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The Efficacy Measurement of Treatment Methods: An Application to Stress-Strength Model

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Abstract: This paper addresses how to use a statistical model to compare two different treatment methods for cystic fibrosis disease. The stress-strength model is used in a parametric form to compare the efficacy of the two treatment methods. Since the available data in most clinical situations are censored, the stress-strength model is investigated in presence of progressively Type II censored data. The kumaraswamy exponential (Kw-E) distribution is used as a parametirc form for the stress-strength model. The statistical inference of the stress–strength reliability $R = P[Y < X]$ is performed when X and Y are independent random variables and follow the Kw-E distribution.

Keywords: Clinical data; Efficacy measurement of treatment methods; Kumaraswamy exponential distribution; Reliability of a stress–strength model; Statistical inference.

1 Introduction

Treatment represents the application of different health care interventions for the cure or reduction of disease related symptoms. There are different treatment methods for the same disease and there is willingness to choose the best treatment. The choice of the best method needs a statistical tool to be used as criteria for comparing the different methods.

In this paper, the stress-strength model is proposed as a good statistical tool to compare the two treatments of cystic fibrosis disease. Basically, the stress-strength model is a mechanical concept, but it can be used in clinical studies, see Johnson [1] and Sharma et al. [2]. Cystic fibrosis is a progressive genetic disease that causes persistent lung infections and limits the ability to breathe over time. Two treatments are applied to two groups of patients: the first group was cured by human enzyme DNase 1(6-MP) and the second group was cured by placebo. Then, lengths of remission (in weeks) for the two groups are registered, see Fuchs et al. [3]

If the lengths of remission of patients in the first group are represented by the random variable X and the lengths of remission of patients in the second group are represented by the random variable Y, according to the famous stress-strength model in the reliability theory, we need to compute the following probabilites $R = P[Y < X]$ or $1 - R = P[X \leq Y]$.

The statistical inference tools, such as maximum likelihood estimation (MLE) and Bayesian estimation, will be used to obtain a good estimator of R, through the two groups of lengths of remission.

It is also assumed that we observe independent progressively type-II censored samples from both Kw-E distributions. Schematically a progressively Type II censored sample can be described as follows: Suppose that n independent items are put on a life test with continuous identically distributed failure times $X_1, X_2, ..., X_n$. Suppose further that a censoring scheme $(r_1, r_2, ..., r_m)$ is previously fixed such that immediately following the first failure X_1, r_1, surviving items are removed from the experiment at random, and immediately following the second failure X_2, r_2, surviving items are removed from the experiment at random. This process continues until, at the time of the m th observed failure X_m, the remaining r_m surviving items are removed from the test. The m ordered observed failure times denoted by $X_{1:m:n}$, $X_{2:m:n}, ..., X_{m:m:n}$ are called progressively Type II right censored order statistics of size m from a sample of size n with progressive censoring

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scheme $(r_1, r_2, ..., r_m)$. It is clear that $n = m + \sum_{i=1}^{m} r_i$. The special case when $r_1 = r_2 = \cdots = r_{m-1} = 0$ so that $r_m = n - m$ is the case of conventional Type-II right censored sampling.
Moreover, when $r_1 = r_2 = \cdots = r_m = 0$, so $m = n$, the progressively Type II right censoring scheme reduces to ordinary order statistics, see Balakrishnan [4]. Adepoju and Chukwu [5] defined the Kw-E distribution as follows:

$$F(x;a,b,\lambda) = 1 - [1 - (1 - e^{-\lambda x})^a]^b, \quad x > 0; a, b, \lambda > 0,$$

where $\lambda > 0$ is a scale parameter and the other positive parameters, a and b, are shape parameters. The corresponding probability density function (PDF) is

$$f(x;a,b,\lambda) = ab\lambda e^{-\lambda x}(1 - e^{-\lambda x})^{a-1}[1 - (1 - e^{-\lambda x})^a]^{b-1}.$$

Let $X \sim$ Kw-E(a, b_1, λ) and $Y \sim$ Kw-E(a, b_2, λ) be independent random variables and

$$R = P(Y < X) = \int_0^\infty \int_0^x f(x)\, g(y)\, dydx = \frac{b_2}{b_1 + b_2}, \tag{1}$$

where $f(x)$ and $g(y)$ are the PDFs of X and Y, respectively. Estimation of the stress–strength parameter has received considerable attention in the statistical literature, starting with the pioneering work of Birnbaum [6], who provided an interesting connection between the classical Mann-Whitney statistic and the stress-strength model.

