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Speckle Contrast of Scattered Quasi-Monochromatic Electromagnetic Waves of Random Amplitudes and Phases

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Abstract: An analytical formula for the speckle contrast is derived. It represents the dependence of the speckle contrast on the random amplitudes and phases of the interfering scattered waves. It shows also the effect of the number of the scattered waves on the speckle contrast. The normalized speckle contrast considering the randomization of both scattered field amplitudes and phases is slightly different from that of random phases only in the range of low root mean square of phases where the speckle pattern is partially developed.

Keywords: Speckle contrast, Phase probability density distribution, Random phase, Random amplitude.

1 Introduction

The basic parameters defining the intensity of the speckle pattern are the point spread function of the imaging system forming it and the randomization of both the phases and the scattered field amplitudes of the interfering waves. The random phases carry information about the microscopic structure of the scattering medium. The random amplitudes give information about the random spatial intensity distribution of the transmitted or reflected waves by the scattering medium. Moser et al [1] derived an expression for amplitude probability density distribution for synthetic aperture radar (SAR) data, to deal with denoising and classification purposes. In medical applications, Motaghiannezam et al [2] formulated a theory to show that the statistics of optical coherence tomography (OCT) signal amplitude and intensity are highly dependent on the sample reflectivity and other parameters to differentiate between regions of motion from static areas. In industrial applications, Feced et al [3] studied the influence of random phase and amplitude fabrication errors on the performance of optical filters based on fiber Bragg gratings. The random phases are the most dominant parameter which define the statistical characteristics of the speckle pattern. This is due to the high sensitivity of the phases to the interfering waves. Therefore, most researchers investigated the statistical characteristic of the speckle contrast and the speckle correlation considering the effect of the random phases that gained by the scattered waves, so for instance has Mansour et al [4] derived an analytical formula for the speckle contrast as a function of the root mean square of the rough surface, the number of the scattering grains and the spectral profile of the illuminating light considering two different phase probability density distributions of the roughness. Xiang [5] developed an expression for the mutual coherence function (MCF) of reflected Gaussian beam. Expression for the mean intensity and the average speckle size based on (MCF) was derived. Tchvialeva et al [6] formulated the speckle contrast as a function of surface roughness, spectral profile and geometry of speckle formation. A calibration speckle contrast curve for blue and red lasers was introduced. Moustafa et al [7] studied theoretically the parameter affecting the visibility of the speckle patterns. Periodic rough surface is considered. Pederson [8] derived a formula for the speckle contrast as a function of root mean square of the roughness. The theoretical result was compared with the available experimental results. Ohtsubo et al [9] studied theoretically and experimentally the properties of speckle patterns at the image plane resulting from coherent light incident on rough surface.

The present study shows the effect of the randomization of both of the scattered amplitudes and phases on the contrast of speckle photography.

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2 Theories

Consider a quasi-monochromatic plane wave incident on a reflecting rough surface at angle θ to obtain speckle pattern at the observation plane, Fig.(1),. The rough surface is considered to be consisting of N numbers of scatterers (grains). Let the backscattered complex amplitude from the j^{th} scatterer at the specular direction θ to be $a_j e^{i\varphi_j}$. a_j and φ_j are its real amplitude and phase, respectively. Both of the amplitudes and phases of the backscattered waves are independent random variables.

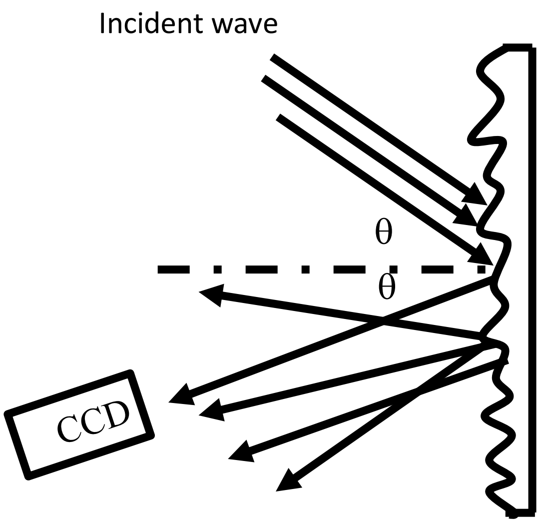


Fig. 1: The Considered configuration for obtaining the speckle pattern.

