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Geometric structure features in $^{72\text{-}80}$Se isotopes

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Abstract: The geometric properties of the interacting boson model, interacting boson model-1and interacting boson model-2 have been used to perform studying of $^{72\text{-}80}$Se isotopes. The low –lying positive party states, energy ratios and the potential energy surface for $^{72\text{-}80}$Se have been investigated by using software package $IBMP\ and\ IBM$ computer code for $IBM-1$ and Neutron Proton Boson $NPBOS$ software package for $IBM-2$. $U(5)$ are the dominant in $^{72-80}Se$ with addition of a small effect of rotational parameters started from ^{74}Se to ^{80}Se isotopes, energy ratios in which ^{72}Se isotopes as the nearest isotopes to typical vibrational limit while $^{74-80}Se$ isotopes tend towards the rotational region lied on $U(5)$-$SU(3)$, leg of "Casten's triangle". ^{72}Se isotope potential energy curve has one minimum point at $\beta = 0$, this is the same as spherical equilibrium shapes (a vibrational-like spectrum). The counter diagram represents as regular concentric, $^{74-80}Se$ isotopes that have evolved the deformation.

Keywords: $IBMP, IBM-1$, and $IBM-2$.

1 Introduction

The interacting boson model has associated with it an intrinsic geometric structure. The natural space for the geometric properties of system with group structure is the so called cost space.[1] Imagining a liquid drop vibrational at high frequency have been gotten a good idea of the physics of nuclear vibration. Although the average shape is spherical. Rotational motion can be observed only in nuclei with no spherical equilibrium shape. These nuclei can have substantial distortions from spherical shape and are called deformed nuclei[2]. There are many studies and research that attempted to understand and explain the behavior of $^{72\text{-}80}$Se nuclei by using different models[3-8] in present study the relation between geometric structure and nuclear structure has been investigated. The $^{72\text{-}80}$Se isotopes have Z=34 then, 3 particle bosons the number of protons and neutrons are lying between 28 and 50 magic shells, ^{72}Se has 38 neutrons that mean 5 particle neutron bosons. While $^{74\text{-}80}$Se have 40-46 neutrons indicating 5-2 hole neutron bosons respectively.

2 Theoretical Parts

In the $IBM-1$, it is assumed that the Hamiltonian operator contains only one body and two body terms thus, introducing creation $(s^{\dagger}, d_m^{\dagger})$ and annihilation (s, d_m)operators where the index $m = 0, \pm1, \pm2$.The most general Hamiltonian ,which includes on-boson terms in boson – boson interaction is [1]:-

$$H = \varepsilon_s(s^{\dagger}s) + \varepsilon_d \sum_m d_m^{\dagger} d_m + \sum_{L=0,2,4} \frac{1}{2}(2L+1)^{\frac{1}{2}} C_L\left[(d^{\dagger}d^{\dagger})^{(L)}.(dd)^{L}\right]^{(0)} + \frac{1}{\sqrt{2}}\upsilon_2\left[(d^{\dagger}d^{\dagger})^{(2)}.(ds)^{(2)} + (d^{\dagger}s^{\dagger})^{(2)}.(dd)^{(2)}\right]^{(0)} + \frac{1}{2}\upsilon_0\left[(d^{\dagger}d^{\dagger})^{(0)}.(ss)^{(0)} + (s^{\dagger}s^{\dagger})^{(0)}.(dd)^{(0)}\right]^{(0)} + \mathrm{u}_2\left[(d^{\dagger}s^{\dagger})^{(2)}.(ds)^{2}\right]^{(0)} + \frac{1}{2}u_0\left[(s^{\dagger}s^{\dagger})^{(0)}.(ss)^{(2)}\right]^{(0)} \quad (1)$$

Where $\varepsilon_d, \varepsilon_s$ are s and d bosons energy and V represent the boson-boson interacting energy. The most commonly used form of $IBM1$ Hamiltonian is [9,10]:-

$$H = \varepsilon n_d + a_0 P^{\dagger}P + a_1 L.L + a_2 Q.Q + a_3 T_3 T_3 + a_4 T_4 T_4 \quad (2)$$

