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A Fuzzy Approach for Solving a Three-Level Linear Programming Problem with Neutrosophic Parameters in the Objective Functions and Constraints

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Abstract: The most widely used actions taken and decisions made in real-world tasks frequently appear as hierarchical systems. The multi-level programming problem is the technique that most often used to address these systems; however, in practical situations, some decisions and performance are imprecise. Neutrosophic sets provide a vital role by considering three independent degrees, specifically, the truth membership degree, the indeterminacy-membership degree, and the falsity membership degree, of every aspect of uncertain decisions. This study focuses on solving neutrosophic multi-level linear programming problems with neutrosophic parameters in the objective functions and constraints. The neutrosophic form of the problem transformed into an equal crisp model in the first stage of the solution methodology to reduce the problem's complexity. In the second stage, a Fuzzy approach is used to obtain the optimal solution among conflicted decision levels.

Keywords: Multi-level programming, linear programming, Neutrosophic set, Trapezoidal neutrosophic number.

1 Introduction

Multi-level programming problems (MLPPs) involve organizational structures that contain multiple levels of decision-making over a feasible region of the solution. MLPPs are primarily important for manufacturing factories, logistics organizations, government units and many other areas. Approaches adopted to solve MLPPs use an objective function with decision variables for every decision maker (DM) and a set of constraints for all the DMs. Every DM independently seeks to address his/her own interest but is influenced by the behaviour of other decision makers [1].
In most cases, the actual decision-making status is not clearly defined and usually causes the decision depend on incomplete or unknown information. In fact, the vagueness of these types of problems is better described as fuzziness rather than randomness [2] .

Fuzzy sets as defined by Zadeh [3] have been used to model the imperfect information especially for uncertainty types of information by representing the membership value between [0, 1].

Attanassov introduced an intuitionistic fuzzy set by generalizing the theory of fuzzy sets and introducing two grades of the membership function, namely the degree of membership function and degree of non-membership function [4] .

In 1998, Smarandache incorporated a new set in mathematical philosophy called neutrosophic sets. He believed that somehow in a life situation, especially for uncertainty condition, there also exist in-between (indeterminacy) opinion or unexpected condition that cannot be controlled [5].

Instead of two grades of the membership function, neutrosophic set introduced in-between (indeterminacy) function where there exists an element which consists of a set of truth membership function (T), indeterminacy function (I) and falsity

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membership function (F) where T, I, and F are standard or non-standard subsets of $]0^{-},1^{+}[$. Compared to other uncertainty theories, the neutrosophic set can deal with indeterminacy situation [5].

There are a large number of studies on MLPPs ([2,6,7]). In [6], Osman et al. proposed an interactive approach for solving multi-level multi-objective fractional programming (ML-MOFP) problems with fuzzy parameters. In the first stage, the ML-MOFP problem's numerical crisp model developed for a particular level of confidence, without changing the fuzzy aspect of the problem.Then,Then, a linear model for the ML-MOFP problem is formulated. The interactive approach simplifies the linear multi-level multi-objective model in the second stage by transforming it into separate multi-objective programming problems. Each separate multi-objective programming problem of the linear model is solved by the ε-constraint method and the concept of satisfactoriness.

Many studies on neutrosophic linear programming problems have been implemented ([2,8,9,**?**,11,12]). In [8], Abdel-Basset presented linear programming models with trapezoidal neutrosophic numbers as parameters and proposed an algorithm for solving them.
Hussian et al. [9] proposed linear programming problems based on a neutrosophic environment. They converted neutrosophic linear programming problems into crisp programming models using neutrosophic set parameters.

Emam et al. [2] focused on solving neutrosophic multi-level linear programming problems; in this case, the problem coefficients considered trapezoidal neutrosophic numbers. To reduce the problem's complexity, the neutrosophic form of the problem transformed into an equal crisp model in the first stage of the solution methodology. In the second stage, an interactive approach was used to reach a solution that involves a compromise between conflicting decision levels.

This paper is arranged as follows: Section 2 presents a preliminary discussion. Section 3 addresses a three-level neutrosophic linear programming problem (TLNLPP) with neutrosophic parameters in the objective functions and constraints. In Section 4, the neutrosophic nature of the problem simplified into an equivalent crisp. In Section 5, a Fuzzy approach for the three-level linear programming problem presented. An algorithm for solving the TLNLPP with neutrosophic parameters in the objective functions and constraints proposed in Section 6. Furthermore, the results and the solution algorithm clarified with a numerical example are introduced in Section 7.Section 8 is devoted to conclusions and suggestions for future work.