Since then, work has been accomplished on the estimation and inference of the stress-strength parameter for different distributions from the frequentist and Bayesian points of view, see review of literature in Kumar and Siju [7], Mahmoud et al. [8] and Mokhlis et al. [9].

The layout of the paper is as follows: Section Two addresses the MLE of R in addition to the corresponding exact confidence interval (CI). In Section Three, the Bayesian estimation of R is computed under the squared error loss (SEL) function. In Section Four, our model is applied to the real data introduced by Fuchs et al. [3]. Simulation study has been presented for illustrative purposes in Section Five. Conclusion is presented in Section Six.

2 Maximum Likelihood Estimation

Suppose $\underline{X} = (X_{1:M}, X_{2:M}, ..., X_{m:M})$ is a progressively Type II censored sample from Kw-E(λ, a, b_1) with censored scheme $\underline{r} = (r_1, r_2, ..., r_m)$ and $\underline{Y} = (Y_{1:N}, Y_{2:N}, ..., Y_{n:N})$ is a progressively Type II censored sample from Kw-E(λ, a, b_2) with censored scheme $\underline{\acute{r}} = (\acute{r}_1, \acute{r}_2, ..., \acute{r}_n)$. Hence, the likelihood function of $b_1$and b_2 is given by

$$L(b_1, b_2 | a, \lambda, \underline{x}, \underline{y}) = c_1 \prod_{i=1}^{m} \{f(x_i)[1 - F(x_i)]^{r_i}\} \times c_2 \prod_{j=1}^{n} \left\{ g(y_j)[1 - G(y_j)]^{\acute{r}_j} \right\}, \tag{2}$$

where

$$c_1 = M(M - 1 - r_1)(M - 2 - r_1 - r_2) \ldots (M - m + 1 - r_1 \ldots - r_{m-1}),$$
$$c_2 = N(N - 1 - \acute{r}_1)(N - 2 - \acute{r}_1 - \acute{r}_2) \ldots (N - n + 1 - \acute{r}_1 \ldots - \acute{r}_{n-1}),$$

for more detials, see Balakrishnan and Aggarwala [10].

Then, $L(b_1, b_2 | a, \lambda, \underline{x}, \underline{y})$ or $L(b_1, b_2)$, for notation simplicity, can be written as follows:

$$L(b_1, b_2) = c_1 c_2 b_1^m b_2^n (\lambda a)^{m+n} \times \prod_{i=1}^{m} \{e^{-\lambda x_i}(1 - e^{-\lambda x_i})^{a-1}[1 - (1 - e^{-\lambda x_i})^a]^{b_1(1+r_i)-1}\} \times \prod_{j=1}^{n} \{e^{-\lambda y_j}(1 - e^{-\lambda y_j})^{a-1}[1 - (1 - e^{-\lambda y_j})^a]^{b_2(1+\acute{r}_j)-1}.\} \tag{3}$$

The log-likelihood function may then be written as follows:

$$\ln L(b_1, b_2) = \ln c_1 + \ln c_2 + (m+n)\ln\lambda + (m+n)\ln a + m\ln b_1 + n\ln b_2 + (a-1)\left[\sum_{i=1}^{m} \ln(1 - e^{-\lambda x_i}) + \sum_{j=1}^{n} \ln(1 - e^{-\lambda y_j})\right] - \lambda \sum_{i=1}^{m} x_i + \sum_{i=1}^{m} [b_1(1 + r_i) - 1] \ln[1 - (1 - e^{-\lambda x_i})^a] - \lambda \sum_{j=1}^{n} y_j + \sum_{j=1}^{n} [b_2(1 + \acute{r}_j) - 1] \ln[1 - (1 - e^{-\lambda y_j})^a].$$

Thus, we have the likelihood equations for b_1 and b_2 respectively, as

$$\frac{\partial \ln L}{\partial b_1} = \frac{m}{b_1} + \sum_{i=1}^{m} (r_i + 1) \ln[1 - (1 - e^{-\lambda x_i})^a] = 0,$$

and

$$\frac{\partial \ln L}{\partial b_2} = \frac{n}{b_2} + \sum_{j=1}^{n} (\acute{r}_j + 1) \ln[1 - (1 - e^{-\lambda y_j})^a] = 0.$$

