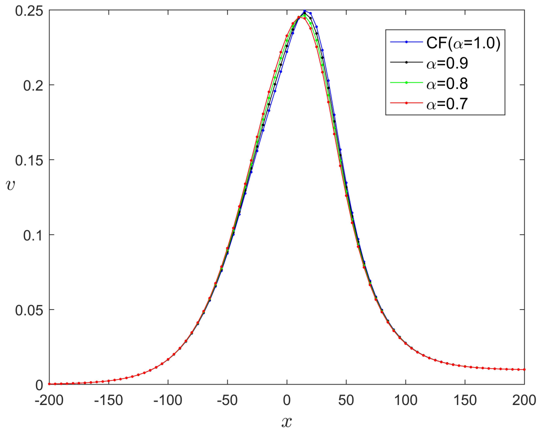


Fig. 2: Graphs of v(x,t) for example 2 at $a = 1$**,** $b = -0.1$**,** $c = 0.5$**,** $C(\alpha) = 1$**,** $t = 60$ **at various values** α**.**

Table 2: The convergence of the NISTM solutions using Caputo, CF and ABC definitions.

α	1			0.9			0.7			0.5		
	Caputo	CF	ABC	Caputo	CF	ABC	Caputo	CF	ABC	Caputo	CF	ABC
R_1	0.0033	0.0033	0.0033	0.0028	0.0030	0.0026	0.0020	0.0025	0.0016	0.0014	0.0019	0.0010
R_2	0.3032	0.3032	0.3032	0.2784	0.2835	0.2607	0.2282	0.2436	0.1874	0.1792	0.2027	0.1314

Example 3. NISTM for fractional KdVB equation using ABC sense

$$^{ABC}D_t^{\alpha} v(x,t) = -avv_x - bv_{2x} - cv_{3x}. \tag{35}$$

With a given initial condition

$$v(x,0) = \frac{12b^2}{25} - \frac{12b^2}{25} Tanh\left(\frac{bx}{5}\right) + \frac{6}{25}\left(Sech\left(\frac{bx}{5}\right)\right)^2. \tag{36}$$

According to the NISTM, we apply Sumudu transform on both sides of equation (35), and we have

$$S_t\left[^{ABC}D_t^{\alpha} v(x,t)\right] = -S_t\left[avv_x + bv_{2x} + cv_{3x}\right]. \tag{37}$$

Using equation (6) and equation (8), we have

$$\frac{AB(\alpha) S_t E_\alpha\left(-\frac{\alpha t^\alpha}{1-\alpha}\right)}{(1-\alpha)} \left(\widetilde{v}(x,u) - v(x,0)\right) = -S_t\left[avv_x + bv_{2x} + cv_{3x}\right].. \tag{38}$$

On simplifying,

$$\widetilde{v}(x,u) = v(x,0) - \frac{(1-\alpha)}{AB(\alpha) S_t E_\alpha\left(-\frac{\alpha t^\alpha}{1-\alpha}\right)} S_t\left[avv_x + bv_{2x} + cv_{3x}\right]. \tag{39}$$

We can write equation (39) in the form

$$\widetilde{v}(x,u) = v(x,0) + \frac{\alpha(1-u^\alpha) - 1}{AB(\alpha)} S_t\left[H(v)\right] \tag{40}$$

where

$$H(v) = avv_x + bv_{2x} + cv_{3x} \quad and \quad S_t E_\alpha\left(-\frac{\alpha t^\alpha}{1-\alpha}\right) = \frac{1-\alpha}{1-\alpha+\alpha u^\alpha}. \quad See\ [37]$$

By comparing both sides of equation (40), we have

$$\widetilde{v}_0(x,u) = v(x,0) = f_0(x) \tag{41}$$

$$\widetilde{v}_{k+1}(x,u) = \frac{\alpha(1-u^\alpha)-1}{AB(\alpha)} S_t\left[H\left(\sum_{r=0}^{k} v_k\right) - H\left(\sum_{r=0}^{k-1} v_{k-1}\right)\right]. \tag{42}$$

By taking inverse Sumudu transform to equation (41), and equation (42), we get

$$v_0(x,t) = v(x,0) = f_0(x), \tag{43}$$

$$v_{k+1}(x,t) = S_t^{-1}\left(\frac{\alpha(1-u^\alpha)-1}{AB(\alpha)} S_t\left[H\left(\sum_{r=0}^{k} v_k\right) - H\left(\sum_{r=0}^{k-1} v_{k-1}\right)\right]\right). \tag{44}$$