2 Preliminary Discussion

This section presents a review of key concepts and definitions related to neutrosophic sets.
Definition 1. (A single-valued neutrosophic set) [**?**]
Let Y be a universe of discourse. A single-valued neutrosophic set N over Y is an object having the form N = $\{ \langle y, T_N(y), I_N(y), F_N(y) \rangle : y \in Y \}$, where $T_N(y): Y \to [0,1]$, $I_N(y): Y \to [0,1]$ and $F_N(y): Y \to [0,1]$ with $0 \leq T_N(y) + I_N(y) + F_N(y) \leq 3$ for all $y \in Y$. The intervals $T_N(y)$, $I_N(y)$ and $F_N(y)$ denote the T, I and F membership functions of y to N, respectively.
Definition 2. [8] The trapezoidal neutrosophic number $\tilde{G}$ is a neutrosophic set in R with the following T, I and F membership functions:

$$T_{\tilde{G}}(X) = \begin{cases} \propto_{\tilde{G}} \left(\frac{x-g_1}{g_2-g_1} \right) & (g_1 \leq x \leq g_2) \\ \propto_{\tilde{G}} & (g_2 \leq x \leq g_3) \\ \propto_{\tilde{G}} & (g_3 \leq x \leq g_4) \\ 0 & otherwise \end{cases} \tag{1}$$

$$I_{\tilde{G}}(X) = \begin{cases} \frac{(g_2 - x + \theta_{\tilde{G}}(X - g'_1)}{(g_2 - g'_1)} & (g'_1 \leq x \leq g_2) \\ \theta_{\tilde{G}} & (g_2 \leq x \leq g_3) \\ \frac{(x - g_3 + \theta_{\tilde{G}}(g'_4 - x))}{(g'_4 - g_3)} & (g_3 \leq x \leq g'_4) \\ 1 & otherwise \end{cases} \tag{2}$$

$$F_{\tilde{G}}(X) = \begin{cases} \frac{(g_2 - x + \beta_{\tilde{G}}(X - g''_1))}{(g_2 - g''_1)} & (g''_1 \leq x \leq g_2) \\ \beta_{\tilde{G}} & (g_2 \leq x \leq g_3) \\ \frac{(x - g_3 + \beta_{\tilde{G}}(g''_4 - x))}{(g''_4 - g_3)} & (g_3 \leq x \leq g''_4) \\ 1 & otherwise \end{cases} \tag{3}$$

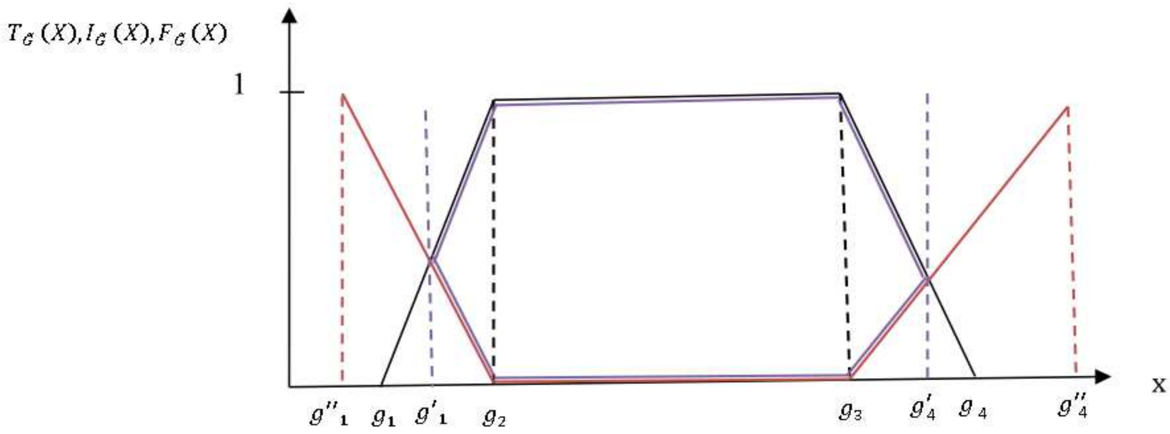

Fig. 1 Truth membership, indeterminacy and falsity membership functions of trapezoidal neutrosophic number

Where $\propto_{\tilde{G}}$, $\theta_{\tilde{G}}$ and $\beta_{\tilde{G}}$ represent the maximum truthiness degree, minimum indeterminacy degree, and minimum falsity degree, sequentially; $\propto_{\tilde{G}}$, $\theta_{\tilde{G}}$; and $\beta_{\tilde{G}}$ $\in [0, 1]$. In addition, $g_1'' \leq g_1 \leq g_1' \leq g_2 \leq g_3 \leq g_4' \leq g_4 \leq g_4''$.