Then,

$$\hat{b}_1 = \frac{-m}{\sum_{i=1}^{m} (r_i + 1) \ln[1 - (1 - e^{-\lambda x_i})^a]},$$

and

$$\hat{b}_2 = \frac{-n}{\sum_{j=1}^{n} (\acute{r}_j + 1) \ln[1 - (1 - e^{-\lambda y_j})^a]}.$$

The MLE of R, say$\hat{R}$, can be written as

$$\hat{R} = \frac{\hat{b}_2}{\hat{b}_1 + \hat{b}_2}. \tag{4}$$

To find the PDF of $\hat{R}$, the following lemma is needed

Lemma 1.*If the random variable* $X \sim Kw - E(a,b,\lambda)$, $T = \ln[1-(1-e^{-\lambda x})^a] \sim Exp(b)$.

*Proof.*The proof is easy to obtain.

Also, the following transformation can be considered:

$$S_1 = MT_1,$$
$$S_2 = (M - R_1 - 1)(T_2 - T_1),$$
$$\vdots$$
$$S_m = (M - R_1 \ldots - R_{m-1} - (m-1))(T_m - T_{m-1}).$$

Balakrishnan and Aggarwala [10] have proved that S_i 's are independent and identically distributed exponential random variables, i.e.$S_i \sim Exp(b), i = 1,\ldots,m$. Furthermore,

$$\sum_{i=1}^{m} S_i = \sum_{i=1}^{m} (R_i + 1) T_i$$
$$= \sum_{i=1}^{m} (R_i + 1) \ln[1-(1-e^{-\lambda x_i})^a] = U.$$

Accordingly, U has a gamma distribution with the shape parameter m and the scale parameter b_1,. Then,

$$\hat{b}_1 = \frac{m}{U} \text{ and } \hat{b}_2 = \frac{n}{V},$$

where V has a gamma distribution with the shape parameter n and the scale parameter b_2. Hence,

$$\hat{R} = \frac{1}{1 + (\frac{m}{n})(\frac{V}{U})} = \frac{1}{1 + (\frac{b_2}{b_1})Z},$$

wher $Z = \frac{mb_1V}{nb_2U}$ has a F distribution with degrees of freedom $2n$ and $2m$,taking into account the independence of the two gamma random variables U and V. The PDF of $\hat{R}$ can be obtained as follows:

$$f_{\hat{R}}(r) = \frac{\left(\frac{n}{m}\right)^n}{Beta(m,n)} \left(\frac{b_1}{b_2}\right)^n \frac{(1-r)^{n-1}}{r^{n+1}\left(1 + \frac{nb_1}{mb_2}\left(\frac{1-r}{r}\right)\right)^{m+n}},$$
$$0 < r < 1. \tag{5}$$

To calculate the variance of $\hat{R}$, the expectation and the second moment of $\hat{R}$ can be obtained, respectively, as follows:

$$E\left[\hat{R}\right] = \frac{m\Gamma(m+n)}{\Gamma(m+n+1)} \left(\frac{m}{n}\right)^m \left(\frac{b_2}{b_1}\right)^m \times$$
$${}_2F_1\left(m+n, m+1; m+n+1; 1 - \frac{mb_2}{nb_1}\right),$$
$$0 < \frac{mb_2}{nb_1} < 2.$$

$$E\left[\hat{R}^2\right] = \frac{m(m+1)\Gamma(m+n)}{\Gamma(m+n+2)} \left(\frac{m}{n}\right)^m \left(\frac{b_2}{b_1}\right)^m \times$$
$${}_2F_1\left(m+n, m+2; m+n+2; 1 - \frac{mb_2}{nb_1}\right),$$
$$0 < \frac{mb_2}{nb_1} < 2.$$

where ${}_2F_1$ is the hypergeometric function given by,

$${}_2F_1(h,q;c;w) = \frac{\Gamma(c)}{\Gamma(q)\Gamma(c-q)} \int_0^1 t^{q-1}(1-t)^{c-q-1}(1-tw)^{-h} dt,$$

see Temme [11]. Hence, the variance of $\hat{R}$ can be calculated.
Since

$$\frac{1-R}{R} \times \frac{\hat{R}}{1-\hat{R}} = \frac{b_1}{b_2} \times \frac{mV}{nU} = Z \sim F(2n, 2m), \tag{6}$$

then $100(1-\gamma)\%$ confidence interval of R is

$$\left[\frac{1-\hat{R}}{(1-\hat{R}) + \hat{R}\, F_{\frac{\gamma}{2}}(2n,2m)}, \frac{1-\hat{R}}{(1-\hat{R}) + \hat{R}\, F_{1-\frac{\gamma}{2}}(2n,2m)}\right]. \tag{7}$$

