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Autocorrelation of non-Circular Statistical Gaussian Speckle Field

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Abstract: Analytical formula for the autocorrelation speckle intensity is derived. The derivation based on the autocorrelation function of the speckle field of non-circular statistics. The autocorrelation function shows its dependence on the lateral correlation length of the roughness, the root mean square of the random roughness heights and also its dependence on the speckles lateral displacement. Experimental validations are carried out through the visibility of interference pattern obtained through mixing two lateral separated speckle patterns. It is investigated for different average roughness heights and for different speckle lateral displacements.

Keywords: Speckle field correlation, non-circular Gaussian distribution, lateral correlation length of rough surface.

1 Introduction

The surface quality of materials mostly expressed in terms of their roughness. The statistical probability distribution of the roughness deviation from its mean value is not enough to describe the surface roughness completely. It does not tell us how the scatterers of the surface are crowded close together or whether they are far apart and how much is their degree of correlation. A second function, the correlation function describes this aspect of the surface. The correlation function is a characteristic property of the second order speckle statistics. It describes how rapidly the surface height varies from point to point on the surface i.e. it analyzes the spatial structure of the surface height. Speckle correlation is one of the important statistical characteristics used as a measure of roughness parameters [1-5].

Ruffing derived theoretical formula to measure the surface roughness through the correlation between two speckle patterns producing by illuminating the surface with coherent light of either two differing wavelengths or of two differing angles of incidence. Most of the previous theoretical studies on speckle correlation were based on treating the statistics of the speckle field obey the central limit theorem. This assumption requires the phases of the elementary scattered waves are completely uniform distributed on an interval of integer number of 2π. It leads to scattered complex field amplitude of circularly Gaussian statistics in the complex plane.

A part from this limited validity, the present study considers the general case of a non-circular complex Gaussian statistics of the speckle field. It represents the dependence of the speckle autocorrelation function on the root mean square of the surface roughness and on its phase correlation length.

2 Theories

It is well known that the autocorrelation of speckle intensities must of necessity arises from the autocorrelation between the contributed fields that give rise to these speckle intensities. It is much easier to derive the autocorrelation function of the speckle intensity through the autocorrelation function of its speckle field.

For simplicity, only the one-dimensional case is treated mathematically here while extension to two – dimensional is straight forward. Through the fourth order moment of the underlying speckle fields at two arbitrary positions x_1 and x_2 on the observation x-plane, the autocorrelation function of speckle intensities $\langle I_s(x_1)I_s(x_2)\rangle$ is given by [6, 7]

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$$\langle I_s(x_1)I_s(x_2)\rangle = \langle I_s(x_1)\rangle\langle I_s(x_2)\rangle + |\langle A(x_1)A^*(x_2)\rangle|^2 + |\langle A(x_1)A(x_2)\rangle|^2 - 2|A(x_1)|^2|A(x_2)|^2 \quad (1)$$

$A(x_1)$ and $A(x_2)$ are the resultant complex field amplitudes at the positions x_1 and x_2 respectively. The angular brackets $\langle\ \rangle$ denote an ensemble average. An asterisk $*$ indicates that A^* is the conjugate of A.

The present study assumed that the probability density function $P(\varphi)$ of the random phase φ, acquired by the scattered waves from the rough surface, is given by the following Gaussian distribution of zero mean value:

$$P(\varphi) = \frac{1}{\sigma_\varphi\sqrt{2\pi}}\exp\left(-\frac{\varphi^2}{2\sigma_\varphi^2}\right) \quad (2)$$

with $\langle\varphi^2\rangle = \sigma_\varphi^2$.

σ_φ is the root mean square of the phase φ where $\sigma_\varphi = \frac{4\pi}{\lambda}\langle h_r\rangle\cos\theta$ with $\langle h_r\rangle$ is the average roughness heights and θ is the specular angle of reflection. The basic factors affecting on the autocorrelation of the speckle field are the normalized phase correlation function μ_φ and the correlation lateral length L_c of the phase variation. It is assumed to be given by the following Gaussian model [8-15].

$$\mu_\varphi(\xi_1-\xi_2) = \frac{\langle\varphi(\xi_1)\varphi(\xi_2)\rangle}{\langle\varphi^2\rangle} = \exp\left(-\frac{|\xi_1-\xi_1|^2}{L_c^2}\right) \quad (3)$$

where ξ_1 and ξ_2 are two separated arbitrary points on the rough surface ξ - plane and $\langle\varphi^2(\xi_1)\rangle = \langle\varphi^2(\xi_2)\rangle = \langle\varphi^2\rangle$.

