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Dynamics of Gaussian Optical Solitons by Collective Variables Method

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This paper studies the classical optical solitons in presence of perturbation terms that arises in various contexts of the propagation of solitons through optical fibers. The adiabatic parameter dynamics of these solitons are laid down by the aid of collective variables method. Gaussian solitons are considered. Finally, the numerical simulations are obtained to complete the study.

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1 Introduction

The study of optical solitons has been a subject of research for the last few decades. This study has made profound progress during that time frame. There have also been many experimental studies during this period in various national labs and in particular, the AT&T

labs. Optical solitons have also become a reality in various parts of Europe and Australia where long distance communications are being carried out by means of optical pulses. There has also been a lot of transformation taking place in other parts of the world where fiber optic cables are being buried for faster communication means. Thus, the study of optical soliton propagation is an important area of research in the modern era.

The dynamics of the propagation of optical solitons, for trans-oceanic and trans-continental distances, is governed by the Nonlinear Schrodinger's equation (NLSE) with Kerr law nonlinearity [1–10]. The dimensionless form of NLSE is given by

$$iq_z + \frac{1}{2}q_{tt} + |q|^2 q = 0 \tag{1.1}$$

where t represents the nondimensional distance along the fiber, while z represents time in dimensionless form, and q represents the wave field. In Eq. (1.1), the first term represents the evolution term, the second term is due to the group velocity dispersion (GVD) and the third term which is the nonlinear term accounts for Kerr law of nonlinearity. It is for this reason that equations of these type fall under the category of nonlinear evolution equations. This is a nonlinear partial differential equation (PDE) that is of S-integrable type. It is therefore integrable by the method of Inverse Scattering Transform (IST) which is the nonlinear analog of Fourier Transform that is used for integrating the linear PDEs.

The fundamental mechanism of soliton formation namely the balanced interplay of GVD and nonlinearity induced self-phase modulation (SPM) is well understood. In the pico-second regime, the nonlinear evolution equation that takes into account this interplay of GVD and SPM and which describes the dynamics of soliton described by the NLSE.

One of the intrinsic properties of the NLSE is that it has infinitely many conserved quantities. However, only the first four of them are listed here which are commonly studied. They are respectively given by the Energy (E), linear momentum (M), Hamiltonian (H) and the Hamiltonian of the next integrable hierarchy of NLSE (H_1).

$$E = \int_{-\infty}^{\infty} |q|^2 dt, \tag{1.2}$$

$$M = \frac{i}{2}\int_{-\infty}^{\infty} (qq_t^* - q^* q_t) dt, \tag{1.3}$$

$$H = \frac{1}{2}\int_{-\infty}^{\infty} (|q_t|^2 - |q|^4) dt, \tag{1.4}$$

$$H_1 = \frac{1}{2}\int_{-\infty}^{\infty} \left\{ q_{tt}q_t^* - q_t q_{tt}^* + 3|q|^2 \left(q^* q_t - qq_t^*\right)\right\} dt. \tag{1.5}$$

The NLSE along with its perturbation terms that will be studied in this paper is given by

$$iq_t + \frac{1}{2}q_{xx} + |q|^2 q = i\epsilon R[q, q^*]. \tag{1.6}$$

Here R is a spatio-differential or integro-differential operator while the perturbation parameter ϵ with $0 < \epsilon \ll 1$ is called the relative width of the spectrum that arises due to quasi-monochromaticity. In this paper, the following perturbation terms that are considered, are all exhaustively studied in the context of fiber optics and optical solitons.

$$\begin{aligned} R = {} & \delta |q|^{2m} q + \alpha q_t + \beta q_{tt} - \gamma q_{ttt} + \lambda \left(|q|^2 q\right)_t + \theta \left(|q|^2\right)_t q \\ & + \rho |q_t|^2 q - i\xi \left(q^2 q_t^*\right)_t - i\eta q_t^2 q^* - i\zeta q^* \left(q^2\right)_{tt} - i\mu_1 \left(|q|^2\right)_t q \\ & - \mu_2 \left(|q|^4\right)_t q - i\chi q_{tttt} - i\psi q_{tttttt} + (\sigma_1 q + \sigma_2 q_t) \int_{-\infty}^{t} |q|^2 ds. \end{aligned} \tag{1.7}$$

In Eq. (1.7), δ is the coefficient of nonlinear damping or amplification [5] depending on its sign and m could be $0, 1, 2$. For $m = 0$, δ is the linear amplification or attenuation according to δ being positive or negative. For $m = 1$, δ represents the two-photon absorption (or a nonlinear gain if $\delta > 0$). If $m = 2$, δ gives a higher order correction (saturation or loss) to the nonlinear amplification-absorption. Also, β is the bandpass filtering term [6]. In Eq. (1.7), λ is the self-steepening coefficient for short pulses [5] (typically ≤ 100 femto seconds), ν is the higher order dispersion coefficient [5]. Here μ_1 is the coefficient of Raman scattering [5, 6] and μ_2 is due to nonlinear dispersion. Also, α is the frequency separation between the soliton carrier and the frequency at the peak of EDFA gain [6]. Moreover, ρ represents the coefficient of nonlinear dissipation induced by Raman scattering [7]. The coefficients of ξ, η and ζ arise due to quasi-solitons [9]. The integro-differential perturbation terms with σ_1 and σ_2 are due to saturable amplifiers [5, 6].

The coefficients of the higher order dispersion terms are respectively given by γ, χ and ψ. It is known that the NLSE, as given by Eq. (1.1), does not give correct prediction for pulse widths smaller than 1 picosecond. For example, in solid state solitary lasers, where pulses as short as 10 femto seconds are generated, the approximation breaks down. Thus, quasi-monochromaticity is no longer valid and so higher order dispersion terms come in. If the group velocity dispersion is close to zero, one needs to consider the third and higher order dispersion for performance enhancement along trans-oceanic and trans-continental distances. Also, for short pulse widths where group velocity dispersion changes within the spectral bandwidth of the signal, the group velocity cannot be neglected; hence, one needs to take into account the presence of higher order dispersion terms. This reasoning leads to the inclusion of the fourth and sixth order dispersion terms that are respectively given by the coefficients of χ and ψ [5].

2 Mathematical Formulation

In the method of collective variables (CV), the solution of the NLSE is assumed to be split into two parts, where the first part represents the soliton solution while the second part represents the residual radiation or the small amplitude dispersive waves. The hypothesis is that the soliton solution depends on a collection of variables that could represent the soliton amplitude, frequency, chirp and others. By introducing the CV increases the phase space of the dynamical system of the soliton parameters. The residual field is approximated to zero. The equations of constraints result in a nonlinear dynamical system of the field variables which can be numerically investigated to study the propagation of solitons through an optical fiber.

