Applied Mathematics & Information Sciences
*An International Journal*

http://dx.doi.org/10.18576/amis/140302

# Soliton Solutions of a Nonlinear Fractional Sasa-Satsuma Equation in Monomode Optical Fibers

*D. Baleanu*[1,2], *M. S. Osman*[3,4,*], *Asad Zubair*[5], *Nauman Raza*[5], *Omar Abu Arqub*[6] *and Wen-Xiu Ma*[7,8,9,10,11,12]

[1]Department of Mathematics, Cankaya University, Öğretmenler Cad. 1406530, Ankara, Türkiye
[2]Institute of Space Sciences, Magurele, Bucharest, Romania
[3]Department of Mathematics, Faculty of Science, Cairo University, Giza, Egypt
[4]Department of Mathematics, Faculty of Applied Science, Umm Alqurah University, Makkah 21955, Saudi Arabia
[5]Department of Mathematics, University of the Punjab, Quaid-e-Azam Campus, Lahore, Pakistan
[6]Department of Mathematics, Faculty of Science, The University of Jordan, Amman 11942, Jordan
[7]Department of Mathematics, Zhejiang Normal University, Jinhua 321004, China
[8]Department of Mathematics, King Abdulaziz University, Jeddah, Saudi Arabia
[9]Department of Mathematics and Statistics, University of South Florida, Tampa, FL 33620-5700, USA
[10]School of Mathematics, South China University of Technology, Guamgazhou 510640, China
[11]College of Mathematics and Systems Science, Shandong University of Science and Technology, Qingdao, Shandong 266590, China
[12]Department of Mathematical Sciences, North-West University, Mafikeng Campus, Private Bag X2046, Mmabatho 2735, South Africa

Received: 22 Dec. 2019 , Revised: 2 Jan. 2020, Accepted: 7 Feb. 2020
Published online: 1 May 2020

**Abstract:** This article is devoted to retrieving soliton solutions of a nonlinear Sasa-Satsuma equation governing the propagation of short light pulses in the monomode optical fibers using the effect of conformable fractional transformation. The Integrability is carried out by incorporating two versatile integration gadgets namely the first integral method and the generalized projective Riccati equation method. The resulting solutions include bright, dark, singular, periodic as well as rational solitons along with their existence criteria. Furthermore, the fractional behavior of the solutions is investigated comprehensively using graphs.

**Keywords:** Nonlinear Sasa-Satsuma equation, Conformable time fractional derivative, First integral method, Generalized projective Riccati equation method, Optical solitons

## 1 Introduction

The propagation of nonlinear waves in optical waveguide has recently grabbed the researchers' interest [1–4]. In dielectric fibers, the dynamic balance between optical Kerr effect and dispersion leads to the generation of optical soliton. The dispersion of the fiber material is used to control the bit rate of transmission. The only factor responsible for the drop of pulse quality is the fiber lose, for details see [5–10].
In optical fibers, solitons have gained much concentration because of their robust nature and powerful applications in all long-distance and optical communications [11–15]. The propagation of optical solitons in a monomode fiber has been explored as the nonlinear Schrödinger equation (NLSE) [16]. However, the propagation of ultrashort pulses in high-bit-rate transmission systems has been described by some higher order linear and nonlinear effects [11], in which the following nonlinear Sasa-Satsuma equation in monomode fibers is of great interest [17–21]:

$$q_x = i\left(a q_{tt} + b\,|\,q\,|^2\,q\right) + c q_{ttt} + d\left(|\,q\,|^2\,q\right)_t + e\left(|\,q\,|^2\right)_t q, \tag{1}$$

where $q$ is the complex valued function and the coefficients of $a,b,c,d$ and $e$ are group velocity dispersion, Kerr effect, third-order dispersion, self-steepening and stimulated Raman scattering, respectively.
Nowadays, fractional calculus is one of the most flourishing areas of mathematical analysis along with fractional operators, such as Caputo, Grunwald-Letnikov

* Corresponding author e-mail: mofatzi@sci.cu.edu.eg

and Riemann-Liouville [22]-[24]. These operators have also been used in hydraulics of dams, waves in liquids and gases, temperature field problem in oil strata, signal processing and diffusion problems [25]-[27]. Conformable time fractional derivative occupied the worthy place in fractional calculus. It was first introduced by khalil et al. [28] in 2014 and developed by Abdeljawad et al. [29] in 2015.

Nonlinear partial differential equations (NPDEs) have been found in the fields of engineering and physics. Exact solutions of NPDEs play an important role in understanding the universe, and there are many direct approaches to exact solutions (see, e.g., [30–32]). The transformed rational function method [31] unifies many existing approaches for constructing traveling wave solutions, which include the tanh function method [33–35], the Jacobi elliptic function method [36–39], the homogeneous balance method [40], the F-expansion method [41], the sub-ODE method [42], the exp function method [43], the unified method [44–46], the $(G'/G)$ expansion method [47], and the generalized unified method [48–50].

The first integral method (FIM) [51], which was first proposed in 2002, is a powerful tool for computing exact traveling wave solutions [52, 53]. For autonomous planar system, FIM gives polynomial first integrals.

Numerous solitary wave solutions of NPDEs [54] can be expressed as a polynomial in two elementary functions that satisfy a projective Riccati system [55]. Recently, Yan [56] has developed Conte's method and presented a generalization of the projective Riccati system, which is known as a generalized projective Riccati equation method (GPREM).

The present paper aims to construct soliton solutions for the following conformable time fractional Sasa-Satsuma equation in monomode optical fibers using FIM and GPREM:

$$q_x = i\left(aD_t^{2\gamma}(q) + b\mid q\mid^2 q\right) + cD_t^{3\gamma}(q) + dD_t^{\gamma}\left(\mid q\mid^2 q\right) + eD_t^{\gamma}\left(\mid q\mid^2\right)q, \tag{2}$$

where $D_t$ is the conformable time fractional operator.

The rest of the paper is organized as follows: In Section Two, we describe the conformable fractional derivative. In Section Three, we apply a traveling wave transformation to our proposed model and extract soliton solutions of Eq. (2) via FIM and GPREM. In Section Four, the physical appearance of the results is addressed.

