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New Approximate Solution of the Time-Fractional Nagumo Equation Involving Fractional Integrals Without Singular Kernel

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Abstract: In this paper, we present an algorithm by using the Adomian Decomposition Method (ADM) in order to solve the time-fractional Nagumo equation (TFNE) based upon the Liouville-Caputo type Caputo-Fabrizio fractional integral (CFCFI) and the Liouville-Caputo type Atangana-Baleanu fractional integral (ABCFI). We compute the residual error function and derive remarkably efficient results. By means of graphical representations, we show the behavior of the time-fractional Nagumo equation (TFNE) for different values of the involved parameters α and μ in our present investigation.

Keywords: Time=fractional Nagumo equation, Adomian Decomposition Method (ADM), Time-Fractional Nagumo equation (TFNE), Caputo-Fabrizio-Caputo fractional integral (CFCFI), Atangana-Baleanu-Caputo fractional integral (ABCFI), Graphical representations.

1 Introduction

In recent years, the theory of fractional calculus has been used to find the solutions to the equations which model real-world problems in mathematical, physical, chemical and engineering sciences. There exist many phenomena in several fields which are modeled by fractional differential equations such as those occurring in fluid mechanics ([1] and [2]), chemistry [3,4,5,6], biology [7], viscoelasticity (see [8] and [9]), engineering, finance, and physics [10,11,12]. This has motivated and encouraged scientists and engineers to give more attention to the fact that fractional derivatives do arise in physical problems. In this regard, we may cite the works [13] and [14]. As a result of the difficulty of finding an analytical solution for the fractional differential equations analytically, the approximate methods were used (see, for example, [15, 16,17,18,19,20,21,22,23,24]).

Here, in our present investigation, we begin by considering the Nagumo equation given by

$$\psi_\eta = \psi_{\zeta\zeta} + \psi(1-\psi)(\psi-\mu) \qquad (0 \leqq \mu < 1). \quad (1)$$

The exact solution of (1) is known as a solitary wave in the following form:

$$\psi(\zeta,\eta) = a + b\tanh\big(\kappa(\varsigma - \nu\eta)\big). \quad (2)$$

There is a family of eight solutions of the form (1) that can be obtained by *Mathematica* (Version 9) or otherwise. We consider the case when

$$a = \frac{1+\mu}{2},\ \nu = \frac{1+\mu}{\sqrt{2}},\ b = \frac{1-\mu}{2} \text{ and } \kappa = \frac{\mu-1}{2\sqrt{2}}. \quad (3)$$

We mention that the solution (2) constrained by

$$\psi(-\infty,\eta) = 1 \qquad \text{and} \qquad \psi(\infty,\eta) = \mu.$$

The Nagumo equation (1) has attracted the attention of many researchers (see, for example, [25,26,27,28,

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29]). This equation has been applied as a model for the transmission of nerve impulses [30,31]. The equation (1) is also an important nonlinear reaction-diffusion equation and has been used in biology and in the areas of population genetics and circuit theory [32].

Adomian derived a new technique, which is called the *Adomian Decomposition Method* (ADM), for computing the solutions of linear and nonlinear equations (see, for details, [33] and [35]). Various authors have studied the convergence of Adomian's method [36,37,38,39]. This method is known to be effective and powerful and it can be applied to ordinary and partial differential equations and integral equations. (see, for example, [40,41,42,43, 44,45,46,47,48]).

Recently, Caputo and Fabrizio proposed and developed a (presumably new) concept of fractional differentiation using the exponential decay as its kernel instead of the power law which occurs in the Liouville-Caputo fractional derivative (see [49] and [50]). On the other hand, Atangana and Baleanu (ABC) proposed a fractional derivative with non-local kernel. It is based on the general Mittag-Leffler function $E_{\alpha,\beta}(z)$. A characteristic of this operator is that it describes many complex physical issues which follow at the same time the power and the exponential decay laws (see, for example, [51] to [69]). This work is organized as follows. Section 2 is devoted to the essential ideas of the fractional calculus. Section 3 concentrates upon the background and preliminaries of the standard ADM. In Section 4, we apply the ADM based upon on the Liouville-Caputo type Caputo-Fabrizio and the Liouville-Caputo type Atangana-Baleanu fractional integrals for modeling and solving the time-fractional Nagumo equation. Conclusions are presented in the last section (Section 5).

