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Nonparametric Test for a Class of Life Time Distribution UBAC(2) Based on Moment Generating Function

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Abstract: In this paper we will prove that if the life time of an item has used better than aged in an increasing concave ordering UBAC(2) ageing property and if the mean life is finite then the moment generating function exists and is finite. A new technique for testing exponentially versus UBAC(2) class of life distribution based on the moment generating function is introduced. Using simulation, critical values for censored and non-censored data, pitman's asymptotic efficiency and power of the test for some commonly used distributions in reliability are calculated. Finally, medical applications are presented as an example to elucidate the use of the proposed test statistic for practical reliability analysis.

Keywords: UBAC(2) class of life distribution, Moment generating function, Testing of hypothesis, Asymptotic to normality, Pitman's efficiency, Monte carlo Method, The power of the test and censoring.

1 Introduction

The nonparametric classes of lifetime distributions plays a vital role in studying the reliability of systems. In reliability analysis, many statisticians and reliability analysts introduced many classes of life distributions and their variations such as IFR, IFRA, NBU, NBUE, DMRL, NBUFR, and NBARFR. For detailed discussions of properties and applications, we refer to Bryson and Siddiqui [1], Rolski [2], Barlow and Proschan [3], Klefsjo [4,5], Ahmad [6] and Mugdadi and Ahmad [7], Abouammoh and Ahmed [8,9] and Alzaid et al. [10].

This article opens up new test statistic based on moment generating function. Moment generating function, moment inequality and laplace transform are different techniques which are widely applied in engineering, biological science, economics, maintenance, etc. In reliability theory, they are utilized to define ageing concepts; for example, Klar and Muller [11] derived the moment generating function order and studied the M class, which was proved to be a more responsible class of life distributions with positive aging. Li [12] also proved that the NBUMG class was closed under the increasing star-shaped transformations, and that the M class was closed under both convex combination and geometric compounding. $NBUE_{mgf}$ was explored by Ahmad and Mugdadi [13]. Some new results on the moment generating function order and related life distribution for NBUMG was addressed by Shuhong and Li [14]. The moment generating function for UBAC class of life distribution was discussed by Abu-Youssef [15]. A new class of life distribution called Exponential Better than Used in moment generating function ordering EBU_{mgf} was derived based on moment generating function ordering by Abbas [16]. Testing ODL_{mgf} class of life distributions based on U-test was introduced by Diab et al. [17].

Suppose the lifetime of a component can be modeled by a nonnegative random variable $X \geq 0$ with a cumulative distribution function $F(x) = P(X \leq t)$, survival function $\overline{F} = 1 - F$ and a finite mean $\mu = E(X) = \int_0^\infty \overline{F}(u)du$.

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Definition 1. *If The distribution function F has finite mean, then F is called UBAC(2) if :*

$$\nu(t)-\nu(x+t)\geq(1-e^{-x})\bar{F}(t). \tag{1}$$

See Ali [18].

It is shown that UBAC(2) includes many classes of life distributions. Willmot and Cai [19] proved that the UBA (used better than aged) class includes the decreasing mean residual life DMRL (decrease mean residual life time) class. Di Crescenzo [20] has shown that UBAE (used better than aged in expectation) class is contained in the HUBAE (harmonic used better than aged in expectation) class. And from Mohi el Din et al. [21], we have:

$$IFR \Rightarrow UBA \Rightarrow UBAC(2)$$

Thus we have

$$\begin{array}{c}\text{IFR} \subset \text{DMRL} \subset \text{UBA} \subset \text{UBAC(2)}\\ \cup \\ \text{HUBAE} \subset \text{UBAE}\end{array}$$

For more definition and properties of these classes, you can see Deshpande et al. [22] and Deshpande and Purohit [23].

In this paper, a new test statistic technique based on moment generating function for testing H_0: F is exponential against H_1: F is UBAC(2) and not exponential is introduced in sec(2). Critical values for complete data are tabulated using Monte Carlo method for sample sizes n = 5(5)100. In sec (3), the PAE and the power estimates of this test are also calculated for some common distributions. In sec (4), we address right-censored data and selected critical values are tabulated for sample sizes n=2(2)20(10)100. In sec (5), we address some applications in medical science by using our proposed test.

