Journal of Statistics Applications & Probability
An International Journal

http://dx.doi.org/10.18576/jsap/090202

Chance Constrained Approach for Treating Multicriterion Inventory Model with Random Variables in the Constraints

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Received: 31 Oct. 2019, Revised: 22 Jan. 2020, Accepted: 7 Feb. 2020
Published online: 1 Jul. 2020

Abstract: This research introduces a new formulation of multicriteria inventory model with random variables in the right hand side of the constraints. The objective functions are minimizing the total inventory cost (ordering cost, holding cost and purchasing cost), the total storage space, and the workload of several items in a single period. The constraints are order quantity that does not exceed annual demand rate, and storage space could not be greater than the warehouse capacity. In addition, budgetary restriction is stochastic parameter. The subset of efficient solutions of stochastic multicriteria inventory model (SMCIM) can be defined using both chance-constrained and kth-objective ε-constraint approaches. A numerical example will be given to clarify validity of proposed approach.

Keywords: Stochastic Inventory model, Chance-constrained, Efficient Solution, Workload, Storage Space, Budgetary Amount, $\varepsilon-$constraint approach.

1 Introduction

Inventory control has an important role in the economic organization by allowing the organization to meet the requirements of the purchasing, production, and sales, which vary according to the internal and external environment effects. Therefore, it is impossible to expect regularity and stability of sales, purchasing, supply and transport of the appropriate quantity on time, which provide customers with their needs to achieve their planned programs. This means that the organization faces difficulty in satisfying customer needs. Inventory management is required to make balance between the total costs and customers' requests. Consequently, the quantitative models that represent a systematic scientific method should be used to optimize inventory management decisions.

The inventory control problem (ICP) faced by organization that must decide how much to order in each time period to meet demand for its product. The ICP can be modelled as a multicriteria programming problem with more than one objective functions. In addition, the inventory problem can be either deterministic or stochastic with some random variables described by probability distributions

Several inventory systems, such as, (Q, r) models were used to handle inventory control as stochastic multiobjective programming problems. Babak et al. [1] provided a mathematical model for multi-item stochastic inventory model, which minimizes the average of an approximate total annual cost. Cheng eta al. [2] used successive approximation approach to solve the tri-objectives inventory models using a tradeoff concept. In Mousavi et al. [3], a multi-product multi-period inventory control was formulated to minimize the total inventory cost, including purchasing, holding, backorder, ordering, and lost sale costs, using a particle swarm optimization algorithm. Parviz [4], formulated the multi-product inventory system as a bi-criteria programming problem. The first objective is to minimize the total cost and the second objective is to maximize the service level, and solve it using different algorithms. Orand et al. [5] presented an inflationary inventory control model under nondeterministic conditions, and they used two approaches to resolve this model: the classic numerical approach and the hybrid approach which combine Simson approximation and particle swarm optimization. In Jiang et al. [6], an optimization model for inventory system was discussed and an algorithm for the optimal inventory costs based on supply-demand balance was suggested to solve this problem. In Mousavi [7], a multi-product multi-period inventory

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model was formulated as a bi-criteria inventory optimization model that solved using tuned Pareto-based algorithms. In Fayez [8], a mathematical formulation a bi-criteria optimization approach for the inventory replenishment problem was discussed and solved.

Agrell [9] presented an interactive approach for moltiobjective framework for inventory control. Roy and Maiti [10] introduced a mathematical model for multicriteria inventory control of deteriorating items with stock dependent demand under restriction of storage space and budgetary cost. Kundu [11] has discussed a multi-product continuous review for inventory model with stochastic parameters and budgetary constraint.

Despite the difficulty of including budget constraints in mathematical programming formulation, budget condition with randomness makes the model more sophisticated, and difficult to deal with.

This paper introduces a new inventory model formulation as a stochastic multicriteria optimization problem. The objective functions are total inventory cost, storage space, and workload, with a random budget constraint.