2 Operators of Fractional Calculus

In this section, we give some basic definitions and properties of fractional calculus theory [1,2,9].

Definition 1. If $\psi(\eta) \in L_1(a,b)$, the set of all integrable functions on (a,b), and $\alpha > 0$, then the Riemann-Liouville fractional integral of order α, denoted by J^{α}_{a+} is defined by

$$J^{\alpha}_{a+}\psi(\eta) = \frac{1}{\Gamma(\alpha)}\int_a^{\eta}(\eta-\varsigma)^{\alpha-1}\psi(\varsigma)d\varsigma. \tag{4}$$

Definition 2. For $\alpha > 0$, the Liouville-Caputo fractional derivative of order α, denoted by ${}^{\mathrm{LC}}D^{\alpha}_{a+}$, is defined by

$$\begin{aligned}{}^{\mathrm{LC}}\mathscr{D}^{\alpha}_{a+}\psi(\eta) &= \frac{1}{\Gamma(n-\alpha)}\int_a^{\eta}(\eta-\varsigma)^{n-\alpha-1}\\ &\quad\cdot\mathscr{D}^n\psi(\varsigma)d\varsigma \qquad (5)\\ (n-1<\alpha<n;\ n\in\mathbb{N}=\{1,2,3,\cdots\}),&\end{aligned}$$

so that

$${}^{\mathrm{LC}}\mathscr{D}^{\alpha}_{a+} = D^{\alpha} \qquad (\alpha\in\mathbb{N}), \tag{6}$$

where $D=\frac{d}{d\eta}$. The Liouville-Caputo fractional derivative on the whole space $\mathbb{R}$ is defined below.

Definition 3. For $\alpha > 0$, the Liouville-Caputo fractional derivative of order α on the whole space, denoted by ${}^{\mathrm{LC}}\mathscr{D}^{\alpha}_{a+}$, is defined by

$$\begin{aligned}{}^{\mathrm{LC}}\mathscr{D}^{\alpha}_{a+}\psi(\zeta) &= \frac{1}{\Gamma(n-\alpha)}\int_{-\infty}^{\zeta}(\zeta-\varsigma)^{n-\alpha-1}\\ &\quad\cdot D^n\psi(\varsigma)d\varsigma \qquad (7)\\ (n-1<\alpha<n;\ n\in\mathbb{N}).&\end{aligned}$$

We next introduce the Caputo-Fabrizio-Caputo fractional integral operator of order $\alpha > 0$ (see [50]).

Definition 4. The Caputo-Fabrizio-Caputo derivative operator (CFC) with respect to η, denoted by ${}^{\mathrm{CFC}}_{0}D^{\alpha}_{\eta}$, is defined by

$$\begin{aligned}{}^{\mathrm{CFC}}_{0}\mathscr{D}^{\alpha}_{\eta}\psi(\eta) &= \frac{M(\alpha)}{n-\alpha}\int_0^{\eta}\exp\left(-\frac{\alpha(\eta-\varsigma)}{n-\alpha}\right)\\ &\quad\cdot\mathscr{D}^n\psi(\varsigma)d\varsigma \qquad (8)\\ (n-1<\alpha<n;\ n\in\mathbb{N}),&\end{aligned}$$

where $M(\alpha)$ is a normalization function such that

$$M(0)=M(1)=1.$$

Definition 5. Losada and Nieto [70] proposed the fractional integral according to the CFCI as follows:

$$\begin{aligned}{}^{\mathrm{CFCFI}}J^{\alpha}_{0}\psi(\eta) &= \frac{2(1-\alpha)}{(2-\alpha)\mathscr{M}(\alpha)}\psi(\eta)\\ &\quad+\frac{2\alpha}{(2-\alpha)\mathscr{M}(\alpha)}\int_0^{\eta}\psi(\varsigma)d\varsigma, \qquad (9)\end{aligned}$$

where

$$\mathscr{M}(\alpha)=\frac{2}{2-\alpha}. \tag{10}$$

The Atangana-Baleanu-Caputo fractional derivative (ABC) of order $\alpha > 0$ is defined as follows (see [71]).

