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A New Exponentiated Distribution with Engineering Science Applications

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Abstract: The article addresses various statistical properties and estimation methods for the exponentiated Garima distribution. The exponentiated Garima distribution has two parameters (scale and shape). Our main focus is on estimation from the frequentist point of view, yet some statistical and reliability characteristics for the model are derived. We briefly describe different estimation procedures namely, the maximum likelihood method, order statistics, entropies and reliability analysis. Finally, the potentiality of the model is analyzed by means of three real lifetime data sets.

Keywords: Exponentiated distribution, Garima distribution, Reliability analysis, Entropies, Order statistics, Maximum likelihood estimator.

1 Introduction

A new family of distributions, namely the exponentiated exponential distribution was introduced by Gupta *et al.* see [1]. The family has two parameters: scale and shape, which are similar to the weibull or gamma family. Later Gupta and Kundu, see [2], studied some properties of the distribution. They observed that many properties of the new family are similar to those of the weibull or gamma family. Hence the distribution can be used as an alternative to a weibull or gamma distribution. The two-parameteric gamma and Weibull are the most popular distributions for analyzing any lifetime data. The gamma distribution has a lot of applications in different fields other than lifetime distributions. The two parameters of gamma distribution represent the scale and the shape parameter and because of the scale and shape parameter, it has quite a bit of flexibility to analyze any positive real data. But one major disadvantage of the gamma distribution is that, if the shape parameter is not an integer, the distribution function or survival function cannot be expressed in a closed form. This makes gamma distribution little bit unpopular as compared to the Weibull distribution, whose survival function and hazard function are simple and easy to study. Nowadays exponentiated distributions and their mathematical properties are widely studied for applied science experimental data sets. Hassan *et al.* [3], discussed exponentiated Lomax geometric distribution with its properties and applications. Maradesa Adeleke [4], discussed exponentiated exponential lomax distribution and its Properties. Nasir *et al.* see [5], obtained the exponentiated Burr XII power series distribution with properties and its applications. Nasiru *et al.* [6], discussed the exponentiated generalized half logistic Burr X distribution and its various statistical properties. Pal *et al.* see [7], studied the exponentiated weibull family as an extension of weibull distribution. Recently, Rather and Subramanian see [8], discussed the exponentiated Mukherjee-Islam distribution which shows more flexibility than the classical distribution. Rodrigues *et al.* [9], studied the exponentiated generalized Lindley distribution.

Garima distribution was introduced by Rama Shanker see [10], which is a newly proposed one parametric lifetime

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model for behavioral and emotional science. The usefulness and importance of the proposed distribution in lifetime data were greater as compared to lindley and the exponential distribution. Furthermore, Garima distribution has been modified and generalized by so many researchers. Rama Shanker [11], discussed the discrete Poisson-Garima distribution. Also Shanker and Shukla see [12], discussed Zero-Truncated Poisson-Garima distribution and its applications. In this paper, we consider a two-parameter exponentiated Garima distribution and study some of its properties. The two parameters of exponentiated Garima distribution represent the shape and the scale parameters. It also has the increasing or decreasing failure rate depending of the shape parameter. The density function varies significantly depending of the shape parameter (see Fig.1). Finally, the three real lifetime data sets have been analyzed, the results are compared over Garima and Exponential distribution and has been found that exponentiated garima distribution provides better fit than Garima and exponential distribution.

2 Exponentiated Garima (EG) Distributions

The probability density function (pdf) of Garima distribution is given by

$$g(x)=\frac{\theta}{\theta+2}(1+\theta+\theta x)e^{-\theta x}\ ; \qquad x>0,\quad \theta>0 \tag{1}$$

and its cumulative distribution function (cdf) is given by

$$G(x)=1-\left(1+\frac{\theta x}{\theta+2}\right)e^{-\theta x}; \qquad x>0,\quad \theta>0 \tag{2}$$