The intensity $I_m(a,\varphi)$ of the speckle pattern due to the interference of number N of strictly monochromatic scattered waves is given by

$$I_m(a,\varphi) = \sum_{i=1}^{N}\sum_{j=1}^{N} a_i a_j cos(\varphi_i - \varphi_j) \tag{1}$$

where $(\varphi_i - \varphi_j)$ is the random phase delay between the scattered waves, given by

$$\varphi_i - \varphi_j = \frac{2\pi}{\lambda}\Delta\eta_{ij}, \tag{2}$$

and $\Delta\eta_{ij}$ is the optical path delay between the i^{th} and j^{th} scattered waves.
For quasi-monochromatic wave of Gaussian spectral distribution $g(\nu,\nu_0) = A_G \exp[-a_G(\nu-\nu_o)^2]$, $A_G = 2\Delta\nu\sqrt{\ln 2/\pi}$, $a_G = \ln 2.(2/\Delta\nu)^2$, $\Delta\upsilon$ is the spectral width of the illuminating light and ν_0 is its central frequency, the speckle intensity $I_q(a,\varphi)$ will be given by the incoherent sum of its spectral distribution [8] .

$$I_q(a,\varphi) = \int_{-\infty}^{\infty} I_m(a,\varphi)\; g(\nu,\nu_0)\, d\nu$$

It gives:

$$I_q(a,\varphi) = \sum_{i=1}^{N}\sum_{j=1}^{N} a_i a_j\, \gamma_{ij} \cos\left(\frac{2\pi\nu_0}{c}\Delta\eta_{ij}\right) \tag{3}$$

With

$$\gamma_{ij} = \exp\left(-\left(\frac{\pi}{c}\Delta\eta_{ij}\right)^2 \frac{1}{\alpha}\right)$$
$$\alpha = \left(\frac{2}{\Delta\upsilon}\right)^2 \ln(2)$$
$$\Delta\eta_{ij} = 2\left(h_i - h_j\right)\cos\theta$$

Where γ_{ij} is the mutual degree of temporal coherence between the interfering beams [4] where $\gamma_{ij} = 1$ for $i = j$ and $\gamma_{ij} = \gamma$ for $i \neq j$ which depends on the type of the beam's spectral broadening.
c is the wave velocity,
θ is the specular direction.
In Eq.(3), (γ_{ij}) is slowly varying than the cosine term so $\Delta\eta_{ij}$ is replaced by $\Delta\eta = \sqrt{\langle\Delta\eta_{ij}{}^2\rangle}$ where $\langle\ldots\rangle$ represents the ensemble average.
Both of the amplitudes and phases of the scattered waves are governed by two independent probability density distributions $P(a)$ and $P(\varphi)$ respectively.
In this case, the average speckle intensity $\langle I_q(a,\varphi)\rangle$ will be given by:

$$\langle I_q(a,\varphi)\rangle = \sum_{i=1}^{N}\sum_{j=1}^{N} \gamma_{ij}\,\langle a_i a_j\rangle\,\langle\cos\left(\varphi_i - \varphi_j\right)\rangle \tag{4}$$

Here the random phase delay, $(\varphi_i - \varphi_j)$, between the scattered waves is given by:
For the independent random variable of the amplitudes we have:

$$\langle a_i a_j\rangle = \begin{cases} \langle a_i\rangle\langle a_j\rangle & for \quad a_i \neq a_j \\ \langle a^2\rangle & for \quad a_i = a_j \end{cases} \tag{5}$$

With $\langle a^\iota\rangle = \int a^\iota\, P(a)da \qquad \iota = 1, 2, 3, \ldots$ (6)

Similarly for the independent random variable φ of the phase we have:

$$\langle\cos\varphi_i\ \cos\varphi_j\rangle = \langle\cos\varphi_i\rangle\langle\cos\varphi_j\rangle = \begin{cases} \langle\cos\varphi\rangle^2 & for\ \varphi_i \neq \varphi_j \\ \langle\cos^2\varphi\rangle & for\ \varphi_i = \varphi_j \end{cases} \tag{7}$$

$$\langle\sin\varphi_i\ \sin\varphi_j\rangle = \langle\sin\varphi_i\rangle\langle\sin\varphi_j\rangle = \begin{cases} \langle\sin\varphi\rangle^2 & for\ \varphi_i \neq \varphi_j \\ \langle\sin^2\varphi\rangle & for\ \varphi_i = \varphi_j \end{cases} \tag{8}$$