Definition 6. The Atangana-Baleanu-Caputo fractional derivative (ABC) of order $\alpha > 0$ is defined by

$$\begin{aligned}{}^{\mathrm{ABC}}_{0}\mathscr{D}^{\alpha}_{t}\psi(\eta) &= \frac{B(\alpha)}{n-\alpha}\int_0^{\eta}E_{\alpha}\left(-\alpha\frac{(\eta-\varsigma)^{\alpha}}{n-\alpha}\right)\\ &\quad\cdot\mathscr{D}^n\psi(\varsigma)d\varsigma \qquad (11)\\ (n-1<\alpha<n;\ n\in\mathbb{N}),&\end{aligned}$$

where $B(\alpha)$ is a normalization function and

$$B(0)=B(1)=1.$$

Definition 7. The Atangana-Baleanu-Caputo fractional integral corresponding to ABC is given by

$$\begin{aligned}{}^{\mathrm{ABCFI}}J^{\alpha}_{0}\psi(\eta) &= \frac{1-\alpha}{B(\alpha)}\psi(\eta)\\ &\quad+\frac{\alpha}{B(\alpha)\Gamma(\alpha)}\int_0^{\eta}(\eta-\varsigma)^{\alpha-1}\psi(\varsigma)d\varsigma.\end{aligned} \tag{12}$$

3 Preliminary Description of the Adomian Decomposition Method (ADM)

In this section, we introduce the preliminaries of the Adomian Decomposition Method (ADM) (see [33] and [34]) by considering the following nonlinear partial differential equation:

$$L\big(\psi(\zeta,\eta)\big)+R\big(\psi(\zeta,\eta)\big)+N\big(\psi(\zeta,\eta)\big)=0, \tag{13}$$

together with the initial condition given by

$$\psi(\zeta,0)=\phi(\zeta), \tag{14}$$

where N denotes a nonlinear operator, R is the remaining linear operator, and L is the highest-order derivative which is assumed to be invertible. By operating on both sides of (13) by the inverse operator L^{-1}, we obtain

$$\psi(\zeta,\eta) = \phi(\zeta)-L^{-1}\Big(R\big(\psi(\zeta,\eta)\big)+N\big(\psi(\zeta,\eta)\big)\Big). \tag{15}$$

Let

$$\psi(\zeta,\eta)=\sum_{m=0}^{\infty}\psi_m(\zeta,\eta) \tag{16}$$

and

$$N(\psi)=\sum_{m=0}^{\infty}\chi_m, \tag{17}$$

where χ_m are the Adomian polynomials which depend upon ψ. In view of the equations (16) to (17), the equation (15) takes the following form:

$$\sum_{m=0}^{\infty}\psi_m(\zeta,\eta) = \phi(\zeta)-L^{-1}\Big(R\big(\psi(\zeta,\eta)\big)+\sum_{m=0}^{\infty}\chi_m\big(\psi(\zeta,\eta)\big)\Big). \tag{18}$$

We now set

$$\psi_0(\zeta,\eta)=\phi(\zeta) \tag{19}$$

and

$$\psi_{m+1}(\zeta,\eta) = -L^{-1}\Big(R\big(\psi(\zeta,\eta)\big)+\sum_{m=0}^{\infty}\chi_m\big(\psi(\zeta,\eta)\big)\Big) \tag{20}$$

$$(m\in\mathbb{N}_0=\mathbb{N}\cup\{0\}),$$

where

$$\chi_m\big(\psi(\zeta,\eta)\big)=\left[\frac{1}{m!}\frac{d^m}{d\lambda^m}N\left(\sum_{m=0}^{\infty}\psi_m(\zeta,\eta)\lambda^m\right)\right]_{\lambda=0}. \tag{21}$$

Hence, clearly, the equations (19) to (20) and (21) lead to the following recurrence relations:

$$\psi_0(\zeta,0)=\phi(\zeta) \tag{22}$$

and

$$\psi_{m+1}(\zeta,\eta) = -L^{-1}\Big(R\big(\psi(\zeta,\eta)\big)+A_m\big(\psi(\zeta,\eta)\big)\Big) \tag{23}$$