2 Testing of hypotheses

In this section, we show that the measure of departure from H_0 moment generating function (mgf) of X exists and is finite for the UBAC(2) class if μ exists. A new test statistic based on moment generating function for testing $H_0 : \overline{F}$ is exponential against $H_1 : \overline{F}$ is UBAC(2) and isn't exponential is studied for a random sample $X_1, X_2, ..., X_n$ from a population with distribution function F is proposed.

We propose the following measure of departure from H_0

$$\begin{aligned}\delta_{U_m} &= \int_0^\infty \int_0^\infty e^{\beta x}[\nu(t)-\nu(x+t)-\bar{F}(t)+e^{-x}\bar{F}(t)]dxdt\\ &= \int_0^\infty \int_0^\infty e^{\beta x}\nu(t)dxdt - \int_0^\infty \int_0^\infty e^{\beta x}\nu(x+t)dxdt - \int_0^\infty \int_0^\infty e^{\beta x}\bar{F}(t)dxdt + \int_0^\infty \int_0^\infty e^{(\beta-1)x}\bar{F}(t)dxdt\\ &= I_1 - I_2 - I_3 + I_4.\end{aligned} \tag{2}$$

where,

$$\begin{aligned}I_1 &= \int_0^\infty \int_0^\infty e^{\beta x}\nu(t)dxdt \qquad ;\beta>0\\ &= \int_0^\infty e^{\beta x}dx \int_0^\infty \nu(t)dt\\ &= \frac{-\mu_{(2)}}{2\beta}\end{aligned} \tag{3}$$

Let $x+t=u$, then

$$\begin{aligned}I_2 &= \int_0^\infty \int_0^\infty e^{\beta x}\nu(x+t)dxdt \qquad ;\beta>0\\ &= \int_0^\infty \int_t^\infty e^{\beta(u-t)}\nu(u)dudt\\ &= \int_0^\infty \int_0^t e^{\beta(t-u)}\nu(t)dudt\\ &= \frac{-\mu_2}{2\beta} - \frac{\mu}{\beta^2} + \frac{1}{\beta^3}(Ee^{\beta X}-1).\end{aligned} \tag{4}$$

$$\begin{aligned} I_3 &= \int_0^\infty \int_0^\infty e^{\beta x}\bar{F}(t)dxdt \qquad ;\beta > 0 \\ &= \frac{-\mu}{\beta}, \end{aligned} \tag{5}$$

and

$$\begin{aligned} I_4 &= \int_0^\infty \int_0^\infty e^{(\beta-1)x}\bar{F}(t)dxdt \\ &= \frac{-\mu}{\beta-1}. \qquad \beta \neq 1. \end{aligned} \tag{6}$$

Then

$$\delta_{U_m} = \frac{-\beta}{\beta^3(\beta-1)}\mu - \frac{1}{\beta^3}(Ee^{\beta X}-1) \qquad ;\beta \neq 1. \tag{7}$$

The empirical estimator $\hat{\delta}_{U_m}$ of our test statistic can be given as follows:

$$\hat{\delta}_{U_m} = \frac{1}{n}\sum_i \left\{ \frac{-\beta}{\beta^3(\beta-1)}X_i - \frac{1}{\beta^3}(e^{\beta X_i}-1) \right\}, \qquad \beta \neq 1. \tag{8}$$

To make the test statistic scale invariant set

$$\hat{\Delta}_{U_m} = \frac{\hat{\delta}_{U_m}}{\overline{X}}. \tag{9}$$

Let us rewrite (8) as follows:

$$\hat{\delta}_{U_m} = \frac{1}{n}\sum_i \phi(X_i), \tag{10}$$

where $\phi(X_i) = \frac{-\beta}{\beta^3(\beta-1)}X_i - \frac{1}{\beta^3}(e^{\beta X_i}-1)$.