The following iteration is then deduced, as follows:

$$v_0(x,t) = f_0 = \frac{12b^2}{25} - \frac{12b^2}{25} Tanh\left(\frac{bx}{5}\right) + \frac{6}{25}\left(Sech\left(\frac{bx}{5}\right)\right)^2, \tag{45}$$

$$v_1(x,t) = \frac{f_1}{AB(\alpha)}\left(\alpha - 1 - \alpha\frac{t^\alpha}{\Gamma(\alpha+1)}\right), \tag{46}$$

$$\begin{aligned} v_2(x,t) = {} & \frac{\alpha-1}{AB(\alpha)} f_2 + \left(\frac{\alpha-1}{AB(\alpha)} f_3 - \frac{\alpha}{AB(\alpha)} f_2\right)\frac{t^\alpha}{\Gamma(\alpha+1)} + \left(a\frac{\alpha^2(1-\alpha)\Gamma(2\alpha+1)}{AB(\alpha)^3\Gamma(\alpha+1)^2} f_1\partial_x f_1 - \frac{\alpha}{AB(\alpha)} f_3\right)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} \\ & - \frac{a\alpha^3\Gamma(2\alpha+1)}{AB(\alpha)^3\Gamma(\alpha+1)^2} f_1\partial_x f_1 \frac{t^{3\alpha}}{\Gamma(3\alpha+1)}, \end{aligned} \tag{47}$$

where

$$f_1 = af_0\partial_x f_0 + b\partial_{x,x} f_0 + c\partial_{x,x,x} f_0,$$

$$f_2 = \frac{\alpha-1}{AB(\alpha)}\left(af_0\partial_x f_1 + af_1\partial_x f_0 + b\partial_{x,x} f_1 + c\partial_{x,x,x} f_1 + \frac{a(\alpha-1)}{AB(\alpha)} f_1\partial_x f_1\right),$$

$$f_3 = \frac{-\alpha}{AB(\alpha)}\left(af_0\partial_x f_1 + af_1\partial_x f_0 + b\partial_{x,x} f_1 + c\partial_{x,x,x} f_1 + \frac{2a(\alpha-1)}{AB(\alpha)} f_1\partial_x f_1\right).$$

And so on. Therefore, we get the solution of equation (35) as

$$v(x,t) = v_0(x,t) + \sum_{m=1}^{\infty} v_m(x,t).$$

See Figures 3, 4(c) and 5.

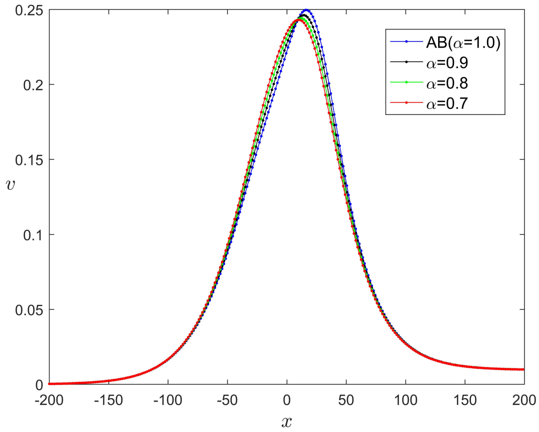
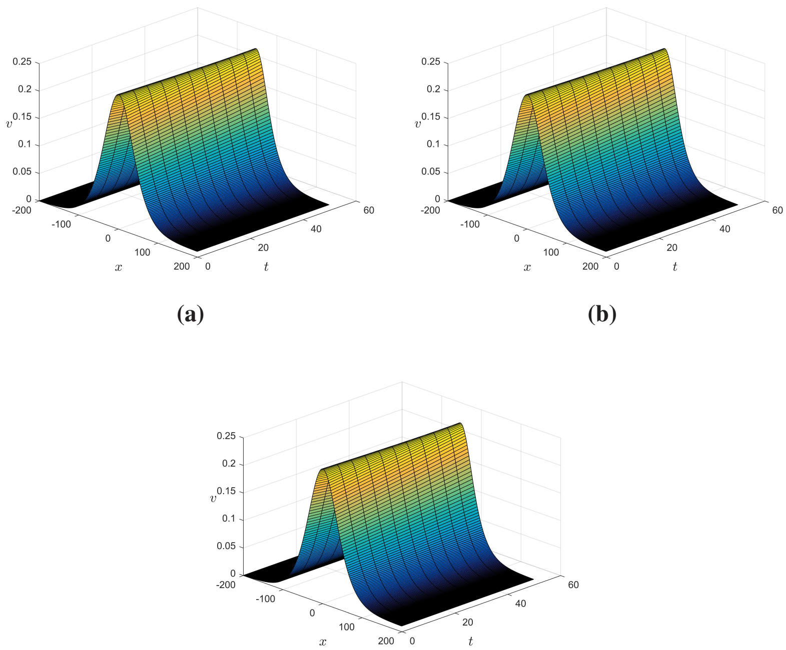