The soliton field $q(z,t)$ is split into the sum of two parts as

$$q(z,t) = f(z,t) + g(z,t), \tag{2.1}$$

where f represents the pulse configuration while g represents the residual field. Now, the soliton field is assumed to be a function of N independent variables x_j for $1 \leq j \leq N$ which are called CVs. Again, each of these CVs in turn are dependent on the variables z and t. Thus one can rewrite Eq. (2.1) as

$$q(z,t) = f(x_1, x_2, \ldots, x_N, t) + g(z,t). \tag{2.2}$$

Thus the introduction of these N CVs for the function f increases the degrees of freedom resulting in the expansion of the available phase-space of the system. In order that the system remains in the original phase space, certain constraints are to be imposed. These constraints are obtained by configuring the function f so that it becomes the best fit for the static solution. The residual free energy (RFE) is given by

$$E = \int_{-\infty}^{\infty} |g|^2 dt = \int_{-\infty}^{\infty} |q - f(x_1, x_2, \ldots, x_N, t)|^2 \, dt. \tag{2.3}$$

From this definition, let C_j denote the rate of change of RFE with respect to the jth CV x_j so that

$$C_j = \frac{\partial E}{\partial x_j} = \frac{\partial}{\partial x_j} \int_{-\infty}^{\infty} |g|^2 dt = \int_{-\infty}^{\infty} \left(\frac{\partial g}{\partial x_j} g^* + \frac{\partial g^*}{\partial x_j} g \right) dt. \tag{2.4}$$

Again from

$$g(z,t) = q(z,t) - f\left\{x_1(z,t), x_2(z,t), \ldots, x_N(z,t), t\right\} \tag{2.5}$$

one can write Eq. (2.4) as

$$C_j = \left\langle \frac{\partial f^*}{\partial x_j} \, g \right\rangle + \left\langle \frac{\partial f}{\partial x_j} \, g^* \right\rangle, \tag{2.6}$$

where the notation

$$\langle u, v \rangle = \int_{-\infty}^{\infty} u(t) v(t) dt \tag{2.7}$$

was used. Then the rate of change of C_j with respect to the normalized distance is defined as

$$\begin{aligned}\dot{C}_j &= \frac{dC_j}{dz} = 2\Re\left\{\frac{d}{dz}\left(\int_{-\infty}^{\infty}\frac{\partial f^*}{\partial x_j}gdt\right)\right\} \\ &= 2\Re\left(\int_{-\infty}^{\infty}\frac{\partial f^*}{\partial x_j}\frac{\partial g}{\partial z}dt + \sum_{k=1}^{N}\int_{-\infty}^{\infty}\frac{\partial^2 f^*}{\partial x_j \partial x_k}\frac{\partial x_k}{\partial z}gdt\right), \end{aligned} \tag{2.8}$$

where $\Re$ represents the real part. Thus,

$$\dot{C}_j = 2\Re\left(\left\langle\frac{\partial f^*}{\partial x_j}, \frac{\partial g}{\partial z}\right\rangle + \left\langle\frac{\partial^2 f^*}{\partial x_j \partial x_k}, \frac{\partial x_k}{\partial z}g\right\rangle\right). \tag{2.9}$$

Now, Dirac's principle states that a function that is approximately zero cannot be set equal to zero until its variations with respect to all its variables are made [2]. Hence the system will evolve such that C_j are a minimum and the equations of the constraints are obtained as

$$C_j \approx 0 \qquad \text{and} \qquad \dot{C}_j \approx 0. \tag{2.10}$$

Substituting Eq. (2.1) into Eq. (1.6), leads to the equation of motion of the residual field $g(z,t)$ which when substituted into Eq. (2.8) leads to

$$-\dot{C}_j = 2\Re\sum_{k=1}^{N}\left[\int_{-\infty}^{\infty}\frac{\partial f^*}{\partial x_j}\frac{\partial f}{\partial x_k}dt - \int_{-\infty}^{\infty}\frac{\partial^2 f^*}{\partial x_j \partial x_k}dt\right]\frac{dx_k}{dt} + R_j \tag{2.11}$$

for $1 \leq j \leq N$. This is equivalent to the matrix equation

$$\dot{\mathbf{C}} = \frac{\partial \mathbf{C}}{\partial \mathbf{X}}\dot{\mathbf{X}} + \mathbf{R}, \tag{2.12}$$

where

$$\mathbf{X} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix} \qquad \text{and} \qquad \mathbf{R} = \begin{bmatrix} R_1 \\ R_2 \\ \vdots \\ R_N \end{bmatrix} \tag{2.13}$$

while the $N \times N$ Jacobian matrix is given by

$$\frac{\partial \mathbf{C}}{\partial \mathbf{X}} = \frac{\partial\left(C_1, C_2, \ldots, C_N\right)}{\partial\left(x_1, x_2, \ldots, x_N\right)} = \left(\frac{\partial C_j}{\partial x_k}\right)_{N\times N} \tag{2.14}$$

with

$$\frac{\partial C_j}{\partial x_k} = 2\Re\left(\int_{-\infty}^{\infty}\frac{\partial f^*}{\partial x_j}\frac{\partial f}{\partial x_k}dt - \int_{-\infty}^{\infty}\frac{\partial^2 f^*}{\partial x_j \partial x_k}dt\right) \tag{2.15}$$

for $1 \leq j, k \leq N$. One can now solve Eq. (2.12) for $\dot{\mathbf{X}}$ to obtain the adiabatic parameter dynamics of the collective variables (soliton parameters) due to the presence of the perturbation terms. It is to be noted that this process of solving for $\dot{\mathbf{X}}$ from Eq. (2.12) requires the inversion of the Jacobian matrix given by Eq. (2.14).

3 Soliton Parameter Dynamics

In this section the adiabatic parameter dynamics of optical solitons will be obtained by virtue of the CV method. For optical solitons, $N = 6$ as will be seen and interpreted. The equations for all the CVs are going to be obtained by the lowest order CV theory [2] that is also known as *bare approximation*. When dressing of the soliton and soliton radiation are negligible, the bare approximation is applied by which the residual field is set to zero. Thus, $g(z,t)$ is set to zero.