A random variable X is said to have an exponentiated distribution if its cumulative distribution function (cdf) is given by

$$F_{\alpha}(x)=\left(G(x)\right)^{\alpha}, \qquad x\in R^{1},\alpha>0 \tag{3}$$

Then X is said to have an exponential distribution.
The probability density function (pdf) of X is given by

$$f_{\alpha}(x)=\alpha\left(G(x)\right)^{\alpha-1}g(x) \tag{4}$$

On substituting (2) in (3), we will get the cumulative density function of exponentiated Garima distribution

$$F_{\alpha}(x)=\left(1-\left(1+\frac{\theta x}{\theta+2}\right)e^{-\theta x}\right)^{\alpha}; \qquad x>0,\theta>0,\alpha>0 \tag{5}$$

and the probability density function of EG distribution can be obtained as

$$f_{\alpha}(x)=\frac{\alpha\theta(1+\theta+\theta x)e^{-\theta x}}{\theta+2}\left(1-\left(1+\frac{\theta x}{\theta+2}\right)e^{-\theta x}\right)^{\alpha-1} \tag{6}$$

3 Reliability Analysis

In this section, we will discuss about the survival function, failure rate, reverse hazard rate and Mills ratio of the EG distribution.
The survival function or the reliability function of the exponentiated Garima distribution is given by

$$S(x)=1-\left(1-\left(1+\frac{\theta x}{\theta+2}\right)e^{-\theta x}\right)^{\alpha}$$

The hazard function is also known as the hazard rate, instantaneous failure rate or force of mortality and is given by

$$h(x)=\left(\frac{\frac{\alpha\theta(1+\theta+\theta x)e^{-\theta x}}{\theta+2}\left(1-\left(1+\frac{\theta x}{\theta+2}\right)e^{-\theta x}\right)^{\alpha-1}}{1-\left(1-\left(1+\frac{\theta x}{\theta+2}\right)e^{-\theta x}\right)^{\alpha}}\right)$$

The reverse hazard rate is given by

$$h_r(x)=\frac{\alpha\theta(1+\theta+\theta x)e^{-\theta x}}{(\theta+2)-(\theta+\theta x+2)e^{-\theta x}}$$

and the Mills ratio of the EG distribution is

$$\text{Mills Ratio}=\frac{(\theta+2)-(\theta+\theta x+2)e^{-\theta x}}{\alpha\theta(1+\theta+\theta x)e^{-\theta x}}$$

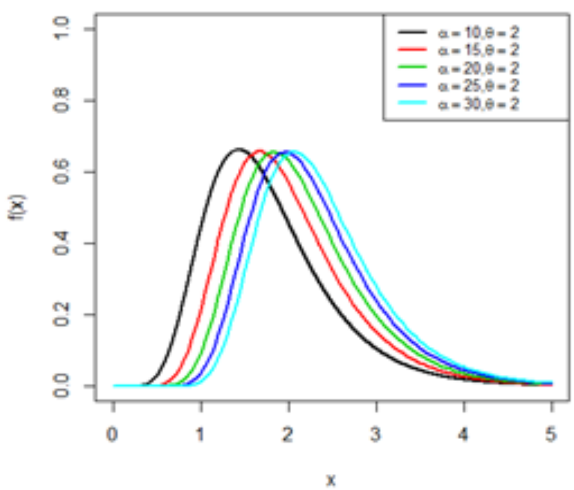

Fig 1: pdf plot of exponentiontiated Gama Distribution.

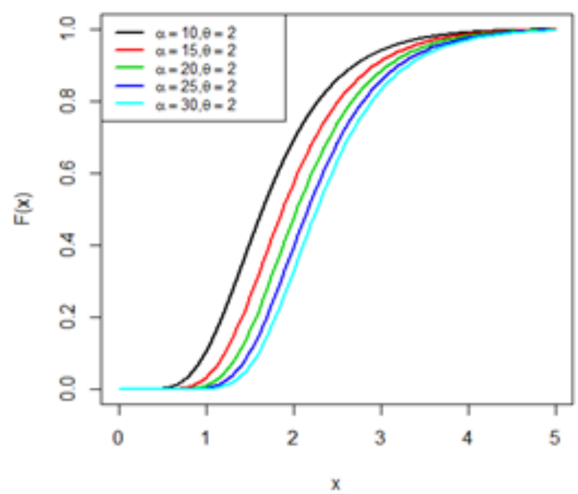

Fig 2: Cdf plot of Exponentiated Garima distribution.

