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Further Study on Sg^*- Closed Sets and Nsg^* Closed Sets in Nano Topological Spaces

R. Parimelazhagan and V. Jeyalakshmi*

Department of Mathematics, RVS Technical Campus, Coimbatore - 641 402, Tamilnadu, India

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Abstract: In this paper, we focused the set Strongly $g^*d_{\#}$, Strongly $\widetilde{d}^{\,0}$, Strongly $\widetilde{d}^{\,1}$, Strongly $\widetilde{d}^{\,2}$, Strongly g^*-kernel of A in topological spaces and studied some of their characters. We also introduce the Nano $Sg*$ closed sets and studied few properties.

Keywords: Strongly $g^*d_{\#}$, Strongly $\widetilde{d}^{\,0}$, Strongly $\widetilde{d}^{\,1}$, Strongly $\widetilde{d}^{\,2}$, Strongly g^*-kernel of A, Nano $Sg*$ closed set.

1 Introduction

Andrijevic [1] presented the thought of b-open sets in a topological space and obtained different properties. El-Etik [2] presented a similar idea for the sake of γ-open sets. El-Etik also introduced the concept of γ-continuous (b-continuous) functions with the guide of b-open sets. In 2004, Ekici and Caldas [3] presented the thought of marginally γ-progression (somewhat b-congruity) which is a debilitated type of b-coherence. In their paper, the author examined essential properties and safeguarding hypotheses of marginally b-continuous capacities. The connections of somewhat b-congruity with other weaker types of coherence have likewise been studied. The idea of generalized closed sets(briefly g-closed) in topological spaces was presented by Levine [4] and a class of topological spaces called $T_{\frac{1}{2}}$ spaces. Arya and Nour [5], Bhattacharya and Lahiri [6], Levine [7], Mashhour [8], Njastad [9] and Andrijevic [10], presented and examined summed up semi-open sets, semi summed up open sets, summed up open sets, semi-open sets, pre-open sets and α-open sets, semi pre-open sets and b-open sets which are portion of the feeble types of open sets and the complements of these sets are known as similar kinds of closed sets.

Tong ([11], [12]) has brought A-sets, B-sets and t-sets. A-sets and B-sets are also weak forms of open sets while t-sets is a susceptible form of a closed sets. Ganster and Reilly [13] have delivered locally closed sets, which are weaker than each open and closed sets. Cameron [14] has brought everyday semi-open sets that are weaker than ordinary open sets.

Generalized open sets play an essential role in eneral Topology. For a subset A of a topological region (X,τ), $Cl(A)$ and $Int(A)$ denote the closure of A and the interior of A, respectively. Wilansky [15] has delivered the concept of US regions. Aull [16] studied some separation axioms among the T_1 and T_2 spaces, specifically S_1 and S_2. Next, S. P. Arya et al.[17] have brought and studied the concept of semi-US areas inside the year 1982 and moreover they made observation of s-convergence, and sequentially semi-closed sets, sequentially s-compact notions. G. B. Navlagi studied P- normal Almost-P-normal and Mildly-P-normal areas. Closedness sets are easy idea for research in gaint topological regions. This concept has been generalized and studied by means of the use of many authors from one of kinds of factors of views. O.Njastad [9] added and defined an α-open and α-closed set. After the work of O.Njastad on α-open sets, numerous mathematicians have grown interests on the generalizations of numerous thoughts in topology with the resource of thinking about semi-open, α-open sets. The idea of g-closed [4], s-open [7] and α-open sets are of enormous position in the generalization of continuity in topological regions. The changed shape of those gadgets and generalized continuity are in addition advanced via many mathematicians ([18], [19], [20], [21], [8]). Many authors have attempted to weaken the situation

* Corresponding author e-mail: parimelazhagankce@gmail.com

closed in this theorem. In 1978, Long and Herrington [22] used opensets because of Singal [23]. Malghan [8] delivered the concept of generalized closed maps in topological areas. Devi [25] brought and studied sg-closed maps ,wg-closed maps and rwg-closed maps which had been brought and studied via Nagavani [26]. Regular closed maps, gpr-closed maps and rg-closed maps were introduced and studied Long [22], Gnanambal [27] and Arockiarani [20] respectively. In 2012, [28] we introduced the ideas of Strongly g^*-closed sets and Strongly g^*-open set in topological areas. Also we have brought the requirement of Strongly g^*-non-prevent abilities, Strongly g^*-irresolute functions, Strongly g^*-open maps and Strongly g^*-closed maps in the paper([29],).