For Gaussian soliton ansatz, one can take the chirped soliton pulse as

$$f\left(x_1,x_2,x_3,x_4,x_5,x_6;t\right) = x_1 e^{-(t-x_2)^2/x_3^2} e^{i\left\{x_4(t-x_2)^2/2+x_5(t-x_2)+x_6\right\}}, \tag{3.1}$$

where x_1 is the soliton amplitude, x_2 is the center position of the soliton, x_3 is the inverse width of the pulse, x_4 is the soliton chirp, x_5 is the soliton frequency and x_6 is the soliton phase. In this case, with $N = 6$,

$$\frac{\partial C}{\partial x} = \begin{bmatrix} \frac{\partial C_1}{\partial x_1} & \frac{\partial C_1}{\partial x_2} & \frac{\partial C_1}{\partial x_3} & \frac{\partial C_1}{\partial x_4} & \frac{\partial C_1}{\partial x_5} & \frac{\partial C_1}{\partial x_6} \\ \frac{\partial C_2}{\partial x_1} & \frac{\partial C_2}{\partial x_2} & \frac{\partial C_2}{\partial x_3} & \frac{\partial C_2}{\partial x_4} & \frac{\partial C_2}{\partial x_5} & \frac{\partial C_2}{\partial x_6} \\ \frac{\partial C_3}{\partial x_1} & \frac{\partial C_3}{\partial x_2} & \frac{\partial C_3}{\partial x_3} & \frac{\partial C_3}{\partial x_4} & \frac{\partial C_3}{\partial x_5} & \frac{\partial C_3}{\partial x_6} \\ \frac{\partial C_4}{\partial x_1} & \frac{\partial C_4}{\partial x_2} & \frac{\partial C_4}{\partial x_3} & \frac{\partial C_4}{\partial x_4} & \frac{\partial C_4}{\partial x_5} & \frac{\partial C_4}{\partial x_6} \\ \frac{\partial C_5}{\partial x_1} & \frac{\partial C_5}{\partial x_2} & \frac{\partial C_5}{\partial x_3} & \frac{\partial C_5}{\partial x_4} & \frac{\partial C_5}{\partial x_5} & \frac{\partial C_5}{\partial x_6} \\ \frac{\partial C_6}{\partial x_1} & \frac{\partial C_6}{\partial x_2} & \frac{\partial C_6}{\partial x_3} & \frac{\partial C_6}{\partial x_4} & \frac{\partial C_6}{\partial x_5} & \frac{\partial C_6}{\partial x_6} \end{bmatrix} \tag{3.2}$$

while

$$\mathbf{X} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} \quad \text{and} \quad \mathbf{R} = \begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_5 \\ R_6 \end{bmatrix}. \tag{3.3}$$

Thus, the spatio-differential/integro-differential operator $\mathbf{R}$ and the nonlinear dynamical system (DS) $\mathbf{X}$ reduce to the following:

$$\begin{aligned} R_1 = \epsilon\Bigg\{ & \frac{\sqrt{2}}{16}\left(\xi+\eta\right)\frac{x_1^3}{x_3}\left(-4+8x_5^2x_3^2+x_4^2x_3^4\right)+\frac{\sqrt{2}}{8}\left(2\rho+\zeta\right)\frac{x_1^3}{x_3}\left(4+8x_5^2x_3^2+x_4^2x_3^4\right) \\ & -\frac{1}{16}\chi\frac{x_1}{x_3^3}\left(48+24x_3^4x_4^2+96x_5^2x_3^2+16x_3^4x_5^4+3x_3^8x_4^4+24x_3^6x_5^2x_4^2\right) \\ & +\frac{1}{64}\psi\frac{x_1}{x_3^5}\left(960+64x_3^6x_5^6+720x_4^2x_3^4+1440x_3^6x_4^2x_5^2+2880x_5^2x_3^2\right. \\ & \left.+960{x_5}^4{x_3}^4+180x_3^8x_4^4+180x_3^{10}x_5^2x_4^4+15x_3^{12}x_4^6+240x_3^8x_5^4x_4^2\right) \\ & -\frac{1}{4}\beta\frac{x_1}{x_3}\left(4+4x_5^2x_3^2+x_4^2x_3^4\right)+\frac{\sqrt{2}\delta x_1^{4m+1}x_3}{\sqrt{4m+2}} \end{aligned}$$

$$+ 2\sigma_1 x_1^3 \int_{-\infty}^{\infty} e^{-2(x-x_2)^2/x_3^2} \int_{-\infty}^{x} exp^{-2(s-x_2)^2/x_3^2} dsdx$$
$$- 4\sigma_2 \frac{x_1^3}{x_3^2} \int_{-\infty}^{\infty} (x-x_2)\, e^{-2(x-x_2)^2/x_3^2} \int_{-\infty}^{x} e^{-2(s-x_2)^2/x_3^2} dsdx \Bigg\}, \tag{3.4}$$

$$R_2 = \frac{1}{8}\frac{x_1^2 x_5}{x_3}\left(12+4x_5^2x_3^2+3x_4^2x_3^4-4\sqrt{2}x_1^2x_3^2\right)+\epsilon\left\{\frac{1}{4}\alpha\frac{x_1}{x_3}\left(-4-4x_1x_5^2x_3^2-x_1x_4^2x_3^2\right)\right.$$
$$+ \frac{1}{16}\gamma\frac{x_1^2}{x_3^3}\left(-96-24x_5^2x_4^2x_3^6-3x_4^2x_3^8-16x_5^4x_3^4-32x_5^2x_3^3+24x_5^2x_3^2\right)$$
$$- \frac{\sqrt{2}}{16}\lambda\frac{x_1^4}{x_3}\left(12-8x_5^2x_3^2-x_4^2x_3^4\right)$$
$$+ \frac{\sqrt{2}}{2}\frac{\mu_1 x_1^4}{x_3} + \frac{4\sqrt{3}}{9}\frac{\mu_2 x_1^6}{x_3} + \frac{\sqrt{2}}{4}(\xi-\eta+\rho)\, x_1^4 x_4 x_5 x_3$$
$$+ 4\sigma_1 \frac{x_1^4}{x_3^2} \int_{-\infty}^{\infty} (x-x_2)\, e^{-2(x-x_2)^2/x_3^2} \int_{-\infty}^{x} e^{-2(s-x_2)^2/x_3^2} dsdx$$
$$\left. - 8\sigma_2 \frac{x_1^4}{x_3^4} \int_{-\infty}^{\infty} (x-x_2)^2\, e^{-2(x-x_2)^2/x_3^2} \int_{-\infty}^{x} e^{-2(s-x_2)^2/x_3^2} dsdx \right\}, \tag{3.5}$$