With

$$\begin{cases} \langle\cos\varphi\rangle = \int \cos\varphi\, P(\varphi)d\varphi = x \\ \langle\sin\varphi\rangle = \int \sin\varphi\, P(\varphi)d\varphi = y \\ \langle\cos 2\varphi\rangle = \int \cos 2\varphi\, P(\varphi)d\varphi = x_2 \\ \langle\sin 2\varphi\rangle = \int \sin 2\varphi\, P(\varphi)d\varphi = y_2 \end{cases} \tag{9}$$

The pervious integrations have to be carried out under limits of integrations which define the validation ranges of the random variables a and φ over which $P(a)$ and $P(\varphi)$ are normalized.
Taking into consideration the pervious conditions, the

averages $\langle I_q(a,\varphi)\rangle$ according to Eq.(4) is obtained for all possible combinations of the mutual interference between the scattered radiations ($i = j$ and $i \neq j$)

$$\langle I_q(a,\varphi)\rangle = N\langle a^2\rangle + N(N-1)\langle a\rangle^2\gamma(x^2+y^2) \quad (10)$$

${I_q}^2(a,\varphi)$ can be written by the following formula

$${I_q}^2(a,\varphi) = \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{m=1}^{N}\sum_{n=1}^{N} a_i a_j \gamma_{ij} \cos(\varphi_i - \varphi_j)\, a_m a_n \gamma_{mn} \cos(\varphi_m - \varphi_n) \quad (11)$$

where

$$\gamma_{ij} = \begin{cases} 1 & for \quad i = j \\ \gamma & for \quad i \neq j \end{cases}$$

$$\gamma_{mn} = \begin{cases} 1 & for \quad m = n \\ \gamma & for \quad m \neq n \end{cases}$$

To calculate $\langle {I_q}^2(a,\varphi)\rangle$ we have to consider the following all possible cases of combinations between the random phases and the real random amplitudes of the waves' complex amplitudes. It is represented in Eq. (11) by the two pairs of sets (i,j) and (m,n).

Case I: $i = j = m = n$

$$\langle {I_q}^2(a,\varphi)\rangle = N\langle a^4\rangle \quad (12)$$

Case II: consists from the following subcases:

(1) $(i = j = m) \neq n$ gives:

$$\langle {I_q}^2(a,\varphi)\rangle = N(N-1)\langle a^3\rangle\langle a\rangle\,\gamma\,(x^2+y^2)$$

(2) $(i = j = n) \neq m$ gives:

$$\langle {I_q}^2(a,\varphi)\rangle = N(N-1)\langle a^3\rangle\langle a\rangle\,\gamma\,(x^2+y^2)$$

(3) $(i = m = n) \neq j$ gives:

$$\langle {I_q}^2(a,\varphi)\rangle = N(N-1)\langle a^3\rangle\langle a\rangle\,\gamma\,(x^2+y^2)$$

(4)$(j = m = n) \neq i$ gives:

$$\langle {I_q}^2(a,\varphi)\rangle = N(N-1)\langle a^3\rangle\langle a\rangle\,\gamma\,(x^2+y^2)$$

The total result of case II is:

$$\langle {I_q}^2(a,\varphi)\rangle = 4N(N-1)\langle a^3\rangle\langle a\rangle\,\gamma\,(x^2+y^2) \quad (13)$$

Case III: consists from the following subcases:

(1) $(i = m) \neq (j = n)$ gives:

$$\langle {I_q}^2(a,\varphi)\rangle = N(N-1)\langle a^2\rangle^2\,\gamma^2\left[\frac{1}{2}(1+x_2^2+y_2^2)\right]$$

(2) $(i = n) \neq (m = j)$ gives similar to the pervious subcase:

$$\langle {I_q}^2(a,\varphi)\rangle = N(N-1)\langle a^2\rangle^2\,\gamma^2\left[\frac{1}{2}(1+x_2^2+y_2^2)\right]$$

(3) $(i = j) \neq (m = n)$ gives:

$$\langle {I_q}^2(a,\varphi)\rangle = N(N-1)\langle a^2\rangle^2$$

The result of case III is:

$$\langle {I_q}^2(a,\varphi)\rangle = N(N-1)\langle a^2\rangle^2\,\left[1+\gamma^2(1+x_2^2+y_2^2)\right] \quad (14)$$

Case IV: consists from the following subcases:

(1) $i \neq j \neq m = n$ gives:

$$\langle {I_q}^2(a,\varphi)\rangle = N(N-1)(N-2)\langle a\rangle^2\langle a^2\rangle\,\gamma\,(x^2+y^2)$$

(2) $m \neq n \neq i = j$ gives similar to the pervious case:

$$\langle {I_q}^2(a,\varphi)\rangle = N(N-1)(N-2)\langle a\rangle^2\langle a^2\rangle\,\gamma\,(x^2+y^2)$$

(3) $i \neq m \neq n = j$ gives:

$$\langle {I_q}^2(a,\varphi)\rangle = \frac{1}{2}N(N-1)(N-2)\langle a\rangle^2\langle a^2\rangle\,\gamma^2\,[x^2+y^2+x_2(x^2-y^2)+2xyy_2]$$

(4) $m \neq j \neq n = i$ gives:

$$\langle {I_q}^2(a,\varphi)\rangle = \frac{1}{2}N(N-1)(N-2)\langle a\rangle^2\langle a^2\rangle\,\gamma^2\,[x^2+y^2+x_2(x^2-y^2)+2xyy_2]$$

(5) $j \neq n \neq i = m$ gives:

$$\langle {I_q}^2(a,\varphi)\rangle = \frac{1}{2}N(N-1)(N-2)\langle a\rangle^2\langle a^2\rangle\,\gamma^2\,[x^2+y^2+x_2(x^2-y^2)+2xyy_2]$$

(6) $i \neq n \neq j = m$ gives also:

$$\langle {I_q}^2(a,\varphi)\rangle = \frac{1}{2}N(N-1)(N-2)\langle a\rangle^2\langle a^2\rangle\,\gamma^2\,[x^2+y^2+x_2(x^2-y^2)+2xyy_2]$$

The total result of case IV is:

$$\langle {I_q}^2(a,\varphi)\rangle = 2N(N-1)(N-2)\langle a\rangle^2\langle a^2\rangle\,\{\gamma(x^2+y^2)+\gamma^2\,[x^2+y^2+x_2(x^2-y^2)+2xyy_2]\} \quad (15)$$

Case V: $i \neq j \neq m \neq n$ gives:

$$\langle {I_q}^2(a,\varphi)\rangle = N(N-1)(N-2)(N-3)\langle a\rangle^4\,\gamma^2\,(x^2+y^2)^2 \quad (16)$$

The final result of $\langle {I_q}^2(a,\varphi)\rangle$ is given by adding all $\langle {I_q}^2(a,\varphi)\rangle$ given by the previous five cases of the possible combinations between the random amplitudes and phases, Eqs. (12-16).

Thus we can write he following formula for the net $\langle {I_q}^2(a,\varphi)\rangle$:

$$\langle {I_q}^2(a,\varphi)\rangle = N\langle a^4\rangle + 4N(N-1)\langle a^3\rangle\langle a\rangle\,\gamma\,(x^2+y^2) + N(N-1)\langle a^2\rangle^2\,[1+\gamma^2(1+x_2^2+y_2^2)] + +2N(N-1)(N-2)\langle a\rangle^2\,\langle a^2\rangle\left[\gamma(x^2+y^2)\;+\gamma^2\,[x^2+y^2+x_2(x^2-y^2)+\;2xyy_2]\right] + N(N-1)(N-2)(N-3)\langle a\rangle^4\,\gamma^2(x^2+y^2)^2 \quad (17)$$

The normalized average speckle contrast C is defined by the ratio of the standard deviation of the speckle intensity to the average speckle intensity and given by [9],

$$C = \frac{\left[\langle {I_q}^2(a,\varphi)\rangle - \langle I_q(a,\varphi)\rangle^2\right]^{1/2}}{\langle I_q(a,\varphi)\rangle} \quad (18)$$

Substituting for $\langle I_q(a,\varphi)\rangle$ and $\langle {I_q}^2(a,\varphi)\rangle$ from Eqs. (10) and (17) into Eq.(18) we get the following formula for the average speckle contrast:

$$C = {}^{D}\!/_{N\langle a^2\rangle + N(N-1)\langle a\rangle^2\gamma(x^2+y^2)} \quad (19)$$

Where

$$D = \left\{N(\langle a^4\rangle - \langle a^2\rangle^2) + N(N-1)[\langle a^2\rangle^2(1+x_2^2+y_2^2) + 2\,(N-2)\,\langle a^2\rangle\,\langle a\rangle^2\,(x^2\;+\;y^2+\;x_2\,(x^2-y^2) + 2\,x\,yy_{\,2}) - (4N-6)\langle a\rangle^4(x^2+y^2)^2]\gamma^2 + 4(\langle a^3\rangle\langle a\rangle - \langle a^2\rangle\langle a\rangle^2)(x^2+y^2)\gamma\right\}^{1/2} \quad (20)$$

To accomplish Eq. (6), the probability density distribution $P(a)$ of the random amplitudes should be specified.