## 2 Conformable fractional derivative

**Definition 2.1** Given a function $V : [0,\infty) \to \mathbb{R}$, then the conformable fractional derivative of $V$ of order $\gamma$ is defined as [28]

$$D_t^{\gamma}V(t) = \lim_{\sigma\to 0}\frac{V\left(t+\sigma t^{1-\gamma}\right) - V(t)}{\sigma}, \ \forall t>0, \gamma\in(0,1]. \tag{3}$$

**Theorem 2.1** [28] Let $\gamma \in (0,1]$, and $U,V$ be $\gamma$-differentiable at a point $t>0$. Then

1. $D_t^{\gamma}(\alpha U + \beta V) = \alpha D_t^{\gamma}(U) + \beta D_t^{\gamma}(V), \quad \forall\ \alpha,\beta\in\mathbb{R}.$
2. $D_t^{\gamma}(t^{\nu}) = \nu t^{\nu-\gamma}, \quad \forall\ \nu\in\mathbb{R}.$
3. $D_t^{\gamma}(UV) = UD_t^{\gamma}(V) + VD_t^{\gamma}(U).$
4. $D_t^{\gamma}\left(\frac{U}{V}\right) = \frac{VD_t^{\gamma}(U) - UD_t^{\gamma}(V)}{V^2}.$

## 3 Traveling wave solutions

We take the traveling wave transformation

$$q(x,t) = U(\xi)\exp(i\phi), \tag{4}$$

where

$$\xi = x - \lambda\frac{t^{\gamma}}{\gamma}, \quad \phi = kx + \omega\frac{t^{\gamma}}{\gamma} + \theta.$$

Applying the above-mentioned transformation to Eq. (2) and separating its real and imaginary parts, we get

$$-c\lambda^3U''' - (3d\lambda + 2e\lambda)U^2U' + (2a\lambda\omega + 3c\lambda\omega^2 - 1)U' = 0, \tag{5}$$

and

$$(a\lambda^2 + 3c\lambda^2)U'' + (b + d\omega)U^3 - (a\omega^2 + c\omega^3 + k)U = 0. \tag{6}$$

Integrating Eq. (5) w. r. t. $\xi$,

$$-c\lambda^3U'' - (3d\lambda + 2e\lambda)U^3 + (2a\lambda\omega + 3c\lambda\omega^2 - 1)U = 0. \tag{7}$$

From Eq. (6) and Eq. (7), we have

$$\frac{-c\lambda}{a\lambda^2 + 3c\lambda^2} = \frac{2a\lambda\omega + 3c\lambda\omega^2 - 1}{-(a\omega^2 + c\omega^3 + k)}, \tag{8}$$

It follows from Eq. (8) that

$$k = \frac{2c\lambda aw^2 - c^2\lambda w^3 + 2a^2\lambda w + 6a\lambda wc + 9c^2\lambda w^2}{c\lambda} - \frac{a+3c}{c\lambda}. \tag{9}$$

Substituting Eq. (9) into Eq. (6), we have

$$(a\lambda^2 + 3c\lambda^2)U'' - \left(a\omega^2 + c\omega^3 + \frac{2c\lambda aw^2 - c^2\lambda w^3 + 2a^2\lambda w + 6a\lambda wc + 9c^2\lambda w^2}{c\lambda} - \frac{a+3c}{c\lambda}\right)U + (b + d\omega)U^3 = 0. \tag{10}$$

### 3.1 Applying FIM

In order to solve Eq. (10) with the help of first integral method, we rewrite Eq. (10) in the following form

$$AU'' - BU + CU^3 = 0, \tag{11}$$

where

$$A = a\lambda^2 + 3c\lambda^2,$$

$$B = \frac{2c\lambda aw^2 - c^2\lambda w^3 + 2a^2\lambda w + 6a\lambda wc + 9c^2\lambda w^2}{c\lambda} - \frac{a+3c}{c\lambda} + a\omega^2 + c\omega^3,$$

$$C = b + d\omega. \tag{12}$$

Let

$$\psi(\xi) = U(\xi), \quad \varphi(\xi) = U'(\xi). \tag{13}$$

From Eq. (11) and Eq. (13), we have

$$\psi'(\xi) = \varphi(\xi), \quad \varphi'(\xi) = \frac{B}{A}\psi - \frac{C}{A}\psi^3. \tag{14}$$

According to the FIM, we suppose that $\psi(\xi)$ and $\varphi(\xi)$ are nontrivial solutions of Eq. (14) and $Q(\psi,\varphi) = \sum_{i=0}^{m} a_i(\psi)\varphi^i(\xi)$ is an irreducible polynomial in the complex domain $C[\psi,\varphi]$, such that

$$Q(\psi,\varphi) = \sum_{i=0}^{m} a_i(\psi)\varphi^i(\xi) = 0, \tag{15}$$

where $a_i(\psi)\,(i = 1,2,\ldots,m)$ are polynomials in $\psi$ and $a_m(\psi) \neq 0$. According to the Division Theorem, there exists a polynomial $[g(\psi) + h(\psi)\varphi(\xi)]$ in $C[\psi,\varphi]$ such that

$$\frac{dQ}{d\xi} = \frac{\partial Q}{\partial\psi}\frac{d\psi}{d\xi} + \frac{\partial Q}{\partial\psi}\frac{d\psi}{d\xi} = \left[g(\psi) + h(\psi)\varphi(\xi)\right] \times \sum_{i=0}^{m} a_i(\psi)\varphi^i(\xi). \tag{16}$$

Here, we will take the case for $m = 2$. For this purpose we will substitute Eq. (14) into Eq. (16), which provides a polynomial in $\varphi^i\,(i = 0,1,2,3)$ whose coefficients yield a set of algebraic equations

$$\varphi^3 : \quad \frac{da_2(\psi)}{d\psi} = h(\psi)a_2(\psi), \tag{17}$$

$$\varphi^2 : \quad \frac{da_1(\psi)}{d\psi} = g(\psi)a_2(\psi) + h(\psi)a_1(\psi), \tag{18}$$

$$\varphi^1 : \quad \frac{da_0(\psi)}{d\psi} + 2a_2(\psi)\left(\frac{B\psi}{A} - \frac{C\psi^3}{A}\right) = g(\psi)a_1(\psi) + h(\psi)a_0(\psi), \tag{19}$$

$$\varphi^0 : \quad g(\psi)a_0(\psi) = a_1(\psi)\left(\frac{B\psi}{A} - \frac{C(\psi)^3}{A}\right). \tag{20}$$

Since $a_i(\psi)\,(i = 0,1,2)$ are polynomials, from Eq. (17) we deduce that $a_2(\psi)$ is a constant and $h(\psi) = 0$. For simplicity we take $a_2(\psi) = 1$. Balancing the degrees of $g(\psi), a_1(\psi)$ and $a_0(\psi)$, we conclude that $\deg(g(\psi)) = 1$. Suppose that

$$g(\psi) = A_1\psi + A_0. \tag{21}$$

From Eq. (18) and Eq. (21), we get

$$a_1(\psi) = \frac{1}{2}A_1\psi^2 + A_0\psi + B_0, \tag{22}$$

and substituting Eq. (21) and Eq. (22) into Eq. (19) and integrating it w. r. t. $\xi$, $\psi(\xi)$ lead to

$$a_0(\psi) = \left(\frac{1}{2}\frac{C}{A} + \frac{1}{8}{A_1}^2\right)\psi^4 + \frac{1}{2}A_0A_1\psi^3 + \left(-\frac{B}{A} + \frac{1}{2}{A_0}^2 + \frac{1}{2}A_1B_0\right)\psi^2 + A_0B_0\psi + l, \tag{23}$$

where $l$ is a constant of integration. Substituting Eqs. (21)-(23) into Eq. (20) yields a polynomial in terms of $\psi^i(\xi)$. Equating the coefficients of the polynomial to zero provides a set of algebraic equations