Definition 3. [8] A ranking function of neutrosophic numbers is function $N(R) \to R$, *where* $N(R)$ is a set of neutrosophic numbers defined as an onset of real numbers that transforms each neutrosophic number into the real line.

Let $\tilde{C} = \langle (c_1, c_2, c_3, c_4); \propto_{\tilde{C}}, \theta_{\tilde{C}}, \beta_{\tilde{C}} \rangle$ and $\tilde{D} = \langle (d_1, d_2, d_3, d_4); \propto_{\tilde{D}}, \theta_{\tilde{D}}, \beta_{\tilde{D}} \rangle$ be two trapezoidal neutrosophic numbers. Then,

1.If $R(\tilde{C}) > R(\tilde{D})$ then $\tilde{C} > \tilde{D}$.
2.If $R(\tilde{C}) < R(\tilde{D})$ then $\tilde{C} < \tilde{D}$.
3.If $R(\tilde{C}) = R(\tilde{D})$ then $\tilde{C} = \tilde{D}$.

3 Problem Formulation and Solution Concept

The TLNLPP with neutrosophic parameters in the objective functions and constraints maybe formulated as follows:
[First Level]

$$\max_{X_1} F_1 \approx \sum_{j=1}^{n} \tilde{c}_{1j} x_j , \tag{4}$$

where x_2, x_3 solves
[Second Level]

$$\max_{X_2} F_2 \approx \sum_{j=1}^{n} \tilde{c}_{2j} x_j , \tag{5}$$

where x_3 solves
[Third Level]

$$\max_{X_3} F_3 \approx \sum_{j=1}^{n} \tilde{c}_{3j} x_j , \tag{6}$$

Subject to
where

$$\sum_{j=1}^{n} a_{ij} x_j \leq \tilde{b}_i \tag{7}$$

Where$i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n,\ x_j \geq 0, \tilde{c}_{1j},\ \tilde{c}_{2j},\ \tilde{c}_{3j}$ are trapezoidal neutrosophic numbers, $b = (\tilde{b}_0, \ldots, \tilde{b}_m)^T$ are trapezoidal neutrosophic numbers, $b = (\tilde{b}_0, \ldots, \tilde{b}_m)^T$ is a $(m+1)$ vector, and $a_{01}, ..., a_{0m}, d_1, ..., d_m$ are constants.

Therefore, $F_i : R^m \rightarrow R, (i = 1,2,3)$ represents the first level, the second level, and the third level objective functions, respectively. In addition, the first level decision maker (FLDM) has x_1 indicating the choice of the first level decision; the second level decision maker (SLDM) and the third level decision maker (TLDM) have x_2 andx_3, respectively, indicating the respective choices of the second level decision and third level decision.

Definition 4. For any $(x_1 \in G_1 = \{x_1 \mid (x_1,\ldots,x_m) \in G\})$ given by the FLDM and $(x_2 \in G_2 = \{x_2 \mid (x_1,\ldots,x_m) \in G\})$ given by the SLDM, if the decision-making variable $(x_3 \in G_3 = \{x_3 \mid (x_1,\ldots,x_m) \in G\})$ is the Pareto optimal solution of the TLDM, then $(x_1,...,x_m)$ is a feasible solution of the TLNLPP.

Definition 5. If $x^* \in R^m$ is a feasible solution of the TLNLPP, then no other feasible solution $x \in G$ exists, such that$F_1(x^*) \leq F_1(x)$, so $x^*$is the Pareto optimal solution of the TLNLPP.

The basic goal in treating the TLNLPP is to use the ranking function to transform each trapezoidal number into its equal crisp number.