Lellis Thivagar [30] introduced Nano topological space with respect to a subset X of a universe which is defined in terms of lower and upper approximations of X. The elements of Nano topology space are called Nano open sets. Bhuvaneswari [31] introduced Nano generalised closed sets in Nano topological space

The main aim of this paper is to introduce, Strongly $g^*d_\#$ set,the spaces called Strongly $\widetilde{d}^{\,0}$, Strongly $\widetilde{d}^{\,1}$, Strongly $\widetilde{d}^{\,2}$ in topological areas, the concept of Strongly g^*-kernel of A and studied some of their characters. Also we introduce few properties of Nsg^* closed sets in Nano topological spaces.

2 Preliminaries

In the course of this paper (X,τ) and (Y,σ) characterize topological spaces on which no separation axioms are assumed until or else observed. For a subset A of a space (X,τ), $cl(A)$, $int(A)$ and A^c denote the closure of A, the interior of A and the complement of A in X, respectively.

Definition 1. *A subset A of a topological space (X,τ) is referred to as*

(a) a preopen set [8] if $A \subseteq int(cl(A))$ and pre-closed set if $cl(int(A)) \subseteq A$.

(b) a semiopen set [7] if $A \subseteq cl(int(A))$ and semi closed set if $int(cl(A)) \subseteq A$.

(c) an α-open set [9] if $A \subseteq int(cl(int(A)))$ and an α-closed set if $cl(int(cl(A))) \subseteq A$.

(d) a semi-preopen set [10] (β-open set) if $A \subseteq cl(int(cl(A)))$ and semi-preclosed set if $int(cl(int(A))) \subseteq A$.

Definition 2. *A space $(X,\ \tau_X)$ is called a $T_{\frac{1}{2}}$-space [4] if every g-closed set is closed.*

Definition 3. *[28] Let $(X,\ \tau)$ be a topological space and A be its subset, then A is Strongly g^*-closed set if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is g-open.*

The complement of Strongly g^*-closed set is called Strongly g^*-open set in $(X,\ \tau)$.

Definition 4. *[29] Let X and Y be topological spaces. A map $f : (X,\ \tau) \longrightarrow (Y,\ \sigma)$ is said to be strongly G^*-continuous (sg^*-continuous) if the inverse image of every open set Y is sg^*-open in X.*

Definition 5. *[29] Let X and Y be topological spaces. A map $f : (X,\ \tau) \longrightarrow (Y,\ \sigma)$ is said to be strongly g^*-irresolute map (sg^*-irresolute map) if the inverse image of every sg^*-open set in Y is sg^*-open in X.*

Definition 6. *[29] Let X and Y be two topological spaces. A bijection map $f : (X,\ \tau) \longrightarrow (Y,\ \sigma)$ from a topological space X into a topological space Y is called strongly g^*-Homeomorphism (sg^*-homeomorphism) if f and f^{-1} are sg^*-continuous.*

Definition 7. *[28] (a) Let X be a topological space and let $x \in X$. A subset N of X is said to be Strongly g^*-nbbd of x if there exists an Strongly g^*-open set G such that $x \in G \subset N$.*

The collection of all Strongly g^*-nbhd of $x \in X$ is called a Strongly g^*-nbhd system at x and shall be denoted by Strongly $g^*N(x)$.

(b) Let X be a topological space and A be a subset of X, A subset N of X is said to be Strongly g^*-nbhd of A if there exists a Strongly g^*-open set G such that $A \in G \subseteq N$.