Where $\varepsilon = \varepsilon_d - \varepsilon_s$ is the boson energy,for simplicity ε_s was set equal to zero only $\varepsilon = \varepsilon_d$ appears, a_0, a_1, a_2, a_3, a_4 designate the strengths of the quadrupole, angular momentum, pairing, octupole, and hexadecapole interacting between bosons respectively. The five component of d boson and the single component of the s boson extended across a six dimensional space. For a fixed number of boson N the group structure of the problem is $U(6)$. Considering the different reductions of $U(6)$, three dynamical symmetries emerge, namely $U(5), SU(3)$ and

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$O(6)$; these symmetries are related to the geometrical idea of the spherical vibrator, deformed rotor and a symmetric($\gamma-$soft) deformed rotor, respectively [8-12]. The general formula for the potential energy surface as a function of geometrical variables β and γ is given by [1, 9,11]:-

$$V(\beta,\gamma)=\frac{N(\varepsilon_s+\varepsilon_d\beta^2)}{1+\beta^2}+\frac{N(N+1)}{(1+\beta^2)^2}(\alpha_1\beta^4+\alpha_2\beta^3 Cos3\gamma+\alpha_3\beta^2+\alpha_4) \quad (3)$$

$$\text{where}\alpha_1=\frac{C_0}{10}+\frac{C_2}{7}+\frac{9C_4}{35},\alpha_2=-\sqrt{\frac{8}{35}}\upsilon_2,\ \alpha_3=\frac{(\upsilon_0+u_2)}{\sqrt{5}},\alpha_4=u_0 \quad (4)$$

where N is the total boson number β is the quadruple deformation parameter operator from $\beta=0-2.4$. γ is the distortion parameter operator or (asymmetry angle) for $0^\circ\le\gamma\le60^\circ$.The variables $\alpha_1,\alpha_2,\alpha_3,\alpha_4$ are related to the parameters C_L,υ_L,u_L which are given in equation (1). The relationships between the variables (α′s) and these parameters have been expressed by F. Iachello [1] as one must take into account the asymmetry angle occurs only in the term $\cos3\gamma$. Thus, the energy surfaces has minima only at $\gamma=0^\circ$and 60°. These expressions give at large N, $\beta_{min}=0,\sqrt{2},1$ for $U(5),SU(3)$,and $O(6)$ respectively. The Hamiltonian operator in $IBM-2$ has been given by[1,9,13]:-

$$H=H_\pi+H_\nu+V_{\pi\nu} \quad (5)$$

A simple schematic Hamiltonian guided by microscopic consideration is given by[14]

$$H=\varepsilon(n_{d\pi}+n_{d\nu})+\kappa Q_\pi.Q_\nu+V_{\pi\pi}+V_{\nu\nu}+M_{\pi\nu} \quad (6)$$

where

$$Q_\rho=(d_\rho^\dagger s_\rho+s_\rho^\dagger d_\rho)_\rho^2+\chi_\rho(d_\rho^\dagger d_\rho)_\rho^2\ \ \rho=\pi,\nu \quad (7)$$

$$V_{\rho\rho}=\sum_{L=0,2,4}\tfrac{1}{2}(2L+1)^{\frac{1}{2}}C_L^\rho\left[(d_\rho^\dagger d_\rho^\dagger)^{(L)}.(d_\rho d_\rho)^{(L)}\right]^{(0)} \quad (8)$$