Set i = 1, then

$$\phi(X_1) = \frac{-\beta}{\beta^3(\beta-1)}X_1 - \frac{1}{\beta^3}(Ee^{\beta X_1}-1), \qquad \beta \neq 1. \tag{11}$$

Then $\hat{\delta}_{U_m}$ is a classical U-statistics, see Lee [24].

Theorem 1.*As $n \to \infty$, $\sqrt{n}(\hat{\delta}_{U_m} - \delta_{U_m})/\sigma$ is asymptotically normal with mean 0 and variance $\sigma^2 = var[\phi(X_1)]$, where $\phi(X_1)$ is given in (11).*
Under H_0

$$\sigma^2(\beta) = \frac{2}{\beta^2(\beta-1)^3(2\beta-1)}, \qquad \beta \neq 1. \tag{12}$$

*Proof.*Using the theory of standard U-statistics and by direct calculations,we can find the mean equal 0 and the variance is given by

$$\sigma^2 = var[\phi(X_1)],$$

Then

$$\sigma^2 = E[\frac{-\beta}{\beta^3(\beta-1)}X_1 - \frac{1}{\beta^3}(e^{\beta X_1}-1)]^2, \qquad \beta \neq 1. \tag{13}$$

Under H_0: $E(\phi(X_1)) = 0$ and

$$\begin{aligned} \sigma_0^2(\beta) &= E[\phi(X_1)]^2 \\ &= \int_0^\infty [\frac{-\beta}{\beta^3(\beta-1)}X_1 - \frac{1}{\beta^3}(e^{\beta X_1}-1)]^2 e^{-x}dx. \end{aligned} \tag{14}$$

Then (12) is given and the Theorem is proved.

when $\beta = 2 \rightarrow \sigma_0^2(2) = \frac{1}{6}$

$$\hat{\Delta}_{U_m}(2) = \frac{1}{n\overline{X}}[\frac{-1}{4}X_1 - (e^{2X_1} - 1)] \tag{15}$$

when $\beta = 0.2 \rightarrow \sigma_0^2(2) = 162.76$

$$\hat{\Delta}_{U_m}(0.2) = \frac{1}{n\overline{X}}[\frac{0.2}{0.0064}X_1 - (e^{0.2X_1} - 1)] \tag{16}$$

To use the above-mentioned test, calculate $\sqrt{n}\hat{\Delta}_{U_m}/\sigma_0$ and reject H_0 if this exceeds the normal variate value $Z_{1-\alpha}$. To illustrate the test, we calculate, using Monte Carlo Method, the empirical critical values of $\hat{\Delta}_{U_m}$ in (16) for sample sizes 5(5)100 and $\beta = 0.2$. Table (2.1) gives the percentile points for 1%, 5%, 10%, 90%, 95%, 99% . The calculations are based on 10000 simulated samples of sizes n=10(5)100.

Table (2.1) Critical Values of $\hat{\Delta}_{U_m}$

n	1%	5%	10%	90%	95%	99%
5	-13.6888	-9.78058	-7.6981	6.90782	8.9903	12.8985
10	-9.71301	-6.94948	-5.47694	4.851	6.32354	9.08707
15	-8.05019	-5.79378	-4.59145	3.84127	5.0436	7.30001
20	-6.72329	-4.76918	-3.72793	3.57502	4.61627	6.57038
25	-6.26087	-4.51306	-3.58175	2.95022	3.88153	5.62934
30	-5.53324	-3.93771	-3.08754	2.8753	3.72547	5.32099
35	-5.29194	-3.81478	-3.02767	2.49285	3.27995	4.75712
40	-4.82885	-3.44709	-2.71082	2.45315	3.18942	4.57119
45	-4.53712	-3.23438	-2.54022	2.32842	3.02258	4.32532
50	-4.32607	-3.09018	-2.43164	2.18716	2.8457	4.08159
55	-4.14732	-2.96895	-2.34106	2.06279	2.69068	3.86906
60	-4.02402	-2.89582	-2.29466	1.92171	2.52287	3.65108
65	-3.94557	-2.86163	-2.28405	1.7669	2.34448	3.42842
70	-3.79696	-2.75244	-2.19587	1.70772	2.26429	3.30881
75	-3.65234	-2.64324	-2.10554	1.66569	2.20338	3.21248
80	-3.55492	-2.57786	-2.05724	1.59424	2.11486	3.09192
85	-3.45424	-2.50636	-2.00128	1.54117	2.04625	2.99413
90	-3.36484	-2.44366	-1.95281	1.48983	1.98068	2.90186
95	-3.30529	-2.40868	-1.93092	1.4199	1.89766	2.79427
100	-3.23648	-2.36257	-1.89691	1.36907	1.83473	2.70863

The Table indicates that the values of the percentiles decrease when the sample size increases.