(c) Let A be a subset of X. A point $x \in A$ is said to be a Strongly g^*-interior point of A, if A is a Strongly $g^*N(x)$. The set of all Strongly g^*-interior points of A is called a Strongly g^*-interior of A and is denoted by $Sg^*INT(A)$.

$Sg^*INT(A) = \bigcup\{G : G \text{ is Strongly } g^*\text{-open},\ G \subset A\}$.

(d) Let A be a subset of X. A point $x \in A$ is said to be a Strongly g^*-closure of A. Then

$Sg^*CL(A) = \bigcap\{F : A \subset F \in \text{Strongly } g^*C(X,\tau)\}$.

Definition 8. *[28] A topological space $(X,\ \tau)$ is said to be*

(a) Strongly-$T_0{}^{g^*}$ if for each pair of distinct points x, y in X, there exists a Strongly g^*-open set U such that either $x \in U$ and $y \notin U$ or $x \notin U$ and $y \in U$.

(b) Strongly-$T_1{}^{g^*}$ if for each pair of distinct points x, y in X, there exist two Strongly g^*-open sets U and V such that $x \in U$ but $y \notin U$ and $y \in V$ but $x \notin V$.

(c) Strongly-$T_2{}^{g^*}$ if for each distinct points x, y in X, there exist two disjoint Strongly g^*-open sets U and V containing x and y respectively.

Definition 9. *[28] A topological space (X,τ) is said to be Strongly g^*-symmetric if for x and y in X, $x \in Sg^*CL(\{y\})$ implies $y \in Sg^*CL(\{x\})$.*

Definition 10. *[28] A function $f : (X,\tau) \longrightarrow (Y,\sigma)$ is called a Strongly g^*-open function if the image of every Strongly g^*-open set in (X,τ) is a Strongly g^*-open set in (Y,σ).*

Definition 11. *A subset A of a topological space (X,τ) is called a generalised closed set (briefly g-closed)[7] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ).*

Definition 12. *[30] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U,R) is said to be the approximation space. Let $X \subseteq U$.*

(i) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by LR(X).

That is, $LR(x) = \cup_{X\in U}\{R(X) : R(X) \subseteq X\}$ A subset A of a topological space (X,τ) is called a generalised closed set (briefly g-closed)[7] if $cl(A) \subseteq U$ whenever A $\subseteq$ U and U is open in (X,τ). where R(X) denotes the equivalence class determined by x.

(ii) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $UR(X)$.
That is,UR(X) = $\cup_{X\in U}\{R(X) : R(X) \cap X \neq \phi\}$

(iii) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not-X with respect to R and it is denoted by BR(x) . That is, BR(x) = UR(X) - L R(X)

Property 1.[32] If (U,R) is an approximation space and X, Y $\subseteq$ U , then

(i) $L_R \subseteq$(X) $\subseteq$ X $\subseteq$ U_R(X)
(ii) $L_R(\emptyset) = U_R(\emptyset)$ and $L_R(U) = U_R(U) = U$
(iii) U_R(XUY) =U_R(X) U U_R(Y)
(iv) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R$ (Y)
(v) L_R (xU Y) $\sqsupseteq$ L_R (x) U L_R (Y)
(vi) L_R (X $\cap$ Y) =L_R (X) $\cap$ L_R (Y)
(vii) L_R (x) $\subseteq$ L_R(Y) and U_R (X) $\subseteq$ U_R (Y) whenever X $\subseteq$ Y
(viii) $U_R(X^c) = [L_R(X)]^C$ and $L_R(X^c) = [U_R(X)]^C$
(ix) U_R U_R(X) = L_RU R(X) = U R(X)
(x) L RL R(X) = URL R(X) = L R(X)

Definition 13.*[30] Let U be the universe, R be an equivalence relation on U and τ R (X) = { U, φ , L R(X) , UR(X), BR(x) } where X ⊆ U . Then by the above property τ R (X) satisfies the following axioms:*

(i) U and ∅ belong to τ R (X)

(ii) The union of the elements of any subcollection of τ R (X) is in τ R (X)

(iii) The intersection of the elements of any finite subcollection of τ R (X) is in τ R (X).