3 Bayesian Estimation of R

The Bayesian approach randomly handles the parameters and uncertainties on the parameters are described by a joint prior distribution, which is developed before the failure data are collected. It is also based on historical data, experience with similar products, design specifications, and experts' opinions. The ability of incorporating prior knowledge in the analysis makes the Bayesian approach very helpful in the reliability analysis because one of the main challenges associated with the reliability analysis is the limited availability of data.

Let the prior knowledge of parameters b_1 and,b_2 be described by the following independent prior distributions:

$$\left.\begin{aligned} \pi(b_1) &= \frac{\lambda_1^{\mu_1}}{\Gamma(\mu_1)} b_1^{\mu_1 - 1} e^{-b_1\lambda_1}, \ b_1, \mu_1, \lambda_1 > 0, \\ \pi(b_2) &= \frac{\lambda_2^{\mu_2}}{\Gamma(\mu_2)} b_2^{\mu_2 - 1} e^{-b_2\lambda_2}, \ b_2, \mu_2, \lambda_2 > 0 \end{aligned}\right\}. \tag{8}$$

Hence, the joint prior of the parametersb_1 and,b_2 can be written follows:

$$\pi(b_1, b_2) = \frac{\lambda_1^{\mu_1}}{\Gamma(\mu_1)} \frac{\lambda_2^{\mu_2}}{\Gamma(\mu_2)} b_1^{\mu_1 - 1} b_2^{\mu_2 - 1} e^{-(b_1\lambda_1 + b_2\lambda_2)}.$$

The joint posterior density function of b_1 and,b_2, denoted by $\pi^*(b_1, b_2|\lambda, a, \underline{x}, \underline{y})$, can be written as

$$\pi^*(b_1, b_2|a, \lambda, \underline{x}, \underline{y}) =$$
$$\frac{L(b_1, b_2|a, \lambda, \underline{x}, \underline{y}) \times \pi(b_1, b_2)}{\int_0^\infty \int_0^\infty L(b_1, b_2|a, \lambda, \underline{x}, \underline{y}) \times \pi(b_1, b_2)\, db_1 db_2}. \tag{9}$$

Then,

$$\pi^*(b_1,b_2|a,\lambda,\underline{x},\underline{y}) \propto b_1^{m+\mu_1-1}b_2^{n+\mu_2-1}e^{-(b_1\lambda_1+b_2\lambda_2)} \times\prod_{i=1}^{m}[1-(1-e^{-\lambda x_i})^a]^{b_1(1+r_i)} \times\prod_{j=1}^{n}[1-(1-e^{-\lambda y_j})^a]^{b_2(1+\acute{r}_j)}.$$

The conditional posterior densities of b_1 and b_2 can be given as

$$\pi_1^*(b_1) \equiv Gamma\left[m+\mu_1,\lambda_1-\sum_{i=1}^{m}\{(r_i+1)\ln[1-(1-e^{-\lambda x_i})^a]\}\right],$$

$$\pi_2^*(b_2) \equiv Gamma\left[n+\mu_2,\lambda_2-\sum_{j=1}^{n}\{(\acute{r}_j+1)\ln[1-(1-e^{-\lambda y_j})^a]\}\right].$$

