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Interactive Approach for Solving Three-Level Linear Programming Problem With Neutrosophic Parameters in the Objective Functions

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Abstract: The most widely actions and decisions of the real-world tasks are frequently appeared as hierarchical systems. To deal with these systems, the multi-level programming problem presents the most flourished technique. However, practical situations involve some impreciseness regarding some decisions and performances. Neutrosophic sets provides a vital role by considering three independent degrees specifically truth membership degree, indeterminacy-membership degree, and falsity membership degree of any aspect of uncertain decision. By preserving the advantages of it, the presented study focuses on solving neutrosophic three-level linear programming problems, taking into account the problem coefficients as trapezoidal neutrosophic numbers. The neutrosophic form of the problem is transformed into an equal crisp model in the first stage of the solution methodology to reduce the problem's complexity. In the second stage, an interactive approach is used to reach a solution compromise between conflicted decision levels. The proposed algorithm is validated by an illustrative example.

Keywords: Multi-level programming; linear programming; Neutrosophic set; Trapezoidal neutrosophic number

1 Introduction

Multi-Level Programming Problems (MLPPs) confronted with organizational structure that contains multiple levels of decision making over a feasible region of the solution. MLPPs have main importance in the areas of manufacturing factories, logistics organizations, government units and multiple other areas. Approaches for solving MLPPs set every Decision Maker (DM) an objective function with decision variables and a set of constraints for the whole DMs. Every DM searches for his/her own interest independently but they are influenced via the different decision maker's behaviors [4].

In most cases, the actual decision-making status isn't obviously defined and usually make the decision depend on incomplete or unknown information. In fact, the vagueness nature is actually fuzziness rather than randomness [8].

To deal with vague and imprecise information, Zadeh introduced Fuzzy Set Theory (FST). However, the FST doesn't efficiently represent ambiguous and inaccurate information, as the truthiness function only takes into account [3].Atanassov extended the FST in 1986 and the intuitionistic FST introduced. This theory presented the truth and falsity functions [8].

However, the intuitive FST also doesn't simulate human decision-making [3]. Intuitionistic FST can only deal with incomplete information not indeterminate one . In 1995, Smarandache introduced neutrosophic theory. The neutrosophic sets can deal with the incomplete and indeterminate information [7].

Neutrosophic sets described by three independent degrees specifically Truth membership degree (T), Indeterminacy-membership degree (I), and Falsity membership degree (F), where T,I,F are standard or non-standard subsets of $]0^-,1^+[$ ([1],[2]).

Most MLPPs studies concentrate on the bi-level problem as a class of MLPPs ([4,10,12,13]). In [4], Emam presented an algorithm for solving bi-level integer multi-objective fractional programming problem. It starts by detecting the convex

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hull of its original set of constraints at firstt, then it simplifies the equal problem by converting it into a separate multi-objective decision-making problem and finally using the ε-constraint method to solve the resulting problem.

Significant studies have been carried out on multi-level programming problems ([9,11]).In[9]Osman et al. proposed an interactive approach for solving multi-level multi-objective fractional programming(ML-MOFP) problems with fuzzy parameters. The proposed interactive approach makes an extended work of Shi and Xia (1997). In the ?rst stage, the numerical crisp model of the ML-MOFP problem has been developed at a con?dence level without changing the fuzzy gist of the problem. Then, the linear model for the ML-MOFP problem is formulated. In the second phase, the interactive approach simplifes the linear multi-level multi-objective model by converting it into separate multi-objective programming problems. Also, each separate multi-objective programming problem of the linear model is solved by theε-constraint method and the concept of satisfactoriness.

Many researches on neutrosophic linear programming problems have been implemented ([3,6]). In [3] Abdel-Basset et al. presented linear programming models where their parameters are trapezoidal neutrosophic numbers and proposed an algorithm for solving them.

Abdel-Baset et al. [2] Introduced Neutrosophic Intger Programming Problem (NIPP) where their parameters are trapezoidal neutrosophic numbers. The degrees of T, F and I membership functions of objectives are taken into account concurrently. The NIPP, using T, F, and I membership functions and single-valued triangular neutrosophic numbers has been converted into a crisp programming model.

Hussian et al. [7] proposed an approach for solving Neutrosophic Linear Fractional Programming (NLFP) problem, which involves triangular Neutrosophic numbers in the cost of the objective function, resources and technological coefficients. Here, the NLFP problem is simplified to an equal crisp Multi-Objective Linear Fractional Programming (MOLFP) problem. The suggested approach reduces the converted MOLFP problem to a single objective Linear Programming (LP) problem that can be easily solved by the most appropriated LP problem algorithm.