That is, $\tau R(X)$ is a topology on U called the Nanotopology on U with respect to X. We call $(U,R(X))$ as the Nanotopological space. The elements of $\tau R(X)$ are called as Nano-open sets.

Remark.[30] If τ R (X) is the Nanotopology on U with respect to X, then the set B = {U,LR(X) , U R(X) }is the basis for τ R (X).

Definition 14.*[30] If (U,τ R (X)) is a Nano topological space with respect to X where X ⊂ U and if A ⊂ U , then the Nano interior of A is defined as the union of all Nano-open subsets of A and it is denoted by NInt(A). That is, N Int(A) is the largest Nano-open subset of A.*

The Nano closure of A is defined as the intersection of all Nano closed sets containing A and it is denoted by NCl(A). That is, NCl(A) is the smallest Nano closed set containing A.

3 On Strongly $g^*d_\#$-sets

On this paper we introduce $sg^*d_\#$ set,the spaces called $\widetilde{sd}^{\,0}$, $\widetilde{sd}^{\,1}$, $\widetilde{sd}^{\,2}$ in topological areas. Also the concept of sg^*-kernel of A and studied some of their characters.

Definition 15. *A subset A of a topological space $(X,\ \tau)$ is called a Strongly g^*-Difference set (briefly, Strongly $g^*d_\#$-set) if there are $U,V \in$ Strongly $g^*O(X,\tau)$ such that $U \neq X$ and $A = U/V$.*

It is true that every Strongly g^*-open set U different from X is a Strongly $g^*d_\#$-set if $A = U$ and $V = \phi$. So, we can observe the following.

Remark. Every proper Strongly g^*-open set is a Strongly $g^*d_\#$-set.

Now we define another set of separation axioms called Strongly $\widetilde{d}^{\,n}$-G^*, for $n = 0,1,2$, by using the Strongly $g^*d_\#$-sets.

Definition 16. *A topological space* $(X,\ \tau)$ *is said to be*

(a) *Strongly* $\widetilde{d}^{\,0}$-G^* *if for any pair of distinct points x and y of X there exists a Strongly* $g^*d_\#$*-set of X containing x but not y or Strongly* $g^*d_\#$*-set of X containing y but not x.*

(b) *Strongly* $\widetilde{d}^{\,1}$-G^* *if for any pair of distinct points x and y of X there exists a Strongly* $g^*d_\#$*-set of X containing x but not y and Strongly* $g^*d_\#$*-set of X containing y but not x.*

(c) *Strongly* $\widetilde{d}^{\,2}$-G^* *if for any pair of distinct points x and y of X there exists disjoint Strongly* $g^*d_\#$*-set G and E of X containing x and y, respectively.*

Remark. For a topological space $(X,\ \tau)$, the following properties hold:

(a) If $(X,\ \tau)$ is Strongly-$T_k{}^{g^*}$, then it is Strongly $\widetilde{d}^{\,k}$-G^*, for $k = 0,1,2$.

(b) If $(X,\ \tau)$ is Strongly $\widetilde{d}^{\,k}$-G^*, then it is Strongly $\widetilde{d}^{\,k-1}$-G^*, for $k = 1,2$.

Proposition 1. *A space X is Strongly* $\widetilde{d}^{\,0}$-G^* *if and only if it is Strongly-*$T_0{}^{g^*}$.

Proof. Suppose that X is Strongly $\widetilde{d}^{\,0}$-G^*. Then for each distinct pair $x,\ y \in X$, at least one of $x,\ y$, say x, belongs to a Strongly $g^*d_\#$-set G but $y \notin G$. Let $G = U_1/U_2$ where $U_1 \neq X$ and $U_1,\ U_2 \in Strongly\ g^*O(X,\ \tau)$. Then $x \in U_1$, and for $y \notin G$ we have two cases:

(a) $y \notin U_1$, (b) $y \in U_1$ and $y \in U_2$.

In case (a), $x \in U_1$ but $y \notin U_1$.

In case (b), $y \in U_2$ but $x \notin U_2$.

Thus in both the cases, we obtain that X is Strongly-$T_0{}^{g^*}$.