The complex field amplitude $A(x)$ at the observation x – plane, in the far field diffraction at distance z apart from the rough surface ξ – plane is given by [16, 17]:

$$A(x) = \frac{1}{i\lambda z}\iint_{-\infty}^{\infty} A_0(\xi)\exp\big(-i\varphi(\xi)\big)\exp\left(\frac{2\pi i}{\lambda z}\xi x\right)d\xi, \quad (4)$$

with λ is the wavelength of the illuminated light. It is the Fourier transform of the complex field amplitude distribution in the rough surface ξ – plane with respect to the variable $\frac{x}{\lambda z}$. The amplitude point spread function characterizing the optical imaging system is replaced here by the amplitude distribution function $A_0(\xi)$ of the illuminating light at the rough surface plane [18, 19]. The rough surface is illuminated by a Gaussian laser beam of spatial amplitude distribution function given by:

$$A_0(\xi) = \exp-\left(\frac{\xi}{w_e}\right)^2 \quad (5)$$

with w_e indicates the effective width of the amplitude distribution of the illuminating light.

The autocorrelation function of the speckle field at two arbitrary points x_1 and x_2 on the observation plane arising from the interference of scattered waves from scattering arbitrary elements ξ_1 and ξ_2 on the rough surface is given by:

$$\langle A(x_1)A^*(x_2)\rangle = \left(\frac{1}{\lambda z}\right)^2\iint_{-\infty}^{\infty}|A_0(\xi)|^2\langle\exp-i\big(\varphi(\xi_1)-\varphi(\xi_2)\big)\rangle\times \exp\frac{2\pi i}{\lambda z}(x_1\xi_1-x_2\xi_2)d\xi_1 d\xi_2 \quad (6)$$

For the assumed Gaussian probability phase density distribution, the following identity holds [15, 20-22]:

$$\langle\exp-i\big(\varphi(\xi_1)-\varphi(\xi_2)\big)\rangle = \exp[-\langle\varphi^2\rangle\big(1-\mu_\varphi(\xi_1-\xi_2)\big)] \quad (7)$$

Substitution of equation (7) into equation (6) and let $\Delta x = x_1 - x_2$ and $\Delta\xi = \xi_1-\xi_2 \geq 0$ yields:

$$\langle A(x_1)A^*(x_2)\rangle = \left(\frac{1}{\lambda z}\right)^2\exp(-\langle\varphi^2\rangle)\int_{-\infty}^{\infty}|A_0(\xi)|^2\exp i\left(\frac{2\pi}{\lambda z}\Delta x\xi_1\right)d\,\xi_1\times \int_{-\infty}^{\infty}\exp[\langle\varphi^2\rangle\mu_\varphi(\Delta\xi)]\exp i\left(\frac{2\pi}{\lambda z}x_2\Delta\xi\right)d\Delta\xi \quad (8)$$

Equation (8) shows the effect of two separated integrands on the complex autocorrelation of the speckle field. The first integrand is the Fourier transform of the aperture illumination function $|A_0(\xi)|^2$. It represents the average size of the speckle in the observation plane. The second integrand is the Fourier transform of the phase correlation function of the scattered waves. It is related to the characteristics of the roughness heights variation.

The first integrand is given by:

$$I_1 = \int_{-\infty}^{\infty}\exp-2\left(\frac{\xi}{w_e}\right)^2\exp i\left(\frac{2\pi}{\lambda z}\Delta x\xi\right)d\xi = \sqrt{\frac{\pi}{2}}w_e\exp-\frac{1}{2}\left(\frac{\pi\Delta x w_e}{\lambda z}\right)^2 \quad (9)$$

The second integrand is written in the form:

$$I_2 = \int_0^{\infty}\sum_{n=0}^{\infty}\frac{(\sigma_\varphi^2)^n}{n!}\exp\left(-n\left(\frac{\Delta\xi}{L_c}\right)^2\right)\exp i\left(\frac{2\pi}{\lambda z}x_2\Delta\xi\right)d\Delta\xi$$

The limits of integration of this integrand is considered to be from zero up to ∞, since $\Delta\xi \geq 0$.