$\varepsilon_\pi,\varepsilon_\nu$ are proton and neutron energy respectively, they assumed to be equal $\varepsilon_\pi=\varepsilon_\nu=\varepsilon$ the last term in Eq. (5) contains the Majorana operator $M_{\pi\nu}$ and it is usually added in order to remove states of mixed proton neutron symmetry ,this term can be written as [15]

$$M_{\pi\nu}=\zeta_2(s_\nu^\dagger d_\pi^\dagger-d_\nu^\dagger s_\pi^\dagger)^{(2)}.(s_\nu d_\pi-d_\nu s_\pi)^{(2)}+\sum_{k=1,3}\zeta_k(d_\nu^\dagger d_\pi^\dagger)^{(k)}-(d_\nu d_\pi)^{(k)} \quad (9)$$

it is possible to obtain spectra which are similar to those of the $IBM-1$with only one kind of boson[15]. The $U(5)$ limit when $\gg\kappa$, $SU(3)$ limit when $\varepsilon\ll\kappa$ and $\chi_\pi=\chi_\nu=-\sqrt{7}/2$, and $O(6)$ limit when $\varepsilon\ll\kappa$ and $\chi_\nu=-\chi_\pi$. Most nuclei do not strictly belong to any of these three limiting cases, but are somewhere between two of them. In the IBM, it is possible to make a smooth transition between the limiting cases for a series of isotopes, as between the vibrational limit and the γ-unstable limit.

3 Results and Discussion

The geometric properties of the interacting boson model, interacting boson model-1and interacting boson model-2 have been used to perform studying of 72-80Se series of isotopes, which that medium mass with (Z=34). The software package IBM, IBMP computer code for $IBM-1$ and Neutron Proton Boson NPBOS software package have been used by estimating set of parameters described in the Hamiltonian operator as they are shown in equations (2) and(6). The parameters estimated for the calculations of excited energy levels for 72-80Se isotopes and geometric interacting boson model are given in table (1).

Table 1: The parameters have been used in the $\boldsymbol{IBM-1}$, IBMP and $\boldsymbol{IBM-2}$ Hamiltonian for even-even $^{72\text{-}80}$Se isotopes (in MeV) except χ , χ_ν and χ_π were unit less.

$IBM-1$parameters in MeV unless χ					
Isotopes	^{72}Se	^{74}Se	^{76}Se	^{78}Se	^{80}Se
N	8	8	7	6	5
ε	0.6002	0.6	0.54	0.6	0.6
a_0	0.0	0.0	0.0	0.0	0.0
a_1	0.02	0.02	0.02	0.02	0.02
a_2	0.0	-0.007	-0.009	-0.038	-0.036
a_3	0.001	0.0	0.0	0.0	0.0
a_4	0.001	0.0	0.0	0.0	0.0
χ	0.0	-1	-0.86	-1	-1
$IBM-2$ Parameters in MeV unless χ , χ_π= -0. 74 , N_π =3					
N_ν	5	5	4	3	2
ε_d	0.99	0.88	0.78	0.78	0.8
κ	-0.076	-0.077	-0.086	-0.076	-0.16
χ_ν	0.88	0.82	0.84	0.84	-0.6
ζ_2	0.002	0.012	0.01	0.03	0.001
$\zeta_{1,3}$	-0.01	-0.02	-0.02	-0.016	-0.01
C_ν^L	-0.99, 0.76,0. 12	-0.9, 0.76,0. 1	-0.41, 0.86,0. 32	0.2, 0.86,0. 5	-0.48, 44,0.69
C_π^L	0.2, 0.76 ,0.12	0.2, 0.76,0. 09	0.2,0.8 0.22	0.22, 0.9,0.5	0.72, 21,0.69
The geometric interacting boson model parameters in MeV					
ε_s	0	-0.035	-0.045	-0.19	-0.18
ε_d	0.723	0.706	0.644	0.682	0.684
α_1	0.001	-0.002	-0.002	0	0
α_2	0	-0.015	-0.017	0.008	-0.008
α_3	0	-0.028	-0.036	-0.152	-0.144
α_4	0	0	0	0	0

Calculated energy ratios of $(E4_1^+/E2_1^+)$, $(E6_1^+/E2_1^+)$ and $(\mathbf{E8_1^+/E2_1^+})$ for 72-80Se isotopes have been indicated in figure (1) as a function of mass numbers. This leads to guess the nearest dynamic symmetries corresponding to the characteristics one of the dynamic symmetries [1] or may possess transitional features between two or more symmetries. The calculated energy levels compared with experimental data [16-20] for 72-80Se isotopes have been shown in figures from (2) to (6).