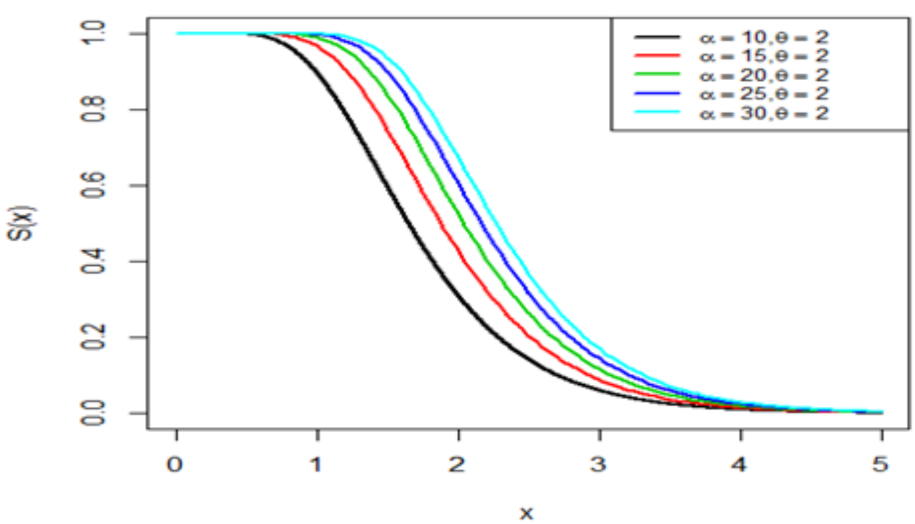

Fig 3: Survival function curves of EG distribution.

4 Statistical Properties

In this section, we will discuss the different structural properties of the proposed exponentiated Garima distribution.

4.1 Moments

Suppose *X* is a random variable following exponentiated Garima distribution with parameters *α* and *θ,* then the r^{th} order moment $E(x^r)$ for a given probability distribution is given by

$$E(X^r)=\mu_r^{'}=\int_0^\infty x^r f_\alpha(x)dx$$

$$=\int_0^\infty x^r\,\frac{\alpha\theta(1+\theta+\theta x)e^{-\theta x}}{\theta+2}\left(1-\left(1+\frac{\theta x}{\theta+2}\right)e^{-\theta x}\right)^{\alpha-1}dx$$

$$=\frac{\alpha\theta}{\theta+2}\int_0^\infty x^r(1+\theta+\theta x)e^{-\theta x}\left(1-\left(1+\frac{\theta x}{\theta+2}\right)e^{-\theta x}\right)^{\alpha-1}dx \qquad (7)$$

Using binomial expansion of $\left(1-\left(1+\frac{\theta x}{\theta+2}\right)e^{-\theta x}\right)^{\alpha-1}=\sum_{i=0}^{\infty}\binom{\alpha-1}{i}\left\{\left(1+\frac{\theta x}{\theta+2}\right)e^{-\theta x}\right\}^i(-1)^i$, equation (7) will become

$$E(X^r)=\frac{\alpha\theta}{\theta+2}\sum_{i=o}^{\infty}(-1)^i\binom{\alpha-1}{i}\int_0^\infty x^r(1+\theta+\theta x)e^{-\theta x(1+i)}\left(1+\frac{\theta x}{\theta+2}\right)^i dx \qquad (8)$$

Again using binomial expansion in equation (8), we get

$$E(X^r)=\alpha\sum_{i=0}^{\infty}\sum_{k=0}^{\infty}(-1)^i\binom{\alpha-1}{i}\binom{i}{k}\left(\frac{\theta}{(\theta+2)}\right)^{k+1}\int_0^\infty x^{r+k}(1+\theta+\theta x)e^{-\theta x(1+i)}dx$$

After simplification, we obtain

$$E(X^r)=\alpha\sum_{i=0}^{\infty}\sum_{k=0}^{\infty}(-1)^i\binom{\alpha-1}{i}\binom{i}{k}\left(\frac{\theta}{(\theta+2)}\right)^{k+1}$$
$$\times\frac{\theta\Gamma(r+k+1)}{\left(\theta(1+i)\right)^{r+k+2}}\left((1+\theta)(1+i)+(r+k+1)\right) \qquad (9)$$

Since equation (9) is a convergent series for all $r\geq 0$, therefore all the moments exist.