Fig. 3: Graphs of v(x,t) for example 3 at $a = 1$, $b = -0.1$, $c = 0.5$, $AB(\alpha) = 1$, $t = 60$ at various values α.

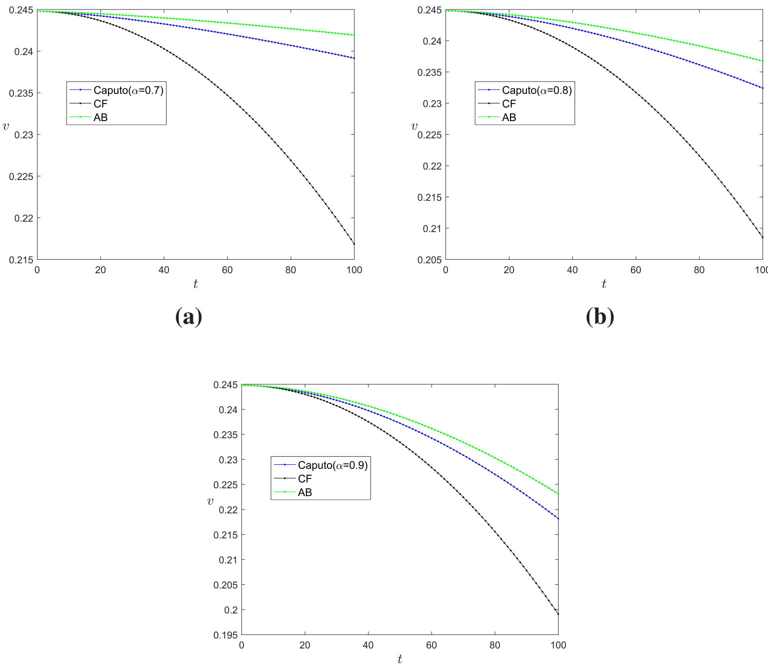


(a) **(b)** **(c)**

Fig. 4: Graphs of NISTM solution v(x,t) for examples 1,2,3 at $a = 1$**,** $b = -0.1$**,** $c = 0.5$**,** $\alpha = 0.7$**.**

Table 3: NISTM solutions for fractional KdV-Burgers equation using Caputo, CF and ABC definitions at $t = 0.1$.

α	0.9			0.8			0.7		
x	Caputo	CF	ABC	Caputo	CF	ABC	Caputo	CF	ABC
0.2	0.244779	0.244809	0.244808	0.244779	0.244805	0.244803	0.244780	0.244802	0.244798
0.4	0.244749	0.244818	0.244818	0.244750	0.244815	0.244814	0.244751	0.244813	0.244810
0.6	0.244712	0.244820	0.244820	0.244713	0.244818	0.244817	0.244715	0.244816	0.244814
0.8	0.244667	0.244814	0.244814	0.244669	0.244813	0.244812	0.244671	0.244812	0.244811
1	0.244615	0.244800	0.244800	0.244617	0.244800	0.244800	0.244619	0.244800	0.244800



(a) (b) (c)

Fig. 5: Graphs of NISTM solution v(x,t) for examples 1,2,3 at $a = 1$, $b = -0.1$, $c = 0.5$, $x = 1$ at various values α.