3 Pitman's asymptotic efficiency

In this section, we calculate PAE for UBAC(2) class of life distribution and compare our proposed test with tests of other well known classes of life distribution on basis of PAE.

Here we choose K^* presented by Hollander and Prochan [25] for (DMRL), $\hat{\delta}_2$ presented by Ahmad [26] for (UBAE) class, $\hat{\Delta}_{U_T}$ presented by Abu-Youssef et al. [27] for (UBACT) class of life distribution based on U-Statistics and Λ_n introduced by Mahmoud et al. [28] for (ODL) .

PAE of $\hat{\Delta}_{U_m}$ is given by :

$$PAE(\Delta_{U_m}(\theta)) = \frac{1}{\sigma_0}\left|\frac{d}{d\theta}\Delta_{U_m}(\theta)\right|_{\theta\rightarrow\theta_0}. \tag{17}$$

Two of the most commonly used alternatives (cf. Hollander and Proschan [29]) are:

(i) Linear failure rate family $: \bar{F}_\theta = e^{-x-\frac{\theta x^2}{2}}, \quad x > 0, \theta > 0$

(ii) Makeham family $: \bar{F}_{2\theta} = e^{-x-\theta(x+e^{-x}-1)}, \quad x > 0, \theta > 0$

The null hypothesis is at $\theta = 0$ for linear failure rate and Makham families. The PAE's of these alternatives of our procedure are, respectively:

$$PAE(\Delta_{U_m}, LFR) = \left|-\frac{1}{\sigma_0}[\frac{\beta-2}{\beta(\beta-1)^3}]\right|, \qquad \beta > 0, \beta \neq 1. \tag{18}$$

$$PAE(\Delta_{U_m}, Makeham) = \left| \frac{1}{\sigma_0} [\frac{-1.5+0.5\beta}{\beta^2(\beta-1)^2(\beta-2)}] \right|, \qquad \beta > 0, \beta \neq 1,2. \tag{19}$$

Table (3.2) PAE of $\hat{\Delta}_{U_m}$

Distribution	K^*	$\hat{\delta}_2$	$\hat{\Delta}_{U_T}$	Λ_n	$\hat{\Delta}_{U_m}$
F_1 Linear failure rate	0.81	0.63	0.748	0.982	1.4
F_2 Makeham	0.29	0.385	0.248	0.218	0.51

Table (3.2) shows that our test statistic $\hat{\Delta}_{U_m}$ is more efficient than K^*, $\hat{\delta}_2$, $\hat{\Delta}_{U_T}$ and Λ_n for both LFR and Makeham families .

Note that: Since $\hat{\Delta}_{U_m}$ defines a class with parameter β of test statistic, we choose β that maximizes the PAE of that alternatives. If we take $\beta = 0.1$ then our test will have more efficiency than others.

Finally, the power of the test statistics $\hat{\Delta}_{U_m}$ is considered for 95% percentiles in Table (3.3) for three of the most commonly used alternatives [see Hollander and Proschan (1975)], they are

(i) Linear failure rate : $\bar{F}_\theta = e^{-x-\frac{\theta x^2}{2}}$, $x > 0, \theta > 0$

(ii) Makeham : $\bar{F}_\theta = e^{-x-\theta(x+e^{-x}-1)}$, $x \geq 0, \theta > 0$

(iii) Weibull : $\bar{F}_\theta = e^{-x^\theta}$, $x \geq 0, \theta > 0$

These distributions are reduced to exponential distribution for appropriate values of θ .