$$R_3 = \frac{1}{2}x_1^2 x_4 \epsilon\Bigg\{ -3\gamma x_1^2 x_4 x_5 + \frac{\sqrt{2}}{64}\xi\frac{x_1^4}{x_3^2}\left(20+8x_5^2x_3^2+3x_4^2x_3^4\right)$$
$$+ \frac{\sqrt{2}}{64}\eta\frac{x_1^4}{x_3}\left(-12+8x_5^2x_3+3x_4^2x_3^3\right) + \frac{\sqrt{2}}{16}\rho\frac{x_1^4}{x_3^2}\left(-8+16x_5^2x_3^2+6x_4^2x_3^4\right)$$
$$+ \frac{1}{32}\chi\frac{x_1^2}{x_3^4}\left(144+96x_5^2x_3^2-24x_4^2x_3^4-16x_5^4x_3^4-72x_3^6x_5^2x_4^2-15x_3^8x_4^2\right)$$
$$+ \frac{1}{128}\psi\frac{x_1^2}{x_3^6}\left(-4800+1440x_4^2x_5^2x_3^6-8640x_5^2x_3^2+900x_3^{10}x_5^2x_4^4-720x_4^2x_3^4\right.$$
$$\left. + 540x_3^8x_4^4-960x_5^4x_3^4+64x_5^6x_3^6+105x_3^{12}x_4^6+720x_3^8x_5^4x_4^2\right)$$
$$+ \frac{1}{8}\beta\frac{x_1^2}{x_3^2}\left(4-4x_5^2x_3^2-3x_4^2x_3^4\right) + \frac{\sqrt{2}}{64}\zeta\frac{x_1^4}{x_3^2}\left(12+8x_5^2x_3^2+3x_4^2x_3^4\right)$$
$$+ \frac{\sqrt{2}\delta x_1^{4m+2}}{(4m+2)^{3/2}} + 4\sigma_1\frac{x_1^4}{x_3^3}\int_{-\infty}^{\infty}(x-x_2)^2\, e^{-2(x-x_2)^2/x_3^2}\int_{-\infty}^{x} e^{-2(s-x_2)^2/x_3^2} dsdx$$
$$- 8\sigma_2\frac{x_1^4}{x_3^5}\int_{-\infty}^{\infty}(x-x_2)^3\, e^{-2(x-x_2)^2/x_3^2}\int_{-\infty}^{x} e^{-2(s-x_2)^2/x_3^2} dsdx\Bigg\}, \tag{3.6}$$

$$R_4 = \frac{1}{64}\left(4x_1^2x_3-4x_1^2x_5^2x_3^3-3x_1^2x_4^2x_3^5+4x_4x_3^3\right)$$
$$+ \epsilon\Bigg\{\frac{1}{8}\alpha x_1^2 x_5 x_3 - 3\gamma x_1^2 x_4 x_5 + \frac{\sqrt{2}}{64}(\xi+3\eta+8\rho)\, x_1^4 x_4 x_3^3$$

$$+\frac{1}{8}\chi\left(-12x_1^2x_4x_3-12x_1^2x_3^3x_4x_5^2-3x_1^2x_3^5x_4^3\right)$$
$$+\frac{1}{64}\psi\frac{x_1^2x_4}{x_3}\left(720+360x_3^6x_4^2x_5^2+1440x_3^2x_5^2+360x_3^4x_4^2\right.$$
$$\left.+45x_3^8x_4^4+240x_3^4x_5^4\right)-\frac{1}{4}\beta x_1^2x_4x_3^3$$
$$\left.+\sigma_2x_1^4\int_{-\infty}^{\infty}(x-x_2)^2(x_5+x_4(x-x_2))e^{-2(x-x_2)^2/x_3^2}\int_{-\infty}^{x}e^{-2(s-x_2)^2/x_3^2}dsdx\right\}, \quad (3.7)$$

$$R_5=-\frac{1}{8}x_1^2x_5x_4x_3^3+\epsilon\left\{\frac{1}{4}\alpha x_1^2x_4x_3^3+\frac{3}{16}\gamma\left(x_1^2x_3^5x_4^3+4x_1^2x_3x_4+4x_1^2x_3^3x_5^2x_4\right)\right.$$
$$+\frac{\sqrt{2}}{16}\lambda x_1^4x_4x_3^3+\frac{\sqrt{2}}{4}\left(\xi+\eta+\frac{1}{4}\rho\right)x_1^4x_3x_5$$
$$-\frac{1}{2}\chi\frac{x_1^2x_5}{x_3}\left(12+3x_4^2x_3^4+4x_5^2x_3^2\right)+\frac{1}{16}\psi\frac{x_1^2x_5}{x_3^3}\left(720+120x_3^6x_5^2x_4^2\right.$$
$$\left.+45x_3^8x_4^4+360x_3^4x_4^2+480x_5^2x_3^2+48x_3^4x_5^4\right)-\beta x_1^2x_5x_3$$
$$\left.+2\sigma_2{x_1}^4\int_{-\infty}^{\infty}(x-x_2)^2(x_5+x_4(x-x_2))e^{-2(x-x_2)^2/x_3^2}\int_{-\infty}^{x}e^{-2(s-x_2)^2/x_3^2}dsdx\right\}, \quad (3.8)$$

$$R_6=\frac{1}{8}\frac{x_1^2}{x_3}\left(-4-4x_5^2x_3^2-x_3^4x_4^2+4\sqrt{2}x_1^2x_3^2\right)+\epsilon\left\{\alpha x_1^2x_5x_3\right.$$
$$+\frac{1}{4}\gamma\frac{x_1^2x_5}{x_3}\left(12+4x_5^2x_3^2+3x_4^2x_3^4\right)+\frac{\sqrt{2}}{2}\lambda x_1^4x_3x_5+\frac{\sqrt{2}}{4}(\xi+\eta)x_1^4x_4x_3$$
$$\left.+2\sigma_2{x_1}^4\int_{-\infty}^{\infty}(x_5+x_4(x-x_2))e^{-2(x-x_2)^2/x_3^2}\int_{-\infty}^{x}e^{-2(s-x_2)^2/x_3^2}dsdx\right\}, \quad (3.9)$$

$$\dot{x}_1=\frac{1}{64}\frac{1}{x_1x_3^6}\epsilon\left\{576\gamma\left(x_1^2x_4x_5x_3^6\right)-\xi\left(3\sqrt{2}x_1^4x_3^8x_4^2-44\sqrt{2}x_1^4x_3^4+40\sqrt{2}x_1^4x_3^6x_5^2\right)\right.$$
$$-\eta\left(40\sqrt{2}x_3^6x_5^2-24\sqrt{2}x_1^4x_3^4+12\sqrt{2}x_1^4x_3^5+3\sqrt{2}x_1^4x_3^8x_4^2\right)$$
$$-\rho\frac{x_1^3}{x_3^6}\left(96\sqrt{2}x_1^4x_3^4+24\sqrt{2}x_1^4x_3^8x_4^2+192\sqrt{2}x_1^4x_3^6x_5^2\right.$$
$$\left.-4\sqrt{2}\left(-4x_1^4x_3^4+8x_1^4x_5^2x_3^6+3x_1^4x_3^8x_4^2\right)\right)$$
$$-\chi\left(-576x_1^2x_3^2-96x_1^2x_3^6x_4^2+30x_1^2x_3^{10}x_4^2-64x_1^2x_3^6x_5^4-18x_1^2x_3^{10}x_4^4-768x_1^2x_3^4x_5^2\right)$$
$$-\psi\left(3840x_1^2+1440x_1^2x_3^4x_4^2+8640x_1^2x_5^2x_3^2+1920x_1^2x_5^4x_3^4\right.$$
$$\left.+64x_1^2x_3^6x_5^6-180x_1^2x_3^{10}x_5^2x_4^4-30x_1^2x_3^{12}x_4^6+1440x_1^2x_3^6x_5^2x_4^2\right)$$
$$-\zeta\left(88\sqrt{2}x_1^4x_3^6x_5^2+9\sqrt{2}x_1^4x_3^8x_4^2+36\sqrt{2}x_1^4x_3^4\right)$$
$$-\beta\left(-64x_1^2x_3^6x_5^2-128x_1^2x_3^4\right)$$