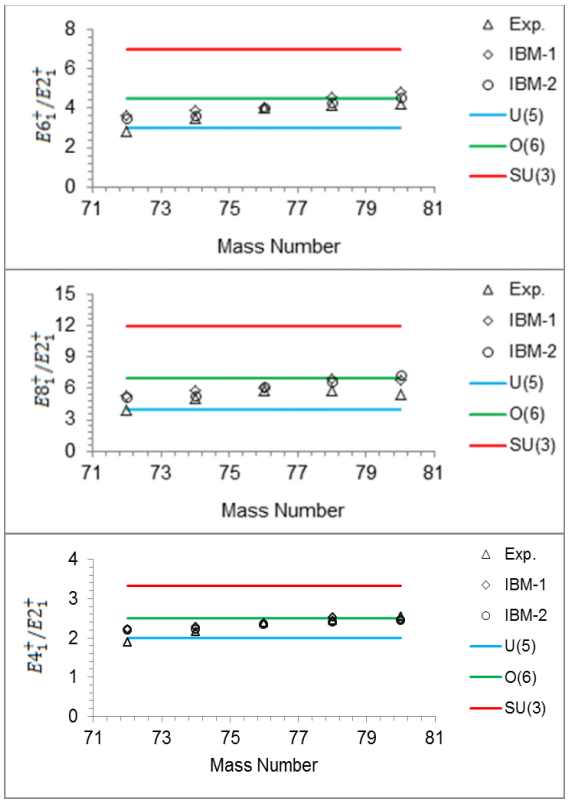


Fig. 1: Show the energy ratios ($E4_1^+/E2_1^+$,$E6_1^+/E2_1^+$, and $E8_1^+/E2_1^+$) as a function of mass numbers for even -even $^{72-80}$Se isotopes.

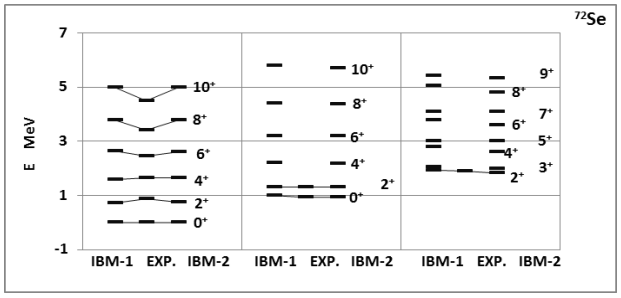


Fig. 2: Comparison between experimental [16] and calculated energy levels for 72Se isotope.

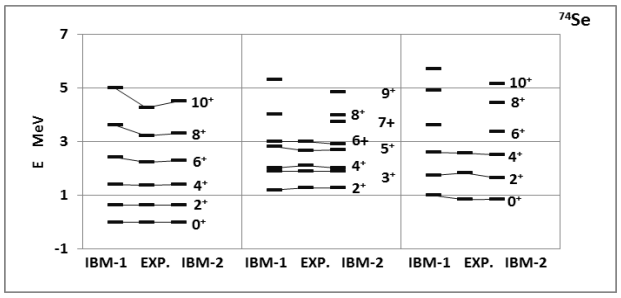


Fig. 3: Comparison between experimental [17] and calculated energy levels for ^{74}Se isotope.

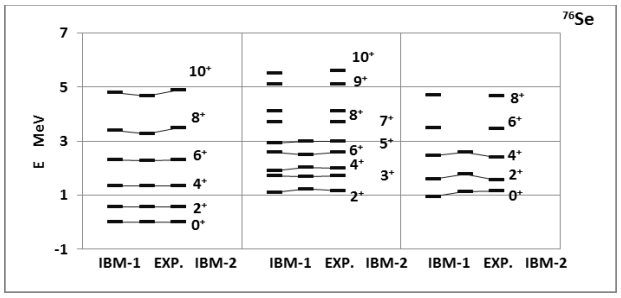


Fig. 4: Comparison between experimental [18] and calculated energy levels for ^{76}Se isotope.