Applying transformation techniques, the posterior PDF of R is

$$f_{R|Data}(r) = K\times\frac{(1-r)^{m+\mu_1-1}}{r^{m+\mu_1+1}}\times \left(1+\frac{\Omega_1(r_i,\lambda_1,x_i)}{\Omega_2(\acute{r}_j,\lambda_2,y_j)}\left(\frac{1-r}{r}\right)\right)^{-(m+n+\mu_1+\mu_2)}, \quad 0<r<1, \tag{10}$$

where

$$\Omega_1(r_i,\lambda_1,x_i) = \lambda_1-\sum_{i=1}^{m}\{(r_i+1)\ln[1-(1-e^{-\lambda x_i})^a]\},$$

$$\Omega_2(\acute{r}_j,\lambda_2,y_j) = \lambda_2-\sum_{j=1}^{n}\{(\acute{r}_j+1)\ln[1-(1-e^{-\lambda y_j})^a]\}$$

and

$$K=\frac{1}{Beta(m+\mu_1,n+\mu_2)}\left(\frac{\Omega_1(r_i,\lambda_1,x_i)}{\Omega_2(\acute{r}_j,\lambda_2,y_j)}\right)^{m+\mu_1}.$$

The Bayes estimate of R using the squared error loss function, say $\hat{R}_{BSEL}$, can be obtained by calculating the posterior mean of R as follows:

$$\begin{aligned}\hat{R}_{BSEL} &= \int_0^1 r f_{R|Data}(r)dr \\ &= K\int_0^1 r\frac{(1-r)^{m+\mu_1-1}}{r^{m+\mu_1+1}} \times\left(1+\frac{\Omega_1(r_i,\lambda_1,x_i)}{\Omega_2(\acute{r}_j,\lambda_2,y_j)}\left(\frac{1-r}{r}\right)\right)^{-(m+n+\mu_1+\mu_2)}dr \\ &= \left(\frac{\Phi_2(\acute{r}_j,\lambda_2,y_j)}{\Phi_1(r_i,\lambda_1,x_i)}\right)^{n+\mu_2}\left(\frac{n+\mu_2}{s}\right) \times{}_2F_1(s,n+\mu_2+1;s+1;1-\frac{\Omega_2(\acute{r}_j,\lambda_2,y_j)}{\Omega_1(r_i,\lambda_1,x_i)}),\end{aligned} \tag{11}$$

where $s=m+n+\mu_1+\mu_2$. In some cases, it is difficult to obtain the estmation of R from (10), so the acceptance rejection principle can be used to obtain the Bayesian point estimates of R and the corresponding credible interval. The acceptance rejection principle is a simulation procedure and is used to generate samples from the posterior distribution. The algorithm of this procedure is introduced and proved by Devroye [12]. The steps for Bayesian estimation of R are described in Saraçoğlu et al. [13].

4 Applications to Clinical Data

First, the following lemma is introduced to make the corresponding known parameters in two populations the same for applying our model to the clinical data.

Lemma 2. If the random variable $T \sim Kw-E(a,b,\lambda)$,then $Y=\left(1-e^{-\lambda t}\right)^a \sim Kw-E(1,b,1)$.

Proof. The proof is easily obtained.

Here, we analyze the clinical data, which was originally reported by Fuchs et al. [3]. It represents the lengths of remission (in weeks) for two groups subjected to two different treatments of cystic fibrosis disease. The data are presented in Table 1, where stars denote censored observations.

Table 1. Lengths of remission (in weeks) for two groups of patients.

6-MP	6	6	6	6*	7	9*	10
Placebo	1	1	2	2	3	4	4
6-MP	10*	11*	13	16	17*	19*	20*
Placebo	5	5	8	8	8	8	11
6-MP	22	23	25*	32*	32*	34*	35*
Placebo	11	12	12	15	17	22	23

Kw-E distribution is fitted to the two data sets, separately. We present the estimated shape, scale parameters, Kolmogorov-Smirnov (K-S) distances between the fitted and the empirical distribution functions, and corresponding p-values in Table 2.

Table 2. Values of a,b,λ,K-S and p - values

Data set	a	b	λ	K-S	p - value
(6-MP)	54.5496	0.1226	0.7066	0.1193	0.8924
(Placebo)	1.2856	0.1001	1.1827	0.1736	0.4972

Table (2) reveals that the Kw-E distribution fits quite well to both the data sets. Because the two parameters a and λ are unequal, we transform the above-mentioned data sets using Lemma 2. For illustrative purposes, we have generated two different progressively censored samples using two different sampling schemes from the transformed data sets. Using progressive censoring schemes $M=N=21$, $m=n=10$ with $\underline{r}=(2,0,0,3,0,0,3,0,3,0)$ and $\underline{\acute{r}}=(2,0,3,0,0,3,0,0,0,3)$. The results on the statistical inference of R is given in Table 3.

Table 3. Statistical inference of R.