If the TLNLPP is in the maximization state, then the ranking function for this trapezoidal neutrosophic number can be stated as follows[1]:

$$R(\tilde{g}) = \left| \left(\frac{-\frac{1}{3}(3g^l - 9g^u) + 2(g^{m1} - g^{m2})}{2} \right) * (T_{\tilde{g}} - I_{\tilde{g}} - F_{\tilde{g}}) \right| \quad (8)$$

If the TLNLPP is in minimization state, then the ranking function for the trapezoidal neutrosophic number can be stated as follows[1]:

$$R(\tilde{g}) = \left(\frac{(g^l + g^u) - 3(g^{m1} + g^{m2})}{-4} \right) * (T_{\tilde{g}} - I_{\tilde{g}} - F_{\tilde{g}}) \quad (9)$$

Where $(\tilde{g} = g^l, g^{m1}, g^{m2}, g^u;\ T_{\tilde{g}}, I_{\tilde{g}}, F_{\tilde{g}})$ is a trapezoidal neutrosophic number and g^l, g^{m1}, g^{m2}, g^u are the lower bound,frst and second median value and upper bound for trapezoidal neutrosophic number, respectively. In addition, $T_{\tilde{g}} - I_{\tilde{g}} - F_{\tilde{g}}$ represent the truth, indeterminacy and falsity degrees of the trapezoidal number.

If the reader deals with a symmetric trapezoidal neutrosophic number,which has the following form:
$\tilde{g} = \langle (g^{m1}, g^{m2}); \alpha, \beta \rangle$, where $\alpha = \beta$ and $\alpha, \beta > 0$,then the ranking function for the neutrosophic number will be defined as follows:

$$R(\tilde{a}) = \left(\frac{(a^{m1} + a^{m2}) + 2(\alpha + \beta)}{2} \right) * T_{\tilde{a}} - I_{\tilde{a}} - F_{\tilde{a}} \quad (10)$$

4 Deterministic Three-Level Linear Programming Problem

Now, after the implementation of the maximization ranking function in Eq(8) , the TLNLPP can be simplified to the next three-level linear problem (TLLPP).

[First Level]

$$\max_{X_1} F_1 = \sum_{j=1}^{n} c_{1j} x_j \,, \quad (11)$$

where x_2, x_3 solves

[Second Level]

$$\max_{X_2} F_2 = \sum_{j=1}^{n} c_{2j} x_j \,, \quad (12)$$

where x_3 solves

[Third Level]

$$\max_{X_3} F_3 = \sum_{j=1}^{n} c_{3j} x_j \,, \quad (13)$$

Subject to

$$\sum_{j=1}^{n} a_{ij}x_j \leq b_i \ , \tag{14}$$

$x_j \geq 0, i = 1,2,\ldots,m,\ j = 1,2,\ldots,n,\ x_j \geq 0$ goals with some leeway

5 A fuzzy approach for solving Three-Level Linear Programming Problem

To solve the TLLPP using a fuzzy method:First, obtain a satisfactory solution that is appropriate to FLDM problem. Then give the FLDM decision variables and goals with some tolerance to the SLDM to find its solution. After that,send the decision variables and objectives of the SLDM to the TLDM to find its satisfactory solution and to find the solution close to the FLDM optimal solution.

The FLDM solves his/her problem as follows:

1. Find individual optimal solution of problem FLDM by finding the best and the worst solutions of the FLDM problem are(z_1^*, z_1^-).

2.To create the membership functions, use this value(z_1^*, z_1^-)

3.

$$\mu_{f_1}\left[f_1(x)\right] = \begin{cases} 1 & if \quad f_1(x) > f_1^*, \\ \frac{f(x)-f_1^-}{f_1^*-f_1^-} & if \quad f_1^- \leq f_1(x) \leq f_1^* \\ 0 & if \quad f_1^- \geq f_1(x). \end{cases} , \tag{15}$$

Now, solving the following Tchebycheff problem, we can get the solution of the FLDM problem.

$$\max \delta \tag{16}$$

Subject to

$$x \in G,$$

$$\mu_{f_1}\left[f_1(x)\right] \geq \delta,$$

$$\delta \in [0,1].$$

[The solution expected to be$x_1^F, x_2^F, x_3^F, f_1^F, \delta^F$ (Satisfactory level)]

The SLDM does the same procedure as the FLDM until it gets the solution assumed to be $x_1^S, x_2^S, x_3^S, f_1^S, \delta^S$(Satisfactory level)].

The TLDM does the same procedure as the FLDM until it gets the solution assumed to be $x_1^T, x_2^T, x_3^T, f_1^T, \delta^T$(Satisfactory level)].