$$\psi^5 : \; {A_1}^3A + 8CA_1 = 0, \tag{24}$$

$$\psi^4 : \; 12CA_0 + 5{A_1}^2AA_0 = 0, \tag{25}$$

$$\psi^3 : \; 8{A_0}^2A_1A + 4A{A_1}^2B_0 - 12BA_1 + 8B_0C = 0, \tag{26}$$

$$\psi^2 : \; -16BA_0 + 12AA_1B_0A_0 + 4A{A_0}^3 = 0, \tag{27}$$

$$\psi^1 : \; 8{A_0}^2B_0A + 8lAA_1 - 8B_0B = 0, \tag{28}$$

$$\psi^0 : \; lA_0 = 0. \tag{29}$$

Solving the system (24)-(29), we have

$$A_1 = 2\sqrt{-\frac{2C}{A}}, \quad A_0 = 0, \quad B_0 = -\frac{B\sqrt{-\frac{2C}{A}}}{C}, \quad l = -\frac{B^2}{2AC}, \tag{30}$$

where $\frac{C}{A} < 0$, which leads to the condition

$$(b + d\omega) < \lambda^2(a + 3c). \tag{31}$$

Using Eq. (30) in Eqs. (22) and (23), we have

$$a_0(\psi) = -\frac{C}{2A}\psi^4 + \frac{B}{A}\psi^2 - \frac{B^2}{2AC}, \tag{32}$$

$$a_1(\psi) = \sqrt{-\frac{2C}{A}}\psi^2 - \frac{B}{C}\sqrt{-\frac{2C}{A}}. \tag{33}$$

From Eq. (15), Eq. (32) and Eq. (33), we have

$$U'(\xi) = -\frac{1}{2}\sqrt{-\frac{2C}{A}}U^2(\xi) + \frac{B}{2C}\sqrt{-\frac{2C}{A}}. \tag{34}$$

Solving Eq. (34), we have the following optical dark soliton solution of Eq. (2):

$$q_1(x,t) = \sqrt{\frac{B}{C}}\tanh\left[\frac{\sqrt{2BC}}{C}\sqrt{-\frac{C}{A}}\left(x - \lambda\frac{t^\gamma}{\gamma} + R_0\right)\right] \times \tag{35}$$

where $R_0$ is a constant of integration.

### 3.2 Applying GPREM

In order to solve Eq. (10) with the help of GPREM, a solution of Eq. (10) is taken as [57]

$$U(\xi) = a_0 + \sum_{j=0}^{N} \mu^{j-1}(\xi)(a_j\mu(\xi) + b_j\nu(\xi)). \tag{36}$$

In Eq. (10), the homogeneous balance principle gives $N = 1$. It follows from Eq. (36) that

$$U(\xi) = a_0 + a_1\,\mu(\xi) + b_1\,\nu(\xi), \tag{37}$$

where $\mu(\xi)$ and $\nu(\xi)$ satisfy the projective Riccati system (PRS)

$$\begin{cases} \mu'(\xi) = \varepsilon\,\mu(\xi)\,\nu(\xi), \\ \nu'(\xi) = \varepsilon\,\nu^2(\xi) - m\,\mu(\xi) + R, \end{cases} \tag{38}$$

and a first integral of PRS is given by

$$\nu^2(\xi) = -\varepsilon\left(R - 2m\,\mu(\xi) + \frac{m^2-1}{R}\mu^2(\xi)\right). \tag{39}$$

Substituting Eqs. (37), (38) and (39) in Eq. (10) provides a polynomial in $(\mu^i(\xi), \nu^j(\xi))$ whose coefficients, being equal to zero, yield a set of algebraic equations

$$\begin{aligned}\mu^3 : &-\frac{6\lambda^3\varepsilon^3c^2a_1\left(m^2-1\right)}{R} + c\lambda\,ba_1{}^3 + c\lambda\,d\omega a_1{}^3 \\ &-\frac{2\lambda^3\varepsilon^3caa_1\left(m^2-1\right)}{R} - \frac{3c\lambda\,ba_1b_1{}^2\varepsilon\left(m^2-1\right)}{R} \\ &-\frac{3c\lambda\,d\omega a_1b_1{}^2\varepsilon\left(m^2-1\right)}{R} = 0,\end{aligned} \tag{40}$$

$$\begin{aligned}\mu^2 : &\ 12\ \lambda^3\varepsilon^3c^2a_1m - \lambda^3\varepsilon\,caa_1m + 4\lambda^3\varepsilon^3caa_1m \\ &+ 6c\lambda\,ba_1b_1{}^2\varepsilon m - \frac{3c\lambda\,ba_0b_1{}^2\varepsilon\left(m^2-1\right)}{R} \\ &+ 3c\lambda\left(ba_0a_1{}^2 + d\omega\,a_0a_1{}^2 + 2d\omega\,a_1b_1{}^2\varepsilon\,m\right) \\ &- \frac{3c\lambda\,d\omega\,a_0b_1{}^2\varepsilon\left(m^2-1\right)}{R} - 3\lambda^3\varepsilon\,c^2a_1m = 0,\end{aligned} \tag{41}$$

$$\begin{aligned}\mu : &\ 3\ \lambda^3\varepsilon\,c^2a_1R - 3c\lambda\,a\omega^2a_1 - 2a^2\lambda\,\omega\,a_1 + a\,a_1 \\ &- 6\lambda^3\varepsilon^3c^2a_1R - 3c\lambda\,ba_1b_1{}^2\varepsilon R - 2\lambda^3\varepsilon^3caa_1R \\ &+ 3ca_1 + \lambda^3\varepsilon\,caa_1R + 6c\lambda\,ba_0b_1{}^2\varepsilon\,m + 3c\lambda\,ba_0{}^2a_1 \\ &- 3c\lambda\,d\omega\,a_1b_1{}^2\varepsilon R + 3c\lambda\,d\omega a_0{}^2a_1 - 6a\lambda\,\omega\,ca_1 \\ &+ 6c\lambda\,d\omega\,a_0b_1{}^2\varepsilon\,m - 9c^2\lambda\,\omega^2a_1 = 0,\end{aligned} \tag{42}$$

$$\begin{aligned}\mu^2\nu : &\ 3\ c\lambda\,ba_1{}^2b_1 - \frac{6\lambda^3\varepsilon^3c^2b_1\left(m^2-1\right)}{R} + 3c\lambda\,d\omega a_1{}^2b_1 \\ &- \frac{c\lambda\,d\omega b_1{}^3\varepsilon\left(m^2-1\right)}{R} - \frac{2\lambda^3\varepsilon^3cab_1\left(m^2-1\right)}{R} \\ &- \frac{c\lambda\,bb_1{}^3\varepsilon\left(m^2-1\right)}{R} = 0,\end{aligned} \tag{43}$$

$$\begin{aligned}\mu\nu : &\ 4\ \lambda^3\varepsilon^3cab_1m + 6c\lambda\,ba_0a_1b_1 + 2c\lambda\,d\omega b_1{}^3\varepsilon\,m \\ &+ 2c\lambda\,bb_1{}^3\varepsilon\,m - 9\lambda^3\varepsilon\,c^2b_1m - 3\lambda^3\varepsilon\,cab_1m \\ &+ 12\lambda^3\varepsilon^3c^2b_1m + 6c\lambda\,d\omega\,a_0a_1b_1 = 0,\end{aligned} \tag{44}$$