Non-Bayesian		
MLE	Exact CI	Length
0.4514	[0.3303, 0.7497]	0.419428
Bayesian		
SEL	CRI	Length
0.4537	[0.0211, 0.9708]	0.9497

The results indicate that the second treatment (Placebo) might be better than the first treatment (6-MP) with a percentage of 40% approximately. Hence, the second treatment does not improve the contribution to curing the cystic fibrosis disease.

5 Simulation Study

In this section, the Monte Carlo simulation is conducted to compare the performances of MLE and the Bayes estimator under different progressive censoring schemes. Two sets of population parameter values (i) $b_1 = 6$ and $b_2 = 4$ and (ii) $b_1 = 4$ and $b_2 = 6$ are considered. For a given M and m, three different progressive censoring schemes are used to generate the progressively censored samples.

Scheme I : $r_1 = n - m$, $r_i = 0$ for $i \neq 1$.

Scheme II : $r_m = n - m$, $r_i = 0$ for $i \neq m$.

Scheme III : All the r_i's, take the same number.

A typical example of the Scheme III is given as $M = 10, m = 5$ and $r_i = 1, i = 1,2,3,4,5$. The comparison between the different schemes has been considered in their mean square error (MSE), as shown in Table 4. In each simulation, the MLE and the boundaries for exact 95% confidence intervals (ECI) are obtained. Bayes estimates are obtained based on the gamma priors with $\mu_1 = \mu_2 = \lambda_1 = \lambda_2 = 0.001$.The average of lengths for excat and credible CIs are calculated based on 1000 simulations, as shown in Table 5.

Table 4 . The average and MSE of the estimates.

				$b_1=6, b_2=4$		$b_1=4, b_2=6$	
M,m	N,n	r	$\acute{r}$	MLE	Bayes	MLE	Bayes
10,5	10,5	I	I	0.4206	0.5012	0.5862	0.5019
				0.0225	0.0190	0.022	0.0185
		I	II	0.4069	0.4998	0.5944	0.4980
				0.0214	0.0189	0.0213	0.0192
		II	I	0.4108	0.5070	0.5920	0.5013
				0.0209	0.0203	0.0237	0.0187
		II	II	0.4110	0.5010	0.5928	0.4971
				0.0216	0.0188	0.0218	0.0203
		III	III	0.4114	0.498	0.5894	0.4969
				0.0217	0.0184	0.0210	0.0189
20,5	10,5	I	I	0.4063	0.4952	0.5891	0.4993
				0.0228	0.0172	0.0214	0.0187
		I	II	0.4055	0.5005	0.5908	0.4985
				0.0217	0.0189	0.0210	0.0192
		II	I	0.4069	0.5006	0.5906	0.5068
				0.0226	0.0192	0.0214	0.0167
		II	II	0.4080	0.5030	0.5911	0.4984
				0.0213	0.0194	0.0207	0.0202
		III	III	0.4095	0.4979	0.5816	0.5016
				0.0207	0.0183	0.0220	0.0182

Table 4. Continued.

				$b_1=6, b_2=4$		$b_1=4, b_2=6$	
M,m	N,n	r	$\acute{r}$	MLE	Bayes	MLE	Bayes
10,5	20,5	I	I	0.4116	0.4941	0.5925	0.5040
				0.0191	0.0182	0.0232	0.0176
		I	II	0.4023	0.4983	0.5954	0.4986
				0.0214	0.0180	0.0212	0.0194
		II	I	0.4094	0.4968	0.5964	0.5002
				0.0219	0.0174	0.0234	0.0183
		II	II	0.4152	0.5019	0.5882	0.5050
				0.0223	0.0193	0.0212	0.0179
		III	III	0.4148	0.4991	0.5905	0.5007
				0.0224	0.0186	0.0204	0.0187
20,5	20,5	I	I	0.4122	0.4937	0.5912	0.4904
				0.021	0.0176	0.0219	0.0209
		I	II	0.4187	0.5009	0.5874	0.4994
				0.022	0.0197	0.0221	0.0192
		II	I	0.4113	0.4984	0.5956	0.5026
				0.0214	0.018	0.0223	0.0183
		II	II	0.4060	0.5017	0.586	0.5007
				0.0213	0.0192	0.0223	0.0187
		III	III	0.4137	0.4999	0.5943	0.4993
				0.0221	0.0192	0.022	0.0191
20,10	10,5	I	I	0.4143	0.4969	0.6023	0.4961
				0.0164	0.0183	0.0156	0.0195
		I	II	0.4145	0.5026	0.6044	0.4976
				0.0170	0.0191	0.0159	0.0199
		II	I	0.4188	0.4983	0.6113	0.5043
				0.0178	0.0183	0.0159	0.0178
		II	II	0.4207	0.5045	0.5977	0.5030
				0.0186	0.0199	0.0166	0.018
		III	III	0.4222	0.5015	0.6035	0.4994
				0.0171	0.019	0.0158	0.0188
10,5	20,10	I	I	0.3960	0.4967	0.5835	0.4998
				0.016	0.0184	0.0174	0.0193
		I	II	0.3926	0.5010	0.5842	0.5022
				0.0151	0.0198	0.0168	0.0182
		II	I	0.3906	0.4957	0.5801	0.4978
				0.0160	0.0181	0.0175	0.0189
		II	II	0.3939	0.5039	0.5797	0.4980
				0.0156	0.0191	0.0183	0.0192
		III	III	0.3918	0.5038	0.5757	0.5050
				0.0164	0.0193	0.0184	0.0181