Now the solution of the three levels decision makers is announced. Nevertheless, three solutions are usually different because of the existence of three objective functions at three levels.

The FLDM recognizes that it is not realistic to use the optimal decisions x_1^F as a control factors for the SLDM. It is more realistic to have some flexibility that helps the SLDM look for the ideal solution and reduce the time of search or interactions. In addition, the SLDM performs the same action with the TLDM.

This means that s, the range of decision variable x_1, x_2 should be around x_1^F, x_2^S with maximum tolerance t_1, t_2 and the following membership function specifies x_1^F, x_2^S as:

$$\mu(x_1) = \begin{cases} \frac{x_1-(x_1^F-t_1)}{t_1} & x_1^F - t_1 \leq x_1 \leq x_1^F, \\ \frac{(x_1^F+t_1)-x_1}{t_1} & x_1^F \leq x_1 \leq x_1^F + t_1 \ , \end{cases} \tag{17}$$

$$\mu(x_2) = \begin{cases} \frac{x_2-(x_2^S-t_2)}{t_2} & x_2^S - t_2 \leq x_2 \leq x_2^S, \\ \frac{(x_2^S+t_2)-x_2}{t_2} & x_2^S \leq x_2 \leq x_2^S + t_2 \ , \end{cases} \tag{18}$$

First, the FLDM goals may reasonably consider all $f_1 \geq f_1^F$ are completely acceptable and all

$f_1 \geq \acute{f}_1 = f_1(x_1^S, x_2^S, x_3^S)$ are completely unacceptable, and that the preference with $[f_1^F, \acute{f}_1]$is linearly increasing. Because the fact that the SLDM got the optimum at(x_1^S, x_2^S, x_3^S), which in turn provides the FLDM the objective function values $\acute{f}_1$, makes any $f_1 \leq \acute{f}_1$ unattractive in practice. The membership functions of the FLDM can be defined as:

$$\acute{\mu}_{f_1}[f_1(x)] = \begin{cases} 1 & if \quad f_1(x) > f_1^F, \\ \frac{f(x) - f_1^-}{f_1^F - f_1^-} & if \quad f_1^- \leq f_1(x) \leq f_1^F \quad , \\ 0 & if \quad f_1(x) \leq f_1^-. \end{cases} \tag{19}$$

Second, the SLDM goals may reasonably consider all $f_2 \geq f_2^S$ are acceptable and all $f_1 \geq \acute{f}_1 = f_1(x_1^F, x_2^F, x_3^F)$ are unacceptable, and that the preference with $[f_2^S, \acute{f}_2]$is linearly increasing. Because of SLDM got the optimum at(x_1^F, x_2^F, x_3^F), which in turn provides the FLDM the objective function values $\acute{f}_2$, makes any $f_2 \leq \acute{f}_2$ unattractive in practice. The membership functions of the SLDM can be defined as:

$$\acute{\mu}_{f_2}[f_2(x)] = \begin{cases} 1 & if \quad f_2(x) > f_2^S, \\ \frac{f(x) - f_2^-}{f_2^S - f_2^-} & if \quad f_2^- \leq f_2(x) \leq f_2^S \quad , \\ 0 & if \quad f_2(x) \leq f_2^-. \end{cases} \tag{20}$$

Third, the TLDM may be willing to create its objective membership function, so it can rate the satisfaction of each potential solution.Thus the TLDM has the following membership function with its objectives:

$$\acute{\mu}_{f_3}[f_3(x)] = \begin{cases} 1 & if \quad f_3(x) > f_3^T, \\ \frac{f(x) - f_3^-}{f_3^T - f_3^-} & if \quad f_3^- \leq f_3(x) \leq f_3^T \quad , \\ 0 & if \quad f_3(x) \leq f_3^-. \end{cases} \tag{21}$$

Where $f_3^- = f_3\left(x_1^S, x_2^S, x_3^S\right)$.
Finally, we can solve the following Tchebycheff problem to produce the satisfactory solution, which is also an optimal Pareto (satisfactory) solution with absolute satisfaction for all DMs.