It gives:

$$I_2 = \frac{1}{2}\sum_{n=0}^{\infty}\frac{(\sigma_\varphi^2)^n}{n!}\sqrt{\frac{\pi}{n}}L_c\exp-\left(\frac{\pi x_2 L_c}{\lambda z\sqrt{n}}\right)^2 \quad (10)$$

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The term corresponding to $n = 0$ in the summation tends to $\lambda z\delta(x_2)$ where $\delta(x_2)$ is a delta function. Substitute for I_1 and I_2 from equations (9) and (10) respectively into equation (8), we get:

$$\langle A(x_1)A^*(x_2)\rangle$$

$$= \left(\frac{1}{\lambda z}\right)^2 \exp\left(-\sigma_\varphi^2\right)\left[\sqrt{\frac{\pi}{2}}w_e \exp -\frac{1}{2}\left(\frac{\pi\Delta x w_e}{\lambda z}\right)^2\right]\times$$

$$\frac{1}{2}\sum_{n=0}^{\infty}\frac{\left(\sigma_\varphi^2\right)^n}{n!}\sqrt{\frac{\pi}{n}}L_c \exp -\left(\frac{\pi x_2 L_c}{\lambda z\sqrt{n}}\right)^2 \quad (11)$$

From the third term of equation (1) we get:

$$\langle A(x_1)A(x_2)\rangle = -\left(\frac{1}{\lambda z}\right)^2 \iint_{-\infty}^{\infty}\langle \exp -i(\varphi(\xi_1) + \varphi(\xi_2))\rangle \exp -2\left(\frac{\xi}{w_e}\right)^2 \exp i\frac{2\pi}{\lambda z}(x_1\xi_1 + x_2\xi_2)\, d\xi_1 d\xi_2 \quad (12)$$

As assumed before, $\varphi(\xi_1)$ and $\varphi(\xi_2)$ are stationary Gaussian random variables of zero mean value, their characteristic function is given by [21, 22]:

$$\langle \exp -i\left(\varphi(\xi_1) + \varphi(\xi_2)\right)\rangle = \exp\left(-\langle\varphi^2\rangle\left(1 + \frac{\langle\varphi(\xi_1)\varphi(\xi_2)\rangle}{\langle\varphi^2\rangle}\right)\right) \quad (13)$$

Equations (12) and (13) give:

$$\langle A(x_1)A(x_2)\rangle = -\left(\frac{1}{\lambda z}\right)^2 \iint_{-\infty}^{\infty} \exp\left(-2\left(\frac{\xi}{w_e}\right)^2\right)\exp\left\{-\langle\varphi^2\rangle\left(1 + \mu_\varphi(\xi_1 - \xi_2)\right)\right\}\exp i\frac{2\pi}{\lambda z}(x_1\xi_1 + x_2\xi_2)\ d\xi_1 d\xi_2 \quad (14)$$

Let $\Delta x = x_1 - x_2$ and $\Delta\xi = \xi_1 - \xi_2 \geq 0$, we get:

$$\langle A(x_1)A(x_2)\rangle = -\left(\frac{1}{\lambda z}\right)^2 \exp\left(-\langle\varphi^2\rangle\right) \int_{-\infty}^{\infty} \exp -2\left(\frac{\xi}{w_e}\right)^2 \exp\left(i\frac{2\pi}{\lambda z}(x_1 + x_2)\xi_1\right) d\xi_1 \int_0^{\infty} \exp\left\{-\langle\varphi^2\rangle \exp -\left(\frac{\Delta\xi}{L_c}\right)^2\right\}\exp i\left(\frac{2\pi}{\lambda z}x_2\Delta\xi\right) d\Delta\xi \quad (15)$$

$$\langle A(x_1)A(x_2)\rangle = -\left(\frac{1}{\lambda z}\right)^2 \exp\left(-\sigma_\varphi^2\right)\sqrt{\frac{\pi}{2}}w_e \exp -\frac{1}{2}\left(\frac{\pi(x_1 + x_2)w_e}{\lambda z}\right)^2 \quad (16)$$

The term corresponding to $n = 0$ in the summation tends to $\lambda z\delta(x_2)$.