$$\begin{aligned}\nu : &-c\lambda\,bb_1{}^3\varepsilon R + 3c\lambda\,d\omega a_0{}^2b_1 + ab_1 - 6\lambda^3\varepsilon^3c^2b_1R \\ &- 3c\lambda\,a\omega^2b_1 - 2a^2\lambda\,\omega\,b_1 + 6\lambda^3\varepsilon\,c^2b_1R \\ &- 2\lambda^3\varepsilon^3cab_1R - 9c^2\lambda\,\omega^2b_1 + 3cb_1 - c\lambda\,d\omega b_1{}^3\varepsilon R \\ &- 6a\lambda\,\omega\,cb_1 + 3c\lambda\,ba_0{}^2b_1 + 2\lambda^3\varepsilon\,cab_1R = 0,\end{aligned} \tag{45}$$

$$\begin{aligned}\text{Const.} : &\ 2\ \lambda\,\omega^2a_0 + c\lambda\,ba_0{}^3 - 3c\lambda\,a\omega^2a_0 + 3ca_0 \\ &- 6a\lambda\,\omega\,ca_0 - 3c\lambda\,ba_0b_1{}^2\varepsilon R + aa_0 \\ &- 3c\lambda\,d\omega\,a_0b_1{}^2\varepsilon R + c\lambda\,d\omega a_0{}^3 = 0.\end{aligned} \tag{46}$$

Solving the system (40)-(46) with the aid of Maple software, we obtain the subsequent cases.

**Case 1.**

$$\begin{aligned}&R = \frac{1}{2}\frac{2a\lambda\,\omega + 3c\lambda\,\omega^2 - 1}{\lambda^3\varepsilon\,c},\ \lambda = \lambda,\ m = 0,\ a_0 = 0, \\ &a_1 = 0,\ b_1 = \sqrt{-\frac{6c+2a}{b+d\omega}}\varepsilon\,\lambda,\ \varepsilon = \pm 1.\end{aligned} \tag{47}$$

**Case 2.**

$$\begin{aligned}&R = 2\frac{2a\lambda\,\omega + 3c\lambda\,\omega^2 - 1}{\lambda^3\varepsilon\,c},\ \lambda = \lambda,\ m = 1,\ a_0 = 0, \\ &a_1 = 0,\ b_1 = \sqrt{\frac{-4\varepsilon^2a + 9c + 3a - 12\varepsilon^2c}{2d\omega + 2b}}\lambda,\ \varepsilon = \pm 1.\end{aligned} \tag{48}$$

**Case 3.**

$$\begin{aligned}&R = -\frac{2a\lambda\,\omega + 3c\lambda\,\omega^2 - 1}{c\left(-1+2\varepsilon^2\right)\lambda^3\varepsilon},\ m = 0,\ b_1 = 0, \\ &a_1 = \sqrt{\frac{\varepsilon^4\lambda^4(4\lambda\,\varepsilon^2ca + 12\lambda\,\varepsilon^2c^2 - 2c\lambda\,a - 6c^2\lambda)}{2a\lambda(\omega b + \omega^2 d) - b - d\omega + 3c\lambda\,\omega^2(b+\omega d)}}, \\ &a_0 = 0,\ \varepsilon = \pm 1.\end{aligned} \tag{49}$$

Using Eqs. (47)-(49), we obtain the subsequent families of soliton solutions of Eq. (2).

*Family 1.* Optical dark and singular soliton solutions

$$q_2(x,t) = \tag{50}$$

$$\sqrt{-\frac{(6c+2a)(2a\lambda\,\omega + 3c\lambda\,\omega^2 - 1)}{2c\lambda}} \times \tanh\left(\sqrt{-\frac{2a\lambda\,\omega + 3c\lambda\,\omega^2 - 1}{2\lambda^3c}}\,\xi\right) \times e^{i\phi}, \tag{51}$$

$$q_3(x,t) = \tag{52}$$

$$\sqrt{-\frac{(6c+2a)(2a\lambda\,\omega + 3c\lambda\,\omega^2 - 1)}{2c\lambda}} \times \coth\left(\sqrt{-\frac{2a\lambda\,\omega + 3c\lambda\,\omega^2 - 1}{2\lambda^3c}}\,\xi\right) \times e^{i\phi}, \tag{53}$$

where $\xi = x - \lambda \frac{t^\gamma}{\gamma}$ and $\phi = kx + \omega \frac{t^\gamma}{\gamma} + \theta$.

*Family 2.* Optical periodic wave solutions

$$q_4(x,t) = \tag{54}$$

$$\sqrt{-\frac{(6c+2a)(2a\lambda\omega+3c\lambda\omega^2-1)}{2c\lambda}} \times \tan\left(\sqrt{\frac{2a\lambda\omega+3c\lambda\omega^2-1}{2\lambda^3 c}}\,\xi\right) \times e^{i\phi}, \tag{55}$$

$$q_5(x,t) = \tag{56}$$

$$-\sqrt{-\frac{(6c+2a)(2a\lambda\omega+3c\lambda\omega^2-1)}{2c\lambda}} \times \cot\left(\sqrt{\frac{2a\lambda\omega+3c\lambda\omega^2-1}{2\lambda^3 c}}\,\xi\right) \times e^{i\phi}, \tag{57}$$

where $\xi = x - \lambda \frac{t^\gamma}{\gamma}$ and $\phi = kx + \omega \frac{t^\gamma}{\gamma} + \theta$.

*Family 3.* Optical soliton solutions

$$q_6(x,t) = \tag{58}$$

$$\sqrt{-\frac{(4a\lambda\omega+6c\lambda\omega^2-1)(4a-9c-3a+12c)}{\lambda c(2d\omega+2b)}} \times \frac{\tanh\left(\sqrt{\frac{4a\lambda\omega+6c\lambda\omega^2-1}{\lambda^3 c}}\,\xi\right)}{\operatorname{sech}\left(\sqrt{\frac{4a\lambda\omega+6c\lambda\omega^2-1}{\lambda^3 c}}\,\xi\right)+1} \times e^{i\phi}, \tag{59}$$

$$q_7(x,t) = \tag{60}$$

$$\sqrt{-\frac{(4a\lambda\omega+6c\lambda\omega^2-1)(4a-9c-3a+12c)}{\lambda c(2d\omega+2b)}} \times \frac{\coth\left(\sqrt{\frac{4a\lambda\omega+6c\lambda\omega^2-1}{\lambda^3 c}}\,\xi\right)}{\operatorname{csch}\left(\sqrt{\frac{4a\lambda\omega+6c\lambda\omega^2-1}{\lambda^3 c}}\,\xi\right)+1} \times e^{i\phi}, \tag{61}$$

where $\xi = x - \lambda \frac{t^\gamma}{\gamma}$ and $\phi = kx + \omega \frac{t^\gamma}{\gamma} + \theta$.