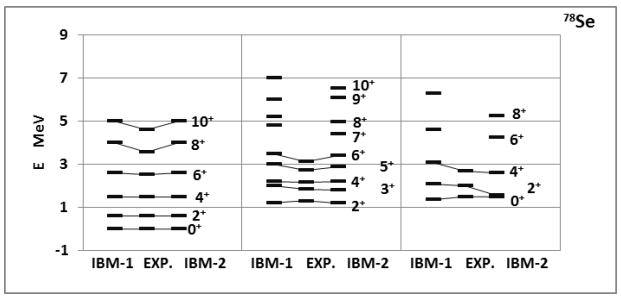


Fig.5: Comparison between experimental [19] and calculated energy levels for ^{78}Se isotope.

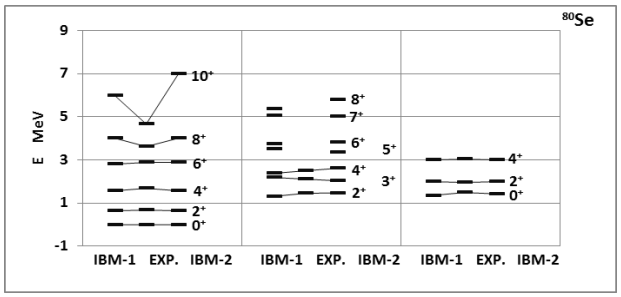
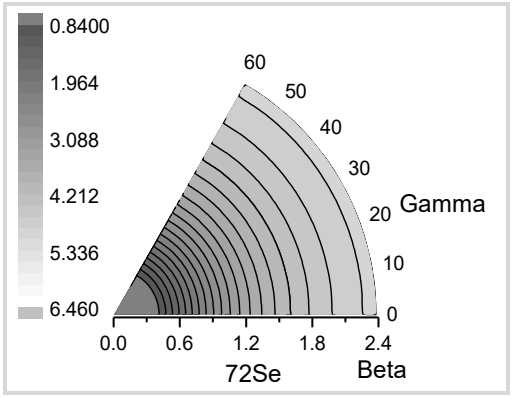
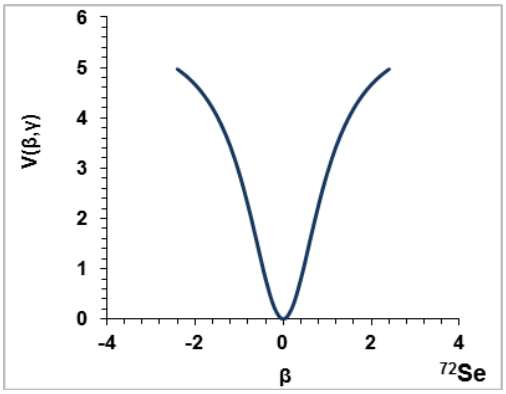
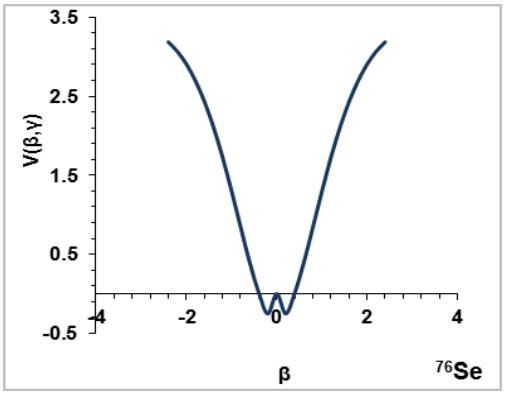


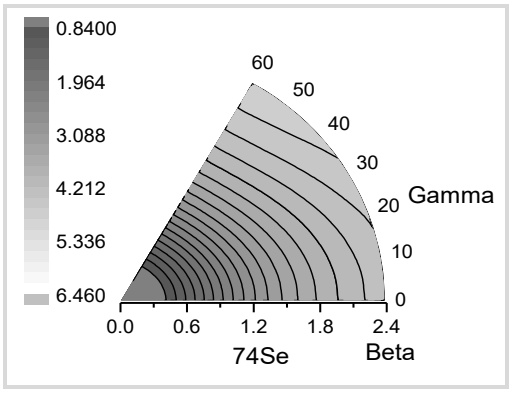
Fig. 6: Comparison between experimental [20] and calculated energy levels for ^{80}Se isotope.