Now we have to derive an expression for $\langle I(x_1)\rangle = \langle A(x_1)A^*(x_1)\rangle$. It represents the correlation function between speckle fields scattered from the arbitrary points ξ_1 and ξ_2 on the scattering surface towards a single arbitrary point on the observation plane. Similarly is the case of $\langle I(x_2)\rangle = \langle A(x_2)A^*(x_2)\rangle$. It represents self correlation function.

Thus by setting $\Delta x = 0$ in equation (11) we get:

$$\langle I(x)\rangle = \left(\frac{1}{\lambda z}\right)^2 \exp\left(-\sigma_\varphi^2\right)\sqrt{\frac{\pi}{2}}w_e \frac{1}{2}\sum_{n=0}^{\infty}\frac{\left(\sigma_\varphi^2\right)^n}{n!}\sqrt{\frac{\pi}{n}}L_c \exp -\left(\frac{\pi x L_c}{\lambda z\sqrt{n}}\right)^2 \quad (17)$$

where x is any arbitrary point on the observation plane.

The average speckle field $\langle A(x_1)\rangle$ scattered from the rough surface towards arbitrary point x_1 on the observation plane is:

$$\langle A(x)\rangle = \frac{1}{i\lambda z}\int_{-\infty}^{\infty}\langle \exp -i\varphi(\xi)\rangle \exp -\left(\frac{\xi}{w_e}\right)^2 \exp\left(\frac{2\pi i}{\lambda z}x\xi\right) d\xi \quad (18)$$

$$\langle A(x)\rangle = \frac{1}{i\lambda z}\exp\left(-\frac{\sigma_\varphi^2}{2}\right) w_e\sqrt{\pi}\exp -\left(\frac{\pi x w_e}{\lambda z}\right)^2 \quad (19)$$

Substitute for $|A(x_1)A^*(x_2)|^2$, $|A(x_1)A(x_2)|^2$, $\langle I_s(x_1)\rangle$, $\langle I_s(x_2)\rangle$, $|A(x_1)|^2$ and $|A(x_2)|^2$ from equations (11), (16), (17) and (19) respectively into equation (1), we get the required formula for the autocorrelation function $\langle I_s(x_1)I_s(x_2)\rangle$ of the speckle intensity.

It shows its dependence on the lateral correlation length L_c of the random phases acquired from the surface roughness, the root mean square of the roughness height expressed in σ_φ, the phase correlation function of the rough surface and the spatial separation Δx of the speckle patterns.

Figs. (1-3) show that the speckle field correlation function increases as the speckle separation Δx decreases, the root mean square of the random phases σ_φ increases and the lateral roughness correlation L_c increases respectively. The correlation reaches a maximum value within a little range of small σ_φ and then decreases with increasing σ_φ. It can be attributed to the phase randomization between the interfering speckle fields become outside the spatial coherence of the speckle fields.

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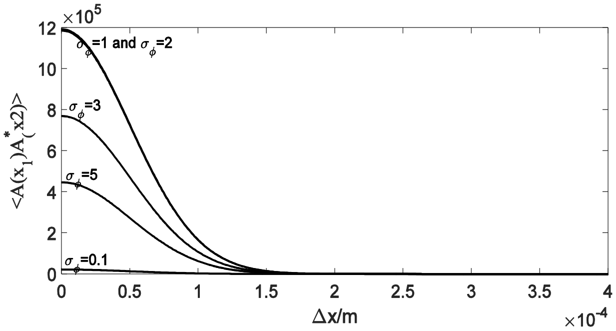


Fig. 1: Autocorrelation function versus Δx with σ_φ as a parameter computed using a laser beam illumination ($\lambda = 6328\ A^o$), $x = 1\ \times 10^{-4}\ m$, $w_e = 0.02\ m$,$z = 0.5\ m$ and $L_c = 100\ \times 10^{-6} m$.