2 Notations and Assumptions

2.1 Notations

The present paper adopts the following notations:

D_i: Demand rate of item i,unit per year
Q_i: Order quantity of item i,(Desicion Variable)
h_i: Inventory holding cost of item i,unit per year
L_i: Lead time of item i,is constant
d_i: Demand during Lead time of item i,(Desicion Variable)
P_i: Purchasing cost per unit of item i
K_i: Setup cost per order of item i
S_i: Storage space per unit of item i
SS_i: Safety Stock per unit of item i
B: Maximum available budget amount (Stochastic parameter)
r_i: Reorder point of item i,(Desicion Variable)
W: Workload
A: Warehouse storage space
ε_1: Upper bound of total inventory cost
ε_2: Upper bound of workload
α: specific small probability that the budget being violated
TOC: Total ordering cost
THC: Total holding cost
TIC: Total inventory cost
ICP: Inventory control problem
SMCIM: Stochastic multicriteria inventory model

2.2 Assumptions

- Demand rates of the items are independent, and constant in a single period.
- Demand rate for each item is fixed.
- The lead time is fixed
- The set up cost is fixed
- The maximum available budget to a purchase item i is random
- Shortage is not allowable
- Storage space of the warehouse is limited

3 Model Description

This paper proposes a new stochastic inventory model with three objective functions, assuming the randomness occurring in the right hand side of the constraint, which is the maximum available budget.

The first objective is to minimize TIC which is formulated as follows:

$$TIC = TOC + THC + TPC \tag{1}$$

Where,

$$TOC = \sum_{i=1}^{n} \frac{D_i K_i}{Q_i} \tag{2}$$

$$THC = \sum_{i=1}^{n} h_i(\frac{Qi}{2} + r_i - d_i) \tag{3}$$

$$TPC = \sum_{i=1}^{n} P_i D_i \tag{4}$$

Thus, the first objective takes the following form:

$$\min TIC = \sum_{i=1}^{n} \left(\frac{D_i K_i}{Q_i} + h_i \left(\frac{Q_i}{2} + r_i - d_i \right) + P_i D_i \right) \tag{5}$$

The second objective function represents the minimization of total storage space,

$$\min V = \sum_{i=1}^{n} (Q_i + r_i - d_i) S_i \tag{6}$$

The third objective function is to minimize the workload, which can be formulated as:

$$\min W = \sum_{i=1}^{n} \left(\frac{D_i}{Q_i} \right) \tag{7}$$

3.1 The Constraints

The first constraint represents that the order quantity of the item i does not exceed the annual demand rate of the item i, i.e.

$$0 \leq Q_i \leq D_i, \qquad i = 1, 2, ..., n \tag{8}$$

The second constraint represents that the reorder point of the item i will exceed the demand during lead time plus safety stock of an item i, i.e.

$$0 \leq d_i \leq r_i - SS, \qquad i = 1, 2, ..., n \tag{9}$$

The third constraint represents that the storage space could not be greater than the warehouse capacity, i.e.

$$\sum_{i=1}^{n} S_i (Q_i + r_i - d_i) \leq A \tag{10}$$

The fourth constraint represents that the purchasing cost does not exceed the maximum available budget, which considered as random variable, i.e.

$$\sum_{i=1}^{n} P_i\left(Q_i + r_i - d_i\right) \leq B \tag{11}$$

3.2 Mathematical Formulation

The SMCIM can be formulated as follows:

$$\begin{cases} \min TIC = \min \sum_{i=1}^{n} \left(\frac{D_i K_i}{Q_i} + h_i \left(\frac{Q_i}{2} + r_i - d_i \right) + P_i D_i \right), \\ \min V = \min \sum_{i=1}^{n} \left(Q_i + r_i - d_i\right) S_i \\ \min W = \min \sum_{i=1}^{n} \left(\frac{D_i}{Q_i} \right) \end{cases} \tag{12}$$

Subject to

$$M = \left\{ \begin{array}{c} (Q,r,d) \in R^n : \ 0 \leq Q_i \leq D_i, \, 0 \leq d_i \leq r_i - SS, \, i = 1,2,...,n, \\ \sum_{i=1}^{n} S_i\left(Q_i + r_i - d_i\right) \leq A, \text{ and } \sum_{i=1}^{n} P_i\left(Q_i + r_i - d_i\right) \leq B \end{array} \right\}$$