From Table 4, it is noted that the MSE of the parameters decreases as the sample size increase.

Table 5. The average lengths of 95% CIs for the estimates.

				$b_1=6, b_2=4$		$b_1=4, b_2=6$	
M,m	N,n	r	$\acute{r}$	ECI	CRI	ECI	CRI
10,5	10,5	I	I	0.528	0.8111	0.5263	0.8081
		I	II	0.525	0.8101	0.5246	0.8117
		II	I	0.5272	0.8078	0.521	0.8141
		II	II	0.5261	0.8075	0.5241	0.8038
		III	III	0.5259	0.8078	0.5273	0.8126
20,5	10,5	I	I	0.5224	0.8112	0.5262	0.8053
		I	II	0.5238	0.8138	0.5264	0.8076
		II	I	0.5228	0.8133	0.5256	0.8133
		II	II	0.5255	0.8172	0.527	0.8044
		III	III	0.5269	0.8096	0.528	0.8074
10,5	20,5	I	I	0.5305	0.798	0.5219	0.8144
		I	II	0.5232	0.8104	0.5244	0.8026
		II	I	0.525	0.8102	0.52	0.8083
		II	II	0.5261	0.8121	0.5271	0.8074
		III	III	0.526	0.8113	0.5275	0.8077
20,5	20,5	I	I	0.5277	0.8139	0.5249	0.8093
		I	II	0.5281	0.812	0.5256	0.8049
		II	I	0.5265	0.8009	0.5223	0.8082
		II	II	0.5247	0.8144	0.5257	0.8015
		III	III	0.5259	0.8033	0.5234	0.8081
20,10	10,5	I	I	0.4653	0.8084	0.4739	0.7979
		I	II	0.4642	0.8131	0.4728	0.8069
		II	I	0.4648	0.8174	0.4708	0.8064
		II	II	0.4642	0.8046	0.4731	0.8151
		III	III	0.4673	0.8089	0.4731	0.8083

Table 5. Continued

				$b_1=6, b_2=4$		$b_1=4, b_2=6$	
M,m	N,n	r	$\acute{r}$	ECI	CRI	ECI	CRI
10,5	20,10	I	I	0.4727	0.8154	0.4644	0.8095
		I	II	0.4733	0.8021	0.4652	0.8052
		II	I	0.4709	0.8098	0.4657	0.8084
		II	II	0.4726	0.8062	0.4645	0.8161
		III	III	0.4707	0.8127	0.466	0.8079

The length of the exact CIs is shorter than the corresponding CRIs in all cases.

6 Conclusion

The present study connects the stress - strength model in reliability theory and the statistical studies in medical research to compare two different treatment methods. When a parametric distribution is fitted to clinical data, more accurate results can be obtained rather than depending on the non-parametric studies. From the computational point of view, the MLEs are the easiest to obtain. Thus, it is suggested to use the MLE for all practical purposes. Our study shows that it is the best used DNase 1(6-MP) treatment method instead of placebo method with probability close to 60% as shown throughout the clinical data and the simulation study. We look forward to developing a statistical tool that compares more than two different treatment methods using the same technique.

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