*Family 4.* Optical periodic wave solutions

$$q_8(x,t) = \tag{62}$$

$$\sqrt{\frac{(4a\lambda\omega+6c\lambda\omega^2-1)(4a-9c-3a+12c)}{\lambda c(2d\omega+2b)}} \times \frac{\tan\left(\sqrt{-\frac{4a\lambda\omega+6c\lambda\omega^2-1}{\lambda^3 c}}\,\xi\right)}{\sec\left(\sqrt{-\frac{4a\lambda\omega+6c\lambda\omega^2-1}{\lambda^3 c}}\,\xi\right)+1} \times e^{i\phi}, \tag{63}$$

$$q_9(x,t) = \tag{64}$$

$$-\sqrt{\frac{(4a\lambda\omega+6c\lambda\omega^2-1)(4a-9c-3a+12c)}{\lambda c(2d\omega+2b)}} \times \frac{\cot\left(\sqrt{-\frac{4a\lambda\omega+6c\lambda\omega^2-1}{\lambda^3 c}}\,\xi\right)}{\csc\left(\sqrt{-\frac{4a\lambda\omega+6c\lambda\omega^2-1}{\lambda^3 c}}\,\xi\right)+1} \times e^{i\phi}, \tag{65}$$

where $\xi = x - \lambda \frac{t^\gamma}{\gamma}$ and $\phi = kx + \omega \frac{t^\gamma}{\gamma} + \theta$.

*Family 5.* Optical bright and formal soliton solutions

$$q_{10}(x,t) = \tag{66}$$

$$\sqrt{\frac{\lambda^2(4a+12c-2a-6c)(2a\lambda\omega+3c\lambda\omega^2-1)}{(2a\lambda\omega(b+\omega d)-b-d\omega+3c\lambda\omega^2(b+\omega d))}} \times \operatorname{sech}\left(\sqrt{\frac{2a\lambda\omega+3c\lambda\omega^2-1}{c\lambda^3}}\,\xi\right) \times e^{i\phi}, \tag{67}$$

$$q_{11}(x,t) = \tag{68}$$

$$\sqrt{\frac{\lambda^2(4a+12c-2a-6c)(2a\lambda\omega+3c\lambda\omega^2-1)}{(2a\lambda\omega(b+\omega d)-b-d\omega+3c\lambda\omega^2(b+\omega d))}} \times \operatorname{csch}\left(\sqrt{\frac{2a\lambda\omega+3c\lambda\omega^2-1}{c\lambda^3}}\,\xi\right) \times e^{i\phi}, \tag{69}$$

where $\xi = x - \lambda \frac{t^\gamma}{\gamma}$ and $\phi = kx + \omega \frac{t^\gamma}{\gamma} + \theta$.

*Family 6.* Optical periodic wave solutions

$$q_{12}(x,t) = \tag{70}$$

$$\sqrt{\frac{\lambda^2(-4a-12c+2a+6c)(2a\lambda\omega+3c\lambda\omega^2-1)}{(2a\lambda\omega(b+\omega d)-b-d\omega+3c\lambda\omega^2(b+\omega d))}} \times \sec\left(\sqrt{-\frac{2a\lambda\omega+3c\lambda\omega^2-1}{c\lambda^3}}\,\xi\right) \times e^{i\phi}, \tag{71}$$

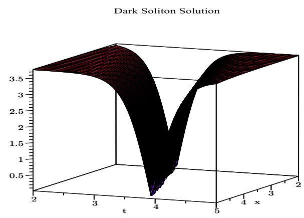
$$q_{13}(x,t) = \tag{72}$$

$$\sqrt{\frac{\lambda^2(-4a-12c+2a+6c\lambda)(2a\lambda\omega+3c\lambda\omega^2-1)}{(2a\lambda\omega(b+\omega d)-b-d\omega+3c\lambda\omega^2(b+\omega d))}} \times \csc\left(\sqrt{-\frac{2a\lambda\omega+3c\lambda\omega^2-1}{c\lambda^3}}\,\xi\right) \times e^{i\phi}, \tag{73}$$

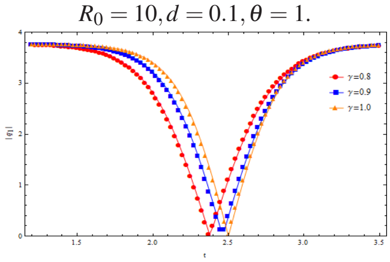
where $\xi = x - \lambda \frac{t^\gamma}{\gamma}$ and $\phi = kx + \omega \frac{t^\gamma}{\gamma} + \theta$.

## 4 Results and Discussion

In this section, the physical aspects of the obtained solutions will be discussed by means of graphical 3D and 2D representations. Fig.1(a) shows the intensity of the dark soliton solution in Eq. (35) obtained by FIM with parameters $a = 0.9, b = -0.9, c = -0.1, \lambda = 4, \omega = 1.5, \gamma = 0.8, R_0 = 10, d = 0.1, \theta = 1$ and Fig.1(b) represents the conformable fractional effects of the solution Eq. (35) at $x = 0$ with the same choice of parameters as in Fig.1(a). In Fig.1(b) various curves are reported against the fractional parametric values 0.8, 0.9, 1.0. The intensity relation of dark soliton obtained by GPREM in Eq. (50) is depicted in Fig.2(a) with parameters $a = 0.5, b = -1, c = -0.1, \lambda = 4, \omega = 3, \gamma = 1, \varepsilon = -1, d = 0.1, \theta = 1$ and the analysis of fractional parameter is carried out with the help of Fig.2(b) for the same choice of parameters as in Fig.2(a).

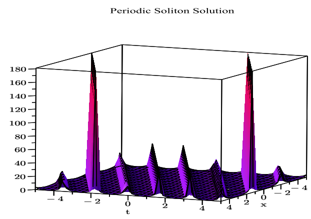



(a) $|q_1|, a = 0.9, b = -0.9, c = -0.1, \lambda = 4, \omega = 1.5, \gamma = 0.8,$ $R_0 = 10, d = 0.1, \theta = 1.$

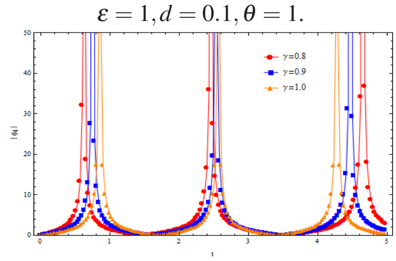



(b) $|q_1|, a = 0.9, b = -0.9, c = -0.1, \lambda = 4, \omega = 1.5, R_0 = 10,$ $d = 0.1, \theta = 1, x = 0.$

**Fig. 1:** 3D and 2D graphical representation of dark soliton solution obtained by FIM.



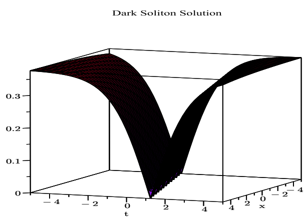

(a) $|q_4|, a = 0.5, b = -1, c = -0.1, \lambda = 2, \omega = 3.5, \gamma = 1,$ $\varepsilon = 1, d = 0.1, \theta = 1.$