The solution $\psi(\zeta,\eta)$ can thus be approximated by the following truncated series:

$$\varphi_k(\zeta,\eta)=\sum_{m=0}^{k-1}\psi_m(\zeta,\eta) \quad\text{and}\quad \lim_{k\to\infty}\varphi_k=\psi(\zeta,\eta). \tag{24}$$

4 The Time-Fractional Nagumo Equation

In this section, we apply the ADM to find the approximation solutions for the time-fractional Nagumo equation. To obtain this equation, we replace u_t by u_t^{α}, where $n-1<\alpha\leqq n$ $(n\in\mathbb{N})$, in the Nagumo equation. We thus obtain the time-fractional Nagumo equation as follows:

$$\psi_{\eta}^{\alpha}=\psi_{\zeta\zeta}+\psi(1-\psi)(\psi-\mu) \qquad (0<\alpha\leqq 1). \tag{25}$$

If we operate upon each member of (25) by ${}^{(\cdot)}J_{\eta}^{\alpha}$, where

$${}^{(\cdot)}J_{\eta}^{\alpha}={}^{(\mathrm{CFCI})}J_{\eta}^{\alpha},$$

or

$${}^{(\cdot)}J_{\eta}^{\alpha}={}^{(\mathrm{ABCI})}J_{\eta}^{\alpha},$$

we find that

$$\psi(\zeta,\eta)=\psi(\zeta,0)+{}^{(\cdot)}J_{\eta}^{\alpha}\big(\psi_{\zeta\zeta}-\mu\psi(\zeta,\eta) +(1+\mu)[\psi(\zeta,\eta)]^2-[\psi(\zeta,\eta)]^3\big). \tag{26}$$

Now the ADM solutions and the nonlinear functions $N\big(\psi(\zeta,\eta)\big)$ can be presented as an infinite series given by

$$\psi(\zeta,\eta)=\psi_0(\zeta,\eta)+\sum_{n=1}^{\infty}\psi_n(\zeta,\eta), \tag{27}$$

and

$$N\big(\psi(\zeta,\eta)\big)=(1+\mu)[\psi(\zeta,\eta)]^2-[\psi(\zeta,\eta)]^3 =\sum_{n=0}^{\infty}\chi_n, \tag{28}$$

where

$$\chi_n=\frac{1}{n!}\left[\frac{d^n}{d\lambda^n}N\left(\sum_{k=0}^{n}\lambda^k\psi_k(\zeta,\eta)\right)\right]_{\lambda=0}, \tag{29}$$

where χ_n are called the Adomian polynomials. Furthermore, the components $\psi_n(\zeta,\eta)$ of the solutions $\psi(\zeta,\eta)$ will be determined by the following recurrence relations:

$$\psi_0(\zeta,\eta)=\psi(\zeta,0) \tag{30}$$

and

$$\psi_{n+1}(\zeta,\eta) =^{(\cdot)} J_\eta^\alpha\Big(\big(\psi_n(\zeta,\eta)\big)_{\zeta\zeta}-\mu\,\psi_n(\zeta,\eta)+\chi_n\Big). \tag{31}$$

In view of (21) and using the software *Mathematica* (Version 9), we evaluate the Adomian polynomials χ_n as follows:

$$\chi_0=(1+\mu)[\psi_0(\zeta,\eta)]^2-[\psi_0(\zeta,\eta)]^3,$$

$$\chi_1=2(1+\mu)\psi_0(\zeta,\eta)\psi_1(\zeta,\eta)-3[\psi_0(\zeta,\eta))]^2\psi_1(\zeta,\eta),$$
$$\vdots \tag{32}$$