$$
\begin{aligned}
&- \sigma_1 \Bigg(192 x_1^4 x_3^5 \int_{-\infty}^{\infty} e^{-2(x-x_2)^2/x_3^2} \int_{-\infty}^{x} e^{-2(-s+x_2)^2/x_3^2} ds dx \\
&- 256 x_1^4 x_3^6 \int_{-\infty}^{\infty} (x-x_2)^2 e^{-2(x-x_2)^2/x_3^2} \int_{-\infty}^{x} e^{-2(-s+x_2)^2/x_3^2} ds dx \Bigg) \\
&- \sigma_2 \Bigg(512 x_1^4 x_3^6 \int_{-\infty}^{\infty} (x-x_2)^3 e^{-2(x-x_2)^2/x_3^2} \int_{-\infty}^{x} e^{-2(-s+x_2)^2/x_3^2} ds dx \\
&- 384 x_1^4 x_3^5 \int_{-\infty}^{\infty} (x-x_2) e^{-2(x-x_2)^2/x_3^2} \int_{-\infty}^{x} e^{-2(-s+x_2)^2/x_3^2} ds dx \Bigg) \\
&- 64\sqrt{2} \frac{1}{\sqrt{4m+2}} \delta x_1^{4m+2} x_3^6 + 32 x_1^2 x_4 x_3^6 \Bigg\}, \qquad (3.10)
\end{aligned}
$$

$$
\begin{aligned}
\dot{x}_2 = \frac{1}{144} \frac{1}{x_3^2 x_1} \epsilon \Bigg\{ & \alpha \left(-144 x_3^2 - 36 x_3^4 x_1 x_4^2 + 36 x_4^2 x_1 x_3^6 \right) \\
&- \gamma \left(-864 x_1 - 27 x_1 x_4^2 x_3^8 + 27 x_4^4 x_1 x_3^8 - 288 x_1 x_5^2 x_3^3 + 648 x_1 x_5^2 x_3^2 + 108 x_3^4 x_1 x_4^2 \right) \\
&- \frac{144\sqrt{2}}{4} (3\lambda + 2\mu_1) x_1^3 x_3^2 - \frac{576\sqrt{3}}{9} \mu_2 x_1^5 x_3^2 - \frac{144\sqrt{2}}{4} (3\xi + \eta + 5\rho) x_3^4 x_1^3 x_4 x_5 \\
&- \chi \left(-864 x_1 x_4 x_5 x_3^2 - 216 x_4^3 x_1 x_5 x_3^6 - 288 x_4 x_1 x_5^3 x_3^4 \right) \\
&- \psi \left(432 x_4 x_1 x_5^5 x_3^4 + 4320 x_4 x_1 x_5^3 x_3^2 + 405 x_4^5 x_1 x_5 x_3^8 \right. \\
&\left. + 1080 x_4^3 x_1 x_5^3 x_3^6 + 6480 x_4 x_1 x_5 + 3240 x_4^3 x_1 x_5 x_3^4 \right) + 144 \beta x_1 x_3^4 x_4 x_5 \\
&- 576 \sigma_1 x_1^3 x_3^3 \int_{-\infty}^{\infty} (x-x_2) e^{-2(x-x_2)^2/x_3^2} \int_{-\infty}^{x} e^{-2(-s+x_2)^2/x_3^2} ds dx \\
&- \sigma_2 \Bigg(288 x_1^3 x_3^3 (x_4 + x_5) \int_{-\infty}^{\infty} (x_5 + x_4 x - x_4 x_2) e^{-2(x-x_2)^2/x_3^2} \\
&\cdot \int_{-\infty}^{x} e^{-2(-s+x_2)^2/x_3^2} ds dx \\
&- 1152 x_1^3 x_3^3 \int_{-\infty}^{\infty} (x-x_2)^2 e^{-2(x-x_2)^2/x_3^2} \int_{-\infty}^{x} e^{-2(-s+x_2)^2/x_3^2} ds dx \Bigg) \\
&- \left(18 x_1 x_5 x_4^2 x_3^6 + 144 x_1 x_5 x_3^2 \right) \Bigg\}, \qquad (3.11)
\end{aligned}
$$

$$
\begin{aligned}
\dot{x}_3 = \frac{1}{32} \frac{1}{x_1^2 x_3^6} \epsilon \Bigg\{ & - \gamma \left(-192 x_1^2 x_4 x_5 x_3^6 \right) \\
&- \xi \left(28\sqrt{2} x_1^4 x_3^4 - 8\sqrt{2} x_1^4 x_3^6 x_5^2 + \sqrt{2} x_1^4 x_3^8 x_4^2 \right) \\
&- \eta \left(\sqrt{2} x_1^4 x_3^8 x_4^2 - 8\sqrt{2} x_1^4 x_3^6 x_5^2 - 12\sqrt{2} x_1^4 x_3^5 + 8\sqrt{2} x_1^4 x_3^4 \right) \\
&- \rho \left(4\sqrt{2} \left(-4 x_1^4 x_3^4 + 8 x_1^4 x_5^2 x_3^6 + 3 x_1^4 x_3^8 x_4^2 \right) - 8\sqrt{2} x_1^4 x_3^8 x_4^2 \right. \\
&\left. - 64\sqrt{2} x_1^4 x_3^6 x_5^2 - 32\sqrt{2} x_1^4 x_3^4 \right)
\end{aligned}
$$