Definition 1.*A point $(Q^*, r^*, d^*) \in M$ is said to be an efficient solution of problem (12), if there is no $(Q,r,d) \in M$ such that $f_j(Q,r,d) \leq f_j(Q^*, r^*, d^*), j = 1,2,3$ with $f_i\ (Q,r,d) < f_i(Q^*, r^*, d^*), i \in \{1,2,3\}$*

4 The Solution Procedure

The above-mentioned problem (12) can be solved through three phases:
Phase I: Transform the SMCIM problem into stochastic single objective programming problem using the $\varepsilon-$constraint approach, which gives:

$$\begin{cases} \min\limits_{(Q,r,d)\in M} \sum_{i=1}^{n} \left(Q_i + r_i - d_i\right) S_i \\ \text{Subject to} \\ \qquad \sum_{i=1}^{n} \left(\frac{D_i K_i}{Q_i} + h_i \left(\frac{Q_i}{2} + r_i - d_i \right) + P_i D_i \right) \leq \varepsilon_1 \\ \qquad \sum_{i=1}^{n} \left(\frac{D_i}{Q_i} \right) \leq \varepsilon_2 \end{cases} \tag{13}$$

Phase II: Using chance-constrained approach to convert problem (13) into a deterministic one. The chance-constrained technique allows the constraints to be violated by a small probability, which is developed by Charnes and Cooper [12].

Assume that the random variable B has a normal distribution with mean μ_B and variance σ_B^2, i.e. $B : N(\mu_B, \sigma_B^2)$, the significance level is α, and let $Z = (B - \mu_B)/\sigma_B$ be a random variable with mean equal zero and variance equal one, i.e. $Z\tilde{}N(0,1)$, the standard cumulative normal distribution $P(Z \leq t) = 1 - \alpha$, i.e. $F_Z(t) = 1 - \alpha, t \in R$.

Where, μ_B, σ_B are the mean and standard deviation of the random variable B, respectively.
Then,

$$P\left(\sum_{i=1}^{n} P_i\left(Q_i + r_i - d_i\right) \le B\right) \ge 1 - \alpha \tag{14}$$

$$\sum_{i=1}^{n} P_i\left(Q_i + r_i - d_i\right) \le \mu_B + z_\alpha \sigma_B \tag{15}$$

Hence, the equivalent deterministic model of SMCIM problem can be rewritten as follows:

$$\begin{cases} \min\limits_{(Q,r,d)\in M} \sum_{i=1}^{n} \left(Q_i + r_i - d_i\right) S_i \\ \text{Subject to} \\ \quad \sum_{i=1}^{n} \left(\frac{D_i K_i}{Q_i} + h_i\left(\frac{Q_i}{2} + r_i - d_i\right) + P_i D_i\right) \le \varepsilon_1 \\ \quad \sum_{i=1}^{n} \left(\frac{D_i}{Q_i}\right) \le \varepsilon_2 \end{cases} \tag{16}$$

Where,

$$M = \left\{ \begin{array}{l} (Q,r,d) \in R^n : \ 0 \le Q_i \le D_i, 0 \le d_i \le r_i - SS, i = 1,2,...,n, \\ \sum_{i=1}^{n} P_i\left(Q_i + r_i - d_i\right) \le \mu_B + z_\alpha \sigma_B, \text{ and } \sum_{i=1}^{n} S_i\left(Q_i + r_i - d_i\right) \le A \end{array} \right\}$$

Phase III: Choose ε_1, ε_2 such that the problem (16) is feasible, then solve the problem using Lingo software package to obtain an optimal solution to (16) which is an efficient solution to the problem (12) [13,14].

Theorem 1._(Q*,r*, d*) is an efficient solution of problem (12) if and only if (Q*, r*, d*) solves P_k (ε*) for every k=1,2,. . . ,n [15]._

Theorem 2._If (Q*, r*, d*) solves P_k (ε*) for some k and if the solution is unique, then (Q*, r*, d*) is an efficient solution of problem (12) [15]._

5 Numerical Example

Apply the above-mentioned approach to solve the inventory problem (16), consider the input data as below:
n=1, D=10000, K=100, P=3, S=2, h=0.25, V=30000, μ_B=20000 , σ_B=150, α=0.05
Table 1. Shows a subset of efficient solutions and corresponding non-dominated solutions for problem (12).