Therefore

$$E(X)=\alpha\sum_{i=0}^{\infty}\sum_{k=0}^{\infty}(-1)^i\binom{\alpha-1}{i}\binom{i}{k}\left(\frac{\theta}{(\theta+2)}\right)^{k+1}\frac{\theta\Gamma(k+2)}{(\theta(1+i))^{k+3}}\big((1+\theta)(1+i)+(k+2)\big)$$

and

$$E(X^2)=\alpha\sum_{i=0}^{\infty}\sum_{k=0}^{\infty}(-1)^i\binom{\alpha-1}{i}\binom{i}{k}\left(\frac{\theta}{(\theta+2)}\right)^{k+1}\frac{\theta\Gamma(k+3)}{(\theta(1+i))^{k+4}}\big((1+\theta)(1+i)+(k+3)\big)$$

Therefore, the variance of X can be obtained as

$$V(X)=E(X^2)-\big(E(X)\big)^2$$

4.2 Harmonic Mean

The harmonic mean for the EG distribution can be obtained as

$$H.M=E\left(\frac{1}{x}\right)=\int_0^\infty\frac{1}{x}f_\alpha(x)dx$$

$$=\int_0^\infty\frac{1}{x}\frac{\alpha\theta(1+\theta+\theta x)e^{-\theta x}}{\theta+2}\left(1-\left(1+\frac{\theta x}{\theta+2}\right)e^{-\theta x}\right)^{\alpha-1}dx$$

$$=\frac{\alpha\theta}{\theta+2}\int_0^\infty\frac{1}{x}(1+\theta+\theta x)e^{-\theta x}\left(1-\left(1+\frac{\theta x}{\theta+2}\right)e^{-\theta x}\right)^{\alpha-1}dx$$

After simplification, we obtain

$$H.M.=\alpha\sum_{i=0}^{\infty}\sum_{k=0}^{\infty}(-1)^i\binom{\alpha-1}{i}\binom{i}{k}\left(\frac{1}{(\theta+2)(1+i)}\right)^{k+1}\big(\theta(1+\theta)(1+i)+\theta k\big)\Gamma k$$

4.3 Moment Generating Function and Characteristic Function

Let X have exponentiated Garima distribution, then the moment generating function of X is obtained as

$$M_X(t)=E(e^{tx})=\int_0^\infty e^{tx}f_\alpha(x)dx$$

Using Taylor's series

$$M_X(t)=\int_0^\infty\left(1+tx+\frac{(tx)^2}{2!}+\cdots\right)f_\alpha(x)dx$$

$$= \int_0^\infty \sum_{j=0}^\infty \frac{t^j}{j!} x^j f_\alpha(x)dx$$

$$= \sum_{j=0}^\infty \frac{t^j}{j!} \mu_j'$$

$$M_X(t) = \alpha \sum_{i=0}^\infty \sum_{j=0}^\infty \sum_{k=o}^\infty (-1)^i \binom{\alpha-1}{i}\binom{i}{k}\left(\frac{\theta}{(\theta+2}\right)^{k+1}$$
$$\times \frac{t^j}{j!} \frac{\theta\Gamma(j+k+1)}{(\theta(1+i))^{j+k+2}}\left((1+\theta)(1+i)+(j+k+1)\right)$$

Similarly, the characteristic function of EG distribution is given by

$$\varphi_X(t) = \alpha \sum_{i=0}^\infty \sum_{j=0}^\infty \sum_{k=o}^\infty (-1)^i \binom{\alpha-1}{i}\binom{i}{k}\left(\frac{\theta}{(\theta+2}\right)^{k+1}$$
$$\times \frac{(mt)^j}{j!} \frac{\theta\Gamma(j+k+1)}{(\theta(1+i))^{j+k+2}}\left((1+\theta)(1+i)+(j+k+1)\right)$$