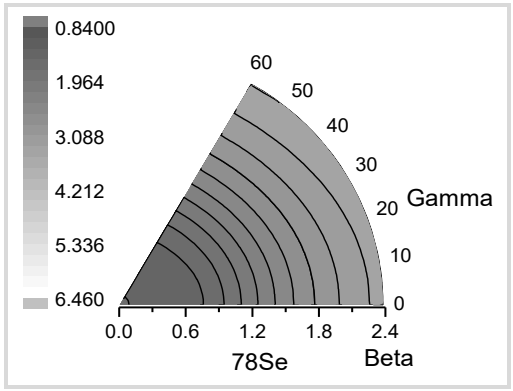
The surface of the potential energy as a function to β with contour diagrams for $^{72-80}Se$ isotopes calculated from equation (3) with $IBMP$ computer code represented in Figure (7).

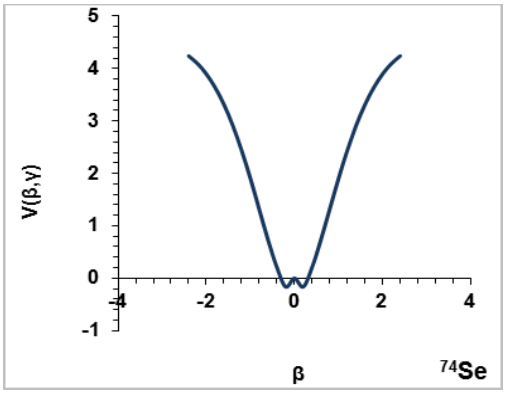


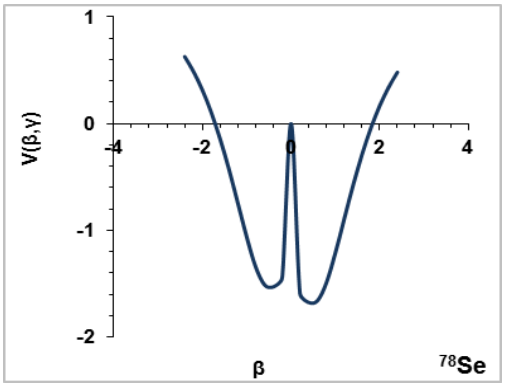


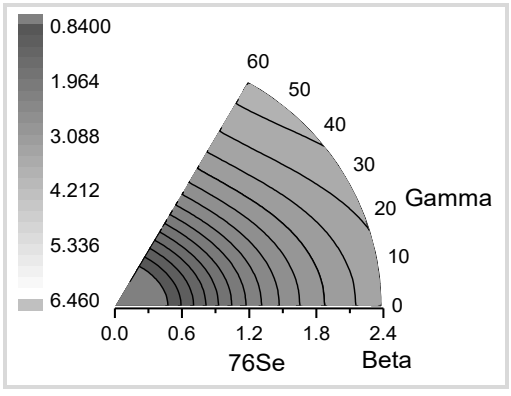


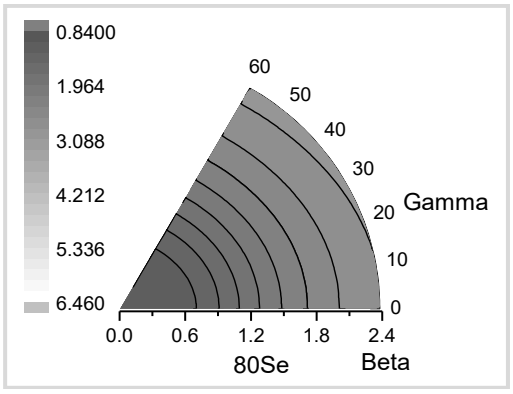












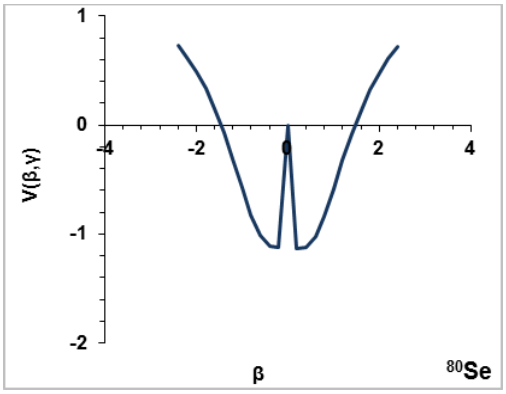


Fig. 7: The surface of the potential energy as a function to β with contour diagrams for even- even $^{72-80}Se$ isotopes.

4 Conclusions

Selenium nuclei have 32 isotopes between mass number 64 to95. According to the interacting boson model, $^{72-80}Se$ has been conducted describing the nuclear structure of even-even nuclei within the $U(6)$ symmetry, possessing the$U(5), SU(3)$, and $O(6)$ limiting dynamical symmetries, appropriate for vibrational, axially deformed, and γ −unstable nuclei respectively. The nucleons distributions of protons and neutrons sub shells are

$$\underbrace{1f_{5/2}^{2}}_{Z=34}\ \underbrace{1f_{5/2}^{6}}_{N=72}\ \underbrace{2p_{1/2}^{2}}_{74}\ \underbrace{1g_{9/2}^{6}}_{76-80}$$

This distribution may support the vibrational sight to these isotopes which indicating that it is arranged as one and two phonon states. The energy of two phonon states has twice the energy of the one – phonon states which are not accurately fulfilled. These energy ratios vary typically between about 1.8 and2.6; in all none closed shell spherical even-even nuclei, the first excited energy state is 2^+ and the next three excited$0^+, 2^+ and 4^+$. In the lighter even-even nuclei, all configurations obtained without exciting nucleons to another shell have even parity, three excited $0^+, 2^+ and 4^+$ most be given a good convergence. In addition