Conversely, if X is Strongly-$T_0{}^{g^*}$, by the previous remark, X is Strongly $\widetilde{d}^{\,0}$-G^*.

Proposition 2. *A space X is Strongly* $\widetilde{d}^{\,1}$-G^* *if and only if it is Strongly* $\widetilde{d}^{\,2}$-G^*.

Proof. *Necessity.* Let $x,\ y \in X$, $x \neq y$. Then there exist Strongly $g^*d_\#$-sets $G_1,\ G_2$ in X such that $x \in G_1$, $y \notin G_1$ and $y \in G_2$, $x \notin G_2$. Let $G_1 = U_1/U_2$ and $G_2 = U_3/U_4$, where $U_1,\ U_2,\ U_3$ and U_4 are Strongly g^*-open sets in X. From $x \notin G_2$, it follows that either $x \notin U_3$ or $x \in U_3$ and $x \in U_4$.

We discuss the two cases separately.

(a) $x \notin U_3$. By $y \notin G_1$ we have two subcases:

(i) $y \notin U_1$. Since $x \in U_1/U_2$, it follows that $x \in U_1/(U_2 \cup U_3)$, and since $y \in U_3/U_4$ we have $y \in U_3/(U_1 \cup U_4)$. Therefore $(U_1/(U_2 \cup U_3)) \cap (U_3/(U_1 \cup U_4)) = \phi$.

(ii) $y \in U_1$ and $y \in U_2$. We have $x \in U_1/U_2$, and $y \in U_2$. Therefore $(U_1/U_2) \cap U_2 = \phi$.

(b) $x \in U_3$ and $x \in U_4$. We have $y \in U_3/U_4$ and $x \in U_4$. Hence $(U_3/U_4) \cap U_4 = \phi$. Therefore X is Strongly $\widetilde{d}^{\,2}$-G^*.

Sufficiency. Obvious.

Corollary 1. *If* $(X,\ \tau)$ *is Strongly* $\widetilde{d}^{\,1}$-G^**, then it is Strongly-*$T_0{}^{g^*}$.

Proof. Follows from the previous proposition.

Definition 17. *A point* $x \in X$ *which has only X as the Strongly* g^**-neighbourhood is called a Strongly* g^**-neat point.*

Proposition 3. *For a Strongly-*$T_0{}^{g^*}$ *topological space* $(X,\ \tau)$ *the followings are equivalent:*

(a) $(X,\ \tau)$ *is Strongly* $\widetilde{d}^{\,1}$-G^*.

(b) $(X,\ \tau)$ *has no Strongly* g^**-neat point.*

Proof. $(a) \Longrightarrow (b)$ Since $(X,\ \tau)$ is Strongly $\widetilde{d}^{\,1}$-G^*, then each point x of X is contained and Strongly $g^*d_\#$-set $A = U/V$ and thus in U. By definition $U \neq X$. This implies that x is not a Strongly g^*-neat point.

$(b) \Longrightarrow (a)$ If X is Strongly-$T_0{}^{g^*}$, then for each distinct pair of points $x,\ y \in X$, at least one of them, x (say) has a Strongly g^*-neighbourhood U containing x and not y. Thus U which is different from X is a Strongly $g^*d_\#$-set. If X has no Strongly g^*-neat point, then y is not a Strongly g^*-neat point. This means that there exists a Strongly g^*-neighbourhood V of y such that $V \neq X$. Thus $y \in V/U$ but not x and V/U is a Strongly $g^*d_\#$-set. Hence X is Strongly $\widetilde{d}^{\,1}$-G^*.

Corollary 2. *A Strongly-*$T_0{}^{g^*}$ *space X is not Strongly* $\widetilde{d}^{\,1}$-G^* *if and only if there is a unique Strongly* g^**-neat point in X.*

Proof. We only prove the uniqueness of the Strongly g^*-neat point. If x and y are two Strongly g^*-neat points in X, then since X is Strongly-$T_0{}^{g^*}$, at least one of x and y, say x, has a Strongly g^*-neighbourhood U containing x but not y. Hence $U \neq X$. Therefore x is not a Strongly g^*-neat point which is a contradiction.