$$
\begin{aligned}
&- \chi \left(6x_1^2 x_3^{10} x_4^4 + 384 x_1^2 x_3^4 x_5^2 + 384 x_1^2 x_3^2 - 30 x_1^2 x_3^{10} x_4^2 - 96 x_1^2 x_3^8 x_5^2 x_4^2\right) \\
&- \psi \left(-2880 x_1^2 + 240 x_1^2 x_3^8 x_5^4 x_4^2 + 180 x_1^2 x_3^8 x_4^4 + 45 x_1^2 x_3^{12} x_4^6\right. \\
&\left.- 5760 x_1^2 x_5^2 x_3^2 + 360 x_1^2 x_3^{10} x_5^2 x_4^4 - 960 x_1^2 x_5^4 x_3^4 - 720 x_1^2 x_3^4 x_4^2\right) \\
&- \zeta \left(-24\sqrt{2} x_1^4 x_3^6 x_5^2 - 4\sqrt{2} x_1^4 x_3^4 - \sqrt{2} x_1^4 x_3^8 x_4^2\right) \\
&- \beta \left(64 x_1^2 x_3^4 - 16 x_1^2 x_3^8 x_4^2\right) \\
&- \sigma_1 \left(-64 x_1^4 x_3^5 \int_{-\infty}^{\infty} e^{-2(x-x_2)^2/x_3^2} \int_{-\infty}^{x} e^{-2(-s+x_2)^2/x_3^2} ds dx\right. \\
&\left.+ 256 x_1^4 x_3^3 \int_{-\infty}^{\infty} (x-x_2)^2 e^{-2(x-x_2)^2/x_3^2} \int_{-\infty}^{x} e^{-2(-s+x_2)^2/x_3^2} ds dx\right) \\
&- \sigma_2 \left(128 x_1^4 x_3^3 \int_{-\infty}^{\infty} (x-x_2) e^{-2(x-x_2)^2/x_3^2} \int_{-\infty}^{x} e^{-2(-s+x_2)^2/x_3^2} ds dx\right. \\
&\left.- 512 x_1^4 x_3 \int_{-\infty}^{\infty} (x-x_2)^3 e^{-2(x-x_2)^2/x_3^2} \int_{-\infty}^{x} e^{-2(-s+x_2)^2/x_3^2} ds dx\right) \\
&\left.+ \frac{64\sqrt{2}}{2m+1} \delta x_1^{4m+2} x_3^6 m - 32 x_1^2 x_4 x_3^6\right\}, \qquad (3.12)
\end{aligned}
$$

$$
\begin{aligned}
\dot{x}_4 = \frac{1}{2} \frac{1}{x_1^2 x_3^6} \epsilon \Big\{ &\alpha \left(-8 x_1^2 x_5 x_3^2 + 8 x_1^2 x_3^4 x_5\right) \\
&+ \gamma \left(8 x_1^2 x_5^3 x_3^4 + 24 x_1^2 x_5 x_3^2 + 192 x_1^2 x_4 x_5 x_3 + 6 x_4^2 x_3^6 x_5 x_1^2\right) \\
&+ 4\sqrt{2} \lambda x_1^4 x_5 x_3^4 + \sqrt{2} (\xi - \eta - 8\rho) x_1^4 x_4 x_3^4 \\
&+ \chi \left(96 x_1^2 x_3^2 x_4 + 24 x_1^2 x_3^6 x_4^3 + 96 x_1^2 x_3^4 x_4 x_5^2\right) \\
&+ \psi \left(-240 x_1^2 x_4 x_5^4 x_3^4 - 1440 x_1^2 x_4 x_5^2 x_3^2 - 360 x_1^2 x_4^3 x_3^6 x_5^2\right. \\
&\left.- 45 x_1^2 x_4^5 x_3^8 - 360 x_1^2 x_4^3 x_3^4 - 720 x_1^2 x_4\right) + 16 \beta x_1^2 x_3^4 x_4 \\
&+ \sigma_2 \left(16 x_1^4 x_3^3 \int_{-\infty}^{\infty} (x_5 + x_4 x - x_4 x_2) e^{-2(x-x_2)^2/x_3^2} \int_{-\infty}^{x} e^{-2(-s+x_2)^2/x_3^2} ds dx\right. \\
&\left.- 64 x_1^4 x_3 \int_{-\infty}^{\infty} (x-x_2)^2 (x_5 + x_4 x - x_4 x_2) e^{-2(x-x_2)^2/x_3^2} \int_{-\infty}^{x} e^{-2(-s+x_2)^2/x_3^2} ds dx\right) \\
&+ \left(4\sqrt{2} x_1^4 x_3^4 - 4 x_4 x_3^4 - 8 x_1^2 x_3^2 + 2 {x_1}^2 x_4^2 x_3^6\right) \Big\}, \qquad (3.13)
\end{aligned}
$$

$$
\begin{aligned}
\dot{x}_5 = \frac{1}{144} \frac{1}{x_1 x_3^6} \epsilon \Big\{ &\alpha \left(-36 x_4^3 x_3^8 x_1 + 36 x_1 x_3^{10} x_4^3 + 144 x_1 x_4 x_3^6 - 144 x_4 x_3^6\right) \\
&- \gamma \left(-27 x_4^3 x_3^{12} x_1 - 288 x_4 x_3^7 x_1 x_5^2 + 1080 x_4 x_3^6 x_1 x_5^2\right. \\
&\left.+ 216 x_4^3 x_3^8 x_1 + 27 x_1 x_3^{12} x_4^5 - 432 x_4 x_3^4 x_1\right) \\
&- 144\sqrt{2} \lambda x_3^6 x_1^3 x_4 - 72\sqrt{2} \mu_1 x_3^6 x_1^3 x_4 - 64\sqrt{3} \mu_2 x_3^6 x_1^5 x_4
\end{aligned}
$$

$$
\begin{aligned}
&-\xi\left(144\sqrt{2}x_1^3x_3^4x_5+108\sqrt{2}x_4^2x_3^8x_1^3x_5\right)-\eta\left(144\sqrt{2}x_1^3x_3^4x_5+36\sqrt{2}x_4^2x_3^8x_1^3x_5\right)\\
&-\rho\left(576\sqrt{2}x_1^3x_3^4x_5+180\sqrt{2}x_4^2x_3^8x_1^3x_5\right)\\
&-\chi\left(-216x_1x_3^{10}x_4^4x_5-288x_1x_3^8x_4^2x_5^3-1728x_1x_5x_3^6x_4^2-3456x_1x_5x_3^2\right.\\
&\left.-1152x_1x_5^3x_3^4\right)-\psi\left(19440x_1x_5x_3^4x_4^2+4860x_1x_5x_3^8x_4^4+432x_1x_3^8x_4^2x_5^5\right.\\
&+8640x_1x_5^3x_3^6x_4^2+1080x_1x_3^{10}x_4^4x_5^3+405x_1x_3^{12}x_4^6x_5+1728x_1x_5^5x_3^4\\
&\left.+17280x_1x_5^3x_3^2+25920x_1x_5\right)-\beta\left(-576x_1x_5x_3^4-144x_1x_3^8x_4^2x_5\right)\\
&-576\sigma_1x_1x_4x_3^5\int_{-\infty}^{\infty}(x-x_2)\,e^{-2(x-x_2)^2/x_3^2}\int_{-\infty}^{x}e^{-2(-s+x_2)^2/x_3^2}dsdx\\
&-\sigma_2\left(288x_3^7x_4^2x_1^3\int_{-\infty}^{\infty}(x-x_2)\,(x_5+x_4x-x_4x_2)\,e^{-2(x-x_2)^2/x_3^2}\right.\\
&\cdot\int_{-\infty}^{x}e^{-2(-s+x_2)^2/x_3^2}dsdx\\
&+288x_4x_3^7x_1^3x_5\int_{-\infty}^{\infty}(x_5+x_4x-x_4x_2)\,e^{-2(x-x_2)^2/x_3^2}\int_{-\infty}^{x}e^{-2(-s+x_2)^2/x_3^2}dsdx\\
&+1152x_1^3x_3^3\int_{-\infty}^{\infty}(x-x_2)\,(x_5+x_4x-x_4x_2)\,e^{-2(x-x_2)^2/x_3^2}\\
&\cdot\int_{-\infty}^{x}e^{-2(-s+x_2)^2/x_3^2}dsdx\\
&\left.-1152x_4x_3^3x_1^3\int_{-\infty}^{\infty}(x-x_2)^2\,e^{-2(x-x_2)^2/x_3^2}\int_{-\infty}^{x}e^{-2(-s+x_2)^2/x_3^2}dsdx\right)\\
&\left.-\left(\frac{72}{x_4}x_3^6x_1x_5+18x_4^3x_3^{10}x_1x_5\right)\right\},
\end{aligned}
\tag{3.14}
$$