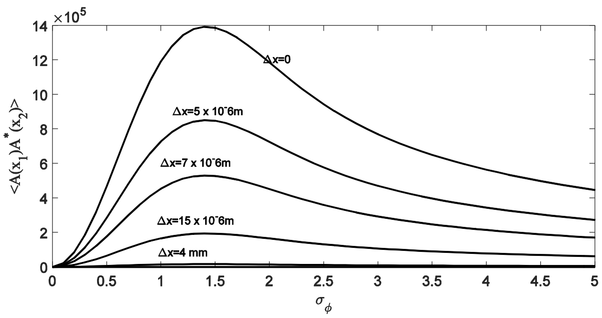


Fig. 2: Autocorrelation function versuσ_φ with Δx as a parameter computed using a laser beam illumination ($\lambda = 6328\ A^o$), $x = 1 \times 10^{-4}\ m$, $w_e = 0.02\ m$,$z = 0.5\ m$ and $L_c = 100 \times 10^{-6} m$.

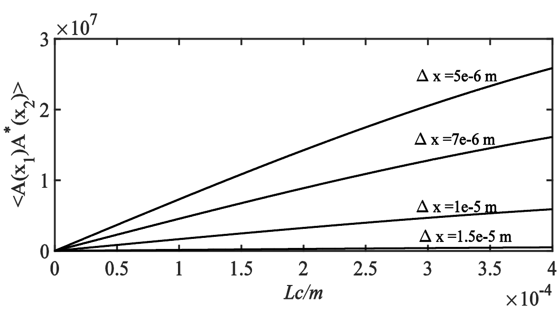


Fig. 3: Autocorrelation function versus L_c with Δx as a parameter computed using a laser beam illumination ($\lambda = 6328\ A^o$), $x = 1\ \times 10^{-4}\ m$, $w_e = 0.02\ m$,$z = 0.5\ m$ and $\sigma_\varphi = 1$.

3 Experimental Validations

Speckle pattern is an interference of scattered waves acquired random phases from the rough surface. Its autocorrelation function arises through the autocorrelation of the interfering scattered waves. From an Interferometric point of view, the autocorrelation function is known as the mutual coherence function of the interfering waves. It is related directly to the visibility of the produced interference fringes [22, 23]. Therefore the autocorrelation function of the speckle pattern is studied here experimentally through the visibility of interference fringes produced from mixing two sets of separated speckle patterns. It is measured as a function of the lateral separation of the speckle patterns Δx. The behaviour of the speckle correlation with the average surface roughness heights $\langle h_r \rangle$ could not be experimentally investigated. It requires various rough surfaces of different values of $\langle h_r \rangle$ but of the same statistical conditions, probability density distributions of roughness heights and the phase correlation function. These required conditions are not available.

Fig. 4 shows the speckle pattern using He-Ne laser $\lambda = 6328\ A^o$ for rough surface of average roughness heights $\langle h_r \rangle = 3.5\ \mu m$.

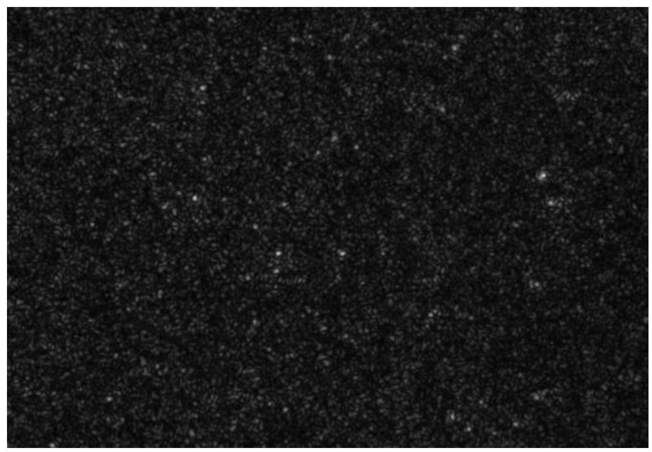
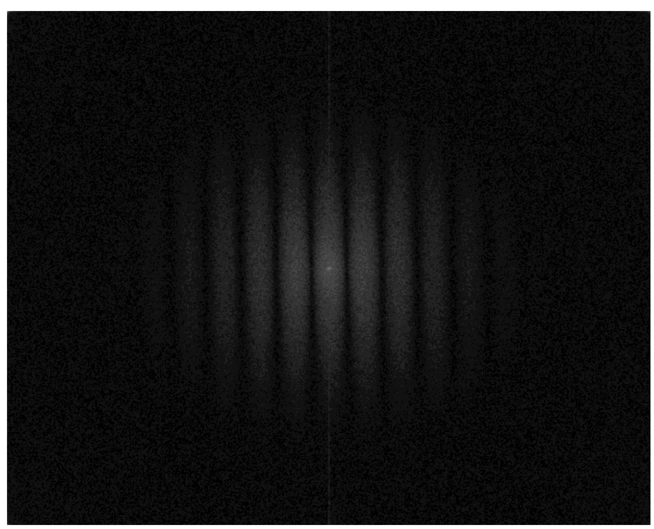