Different types of distributions are considered namely uniform, Simpson, and Rayleigh probability distribution.
For uniform amplitude probability density distribution

$$P(a) = \frac{1}{A} \qquad 0 < A < 1 \qquad \text{A is constant} \tag{21}$$

$$\langle a^\ell \rangle = \frac{1}{(\ell+1)} A^\ell \tag{22}$$

Where $\ell = 1,2,3,4$.
The Simpson amplitude probability density distribution is given by

$$P(a) = \begin{cases} \frac{a}{b^2} & 0 < a < b \\ \frac{2b-a}{b^2} & b < a < 2b \end{cases} \qquad b = 0.5 \tag{23}$$

$$\langle a^\ell \rangle = \begin{cases} \langle a^\ell \rangle = \frac{2^{(\ell+2)}-2}{(\ell+1)(\ell+2)} \; b^\ell & \mathit{for\ even}\ \ell \\ \langle a^\ell \rangle = \frac{\ell}{(\ell-1)!} b^\ell & \mathit{for\ odd}\ \ell \end{cases} \tag{24}$$

Similarly for the normalized Rayleigh amplitude probability density distribution

$$P(a) = \frac{a}{K\sigma^2} \exp\left(-\frac{a^2}{2\sigma^2}\right) \qquad 0 < a < 1 \tag{25}$$

Where $K = \left[1 - e^{-1/2\sigma^2}\right]$ is the normalization factor, then the first, second, third and fourth moment of Rayleigh distribution can be written as

$$\langle a^\ell \rangle = \begin{cases} K\left[2^{\ell-1}\sigma^\ell - \left(1 + \ell\sigma^2 + 2\ell\sigma^\ell \; U(\ell-3)\right) e^{-1/2\sigma^2}\right] \\ K\left[\ell\sqrt{\frac{\pi}{2}} \; \sigma^\ell \; erf\left(\sqrt{1/2\sigma^2}\right) - (1+3\sigma^2)^{(\ell-1)/2} \; e^{-1/2\sigma^2}\right] \end{cases} \tag{26}$$

Where $U(x)$ and $erf(x)$ are the Heavyside function and the error function at the point x.

It is remarkable that; if the randomization of the reflected amplitudes is not considered, thus $\langle a^\ell \rangle$ in Eq.(19) will be constant and one gets the developed equation of the speckle contrast given in [10].

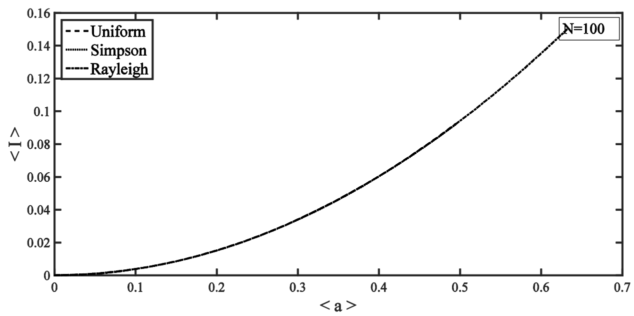
3 Results and Discussion

Throughout the calculations, the random phase probability distribution for the rough surface $P(\varphi)$is chosen to be Gaussian with zero mean value

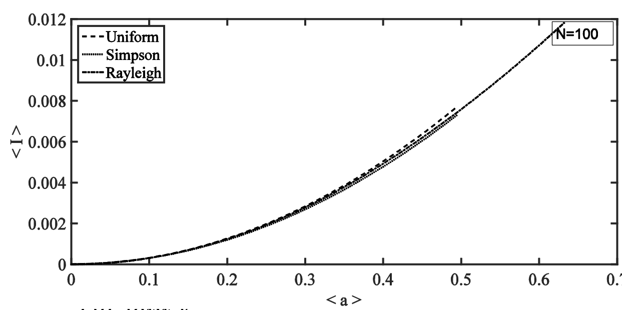
$$P(\varphi) = \frac{1}{2\sigma_\varphi\sqrt{\pi}} e^{-\frac{\varphi^2}{2\sigma_\varphi^2}}$$