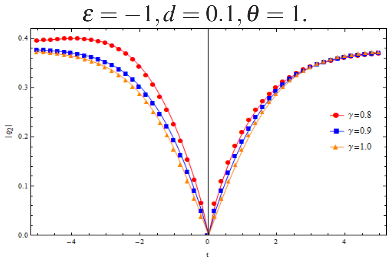

(b) $|q_4|, a = 0.5, b = -1, c = -0.1, \lambda = 2, \omega = 3.5, \varepsilon = 1,$ $d = 0.1, \theta = 1, x = 0.$

**Fig. 3:** 3D and 2D graphical representation of periodic soliton solution obtained by GPREM.

The intensity profiles of periodic, traveling wave and bright solutions obtained by GPREM given in Eqs. (54), (58), (62), (66) and (70) are shown Fig.3(a), Fig.4(a), Fig.5(a), Fig.6(a) and Fig.7(a) respectively and the effects of conformable fractional parameter on these solutions with $x = 0$ is highlighted through Fig.3(b), Fig.4(b), Fig.5(b), Fig.6(b) and Fig.7(b).

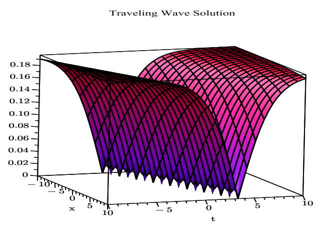



(a) $|q_2|, a = 0.5, b = -1, c = -0.1, \lambda = 4, \omega = 3, \gamma = 1,$ $\varepsilon = -1, d = 0.1, \theta = 1.$

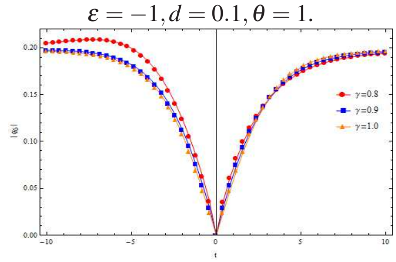



(b) $|q_2|, a = 0.5, b = -1, c = -0.1, \lambda = 4, \omega = 3, \varepsilon = -1,$ $d = 0.1, \theta = 1, x = 0.$

**Fig. 2:** 3D and 2D graphical representation of dark soliton solution obtained by GPREM.

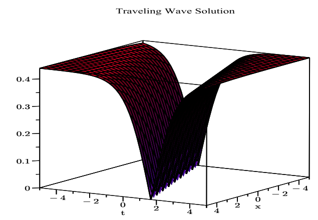



(a) $|q_6|, a = 0.5, b = -1, c = -0.1, \lambda = 3, \omega = 0.4, \gamma = 1,$ $\varepsilon = -1, d = 0.1, \theta = 1.$

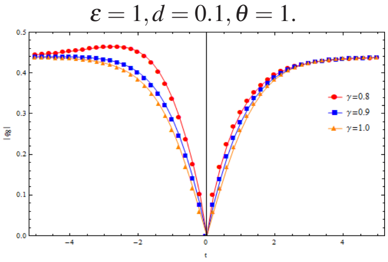



(b) $|q_6|, a = 0.5, b = -1, c = -0.1, \lambda = 3, \omega = 0.4, \varepsilon = -1,$ $d = 0.1, \theta = 1, x = 0.$

**Fig. 4:** 3D and 2D graphical representation of traveling wave solution obtained by GPREM.

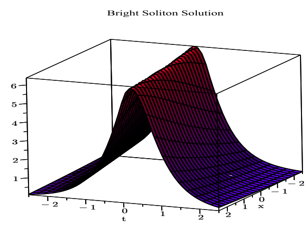



(a) $|q_8|, a = 0.5, b = -1, c = -0.1, \lambda = 3, \omega = 0.5, \gamma = 1, \varepsilon = 1, d = 0.1, \theta = 1.$

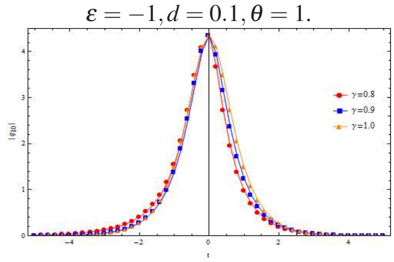



(b) $|q_8|, a = 0.5, b = -1, c = -0.1, \lambda = 3, \omega = 0.5, \varepsilon = 1, d = 0.1, \theta = 1, x = 0.$

**Fig. 5:** 3D and 2D graphical representation of traveling wave solution obtained by GPREM.

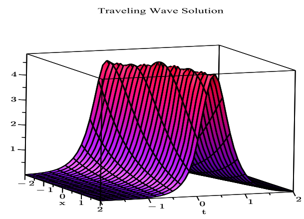



(a) $|q_{10}|, a = 0.5, b = 1, c = -0.1, \lambda = 8, \omega = 3.5, \gamma = 0.9, \varepsilon = -1, d = 0.1, \theta = 1.$

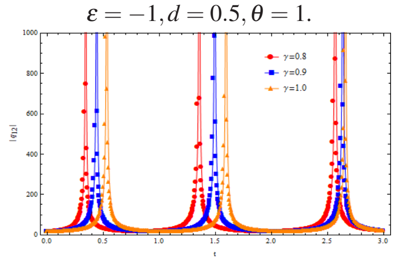



(b) $|q_{10}|, a = 0.5, b = 1, c = -0.1, \lambda = 8, \omega = 3.5, \varepsilon = -1, d = 0.1, \theta = 1, x = 0.$

**Fig. 6:** 3D and 2D graphical representation of bright soliton solution obtained by GPREM.




(a) $|q_{12}|, a = 0.7, b = 1, c = 1, \lambda = 9, \omega = 1.5, \gamma = 1, \varepsilon = -1, d = 0.5, \theta = 1.$




(b) $|q_{12}|, a = 0.7, b = 1, c = 1, \lambda = 9, \omega = 1.5, \varepsilon = -1, d = 0.5, \theta = 1, x = 0.$

**Fig. 7:** 3D and 2D graphical representation of periodic soliton solution obtained by GPREM.

**Remark.** By thoroughly looking at the literature, our findings are all novel and not previously published.

## 5 Conclusion

A conformable fractional nonlinear Sasa-Satsuma equation governing the propagation of short light pulses in the monomode optical fibers was investigated using FIM and GPREM. FIM was utilized to extract dark soliton solutions and GPREM provides dark, singular, and periodic soliton solutions. Those physical features of the resulting solutions were studied by their graphical portraits. The conformable time fractional effect on the dispersion was also analyzed. It is expected that the results of this paper would be useful in the empirical application of fiber optics.

## References

[1] M.S. Osman, D. Lu, M.M.A. Khater, R.A.M. Attia, Complex wave structures for abundant solutions related to the complex Ginzburg-Landau model, *Optik*, **192**, 162927 (2019).

[2] J.G. Liu, M.S. Osman, W.H. Zhu, L. Zhou, G.P. Ai, Different complex wave structures described by the Hirota equation with variable coefficients in inhomogeneous optical fibers, *Appl. Phys. B*, **125**, 175 (2019).