$$
\begin{aligned}
\dot{x}_6=\frac{1}{144}\frac{1}{x_1^2x_3^4}\epsilon&\left\{\alpha\left(216x_1^2x_3^4x_5+36x_4^2x_3^8x_5x_1^2-144x_1x_5x_3^4-36x_4^2x_3^6x_5x_1^2-72x_1^2x_5x_3^2\right)\right.\\
&-\gamma\left(864x_1^2x_5^3x_3^4+27x_4^4x_3^{10}x_5x_1^2-27x_1^2x_5x_3^{10}x_4^2+1728x_1^2x_4x_5x_3+270x_4^2x_3^6x_5x_1^2\right.\\
&\left.-216x_1^2x_5x_3^2-288x_1^2x_5^3x_3^5\right)-72\sqrt{2}\,(3\lambda+\mu_1)\,x_1^4x_3^4x_5-64\sqrt{3}\mu_2x_3^4x_1^6x_5\\
&-\xi\left(108\sqrt{2}x_1^4x_5^2x_3^6x_4+45\sqrt{2}x_1^4x_4x_3^4\right)\\
&-\eta\left(27\sqrt{2}x_1^4x_4x_3^4+36\sqrt{2}x_1^4x_5^2x_3^6x_4\right)\\
&-\rho\left(-72x_1^4x_4x_3^4\sqrt{2}+180x_1^4x_5^2x_3^6x_4\sqrt{2}\right)\\
&-\chi\left(-216x_4^3x_3^8x_5^2x_1^2+864x_1^2x_3^2x_4-288x_4x_3^6x_5^4x_1^2+216x_1^2x_3^6x_4^3\right)\\
&-\psi\left(-3240x_1^2x_4^3x_3^4-6480x_1^2x_4+432x_4x_3^6x_5^6x_1^2-405x_1^2x_4^5x_3^8\right.\\
&\left.+2160x_1^2x_4x_5^4x_3^4-6480x_1^2x_4x_5^2x_3^2+405x_4^5x_3^{10}x_5^2x_1^2+1080x_4^3x_3^8x_5^4x_1^2\right)\\
&-\beta\left(144x_1^2x_4x_3^4-144x_4x_3^6x_5^2x_1^2\right)
\end{aligned}
$$

$$- 576\sigma_1 x_1^4 x_5 x_3^5 \int_{-\infty}^{\infty} (x - x_2)\, x_3^{-2} e^{-2(x-x_2)^2/x_3^2} \int_{-\infty}^{x} e^{-2(-s+x_2)^2/x_3^2} ds dx$$

$$- \sigma_2 \Bigg(-576 x_1^4 x_3 \int_{-\infty}^{\infty} (x - x_2)^2 (x_5 + x_4 x - x_4 x_2)\, e^{-2(x-x_2)^2/x_3^2}$$

$$\cdot \int_{-\infty}^{x} e^{-2(-s+x_2)^2/x_3^2} ds dx$$

$$+ 432 x_1^4 x_3^3 \int_{-\infty}^{\infty} (x_5 + x_4 x - x_4 x_2)\, e^{-2(x-x_2)^2/x_3^2} \int_{-\infty}^{x} e^{-2(-s+x_2)^2/x_3^2} ds dx$$

$$- 1152 x_1^4 x_5 x_3 \int_{-\infty}^{\infty} (x - x_2)^2 e^{-2(x-x_2)^2/x_3^2} \int_{-\infty}^{x} e^{-2(-s+x_2)^2/x_3^2} ds dx$$

$$+ 288 x_1^4 x_3^5 x_5^2 \int_{-\infty}^{\infty} (x_5 + x_4 x - x_4 x_2)\, e^{-2(x-x_2)^2/x_3^2} \int_{-\infty}^{x} e^{-2(-s+x_2)^2/x_3^2} ds dx$$

$$+ 288 x_4 x_3^5 x_5 x_1^4 \int_{-\infty}^{\infty} (x - x_2)(x_5 + x_4 x - x_4 x_2)\, e^{-2(x-x_2)^2/x_3^2}$$

$$\cdot \int_{-\infty}^{x} e^{-2(-s+x_2)^2/x_3^2} ds dx \Bigg)$$

$$- \left(108\sqrt{2} x_1^4 x_3^4 - 36 x_4 x_3^4 - 144 x_1^2 x_3^2 + 18 x_1^2 x_5^2 x_3^8 x_4^2 + 72 x_1^2 x_5^2 x_3^4 \right) \Bigg\}. \tag{3.15}$$

4 Numerical Simulation

In order to understand the nonlinear DS (3.10)- (3.15), one needs to carry out its numerical simulation in order to study the evolution of the pulse parameters along the propagation distance z. This was carried out by the fourth order Runge-Kutta method to integrate the system of ordinary differential equations that forms the nonlinear DS. In all cases the choice $\epsilon = 0.1$ was made. The initial conditions were x_1 = 1, x_2 = 0, x_3 = 1, $x_4 = x_5 = x_6$ = 0.