Now, in the first iteration, we have

$$\psi_1^{(\mathrm{CFC})}(\zeta,\eta)=\frac{2(1-\alpha)}{(2-\alpha)\mathscr{M}(\alpha)}\big(\psi_0(\zeta,\eta)\big)_{\zeta\zeta}-\mu\,\psi_0(\zeta,\eta)+\chi_0 + \frac{2\alpha}{(2-\alpha)\mathscr{M}(\alpha)}\int_0^\eta\Big[\big(\psi_0(\zeta,\varsigma)\big)_{\zeta\zeta}-\mu\,\psi_0(\zeta,\varsigma)+\chi_0\Big]d\varsigma \tag{33}$$

and

$$\psi_1^{(\mathrm{ABC})}(\zeta,\eta)=\frac{1-\alpha}{B(\alpha)}\big(\psi_0(\zeta,\eta)\big)_{\zeta\zeta}-\mu\,\psi_0(\zeta,\eta)+\chi_0 + \frac{\alpha}{B(\alpha)\Gamma(\alpha)}\int_0^\eta\Big[(\eta-\varsigma)^{\alpha-1}\,\big(\psi_0(\zeta,\varsigma)\big)_{\zeta\zeta}-\mu\,\psi_0(\zeta,\varsigma)+\chi_0\Big]d\varsigma. \tag{34}$$

The initial condition is then taken by setting $\eta=0$ in (2). We thus find that

$$\psi(\zeta,0)=\frac{1}{2}\left(1+\mu-(\mu-1)\tanh\left[\frac{(\mu-1)\zeta}{2\sqrt{2}}\right]\right). \tag{35}$$

In view of (30) and (33) to (35), we obtain the first three approximations as follows:

$$\psi_1^{(\mathrm{CFC})}(\zeta,\eta)=\frac{\alpha(\eta-1)+1}{4(2-\alpha)\mu}(\mu-1)^2(\mu+1)\cdot\operatorname{sech}^2\left(\frac{(\mu-1)\zeta}{2\sqrt{2}}\right), \tag{36}$$

$$\psi_2^{(\mathrm{CFC})}(\zeta,\eta)=\frac{\alpha^2\left(\eta^2-4\eta+2\right)+4\alpha(\eta-1)+2}{(\alpha-2)^2[M(\alpha)]^2}\cdot(\mu-1)^3(\mu+1)^2\sinh^4\left(\frac{(\mu-1)\zeta}{2\sqrt{2}}\right)\operatorname{csch}^3\left(\frac{(\mu-1)\zeta}{\sqrt{2}}\right), \tag{37}$$