$$\max \delta \tag{22}$$

Subject to

$$\begin{gathered} x \in G, \\ \frac{x_1 - \left(x_1^F - t_1\right)}{t_1} \geq \delta I, \\ \frac{\left(x_1^F + t_1\right) - x_1}{t_1} \geq \delta I, \\ \frac{x_2 - \left(x_2^S - t_2\right)}{t_2} \geq \delta I, \\ \frac{\left(x_2^S + t_2\right) - x_2}{t_2} \geq \delta I, \\ \acute{\mu}_{f_1}[f_1(x)] \geq \delta I, \\ \acute{\mu}_{f_2}[f_2(x)] \geq \delta I, \\ \acute{\mu}_{f_3}[f_3(x)] \geq \delta I, \\ x_1, x_2, x_3 \in G, \\ t_1, t_2 > 0, \\ \delta \in [0, 1]. \end{gathered}$$

6 Numerical example

[First Level]

$$\max_{X_1} F_1 \approx (10,30,43,60)X_1 + (8,20,24,35)X_2 - (3,5,14,16)X_3$$

Where x_2, x_3 solve
[Second Level]

$$\max_{X_2} F_2 \approx (9,20,26,43)X_1 + (9,20,26,43)X_2 + (8,18,23,32)X_3$$

Where x_3 solves
[Third Level]

$$\max_{X_3} F_3 \approx (10,17,23,36)X_1 + (10,25,34,50)X_2 + (10,21,30,54)X_3$$

Subject to

$3x_1 + 2x_2 + x_3 \tilde{\leq} (10,50,73,120),$
$x_1 + x_3 \tilde{\leq} (12,22,30,56),$
$2x_1 + x_2 \tilde{\leq} (10,40,58,70),$
$x_1, x_2, x_3 \geq 0.$

In this example, we solve a TLLP problem with trapezoidal neutrosophic numbers in both objective function and constrains. The order of element for trapezoidal neutrosophic numbers is : lower, first median value, second median value and finally the upper bound. The decision makers' confirmation degree about each value of trapezoidal neutrosophic number is (0.8, 0.1, and 0.1) for the first level and (0.9, 0.2, 0.1) for the second level and (0.8, 0.2, 0.1) for the third level. The decision makers' confirmation degree about each value of trapezoidal neutrosophic number in constrains is (0.8, 0.2, 0.1).

First step:
Since this NLP problem is a problem with maximisation, any trapezoid number will be transformed using the Eq(8) into its corresponding crisp number.

$$R(\tilde{a}) = \left| \left(\frac{-\frac{1}{3}(3a^{l} - 9a^{u}) + 2(a^{m_1} - a^{m_2})}{2} \right) * (T_{\tilde{a}} - I_{\tilde{a}} - F_{\tilde{a}}) \right|$$

Then, the previous problem's crisp model is:
[First Level]

$$\max_{X_1} F_1 = 43.2\, x_1 + 26.7\, x_2 - 8.1\, x_3$$

Where x_2, x_3 solve
[Second Level]

$$\max_{X_2} F_2 = 22.4\, x_1 + 37.8\, x_2 + 27.3\, x_3$$

Where x_3 solves
[Third Level]

$$\max_{X_3} F_3 = 21.5\, x_1 + 30.5\, x_2 + 33.5\, x_3$$

Subject to

$2x_1 + 2x_2 + x_3 \leq 76,$
$2x_1 + 2x_3 \leq 35,$
$x_1 + x_2 \leq 41,$
$x_1, x_2, x_3 \geq 0.$

1.To solve a problem of three-Level linear programming with Fuzzy Approach

First, the first level solves its problem as follows:

$$\max_{X_1} F_1 = 43.2\, x_1 + 26.7\, x_2 - 8.1$$

Subject to

$2x_1 + 2x_2 + x_3 \leq 76,$
$2x_1 + 2x_3 \leq 35,$
$x_1 + x_2 \leq 41,$
$x_1, x_2, x_3 \geq 0.$

1. Seek an individual optimal first-level solution by finding the best and worst solutions for the first-level problem:

$$f_1^* = 1303.35 \quad \overline{f_1} = -143.5$$

These values f_1^* , $\overline{f_1}$ are used to build the membership functions then the membership function of fuzzy set theory (14) and (15).