where $\sigma_\varphi = [\langle\varphi^2\rangle]^{\frac{1}{2}}$ represents the root mean square of phase deviation. Under this assumption, the normalized speckle contrast is calculated using Eq.(19) by considering Eq.(9) for the values of x , y, x_2 and y_2. The values of the average amplitudes $\langle a^\iota \rangle$ are considered from Eq. (22), (24) and (26)) for uniform, Simpson and Rayleigh probability distributions. The calculations are carried out for monochromatic light where $\gamma = 1$.

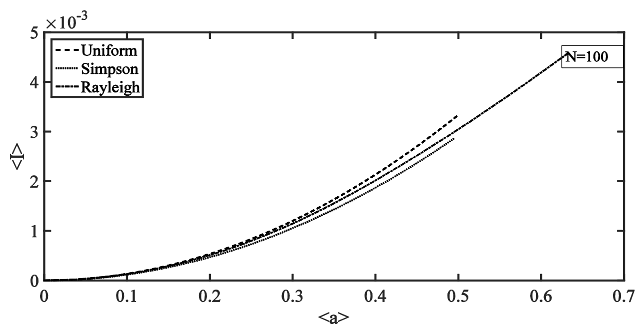
Figure 2 (a-c) represents the dependence of the average speckle intensity on the average scattered field amplitude computed for $\sigma_\varphi = 1, 2, 5$ respectively. In each figure the computation is performed for the considered random field amplitude distributions (uniform Simpson and Rayleigh).



(a)



(b)



(c)

Fig. 2: The normalized average speckle intensity versus the average random scattered field amplitude for uniform, Simpson and Rayleigh probability distribution computed for $N = 100$. (a) $\sigma_\varphi = 1$, (b) $\sigma_\varphi = 2$ and (c) $\sigma_\varphi = 5$.

Figure 3(a-c) represents the average squared intensity versus the average random field amplitude computed for $\sigma_\varphi = 1, 2, 5$ respectively. In each figure the computation is performed for the considered random field amplitude distributions.

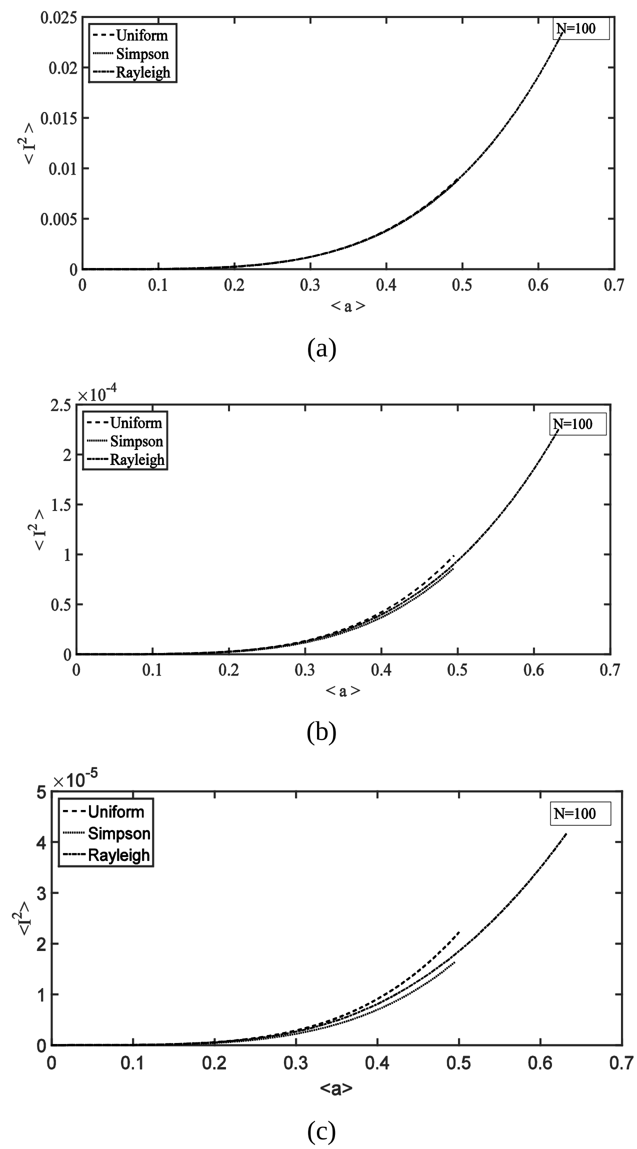


Fig. 3: The normalized average of the squared speckle intensity versus the average random scattered field amplitude for uniform, Simpson and Rayleigh probability distribution calculated for $N = 100$. (a) $\sigma_\varphi = 1$, (b) $\sigma_\varphi = 2$ and (c) $\sigma_\varphi = 5$.