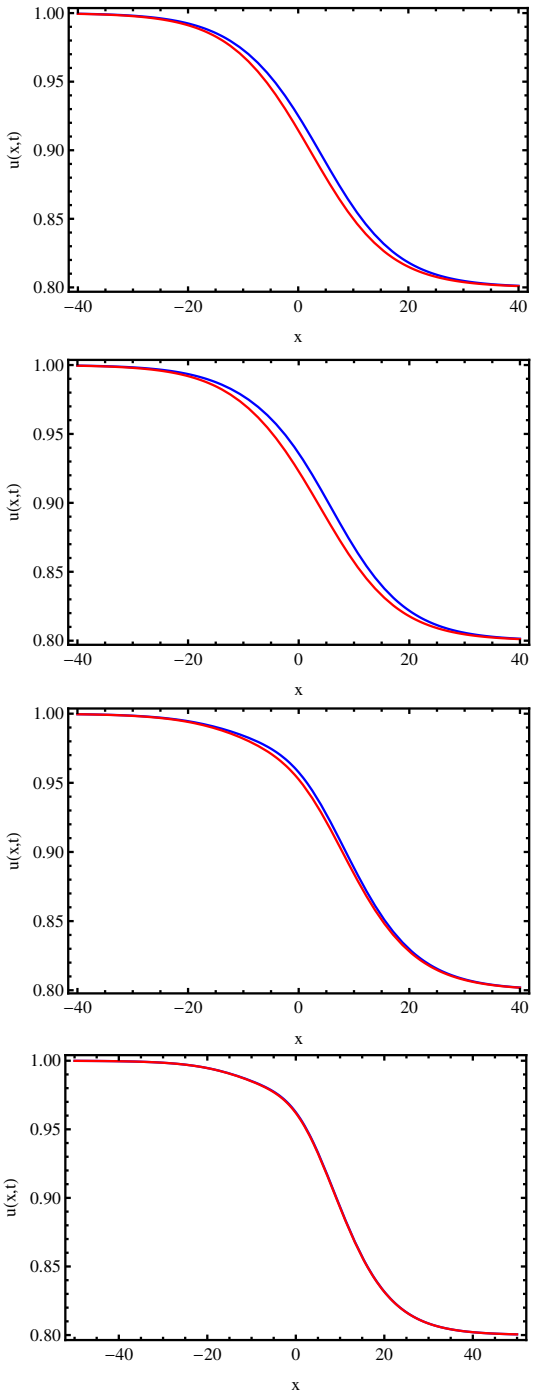

Fig. 1: The plot of the first 4 terms of the CFC and ABC solutions with $\mu=0.8$ and $\eta=7.0$: (a) $\alpha=0.3$; (b) $\alpha=0.5$; (c) $\alpha=0.9$; (d) $\alpha=0.99$. Blue color: (CFC); Red color: (ABC).

$$\psi_1^{(\mathrm{ABC})}(\zeta,\eta)=\frac{(-\alpha\Gamma(\alpha)+\Gamma(\alpha)+\eta^\alpha)}{8B(\alpha)\Gamma(\alpha)}\cdot(\mu-1)^2(\mu+1)\,\operatorname{sech}^2\left(\frac{(\mu-1)\zeta}{2\sqrt{2}}\right) \tag{38}$$

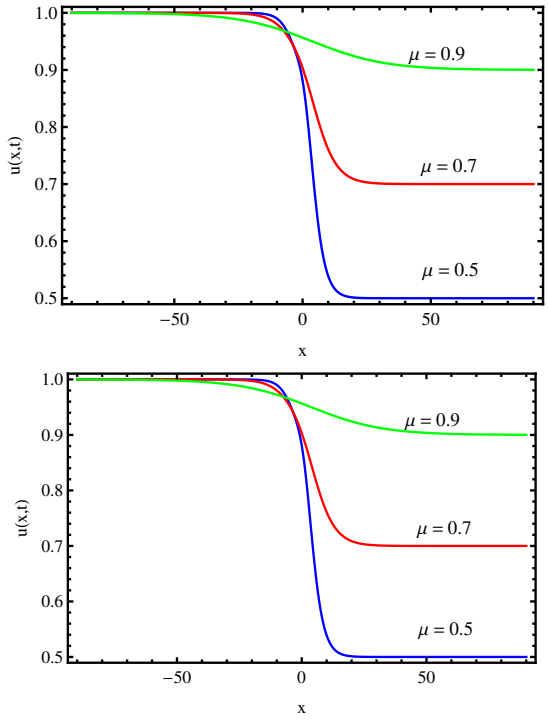

Fig. 2: The plot of the first 4 terms of the CFC and ABC solutions with $\alpha = 0.5$ and $\eta = 7.0$: (a) CFC; (b) ABC. Blue color: $(\mu = 0.5)$; Red color: $(\mu = 0.7)$; Green color: $(\mu = 0.9)$.

and

$$\psi_2^{(\mathrm{ABC})}(\zeta,\eta) = \frac{2^{-2\alpha-1}\left(\sqrt{\pi}\,\alpha\eta^{2\alpha} + 4^{\alpha}(\alpha-1)\Gamma\left(\alpha+\frac{1}{2}\right)\left[(\alpha-1)\Gamma(\alpha) - 2\eta^{\alpha}\right]\right)}{[B(\alpha)]^2\Gamma(\alpha)\Gamma\left(\alpha+\frac{1}{2}\right)}$$
$$\cdot(\mu-1)^3(\mu+1)^2\,\sinh^4\left(\frac{(\mu-1)\zeta}{2\sqrt{2}}\right)\mathrm{csch}^3\left(\frac{(\mu-1)\zeta}{\sqrt{2}}\right). \quad (39)$$