$\max \delta,$
Subject to
$x \varepsilon G,$
$43.2\, x_1 + 26.7\, x_2 - 8.1 - 143.5 - 1159.85\, \delta \geq 0,$
$\delta \varepsilon [0,1].$
Whose solution is:
$(x_1^f, x_2^f, x_3^f) = (0, 18.4, 0), F_1^f = 491.28 \delta = 0.3$

Second, the second level solves its problem as follows:

$$\max_{X_2} F_2 = 22.4\, x_1 + 37.8\, x_2 + 27.3\, x_3$$

Subject to
$2x_1 + 2x_2 + x_3 \leq 76,$
$2x_1 + 2x_3 \leq 35,$
$x_1 + x_2 \leq 41,$
$x_1, x_2, x_3 \geq 0.$

1. Find individual optimal solution of second level by finding the best and the worst solutions of the second level problem, to get:

$$f_2^* = 1583.4 \quad \overline{f_2} = 0$$

Use this value of f_2^* , $\overline{f_2}$ and build the membership functions then solve as follows: use the membership function of fuzzy set theory (14) and (15).
$\max \delta,$
Subject to
$x \varepsilon G,$
$22.4\, x_1 + 37.8\, x_2 + 27.3\, x_3 - 1583.4 \delta \geq 0,$
$\delta \varepsilon [0,1].$
$x_1, x_2, x_3 \geq 0$
Whose solution is:

$$(x_1^s, x_2^s, x_3^s) = (0, 20.9, 0), F_2^s = 790.02 \delta = 0.5$$

The third level solves its problem as follows:

$$\max_{X_3} F_3 = 21.5\, x_1 + 30.5\, x_2 + 33.5\, x_3$$

Subject to
$2x_1 + 2x_2 + x_3 \leq 76,$
$2x_1 + 2x_3 \leq 35,$
$x_1 + x_2 \leq 41,$
$x_1, x_2, x_3 \geq 0.$

1. Find individual optimal solution of third level by obtaining the best and the worst solutions of the third level problem, to get:

$$f_3^* = 1478.37 \quad \overline{f_3} = 0$$

Use this value of f_3^* , $\overline{f_3}$ and build the membership functions then solve as follows: use the membership function of fuzzy set theory (14) and (15).

$\max\delta$,
Subject to
$x\varepsilon G$,
$21.5\, x_1 + 30.5\, x_2 + 33.5\, x_3 - 1478.37\, \delta \geq 0$,
$\delta\varepsilon\, [0,1]$.

$$x_1, x_2, x_3 \geq 0$$

Whose solution is:
$(x_1^T, x_2^T, x_3^T) = (0,0,4.4), F_3^T = 147.4\delta = 0.1$
Finally,
1. We assume the FLDMs control decision x_1^f is around 0 with tolerance 1.
2. We assume the SLDMs control decision x_2^s is around 0 with the tolerance 1.
3. Using (17)-(21) the TLDM solves the following problem of (22) as follows:
$\max\delta$,
Subject to
$2x_1 + 2x_2 + x_3 \leq\ 76$,
$2x_1 + 2x_3 \leq\ 35$,
$x_1 + x_2 \leq\ 41$,
$43.2\, X_1 + 26.7\, X_2 - 8.1\, X_3 - 558.03 + 66.75\, \delta \geq 0$,
$22.4\, X_1 + 37.8\, X_2 + 27.3\, X_3 - 695.52 - 94.45\delta \geq 0$,
$21.5\, X_1 + 30.5\, X_2 + 33.5\, X_3 - 637.45 + 490.05\, \delta \geq 0$,
$x_1, x_2, x_3 \geq 0$
$\delta\varepsilon\, [0,1]$.
Whose compromise solution is $X_0 = (0,20.9,8.14)$; *and* $\delta = 0.9999$ *(overall satisfaction for all DMs). And* $F_1 = 491.282\ F_2 = 1012.242\ F_3 = 910.14$

7 Conclusion and Suggestions for Future Research

This paper introduced a solution algorithm for solving the TLNLPP with neutrosophic parameters in the objective functions and constrain.To decrease the complexity, the neutrosophic nature of the problem in the first step of the solution algorithm converted into the corresponding crisp model. The second phase was based on a fuzzy approach to achieve a compromised TLLPP solution. Finally, accuracy of the proposed solution algorithm was illustrated with a numerical illustration.

However, a number of issues will be open to future discussion and need to be addressed in multi-level neutrosophic optimization:

1. Multi-level quadratic decision-making problems with neutrosophic parameters in both objective functions and constraints.
2. Multi-level fractional large-scale decision-making problems with neutrosophic parameters in both objective functions and constraints and with integrity conditions.

1. Multi-level fractional decision-making problems with neutrosophic parameters in both objective functions and constraints and with integrity conditions.

Conflict of Interest

The authors declare that they have no conflict of interest.

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