5 Information Measures of EG Distribution

5.1 Renyi Entropy

Entropies quantify the diversity, uncertainty, or randomness of a system. The Renyi entropy is named after Alfred Renyi in the context of fractual dimension estimation, the Renyi entropy forms the basis of the concept of generalized dimensions. The Renyi entropy is important in ecology and statistics as index of diversity. The Renyi entropy is also important in quantum information, where it can be used as a measure of entanglement. For a given probability distribution, Renyi entropy is given by

$$e(\beta) = \frac{1}{1-\beta} \log\left(\int_0^\infty f^\beta(x)dx\right)$$

where, $\beta > 0$ and $\beta \neq 1$

$$= \frac{1}{1-\beta} \log\left[\frac{\alpha\theta}{\theta+2}\int_0^\infty \left\{e^{-\beta\theta x}(1+\theta+\theta x)^\beta \left(1-\left(1+\frac{\theta x}{\theta+2}\right)e^{-\theta x}\right)^{\beta(\alpha-1)}\right\}\right]dx$$

After simplification, we get

$$e(\beta) = \frac{1}{1-\beta} \log\left[\left(\frac{\alpha\theta}{\theta+2}\right)^\beta \sum_{i=0}^\infty \sum_{k=0}^\infty \sum_{j=0}^\infty (-1)^i (1+\theta)^{\beta-j} \binom{\beta(\alpha-1)}{i}\binom{i}{k}\binom{k}{j}\left(\frac{\theta}{\theta+2}\right)^k \theta^j \int_0^\infty e^{-\theta(\beta+i)x} x^{(j+k+1)-1} dx\right]$$

$$e(\beta)=\frac{1}{1-\beta}\log\left[\alpha^{\beta}\sum_{i=0}^{\infty}\sum_{k=0}^{\infty}\sum_{j=0}^{\infty}(-1)^{i}(1+\theta)^{\beta-j}\binom{\beta(\alpha-1)}{i}\binom{i}{k}\binom{k}{j}\frac{\theta^{\beta-1}}{(\theta+2)^{\beta+k}}\frac{\Gamma(j+k+1)}{(\beta+i)^{j+k+1}}\right]$$

5.2 Tsallis Entropy

A generalization of Boltzmann-Gibbs (B-G) statistical mechanics initiated by Tsallis has gained a great deal of attention. This generalization of B-G statistics was proposed firstly by introducing the mathematical expression of Tsallis entropy, see [13], for a continuous random variable, it is defined as follows

$$S_{\lambda}=\frac{1}{\lambda-1}\left(1-\int_{0}^{\infty}f^{\lambda}(x)dx\right)$$

$$=\frac{1}{\lambda-1}\left[1-\frac{\alpha\theta}{\theta+2}\int_{0}^{\infty}\left\{e^{-\lambda\theta x}(1+\theta+\theta x)^{\lambda}\left(1-\left(1+\frac{\theta x}{\theta+2}\right)e^{-\theta x}\right)^{\lambda(\alpha-1)}\right\}\right]dx$$

After simplification, we get

$$S_{\lambda}=\frac{1}{\lambda-1}\left[1-\left(\frac{\alpha\theta}{\theta+2}\right)^{\lambda}\sum_{i=0}^{\infty}\sum_{k=0}^{\infty}\sum_{j=0}^{\infty}(-1)^{i}(1+\theta)^{\lambda-j}\binom{\lambda(\alpha-1)}{i}\binom{i}{k}\binom{k}{j}\left(\frac{\theta}{\theta+2}\right)^{k}\theta^{j}\int_{0}^{\infty}e^{-\theta(\lambda+i)x}x^{(j+k+1)-1}dx\right]$$

$$S_{\lambda}=\frac{1}{\lambda-1}\left[1-\alpha^{\lambda}\sum_{i=0}^{\infty}\sum_{k=0}^{\infty}\sum_{j=0}^{\infty}(-1)^{i}(1+\theta)^{\lambda-j}\binom{\lambda(\alpha-1)}{i}\binom{i}{k}\binom{k}{j}\frac{\theta^{\lambda-1}}{(\theta+2)^{\lambda+k}}\frac{\Gamma(j+k+1)}{(\lambda+i)^{j+k+1}}\right]$$