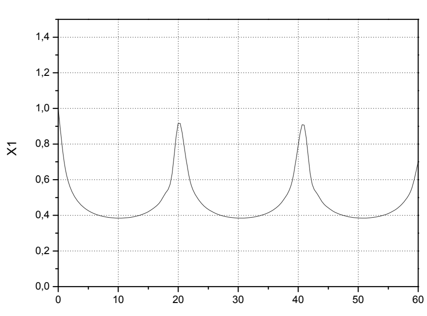

Figure 4.1: Amplitude variation

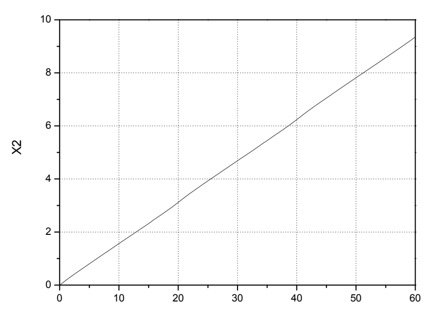

Figure 4.2: Center of soliton change

In Figures 4.1 – 4.6, γ = 0.1, λ = 0.2, while μ_2 = 0.01. As the soliton moves, its amplitude, width, frequency and chirp vary periodically. The center position of the soliton monotonically increases with the distance of propagation.

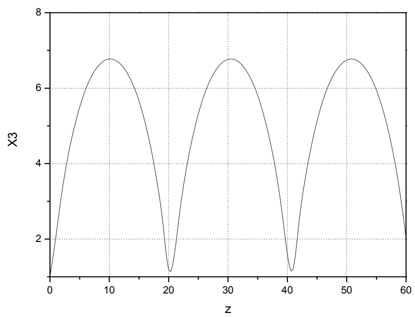

Figure 4.3: Variation of soliton width

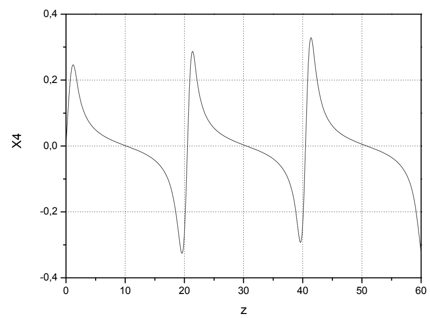

Figure 4.4: Variation of soliton chirp

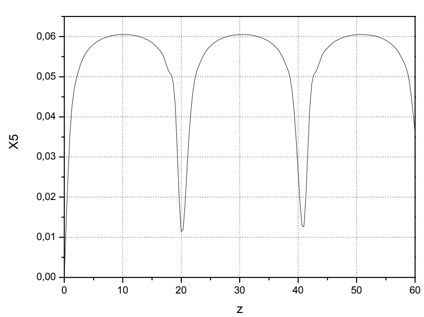

Figure 4.5: Soliton frequency variation

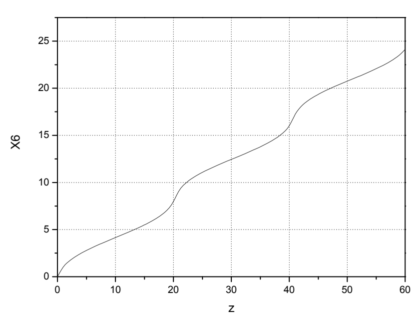

Figure 4.6: Variation of soliton phase

In Figures 4.7 – 4.12, $\gamma = 0.1$, $\lambda = 0.15$, while $\mu_2 = 0.02$. In this case, similar observations are made.

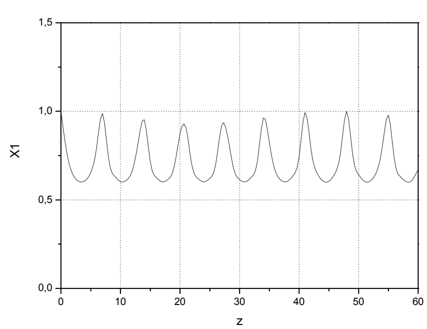

Figure 4.7: Amplitude variation

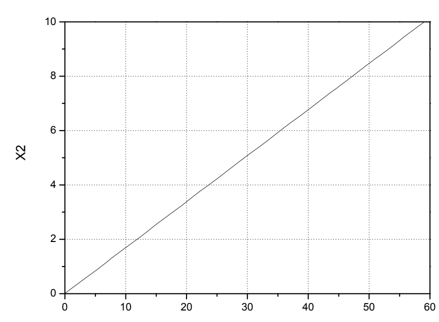

Figure 4.8: Center of soliton change

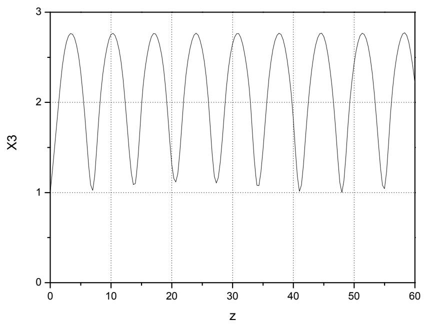

Figure 4.9: Variation of soliton width

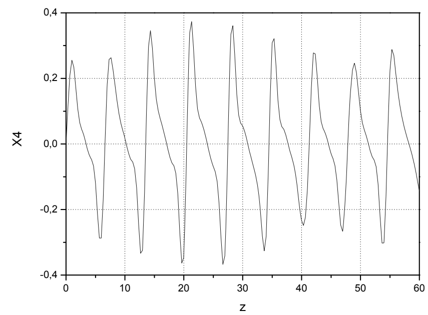

Figure 4.10: Variation of soliton chirp

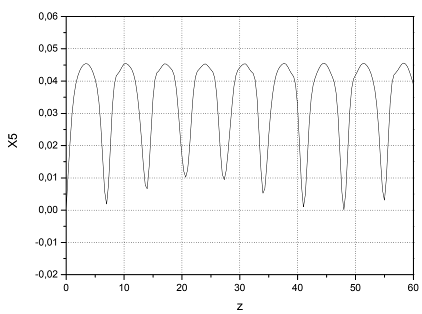

Figure 4.11: Soliton frequency variation

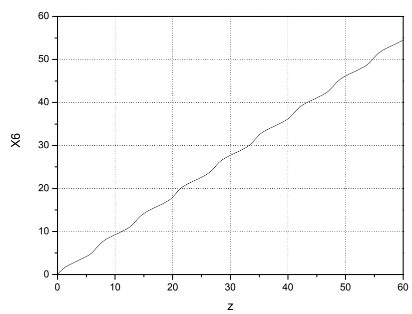

Figure 4.12: Variation of soliton phase

5 Conclusion

This paper talks about the adiabatic parameter dynamics of Gaussian optical solitons in presence of perturbation terms that are both local as well as non-local. In the local ones, both Hamiltonian as well as non-Hamiltonian type perturbations are taken into account.

These results are going to be used for further study of dispersion-managed solitons. One immediate application of this is in the study of intra-channel collision of optical solitons by virtue of quasi-particle theory. Another application of this is in the issue of Gabitov-Turitsyn equation in presence of perturbation terms. Such applications will be studied in the future and the results of this research will be reported in future publications

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