Table (3.3) Power Estimate of $\hat{\Delta}_{U_m}$

Distribution	θ	Sample Size n=10	n=20	n=30
F_1 Linear failure rate	3	0.8686	0.8159	0.7704
	4	0.8729	0.8264	0.7823
	5	0.9719	0.8340	0.7939
F_2 Makham	3	0.8643	0.8159	0.7673
	4	0.8708	0.8238	0.7764
	5	0.9332	0.8289	0.7881
F_3 Weibull	3	0.8051	0.7054	0.6141
	4	0.7967	0.6879	0.5910
	5	0.7910	0.6772	0.5793

Power estimates increase when the size of the sample n decrease. When θ increase, the power increase for LFR and makeham families.

4 Testing for Censored Data.

In this section, a test statistic is proposed to test $H_0 : \overline{F}$ is exponential distribution with mean μ versus, $H_1 : \overline{F}$ is $UBAC(2)$ and not exponential distribution, with randomly right-censored data (RR-C)in many experiments . Censored data is usually the only information available in a life-testing model or in a clinical study where patients may be lost (censored) before the completion of a study. This experimental situation can be modeled, as follows:

Suppose n items are put on test, and $X_1, X_2, ..., X_n$ are independent and identically distributed (i.i.d) r.vs according to a continuous life distribution F which denote their true life time. Let $Y_1, Y_2, ..., Y_n$ be (i.i.d) according to a continuous life distribution G and assume that X's and Y's are independent. In the randomly right-censored model, we observe the pairs (Z_i, δ_i), $i = 1, ..., n$, where $Z_i = \min(X_i, Y_i)$ and

$$\delta_i = \begin{cases} 1, & if\ Z_i = X_i \quad (i-th\ observation\ is\ uncensored) \\ 0, & if\ Z_i = Y_i \quad (i-th\ observation\ is\ censored) \end{cases} \tag{20}$$

Let $Z_{(0)} < Z_{(1)} < ... < Z_{(n)}$ denoted the ordered of Z's and δ_i is the δ corresponding to $Z_{(i)}$, respectively. Using the Kaplan and Meier estimator [30] in the case of censored data (Z_i, δ_i), $i = 1, 2, ..., n$, then the proposed test statistic given by (7) can be written using right censored data as

$$\hat{\delta}^c_{U_m} = \frac{-\beta}{\beta^3(\beta-1)}\eta - \frac{1}{\beta^3}(\zeta - 1) \tag{21}$$

where

$$\eta = \sum_{k=1}^{n}[\prod_{m=1}^{k-1} C_m^{\delta(m)}(Z_{(k)} - Z_{(k-1)})]$$

$$\zeta = \sum_{j=1}^{n} e^{-sZ_{(j)}}[\prod_{p=1}^{j-2} C_p^{\delta(p)} - \prod_{p=1}^{j-1} C_p^{\delta(p)}]$$

and

$$dF_n(Z_j) = \bar{F}_n(Z_{j-1}) - \bar{F}_n(Z_{j-2}),$$

$$c_k = \frac{n-k}{n-k+1}$$

To make the test invariant, let

$$\hat{\Delta}^c_{U_m} = \frac{\hat{\delta}^c_{U_m}}{\bar{Z}}, \bar{Z} = \frac{1}{n}\sum_{i=1}^{n} Z_i. \tag{22}$$

Table (4.1) shows the critical values percentiles of $\hat{\Delta}^c_{U_m}$ for sample size n=2(2)20(10)100.

Table (4.1) Critical Values of $\hat{\Delta}^c_{U_m}$

n	90%	95%	99%
2	0.349745	0.455112	0.652854
4	0.241535	0.31604	0.455865
6	0.193607	0.25444	0.368607
8	0.165046	0.217729	0.3166
10	0.145565	0.192687	0.28112
12	0.131198	0.174213	0.254941
14	0.120046	0.159871	0.234611
16	0.111078	0.14833	0.218243
18	0.103679	0.138801	0.204716
20	0.0974745	0.130794	0.193326
30	0.0754403	0.102646	0.153703
40	0.0626584	0.086219	0.130436
50	0.0531155	0.0741888	0.113737
60	0.0471831	0.0664202	0.102523
70	0.0422858	0.060096	0.0935206
80	0.0383029	0.0549628	0.0862286
90	0.0349919	0.050699	0.0801767
100	0.0321855	0.0470866	0.0750516

The Table shows that the values of the percentiles decrease when the sample size increases.