6 Bonferroni And Lorenz Curves

The Bonferroni and the Lorenz curves are not only used in economics in order to study the income and poverty. It has also various applications in other fields like reliability, medicine, insurance and demography. The Bonferroni and Lorenz curves are given by

$$B(p)=\frac{1}{p\mu_1'}\int_{0}^{q}xf(x)dx$$

and

$$L(p)=PB(p)=\frac{1}{\mu_1'}\int_{0}^{q}xf(x)dx$$

and $q=F^{-1}(p)$

Where $\mu_1'=\alpha\sum_{i=0}^{\infty}\sum_{k=0}^{\infty}(-1)^{i}\binom{\alpha-1}{i}\binom{i}{k}\left(\frac{\theta}{(\theta+2}\right)^{k+1}\frac{\theta\Gamma(k+2)}{(\theta(1+i))^{k+3}}\left((1+\theta)(1+i)+(k+2)\right)$

$$B(p)=\frac{1}{p\mu_1'}\int_{0}^{q}x\frac{\alpha\theta(1+\theta+\theta x)e^{-\theta x}}{\theta+2}\left(1-\left(1+\frac{\theta x}{\theta+2}\right)e^{-\theta x}\right)^{\alpha-1}dx$$

After simplification, we get

$$B(p)=\frac{1}{p}\sum_{i=0}^{\infty}\sum_{i=0}^{\infty}\binom{\alpha-1}{i}\left(\frac{(1+\theta)(1+i)\gamma\big((k+2),\theta(1+i)q\big)+\gamma\big((k+3),\theta(1+i)q\big)}{\big((1+\theta)(1+i)+(k+2)\big)\Gamma(k+2)}\right)$$

and

$$L(p)=PB(p)=\sum_{i=0}^{\infty}\sum_{i=0}^{\infty}\binom{\alpha-1}{i}\left(\frac{(1+\theta)(1+i)\gamma\big((k+2),\theta(1+i)q\big)+\gamma\big((k+3),\theta(1+i)q\big)}{\big((1+\theta)(1+i)+(k+2)\big)\Gamma(k+2)}\right)$$

7 Maximum Likelihood Estimation

In this section, we will discuss the maximum likelihood estimators of the parameters of EG distribution.

Let $X_1, X_2, \ldots, X_n$ be the random sample of size n from the EG distribution, then the likelihood function can be written as

$$L(\alpha,\theta)=\frac{(\alpha\theta)^n}{(\theta+2)^n}\prod_{i=1}^{n}\left((1+\theta+\theta x)e^{-\theta x}\left(1-\left(1+\frac{\theta x}{\theta+2}\right)e^{-\theta x}\right)^{\alpha-1}\right)$$

The log likelihood function is

$$\log L(\alpha,\theta)=n\log\alpha+n\log\theta-n\log(\theta+2)+\sum_{i=1}^{n}\log(1+\theta+\theta x_i)$$
$$-\theta\sum_{i=1}^{n}x_i+(\alpha-1)\sum_{i=1}^{n}\log\left(1-\left(1+\frac{\theta x_i}{\theta+2}\right)e^{-\theta x_i}\right) \qquad (10)$$

The maximum likelihood estimates of α, θ which maximize (10), must satisfy the normal equations given by

$$\frac{\partial\log L}{\partial\alpha}=\frac{n}{\alpha}+\sum_{i=1}^{n}\log\left(1-\left(1+\frac{\theta x_i}{\theta+2}\right)e^{-\theta x_i}\right)=0$$

$$\Rightarrow\hat{\alpha}=\frac{n}{\sum_{i=1}^{n}\log\left\{\left(1+\frac{\theta x_i}{\theta+2}\right)e^{-\theta x_i}+1\right\}}$$

and

$$\frac{\partial\log L}{\partial\theta}=\frac{n}{\theta}-\frac{n}{\theta+2}-\sum_{i=1}^{n}x_i+\sum_{i=1}^{n}\left(\frac{1+x_i}{1+\theta+\theta x_i}\right)+(\alpha-1)\psi\left(1-\left(1+\frac{\theta x_i}{\theta+2}\right)e^{-\theta x_i}\right)=0$$

Where ψ(.) is the digamma function.