5 Analysis and discussion

We investigate the convergence of the solutions of problems (9), (22) and (35) by the NISTM. In view of theorem (1), then we have,

$$\|v_n(x,t)\| = \sqrt{\int_{-5}^{5}\int_{0}^{10} |v_n(x,t)|^2 \mathrm{dt}\,\mathrm{dx}},$$

We note that NISTM solutions converge because all the values of R_1 and R_2 at different values of α are less than 1, see Table 2. Comparing the values of R_1 and R_2, we note that the values of R_1 and R_2 with ABC fractional derivative are lesser than the values of R_1 and R_2 for the same value of Caputo and CF fractional derivatives. It is found that the model representing of Mittag-Leffler type kernel (ABC) has the highest convergence, particularly in comparison with other fractional derivatives. It is also shown that ABC fractional derivative is helpful for real world applications. For conventional case $\alpha = 1$, the comparisons between a NISTM solution, HAM [6] and an equivalent solution are shown in Table 1. We notice that the three fractional derivatives give the same results. The results obtained through the current method were found to be very precise. Figures (1-5) and Table 3 show the new fractional KdVB geometric component solution at different values α using three fractional derivatives (Caputo-CF-ABC). For fractional case, the ABC fractional derivatives are affected more by the past than Caputo fractional derivative and CF fractional derivative.

6 Conclusion

As has been quite apparent, we implemented the NISTM to solve fractional KdVB by using three fractional derivatives (Caputo, CF and ABC). Prior studies arrived at an exact solution to the problem whereas the present study arrived at a close to the exact solution. It is noted that NISTM is a very effective approach for solving nonlinear fractional problems. It offers a range of solutions, and therein lies its strength. The present paper offers solutions that are more realistic. It offers serious solutions that converge in real physical problems very quickly, in general. The findings can certainly be of great use in situations where an exact solution is not needed, and where complexities in finding one are to be avoided. Yet, it is to be asserted that further studies on the topic may open new avenues for research, leading to more refined conclusions and highly productive results.

Conflicts of Interests
The authors declare that they have no conflicts of interests.

References

[1] G. L. Lamb Jr., *Elements of soliton theory*, John Wiley Sons, 1980.
[2] J. L. Bona and M. E. Schonbek, Travelling-wave solutions to the Korteweg–de Vries–Burgers equation, *Proc. R Soc. Edinb. Sect. A Math.* **101**, 207–226 (1985).
[3] A. El-Ajou, O. AbuArqub and S. Momani, Approximate analytical solution of the nonlinear fractional KdV–Burgers equation: A new iterative algorithm, *J. Comput. Phys.* **293**, 81–95 (2015).
[4] A. K. Gupta and S. Saha Ray, On the solution of time-fractional KdV–Burgers equation using Petrov–Galerkin method for propagation of long wave in shallow water, *Chaos Solit. Fract.* **116**, 376–380 (2018).
[5] Q. Wang, Numerical solutions for fractional KdV–Burgers equation by Adomian decomposition method, *Appl. Math. Comput.* **182**, 1048–1055 (2006).
[6] K. M. Saad, E. H. F. AL-Shareef, A. K. Alomari, D. Baleanu and J. F. Gómez-Aguilar, On exact solutions for time-fractional Korteweg-de Vries and Korteweg-de Vries-Burger's equations using homotopy analysis transform method, *Chin. J. Phys.* **63**, 149–162 (2020).
[7] S. Das, *Functional fractional calculus*, Springer-Verlag, Heidelberg, 2011.
[8] B. I. Henry and S. L. Wearne, Fractional reaction–diffusion. *Physica A* **276**, 448–455 (2000).
[9] I. Podlubny, *Fractional differential equations*, Academic Press, San Diego, 1999.
[10] H. Weitzner and G. M. Zaslavsky, Some applications of fractional equations, *Commun. Nonlin. Sci. Numer. Simul.* **8**, 273–281 (2003).
[11] S. Rida, A. Arafa, A. Abedl-Rady and H. Abdl-Rahaim, Fractional physical differential Equations via natural transform, *Chin. J. Phys.* **55**, 1569-1575 (2017).
[12] Z. Odibat and D. Baleanu, Numerical simulation of initial value problems with generalized Caputo-type fractional derivatives, *Appl. Numer. Math.* **156**, 94-105 (2020).
[13] N. Sweilam, S. Al-Mekhlafi, S. Shatta and D. Baleanu, Numerical study for two types variable-order Burgers' equations with proportional delay, *Appl. Numer. Math.* **156**, 364-376 (2020).
[14] B. Shiri, G. Wu and D. Baleanu, Collocation methods for terminal value problems of tempered fractional differential equations, *Appl. Numer. Math.* **156**, 385-395 (2020).
[15] N. H. Tuan, D. Baleanu, T. N. Thach, D. O'Regan and N. H. Can, Final value problem for nonlinear time fractional reaction-diffusion equation with discrete data, *J. Comput. Appl. Math.* **376**, 112883 (2020).
[16] A. A. M. Arafa, S. Z. Rida and H. Mohamed, Homotopy analysis method for solving biological population model, *Commun. Theor. Phys.* **56**, 797–800 (2011).
[17] A. A. M. Arafa and A. M. S. Hagag, A new analytic solution of fractional coupled Ramani equation, *Chin. J. Phys.* **60**, 388-406 (2019).
[18] A. A. M. Arafa, S. Z. Rida, A. A. Mohammadein and H. M. Ali, Solving nonlinear fractional differential equation by generalized Mittag-Leffler function method, *Commun. Theor. Phys.* **59**, 661–663 (2013).
[19] S. Rida, A. Arafa, A. Abedl-Rady and H. Abdl-Rahaim, Fractional physical differential equations via natural transform, *Chin. J. Phys.* **55**, 1569-1575 (2017).
[20] A. Arafa A and G. Mahdy, Application of Residual Power series method to fractional coupled physical equations arising in fluids flow, *Int. J. Differ.* **10**, Article ID 7692849 (2018).
[21] A. A. M. Arafa and S. H. Hagag, Q -homotopy analysis transform method applied to fractional Kundu-Eckhaus equation and fractional massive Thirring model arising in quantum field theory, *Asian Eur. J. Math.* **12**, 1950045 (2019).