Definition 18. *(a) The intersection of all Strongly g^*-open subsets of $(X,\ \tau)$ containing A is called the Strongly g^*-kernel of A (briefly, S-g^*-$K_{\#}(A)$).*
S-g^*-$K_{\#}(A) = \cap\{G \in Strongly\ g^*(X,\ \tau) : A \subseteq G\}$.

(b) Let $x \in X$. Then Strongly g^*-kernel of x is denoted by S-g^*-$K_{\#}(\{x\}) = \cap\{G \in Strongly\ g^*(X,\ \tau) : x \in G\}$.

Theorem 1. *Let X be a topological space.Then for any nonempty subset A of X,* S-g^*-$K_{\#}(A) = \{x \in X : Sg^*CL(\{x\}) \cap A \neq \phi\}$.

*Proof.*Let $x \in S$-g^*-$K_{\#}(A)$. Suppose that $Sg^*CL(\{x\}) \cap A = \phi$. Then $A \subseteq X - Sg^*CL(\{x\})$ and $X - Sg^*CL(\{x\})$ is Strongly g^*-open set containing A but not x, which is a contradiction.

Conversely, let us assume that $x \notin S$-g^*-$K_{\#}(A)$ and $Sg^*CL(\{x\}) \cap A \neq \phi$. Then there exist a Strongly g^*-open set D containing A but not x and $y \in Sg^*CL(\{x\}) \cap A$. Hence a Strongly g^*-closed set $X - D$ contains x, and $\{x\} \subset X - D$, $y \notin X - D$. This is a contradiction to $y \in Sg^*CL(\{x\}) \cap A$. Therefore $x \in S$-g^*-$K_{\#}(A)$.

Proposition 4. *If a singleton $\{x\}$ is a Strongly $g^*d_{\#}$-set of (X,τ), then S-g^*-$K_{\#}(\{x\}) \neq X$.*

*Proof.*Since $\{x\}$ is a Strongly $g^*d_{\#}$-set of (X,τ), then there exist two subsets U_1, $U_2 \in \alpha^m O(X,\tau)$ such that $\{x\} = U_1/U_2$, $\{x\} \subseteq U_1$ and $U_1 \neq X$. Thus, we have that S-g^*-$K_{\#}(\{x\}) \subseteq U_1 \neq X$ and so S-g^*-$K_{\#}(\{x\}) \neq X$.

Remark. For a Strongly g^*-symmetric space (X,τ), the followings are equivalent:

(a) (X,τ) is Strongly-$T_0{}^{g^*}$.

(b) (X,τ) is Strongly $\widetilde{d}^{\,1}$-G^*.

(c) (X,τ) is Strongly-$T_1{}^{g^*}$.

Proposition 5. *The following properties hold for the subsets A, B of a topological space (X,τ).*

(a) $A \subseteq S$-g^*-$K_{\#}(A)$.

(b) $A \subseteq B$ implies that S-g^*-$K_{\#}(A) \subseteq S$-g^*-$K_{\#}(B)$.

(c) If A is Strongly g^*-open in (X,τ), then $A = S$-g^*-$K_{\#}(A)$.

(d) S-g^*-$K_{\#}(S$-g^*-$K_{\#}(A)) = S$-g^*-$K_{\#}(A)$.

Proof. (a), (b) and (c) are immediate consequences of Definition. To prove (d), we firstly observe that by (a) and (b), we have S-g^*-$K_{\#}(A) \subseteq S$-g^*-$K_{\#}(S$-g^*-$K_{\#}(A))$. If $x \notin S$-g^*-$K_{\#}(A)$, then there exists $U \in S$-g^*-$K_{\#}O(X,\tau)$ such that $A \subseteq U$ and $x \notin U$. Hence S-g^*-$K_{\#}(A) \subseteq U$, and so we have $x \notin S$-g^*-$K_{\#}(S$-g^*-$K_{\#}(A))$. Thus S-g^*-$K_{\#}(S$-g^*-$K_{\#}(A)) = S$-g^*-$K_{\#}(A)$.