Table 1. The subset of efficient and non-dominated solutions

ε_1	ε_2	Efficient Solutions			Non-dominated Solution		
		Q	r	d	TIC	V	W
50000	5	1999.999	103.9071	3.907115	30775.00	4199.998	5.00
50000	10	1000.000	159.6951	59.69509	31150.00	2200.000	10.00
50000	12	833.3331	500.3488	100.3488	31329.17	1866.666	12.00
50000	18	555.5554	555.5554	455.554	31894.44	1311.111	18.00
50000	20	499.9997	345.4705	245.4705	32087.50	1199.999	20.00
50000	24	416.6666	114.8858	14.88575	32477.08	1033.333	24.00
50000	36	277.7776	208.4900	108.4900	33659.72	755.5552	36.00
50000	52	192.3076	153.1458	53.14585	35249.04	584.6151	52.00

It is noted that the efficient solution will be the same with varying the values of ε_1, which indicate that the problem is stable. The non-dominated solutions of problem (12) using the proposed procedure are also plotted in Figure 1 to reinforce the readability of the result analysis.

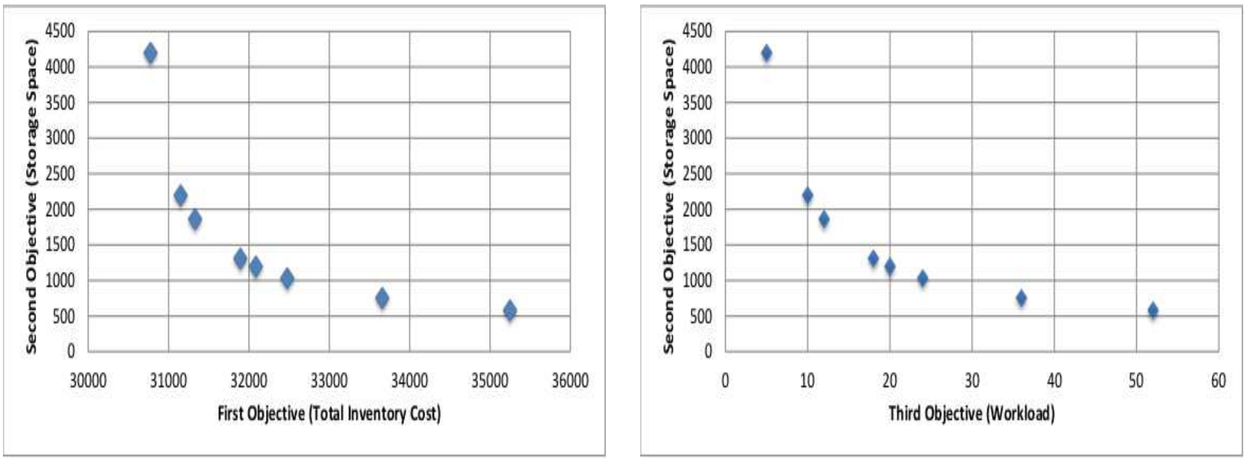

Fig. 1: Non-dominated solutions for the problem (12)

6 Conclusion

In this work, a new stochastic multicriteria inventory model is constructed with randomness parameter in the right hand side of the budget constraint. The mathematical model consists of three objective functions: the total inventory cost, storage space, and the workload, under restriction of limited storage space of the warehouse, and uncertain budget.

The solution procedure of the above-mentioned model was handled through three phases: The first was converting the SMCIPP into a single stochastic inventory model, the second was using chance-constrained approach to transform the stochastic model to a deterministic one, and the third phase was choosing ε_1, ε_2 with feasibility condition and solving the problem using Lingo package to generate both the set of all efficient solutions of SMCIPP, and the corresponding non-dominated solutions.

7 Suggestion

For further work, we are going to characterize and determined some of stability notions in mathematical programming for problem (12). These notions are the feasible parameters set, the set of solvability, and the stability set of the first kind. In addition, a decomposition of the parametric space will be considered with stochastic budget in the constraint.

Acknowledgment
"The author extend his appreciation to the Deanship of Scientific Research at Qassim University for funding this work through the Small Sustainable Grants, Research Project no. (cos-2018-1-14-S-5049) ".
The author is grateful to the Editor and the anonymous referee for a careful checking of the details and for valuable comments that improved the quality of this paper.

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