Fig. 4: The recorded speckle pattern for a rough surface of average heights $\langle h_r \rangle = 3.5\ \mu m$ using He-Ne laser ($\lambda = 6328\ A^o$).

Figure 5 (a-b) show the interference fringes of two separated speckle patterns $\Delta x = 100\ \mu m$ and $\Delta x = 200\ \mu m$ respectively and $\langle h_r \rangle = 3.5\ \mu m$.



(a)

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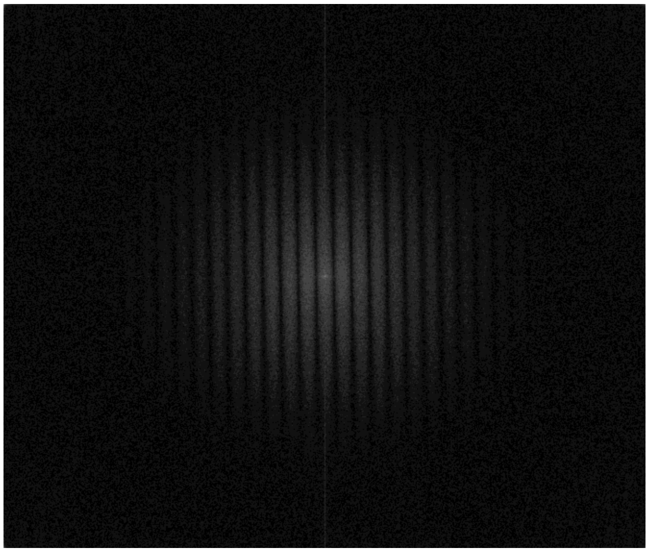


(b)

Fig. 5: The interference fringes of two speckle patterns obtained for a rough surface of average heights $\langle h_r \rangle = 3.5\ \mu m$ separated by (a) $\Delta x = 100\ \mu m$ and (b) $\Delta x = 200\ \mu m$ using He-Ne laser ($\lambda = 6328\ A^o$).

Figure 6 shows the speckle pattern using He-Ne laser ($\lambda = 6328\ A^o$) for rough surface of average roughness heights $\langle h_r \rangle = 6.5\ \mu m$.