At this point, it is important to highlight that the analytical solution of the above system of non-linear equation is unknown. Algebraically it is very difficult to solve the complicated form of likelihood system of nonlinear equations. Therefore we use R and wolfram mathematics for estimating the required parameters.

8 Order Statistics

Order statistics has been widely used in the field of reliability and life testing. There is also an extensive role of order statistics in several aspects of statistical inference. Let $X_{(1)}, X_{(2)}, \ldots, X_{(n)}$ be the order statistics of a random sample $X_1, X_2, \ldots, X_n$ drawn from the continuous population with probability density function $f_x(x)$ and cumulative density function with $F_x(x)$, then the pdf of r^{th} order statistics $X_{(r)}$ can be written as

$$f_{X(r)}(x)=\frac{n!}{(r-1)!(n-r)!}f_X(x)\left[F_X(x)\right]^{r-1}\left[1-F_X(x)\right]^{n-r} \tag{11}$$

Substitute the values of (5) and (6) in equation (11), we will get the pdf of r^{th} order statistics $X_{(r)}$ for exponentiated Garima distribution and is given by

$$f_{X(r)}(x)=\frac{n!}{(r-1)!(n-1)!}\frac{\alpha\theta(1+\theta+\theta x)e^{-\theta x}}{\theta+2}\left(1-\left(1+\frac{\theta x}{\theta+2}\right)e^{-\theta x}\right)^{\alpha-1}$$
$$\times\left(1-\left(1+\frac{\theta x}{\theta+2}\right)e^{-\theta x}\right)^{\alpha(r-1)}\left(1-\left(1-\left(1+\frac{\theta x}{\theta+2}\right)e^{-\theta x}\right)^{\alpha}\right)^{n-r}$$

The probability density function of higher order statistics $X_{(n)}$ can be obtained as

$$f_{X(n)}(x)=n\frac{\alpha\theta(1+\theta+\theta x)e^{-\theta x}}{\theta+2}\left(1-\left(1+\frac{\theta x}{\theta+2}\right)e^{-\theta x}\right)^{\alpha-1}\left(1-\left(1+\frac{\theta x}{\theta+2}\right)e^{-\theta x}\right)^{\alpha(n-1)}$$

Similarly, the pdf of 1st order statistic $X_{(1)}$can be obtained as

$$f_{X(1)}(x)=n\frac{\alpha\theta(1+\theta+\theta x)e^{-\theta x}}{\theta+2}\left(1-\left(1+\frac{\theta x}{\theta+2}\right)e^{-\theta x}\right)^{\alpha-1}\left(1-\left(1-\left(1+\frac{\theta x}{\theta+2}\right)e^{-\theta x}\right)^{\alpha}\right)^{n-1}$$

9 Data Analysis

In this section, we have used three real lifetime data sets in exponentiated Garima distribution and the model has been compared with Garima and Exponential distribution.

Data set 1: The data set is reported by (see [14]), which is related with strength data of window glass of the aircraft of 31 windows. The data are

18.83	20.80	21.657	23.03	23.23	24.05	24.321	25.50
25.52	25.80	26.69	26.77	26.78	27.05	27.67	29.90
31.11	33.20	33.73	33.76	33.89	34.76	35.75	35.91
36.98	37.08	37.09	39.58	44.045	45.29	45.431	

Data set 2: The data set is reported by (see [15]). The following data are related to failure times of the air conditioning system of an airplane. The data are given below:

23,	261,	87,	7,	120,	14,	62,	47,	225,	71,	246,	21,	42,
20,	5,	12,	120,	11,	3,	14,	71,	11,	14,	11,	16,	90,
1,	16,	52,	95									

Data set 3: The data set represents the tensile strength, measured in GPa, of 69 carbon fibers tested under tension at gauge lengths of 20mm which were originally reported by (see [16]), the data are given as:

1.312,	1.314,	1.479,	1.552,	1.700,	1.803,
1.861,	1.865,	1.944,	1.958,	1.966,	1.997,
2.006,	2.021,	2.027,	2.055,	2.063,	2.098,
2.14,	2.179,	2.224,	2.240,	2.253,	2.270,

2.272,	2.274,	2.301,	2.301,	2.359,	2.382,
2.382,	2.426,	2.434,	2.435,	2.478,	2.490,
2.511	2.514,	2.535,	2.554,	2.566,	2.57,
2.586,	2.629,	2.633,	2.642,	2.648,	2.684,
2.697,	2.726,	2.770,	2.773,	2.800,	2.809,
2.818,	2.821,	2.848,	2.88,	2.954,	3.012,
3.067,	3.084,	3.090,	3.096,	3.128,	3.233,
3.433,	3.585,	3.585.			