The following components:

$$\psi_3^{(\mathrm{CFC})}(\zeta,\eta), \quad \psi_4^{(\mathrm{CFC})}(\zeta,\eta), \quad \psi_3^{(\mathrm{ABC})}(\zeta,\eta), \quad \psi_4^{(\mathrm{ABC})}(\zeta,\eta), \cdots$$

were also determined and will be used, but (for the sake of brevity) are not listed here. The general form of the approximations is given by (27), that is, by

$$\psi^{(\mathrm{CFC})}(\zeta,\eta) = \psi_0^{(\mathrm{CFC})}(\zeta,\eta) + \psi_1^{(\mathrm{CFC})}(\zeta,\eta) + \psi_2^{(\mathrm{CFC})}(\zeta,\eta) + \cdots \quad (40)$$

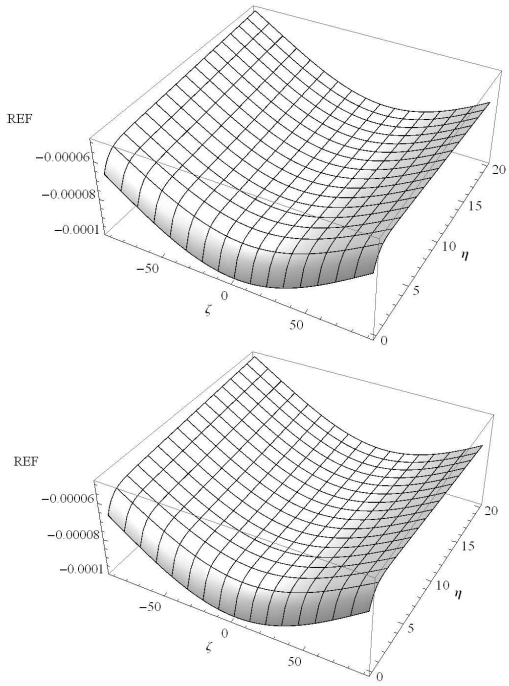

Fig. 3: The plotting of the REF for the first 4 terms of the CFC and ABC solutions with $\alpha = 0.6$ and $\mu = 0.98$: (a) CFC; (b) ABC.

and

$$\psi^{(\mathrm{ABC})}(\zeta,\eta) = \psi_0^{(\mathrm{ABC})}(\zeta,\eta) + \psi_1^{(\mathrm{ABC})}(\zeta,\eta) + \psi_2^{(\mathrm{ABC})}(\zeta,\eta) + \cdots. \quad (41)$$

In Figure 1, we have shown the first 4 terms of the CFC and ABC solutions for $\alpha = 0.3, 0.5, 0.9$ and $\alpha = 0.99$ with $\mu = 0.8, \eta = 7.0$ in (a) to (d), respectively. In this figure, as $\alpha \to 1$, the CFC and ABC solutions coincide with each other. These solutions agree with the known LC solutions in [72].

In Figure 2, we have shown the effect of the parameter μ in the CFC and ABC solutions with $\alpha = 0.3$ and $\eta = 7.0$. The solutions still between 1 and μ as we mentioned in the introduction. Now, for further illustrating our results, we present the residual error function [72]. The REF is given by

$$\mathrm{REF}(\cdot) := \sum_{j=0}^{n} \psi_{j,\eta} - \psi_{j,\zeta\zeta} - \psi_j(1-\psi_j)(\psi_j - \mu). \quad (42)$$

In Figure 3, we have plotted the REF for the CFC and ABC solutions with $n = 3$, $\alpha = 0.6$ and $\mu = 0.98$. It can be seen from Figure 3 that the order of the REF is 10^{-5} for the proposed fractional integrals. By increasing the terms of the CFC and ABC solutions, we get accurate solutions.

5 Perspective

In this paper, the Adomian Decomposition Method (ADM) has been successfully applied efficiently based upon the Caputo-Fabrizio and the Atangana-Baleanu fractional integrals in order to obtain the approximate solutions of the time-fractional Nagumo equation. We have studied the residual error function (REF) that applies to the time-fractional Nagumo equation. The residual error function has been found to be remarkably satisfactory. Furthermore, the results derived by us demonstrate that the ADM is sufficiently accurate for solving the time-fractional Nagumo equation. We can get any desired accuracy by appropriately increasing the number of terms. In this paper, we have fruitfully used *Mathematica* (Version 9) in all of our calculations.

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