4 Nano Sg^* closed sets

Throughout this paper (U, τ R (X)) is a Nano topological space with respect to X where X ⊂ U , R is an equivalence Relation on U , U/R denotes the family of equivalence classes of U by R

Definition 19.
Let (U, τ R (X)) be a Nano topological space. A subset A of (U, τ R (X)) is called Nano strongly g^* closed set (briefly Nsg- closed) if NCl(Nint(A)) ⊆ V where A ⊆ V and V is Nano open.*

Theorem 2. *Every closed set is nano strongly g^*-closed sets.*

Proof. The proof is true from the definition of closed set.

*Example 1.*Let $U = \{a,b,c\}$ with $U/R = \{\{a\},\{b,c\}\}$ and $X = \{a,c\}$.Then the Nanotopology $\tau R(X) = \{U,\emptyset,\{a\},\{b,c\}\}$. Nano closed sets are $\{\emptyset,U,\{b,c\},\{a\}\}$. Let $V = \{b,c\}$ and $A = \{b\}$.Then $NCl(Nint(A)) = \{b,c\} \subseteq V$. A is said to be $Nsg*$- closed in $(U,\tau R(X))$

Theorem 3. *A subset A of (U,τ R (X)) is Nsg*-closed if NCl(Nint(A))- A contains no nonempty Nsg*- closed set.*

Proof. Suppose if A is Ng- closed. Then $NCl(Nint(A)) \subseteq V$ where $A \subseteq V$ and V is Nano open. Let Y be a Nanoclosed subset of $NCl(Nint(A))$- A . Then $A \subseteq Y^c$ is Nano open. Since A is $Nsg*$- closed, $NCl(Nint(A)) \subseteq Y^c$ or $Y \subseteq [Ncl(Nint(A))^C$. That is $Y \subseteq NCl(Nint(A))$ and $Y \subseteq [Ncl(Nint(A))^C$ implies that $Y \subseteq \emptyset$. So Y is empty

Theorem 4. *The Intersection of two Nsg∗- closed sets is again an Nsg∗-closed set which is shown in the following example.*

Proof. Let U = $\{a,b,c\}$, $X = \{a,b\}$, $U/R = \{\{a,b\},\{b,a\},\{c\}\}$, $\tau R(X) = \{U,\emptyset,\{a,b\}\}$.Let $A = \{b,c\}$, $B = \{a,c\}$ and $A \cap B = \{c\}$. Here $NCl(Nint(A \cap B)) \subseteq V$ when $(A \cap B) \subseteq V$ and V is Nano open.

5 Conclusion

we have studied the set Strongly $g^*d_{\#}$, Strongly $\widetilde{d}^{\,0}$, Strongly $\widetilde{d}^{\,1}$, Strongly $\widetilde{d}^{\,2}$ and we have also introduced Strongly g^*-kernel of A in topological spaces and studied some of their properties. In addition, we have provided further investigation on Nano strongly $g*$ -closed sets.

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R. Parimelazhagan is currently working as a Professor & Head in the Department of Mathematics, RVS Technical Campus - Coimbatore, Tamil Nadu, India. He has 25 years of Teaching Experience and 20 years of Research Experience. His area of specialization is Topology. He has published 65 research articles in reputed SCI, SCIE and ANNEXURE– I journals. He has produced 8 P.hd research scholars and is guiding 7 research scholars. He has been members of many technical and advisory committees for many national conferences/workshops/seminars. He has visited Turkey, Thailand, and Malaysia purpose of research. AL-al- Bayt University has nominated to serve as member of the Scientific Committee. He has evaluated more than 10 PhD thesis and reviewed many research papers in some SCIE and SCI journals.

V. Jeyalakshmi is currently working as an Assistant Professor in the Department of Mathematics, RVS Technical Campus - Coimbatore, Tamil Nadu, India. She has 5 years of teaching experience. Her area of specialization is Topology. She has published 5 papers on various journals. She is a research scholar in Anna University, Chennai.