In order to compare the exponentiated Garima distribution with Garima and Exponential distribution, we consider the criteria like Bayesian Information Criterion *(BIC),* Akaike Information Criterion *(AIC),* Akaike Information Criterion Corrected *(AICC)* and *-2 logL*. The better distribution is which corresponds to lesser values of *AIC, BIC, AICC* and *– 2 log L*. For calculating *AIC, BIC, AICC* and *-2 logL* can be evaluated by using the formulas as follows:

$$AIC = 2K - 2logL, \qquad BIC = klogn - 2logL, \qquad AICC = AIC + \frac{2k(k+1)}{(n-k-1)}$$

Where *k* is the number of parameters, *n* is the sample size and *-2 logL* is the maximized value of log likelihood function and they are showed in Table 1.

Table 1: Fitted distributions of the three data sets and criteria for comparison.

Data Sets	Distribution	MLE	S.E	-2 logL	AIC	BIC	AICC
1	Exponentiated Garima	$\hat{\theta}$=0.19028545 $\hat{\alpha}$=54.28826739	$\hat{\theta}$=0.02522012 $\hat{\alpha}$=31.57442845	208.2141	212.2141	215.082	212.642671
	Garima	$\hat{\theta}$=0.051574492	$\hat{\theta}$=0.007885046	267.278	269.278	270.712	269.415931
	Exponential	$\hat{\theta}$=0.032461087	$\hat{\theta}$=0.005824644	274.5289	276.5289	277.9629	276.666831
2	Exponentiated Garima	$\hat{\theta}$=0.019286347 $\hat{\alpha}$=0.689518572	$\hat{\theta}$=0.004468638 $\hat{\alpha}$=0.153184409	305.7035	309.7035	311.5059	309.7035
	Garima	$\hat{\theta}$=0.024054339	$\hat{\theta}$=0.003909228	308.7266	310.7266	312.1277	310.869457
	Exponential	$\hat{\theta}$=0.016793310	$\hat{\theta}$=0.003055108	315.2594	317.2594	318.6606	317.402257
3	Exponentiated Garima	$\hat{\theta}$=2.2829702 $\hat{\alpha}$=70.0895927	$\hat{\theta}$=0.1867621 $\hat{\alpha}$=26.0879240	108.4583	112.4583	116.9265	112.640118
	Garima	$\hat{\theta}$=0.58724915	$\hat{\theta}$=0.05988194	251.0992	253.0992	255.3333	253.158901
	Exponential	$\hat{\theta}$=0.40794208	$\hat{\theta}$=0.04911016	261.7352	263.7352	265.9693	263.794901

Where, $AIC = 2k - 2\log L$, $BIC = k\log n - 2\log L$ and $AICC = AIC + \frac{2k(k+1)}{(n-k-1)}$

It can be easily seen from Table 1, that the exponentiated Garima distribution has the lesser *AIC, BIC, AICC* and *-2 logL* values as compared to Garima and Exponential distributions. Hence, we can conclude that the exponentiated Garima distribution leads to better fit than the Garima and Exponential distributions.

10 Conclusions

In the present study, we have introduced a new generalization of the Garima distribution namely as exponentiated Garima distribution with two parameters (scale and shape). The subject distribution is generated by using the exponentiated technique and the parameters have been obtained by using maximum likelihood estimator. Some mathematical properties along with reliability measures are discussed. The new distribution with its applications in real life-time data has been demonstrated. Finally, the results of three real lifetime data sets have been compared over Garima and exponential distributions and have been found that exponentiated Garima distribution provides better fit than Garima and exponential distributions.

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