The figures show that, the effect of the considered random scattered field amplitude distributions is remarkable. $\langle I \rangle$ and $\langle I^2 \rangle$ decrease with increasing σ_φ. They increase with increasing $\langle a \rangle$ due to the increase of the reflected scattered field amplitude from the rough surface.

Figure 4(a-c) represents the normalized speckle contrast versus σ_φ for $N = 10, 100$ and 1000. Each figure is computed for the considered probability density distributions of the random scattered field amplitude. As a comparison the normalized speckle contrast considering that the scattered waves are of the same value of amplitude (only the random phases is considered) is represented.

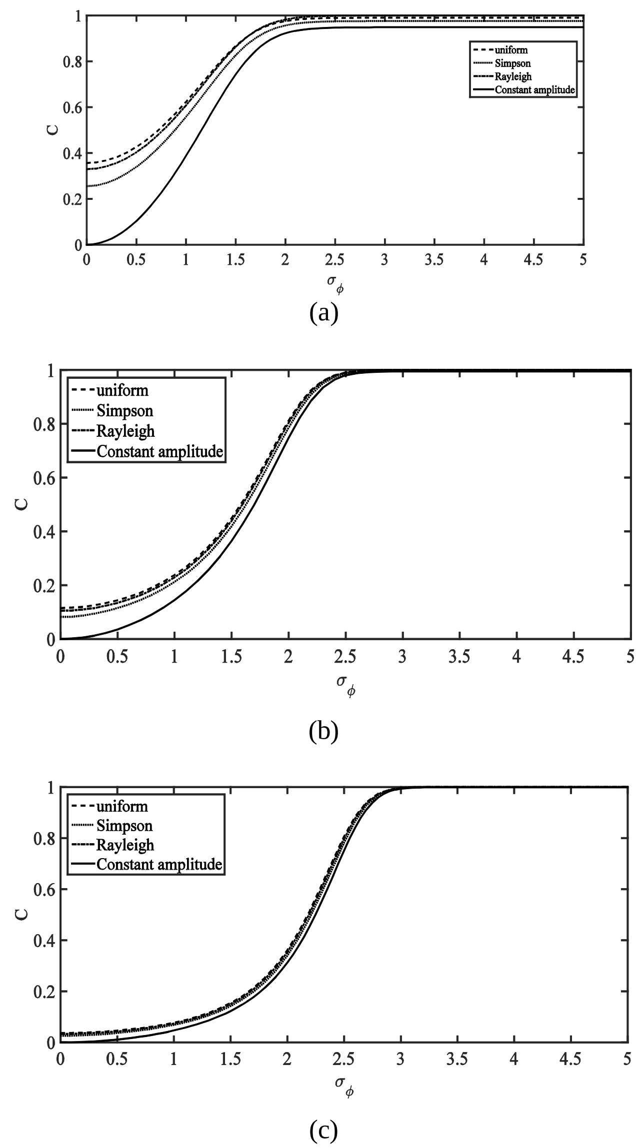


Fig. 4: The average normalized speckle contrast versus the root mean square of phase deviation considering randomization of phases only (solid line) and randomization of both phases and amplitudes (dash lines). (a) $N = 10$, (b) $N = 100$ and (c) $N = 1000$.

The comparison show that, the random phases of the scattered waves acquired from the roughness and their probability distribution are the basic factors which affect the behavior of the normalized speckle contrast. The scattered random amplitudes of the waves affect the speckle contrast in the range of slightly low roughness where the speckle pattern is partially developed. As the roughness increases, the acquired random phases of the scattered waves dominate the behavior of the speckle contrast. The mean speckle intensity and its variance increases markedly with increasing the mean value of the contributing random amplitudes of the scattered waves.

4 Conclusions

The effect of the random scattered field amplitude on the contrast is only considerable for small and moderate roughness (partially developed speckle pattern).

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