[22] A. El-Ajou, O. Abu Arqub, S. Momani, D. Baleanu and A. Alsaedi, A novel expansion iterative method for solving linear partial differential equations of fractional order, *Appl. Math. Comput.* **257**, 119–133 (2015).
[23] S. A. El-Wakil, A. Elhanbaly and M. A. Abdou, Adomian decomposition method for solving fractional nonlinear differential equations, *Appl. Math. Comput.* **182**, 313-325 (2006).
[24] M. A. Gondal, A. S. Arife, M. Khan and I. Hussain, An efficient numerical method for solving linear and nonlinear partial differential Equations by combining homotopy analysis and transform method, *World Appl. Sci. J.* **15**, 1786-1791 (2011).
[25] G. Wu, A fractional variational iteration method for solving fractional nonlinear differential Equations, *Comput. Math. Appl.* **61**, 2186-2190 (2011).
[26] Z. Odibat and S. Momani, A generalized differential transform method for linear partial differential Equations of fractional order, *Appl. Math. Lett.* **21**, 195-199 (2008).
[27] K. Seki, M. Wojcik and M.Tachiya, Fractional reaction–diffusion equation, *J. Chem. Phys.* **119**, 2165 (2003).
[28] P. K. Gupta and M. Singh Homotopy perturbation method for fractional Fornberg–Whitham Equation, *Comput. Math. Appl.* **61**, 250-255 (2011).
[29] A. Prakash, M. Kumar and D. Baleanu, A new iterative technique for a fractional model of nonlinear Zakharov–Kuznetsov equations via Sumudu transform, *Appl. Math. Comput.* **334**, 30–40 (2018).
[30] A. Atangana and D. Baleanu, New fractional derivatives with nonlocal and non-singular kernel: theory and application to heat transfer model, *Therm. Sci.* **20**, 763–769 (2016).
[31] T. Abdeljawad and D. Baleanu, Integration by parts and its applications of a new nonlocal fractional derivative with Mittag–Leffler nonsingular kernel, *J. Nonlin. Sci. Appl.* **10**, 1098–1107 (2016).
[32] A. Atangana and I. Koca, Chaos in a simple nonlinear system with Atangana–Baleanu derivatives with fractional order, *Chaos Solit. Fract.* **89**, 447–454 (2016).
[33] F. B. M. Belgacema and R. Silambarasan, Distinctive Sumudu treatment of trigonometric functions, *J. Comput. Appl. Math.* **312**, 74–81 (2017).
[34] A. Kılıcmana and H. E. Gadainb, On the applications of Laplace and Sumudu transforms, *J. Franklin Inst.* **347**, 848–862 (2010).
[35] R. K. Pandey and H. K. Mishra, Numerical simulation for solution of space–time fractional telegraphs equations with local fractional derivatives via HAFSTM, *New Astron.* **57**, 82–93 (2017).
[36] D. Braess, *Finite elements, theory, fast solvers and applications in elasticity theory*, Cambridge University Press, 2007.