[3] M.S. Osman, D. Lu, M.M. Khater, A study of optical wave propagation in the nonautonomous Schrödinger-Hirota equation with power-law nonlinearity, *Results Phys.*, **13**, 102157 (2019).

[4] Y. Ding, M.S. Osman, A.M. Wazwaz, Abundant complex wave solutions for the nonautonomous Fokas-Lenells equation in presence of perturbation terms, *Optik*, **181**, 503–513 (2019).

[5] M. Gedalin, T.C. Scott, Y.B. Band, Optical solitary waves in the higher order nonlinear schrödinger equation, *Phys. Rev. Lett.*, **78**, 448–451 (1997).

[6] A.R. Seadawy and D. Lu, Bright and dark solitary wave soliton solutions for the generalized higher order nonlinear Schrödinger equation and its stability, *Results Phys.*, **7**, 43–48 (2017).

[7] H. Zhong, B. Tian, M. Li, W.R. Sun, H.L. Zhen, Stochastic dark solitons for a higher-order nonlinear Schrödinger equation in the optical fiber, *J. Mod. Opt.*, **60**, 1644–1651 (2013).

[8] H. Kumar and F. Chand, Dark and bright solitary wave solutions of the higher order nonlinear Schrödinger equation with self-steepening and self-frequency shift effects, *J. Nonlinear Opt. Phys.*, **22**, 1350001 (2013).

[9] A.R. Seadawy, Exact solutions of a two-dimensional nonlinear Schödinger equation, *Appl. Math. Lett.*, **25**, 687–691 (2012).

[10] S.F. Tian, The mixed coupled nonlinear Schrödinger equation on the half-line via the Fokas method, *P. Roy. Soc. A-Math. Phy.*, **472**, 20160588 (2016).

[11] G.P. Agarwal, Nonlinear fiber optics. In Nonlinear Science at the Dawn of the 21st Century, *Springer*, Berlin, Heidelberg, 195-211, (2000).

[12] A. Zubair, N. Raza, M. Mirzazadeh, W. Liu, Q. Zhou, Analytic study on optical solitons in parity-time-symmetric mixed linear and nonlinear modulation lattices with non-Kerr nonlinearities, *Optik*, **173**, 249–262 (2018).

[13] N. Raza and A. Zubair, Optical dark and singular solitons of generalized nonlinear Schrödinger's equation with anti-cubic law of nonlinearity, *Mod. Phys. Lett. B*, **65**, 1950158 (2019).

[14] N. Raza and A. Zubair, Dipole and Combo Optical Solitons in Birefringent Fibers in the Presence of Four-Wave Mixing, *Commun. Theor. Phys.*, **71**, 723-730 (2019).

[15] N. Raza and A. Javid, Generalization of optical solitons with dual dispersion in the presence of Kerr and quadratic-cubic law nonlinearities, *Mod. Phys. Lett. B*, **33**, 1850427 (2019).

[16] A. Hasegawa and F. Tappert, Trnsmission of stationary nonlinear optical pulses in dispersive dielectric fibers. II. Normal dispersion, *Appl. Phys. Lett.*, **23**, 171 (1973).

[17] K.U. Tariq and A.R. Seadawy, Optical soliton solutions of higher order nonlinear Schrödinger equation in monomode fibers and its applications, *Optik*, **154**, 785–798 (2018).

[18] A.R. Seadawy, Modulation instability analysis for the generalized derivative higher order nonlinear Schrödinger equation and its the bright and dark soliton solutions, *J. Electromagnet. Wave*,**31**, 1353–1362 (2017).

[19] D.J. Kaup and A.C. Newell, An exact solution for a derivative nonlinear Schrödinger equation, *J. Math. Phys.*, **19**, 798–801 (1978).

[20] M.J. Ablowitz and Z.H. Musslimani, Inverse scattering transform for the integrable nonlocal nonlinear Schrödinger equation, *Nonlinearity*, **29**, 9–15 (2016).

[21] A. Seadawy, The generalized nonlinear higher order of kdV equations from the higher order nonlinear Schrödinger equation and its solutions, *Optik*, **139**, 31–43 (2017).

[22] S. Jahanshahi, E. Babolian, D.F.M. Torres, A. Vahidi, Solving Abel integral equations of first kind via fractional calculus, *J. King Saud Univ.*, **27**, 161–167 (2015).

[23] J.T. Machado, V. Kiryakova, F. Mainardi, Recent history of fractional calculus, *Commun. Nonlinear Sci.*, **16**, 1140–1153 (2011).

[24] V.E. Tarasov, Lattice fractional calculus, *Appl. Math. Comput.*, **257**, 12–33 (2015).

[25] N. Benkhettou, A.M.C. Brito da Cruz, D.F.M. Torres, A fractional calculus on arbitrary time scales: fractional differentiation and fractional integration, *Signal Process.*, **107**, 230–237 (2015).

[26] L. Boyadjiev and R. Scherer, Fractional extensions of the temperature field problem in oil strata, *Kuwait J. Sci. Eng.*, **31**, 15–32 (2004).

[27] W.R. Schneider and W. Wyss, Fractional diffusion and wave equations, *J. Math. Phys.*, **30**, 134–144 (1989).

[28] R. Khalil, M. Al Horani, A. Yousef, M. Sababheh, A new definition of fractional derivative, *J. Comput. Appl. Math.*, **264**, 65–70 (2014).

[29] T. Abdeljawad, On conformable fractional calculus, *J. Comput. Appl. Math.*, **279**, 57–66 (2015).

[30] W.X. Ma and B. Fuchssteiner, Explicit and exact solutions to a Kolmogorov-Petrovskii-Piskunov equation, *Int. J. Nonlin. Mech.*, **31**, 329–338 (1996).

[31] W.X. Ma and J.H. Lee, A transformed rational function method and exact solutions to the 3+ 1 dimensional JimboMiwa equation, *Chaos Soliton. Fract.*, **42**, 1356–1363 (2009).

[32] W.X. Ma., T. Huang, Y. Zhang, A multiple exp-function method for nonlinear differential equations and its application, *Phys. Scr.*, **82**, 065003 (2010).

[33] E.J. Parkes and B.R. Duffy, An automated tanh-function method for finding solitary wave solutions to nonlinear evolution equation, *Comput. Phys. Commun.*, **98**, 288–300 (1996).

[34] E.G. Fan, Extended tanh-function method and its applications to nonlinear equations, *Phys. Lett. A*, **277**, 212–218 (2000).

[35] Z.Y. yan, New explicit traveling wave solutions for two new integrable coupled nonlinear evolution equations, *Phys. Lett. A*, **292**, 100–106 (2001).

[36] S. Liu and Z. Fu, Jacobi elliptic function expension method and periodic wave solutions of nonlinear wave equations, *Phys. Lett. A*, **289**, 69–74 (2001).

[37] Z.Y. Yan, Abundant families of Jacobi elliptic function solutions of the (2+1)-dimensional integrable Devey-Stewartson-type equation via a new method, *Chaos Soliton. Fract.*, **18**, 299–309 (2003).