5 Applying the test

5.1 Applications for Complete Data.

*Example 1.*The following data represent 39 liver cancers patients taken from El Minia Cancer Center Ministry of Health Egypt A. F. Attia[31]. The ordered life times (in days) are:

107, 18, 74, 20, 23, 20, 23, 24, 52, 105, 60, 31, 75, 107, 71, 107, 14, 49, 10, 15, 30, 26, 14, 87, 51, 17, 116, 67, 20, 14, 40, 14, 30, 96, 20, 20, 61, 150, 14.

Using equation (16), the value of test statistics, based on the above data is $\hat{\Delta}_{U_m} = -4.25723$. The critical value at $\alpha = 0.05$ is 3.24702, then we accept H_0 at $\alpha = 0.05$. Thus, the data does not have UBAC(2) Property.

*Example 2.*Consider the data in Abouammoh et al. [32]. These data represent set of 40 patients suffering from blood cancer (Leukemia) from The Ministry of Health hospital in Saudi Arabia. The ordered life times (in day) are:

0.315, 0.496, 0.699, 1.145, 1.208, 1.263, 1.414, 2.025, 2.036, 2.162, 2.211, 2.370, 2.532, 2.693, 2.805, 2.910, 2.912, 3.192, 3.263, 3.348, 3.348, 3.427, 3.499, 3.534, 3.718, 3.751, 3.858, 3.986, 4.049, 4.244, 4.323, 4.323, 4.381, 4.392, 4.397, 4.647, 4.753, 4.929, 4.973, 5.074.

The value of test statistics based on the above data is $\hat{\Delta}_{U_m} = 1.32206$. The critical value at $\alpha = 0.05$ is 3.18942 This value leads to the acceptance of H_0 at the significance level $\alpha = 0.05$. Hence, the data does not have UBAC(2) Property.

*Example 3.*In an experiment at Florida State University to study the effect of methyl mercury poisoning on the life lengths of fish goldfish, they were subjected to various dosages of methyl mercury (Kochar [33]). At one dosage level the ordered times to death in week are:

$$6, 6.143, 7.286, 8.714, 9.429, 9.857, 10.143, 11.571, 11.714, 11.714.$$

The value of test statistics based on the above data is $\hat{\Delta}_{U_m} = 0.731805$. The critical value at $\alpha = 0.05$ is 6.32354. Then H_0 at the significance level $\alpha = 0.05$ is accepted. Therefore the data does not have UBAC(2) Property.

5.2 Application for Censored Data.

Consider the following data in Mahmoud and Abdul Alim [34] that represent 51 liver cancers patients taken from the Elminia Cancer Center Ministry of Health in Egypt. (39) represent whole life times (non-censored data) and the others represent censored data. The ordered life times (in days) are:

(i) Non-censored data

$$10, 14, 14, 14, 14, 14, 15, 17, 18, 20, 20, 20, 20, 20, 23, 23, 24, 26, 30, 30, 31,$$
$$40, 49, 51, 52, 60, 61, 67, 71, 74, 75, 87, 96, 105, 107, 107, 107, 116, 150.$$

(ii) Censored data

$$30, 30, 30, 30, 30, 60, 150, 150, 150, 150, 150, 185.$$

It is noticeable that the test statistic for the set of data $\delta^{\hat{c}}_{UK} = 1.79554 * 10^8$. The critical value is 0.0733917, so we reject H_0 which states that the set of data have UBAC(2) Property under significant level $\alpha = 0.5$.

6 Conclusion

We derived a new test statistic technique for testing the exponentially against the UBAC(2) class of life distributions, which are not exponential. It was shown that our proposed test is more efficient than other well known classes of life distributions and has high power.

Conflict of Interest

The authors declare that they have no conflict of interest.

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