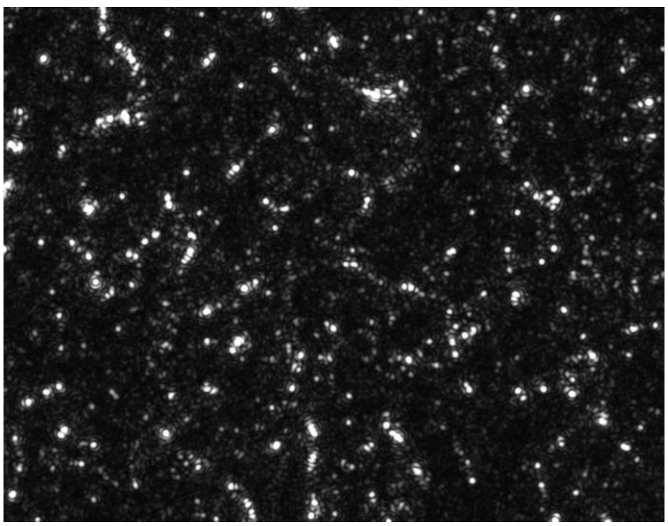
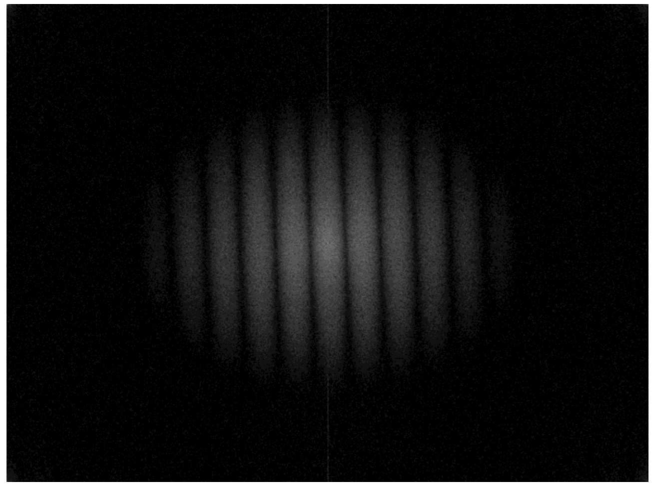
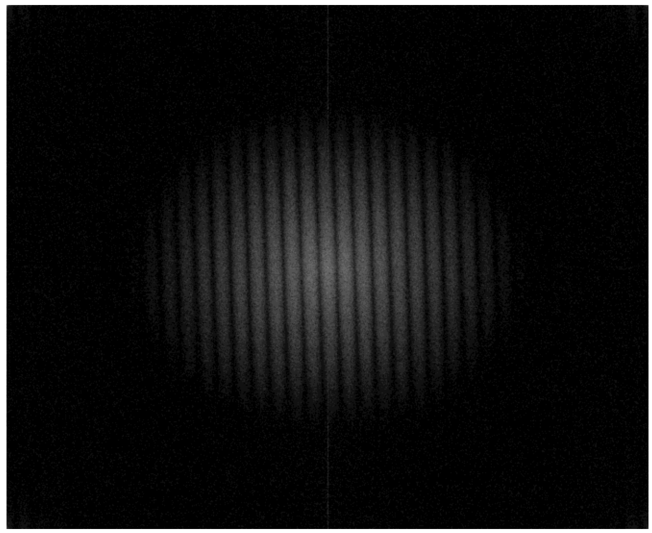


Fig. 6: The recorded speckle pattern for a rough surface of average heights $\langle h_r \rangle = 6.5\ \mu m$ using He-Ne laser ($\lambda = 6328\ A^o$).

Figure 7 (a-b) show the interference fringes of two separated speckle patterns $\Delta x = 100\ \mu m$ and $\Delta x = 200\ \mu m$ respectively and $\langle h_r \rangle = 6.5\ \mu m$.



(a)



(b)

Fig. 7: The interference fringes of two speckle patterns obtained for a rough surface of average heights $\langle h_r \rangle = 6.5\ \mu m$ separated by (a) $\Delta x = 100\ \mu m$ and (b) $\Delta x = 200\ \mu m$ using He-Ne laser ($\lambda = 6328\ A^o$).

Figure 8 shows that the visibility of the interference fringes, in turn the speckle correlation, decreases with increasing the separated distance Δx between the speckle patterns.

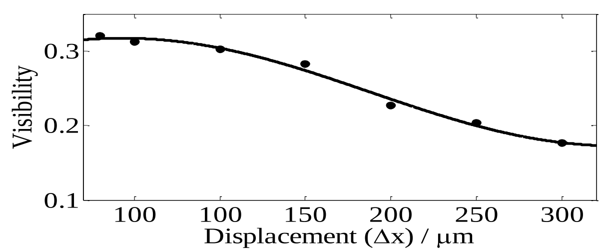


Fig. 8: Fringes visibility versus lateral displacement obtained for a rough surface of average heights$\langle h_r \rangle = 3.5\ \mu m$ using He-Ne laser ($\lambda = 6328\ A^o$)

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