[38] Z.T. Fu and S.K. Liu, A new approach to solve nonlinear wave equations, *Commun. Theor. Phys.*, **39**, 27–30 (2003).

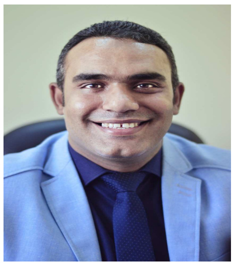
[39] N. Raza and A. Zubair, dark and dark-singular soliton solutions of nonlinear Schra"odinger's equation with spatio-temporal dispersion, *Mod. Phys. Lett. B*, **33**, 1975-1982 (2018).

[40] M.L. Wang, Solitary wave solutions for variant Boussinesq equations, *Phys. Lett. A*, **199**, 169–172 (1995).

[41] M.L. Wang and Y B. Zhou, The periodic wave solutions for the Klein-Gordon-Schrödinger equations, *Phys. Lett. A*, **318**, 84–92 (2003).

[42] X.Z. Li and M.L. Wang, A sub-ODE method for finding exact solutions of a generalized KdV ans mkdV equation with high order nonlinear terms, *Phys. Lett. A*, **361**, 115–118 (2007).
[43] J.H. He and X.H. Wu, Exp-function ethod for nonlinear wave equations, *Chaos Soliton. Fract.*, **30**, 700–708 (2006).
[44] M.S. Osman, B. Ghanbari, J.A.T. Machado, New complex waves in nonlinear optics based on the complex Ginzburg-Landau equation with Kerr law nonlinearity, *Eur. Phys. J. Plus*, **134**, 20 (2019).
[45] H.I. Abdel-Gawad, M. Tantawy, M.S. Osman, Dynamic of DNA's possible impact on its damage, *Math. Meth. Appl. Sci.*, **39**, 168–176 (2016).
[46] H.I. Abdel-Gawad and M. Osman, On shallow water waves in a medium with time-dependent dispersion and nonlinearity coefficients, *J. Adv. Res.*, **6**, 593–599 (2015).
[47] M. Wang and J. Zhang, X. Li, The $(G'/G)$-expension method and traveling wave solutions ofnonlinear evolution equations in mathematical physics, *Phys. Lett. A*, **372**, 417–423 (2008).
[48] M.S. Osman, One-soliton shaping and inelastic collision between double solitons in the fifth-order variable-coefficient Sawada-Kotera equation, *Nonlinear Dyn.*, **96**, 1491–1496 (2019).
[49] A. Javid, N. Raza, M.S. Osman, Multi-solitons of Thermophoretic Motion Equation Depicting the Wrinkle Propagation in Substrate-Supported Graphene Sheets, *Commun. Theor. Phys.*, **71**, 362 (2019).
[50] M.S. Osman, Multi-soliton rational solutions for quantum Zakharov-Kuznetsov equation in quantum magnetoplasmas, *Waves Random Complex Media*, **26**, 434–443 (2016).
[51] Z.S. Feng, The first integral method to study the Burgers-KdV equation, *J. Phys. A-Math. Theor.*, **35**, 343–349 (2002).
[52] B. Zheng, Traveling wave solution for some nonlinear evolution equations by the first integral method, *WSEAS Trans. Math.*, **8**, 249–258 (2011).
[53] N. Taghizadeh, M. Mirzazadeh, F. Farahrooz, *J. Math. Anal. Appl.*, **374**, 549–553 (2011).
[54] R. Conte and M. Musette, *J. Phys. A-Math. Theor.*, **25**, 5609–5623 (1992).
[55] T.C. Bountis, V. Papageorgiou, P. Winternitz, Exact solutions of the nonlinear Schrödinger equation by the first integral method, *J. Math. Phys.*, **27**, 1215 (1986).
[56] Y. Chen and B. Li, Link between solitary waves and projective Riccati equations, *Chaos Soliton. Fract.*, **19**, 977–984 (2004).
[57] H. Rezazadeh, A. Korkmaz, M. Eslami, J. Vahidi, R. Asghari, On the integrability of nonlinear ODEs with Superposition Principles, *Opt. Quant. Electron.*, **50**, 150 (2018).

**D. Baleanu** received a B.Sc. degree in Physics from the University of Craiova, Romania, in 1988, an M.Sc. degree from the University of Bucharest, Romania, in 1989, and a Ph.D. degree from the Institute of Atomic Physics, Romania, in 1996. He is Professor at the Institute of Space Sciences, Romania, and since 2000 he is visiting staff member at Cankaya University, Turkey. He published more than 1000 papers in journals indexed in SCI. He is a co-editor of many books published by Springer. He is coauthor of many books published by Elsevier and World Scientific. He is an editorial board member of many ISI journals and is on the 2015-2019 Highly Cited Researcher list in mathematics. His Hirsch index is 61.

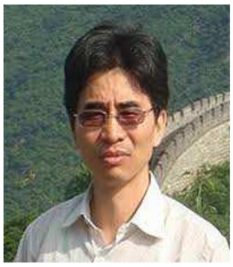


**M. S. Osman** received a B.Sc. degree in Mathematics from Cairo University, Egypt, in 1997 and M.Sc in 2006. He is an Associate Professor of Applied Mathematics at Department of Mathematics, Faculty of Science, Cairo University, Egypt. He published more than 50 papers in journals indexed in SCI.

**A. Zubair** is a PhD student in Department of Mathematics, University of the Punjab, Quaid-e-Azam Campus, Lahore, Pakistan. He published more than 5 papers in journals indexed in SCI.

**N. Raza** is an Assistant Professor in the Department of Mathematics at the University of the Punjab. He has M.Sc. degrees in Mathematics and Telecommunications and a Ph.D. in Mathematics. He also worked as a postdoctoral fellow in Department of Mathematics and Statistics McMaster University, Canada. He has also served the University of Engineering and Technology and University of Management and Technology as Assistant Professor. Dr. Nauman's research interests include scientific computing, computational fluid dynamics and nonlinear wave phenomenon. His research has been published in high impact journals including Journal of Computational Physics, Computers and Mathematics with Applications, Applied mathematics and Computation. He has participated and presented his research papers in many International and National conferences in his field. He has been acting as a referee of some reputed journals like Journal of Computational Physics, International Journal of Computational Methods etc.

**O. Abu-Arqub** is Professor of Mathematics in Department of Mathematics, Faculty of Science, The University of Jordan, Amman 11942, Jordan. He received his Ph.D. degree in the Applied Mathematics from the University of Jordan (Jordan) in 2009. His research interests focus on the following subjects: Numerical Analysis, Computational Optimization, Inverse Problems, Differential Inequalities, Fractional Calculus Theory, Fuzzy Calculus Theory, Stochastic Calculus Theory, Optimal Control Theory, Signal Processing Theory.



**W. Ma** Department of Mathematics and Statistics, University of South Florida, USA, Professor. Qualifications: 1990 Ph.D., Chinese Academy of Sciences, Beijing, Mathematical Physics. 1985 M.Sc., Chinese Academy of Sciences, Beijing, Applied Mathematics. 1982 B.Sc., University of Science and Technology, Hefei, Computational Mathematics