to convergence three-phonon excitations states with spins and parity of $0^+, 2^+, 3^+, 4^+ and 6^+$, nuclei whose level schemes contained sets with the characteristic properties belong to the $U(5)$ vibrational subgroup of the interacting boson model (IBM). However, the energy levels sequence of $^{74\text{-}80}$Se isotopes moved away from the typical phonons interactions towards rotational one. In $IBM1, \varepsilon$ parameter is the dominant in $^{72-80}Se$ with addition a small effect of a_2 parameter started from ^{74}Se to ^{80}Se isotopes. In $IBM2$ parameters, the $U(5)$ limit expectation when$\varepsilon \gg \kappa$, energy ratios $(E4_1^+/E2_1^+)$,$(E6_1^+/E2_1^+)$ and $(E8_1^+/E2_1^+)$, explains ^{72}Se isotope $(1.89, 2.86 and\ 3.97)$ as the nearest isotope to typical vibrational limit while $^{74-80}Se$ isotopes tend towards a rotational region lied on $U(5)$- $SU(3)$, leg of "Casten's triangle". The nuclear shape for even-even $^{72-80}Se$ has been tested in term of potential energy surface, see shape (7). ^{72}Se isotope potential energy curve has one minimum point at $\beta = 0$, this is the same as spherical equilibrium shapes (a vibrational-like spectrum). The counter diagram represents as regular concentric, $^{74-80}Se$ isotopes that have evolved the deformation as illustrated in figures (7). The couture diagrams appeared the deformation circulars with two equal minimum potential energy surfers $V(\beta, \gamma)$ equals to$(-0.1639\ and\ -0.2508\ at\ 0.2\ and -0.2)$ for $^{74,76}Se$ respectively, and with two not equal minimum potential energy surfers $V(\beta, \gamma)$ that equal to $(-1.588\ and -1.441\ at\ 0.2 and\ -0.2)$ and $(-1.127\ at\ \beta = 0.2\ and -1.116\ at\ \beta = -0.2)$ for $^{78,80}Se$ respectively.

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