Abdelbaset et al. [1] introduced two models for solving Neutrosophic Goal Programming Problem (NGPP), to minimize the sum of the deviation (the I^{st} model) on the one hand and to convert NGPP to a crisp programming model on the other hand, using T , F, and I membership functions (the II^{nd} model). To prove the efficiency of the presented models, an industrial design problem has been raised. The results obtained in the I^{st} model and the II^{nd} model are compared with other techniques.

Hezam et al. [5] presented a Taylor series for solving Neutrosophic Multi-Objective Programming Problem (NMOPP). In the suggested approach, the T, F, I membership functions related with each objective of multi-objective programming problems are transformed into a single objective LP problem by means of a first-order Taylor polynomial series.

This paper is arranged as follows: Section 2 provides some preliminaries. In Section 3, a Three-Level Neutrosophic Linear Programming Problem (TLNLPP) with neutrosophic parameters in the objective functions is formulated. In Section 4, the neutrosophic nature of the problem is simplified into an equivalent crisp. In Section 5, an interactive model for the three-level linear programming problem is presented. An algorithm for solving TLNLPP with neutrosophic parameters in the objective functions is proposed in Section 6. Furthermore, the results and the solution algorithm are clarified with a numerical example in Section 7. Finally, Section 8 contains the conclusions and future works.

2 Preliminaries

This section presents a review of key neutrosophic set concepts and definitions.

Definition 1._(A single-valued neutrosophic set) [5]_

Let Y be a universe of discourse. A single-valued neutrosophic set N over Y is an object having the form $N = \{ \langle y , T_N(y), I_N (y),F_N(y) \rangle : y\in Y \}$, _where_ $T_N (y): Y \rightarrow [0,1]$, $I_N (y): Y \rightarrow [0,1]$ _and_ $F_N(y): Y \rightarrow [0,1]$ _with_ $0\leq T_N(y)+ I_N(y)+F_N(y)\leq 3$ _for all_ $y\in Y$. _The intervals_ $T_N (y)$, $I_N(y)$ _and_ $F_N(y)$ _denote T, I and the F membership functions of y to N , respectively._

Definition 2(3).

The trapezoidal neutrosophic number $\tilde{G}$ _is a neutrosophic set in R with the following T, I and F membership functions:_

$$T_{\tilde{G}}(X) = \begin{cases} \propto_{\tilde{G}} \left(\frac{x-g_1}{g_2-g_1}\right) & (g_1 \leq x \leq g_2) \\ \propto_{\tilde{G}} & (g_2 \leq x \leq g_3) \\ \propto_{\tilde{G}} & (g_3 \leq x \leq g_4) \\ 0 & otherwise \end{cases} \quad (1)$$

$$I_{\tilde{G}}(X)=\begin{cases}\frac{(g_2-x+\theta_{\tilde{G}}(X-g_1'))}{(g_2-g_1')} & (g_1'\le x\le g_2)\\ \theta_{\tilde{G}} & (g_2\le x\le g_3)\\ \frac{(x-g_3+\theta_{\tilde{G}}(g_4'-x))}{(g_4'-g_3)} & (g_3\le x\le g_4')\\ 1 & otherwise\end{cases} \tag{2}$$

$$F_{\tilde{G}}(X)=\begin{cases}\frac{(g_2-x+\beta_{\tilde{G}}(X-g_1''))}{(g_2-g_1')} & (g_1''\le x\le g_2)\\ \beta_{\tilde{G}} & (g_2\le x\le g_3)\\ \frac{(x-g_3+\beta_{\tilde{G}}(g_4''-x))}{(g_4''-g_3)} & (g_3\le x\le g_4'')\\ 1 & otherwise\end{cases} \tag{3}$$

Where $\propto_{\tilde{G}},\theta_{\tilde{G}}$ *and* $\beta_{\tilde{G}}$ *represent the maximum truthiness degree, minimum indeterminacy degree, minimum falsity degree, sequentially,* $\propto_{\tilde{G}}$, $\theta_{\tilde{G}}$ *and* $\beta_{\tilde{G}}$ $\in$ *[0, 1]. Also,* $g_1''\le g_1\le g_1'\le g_2\le g_3\le g_4'\le g_4\le g_4''$.

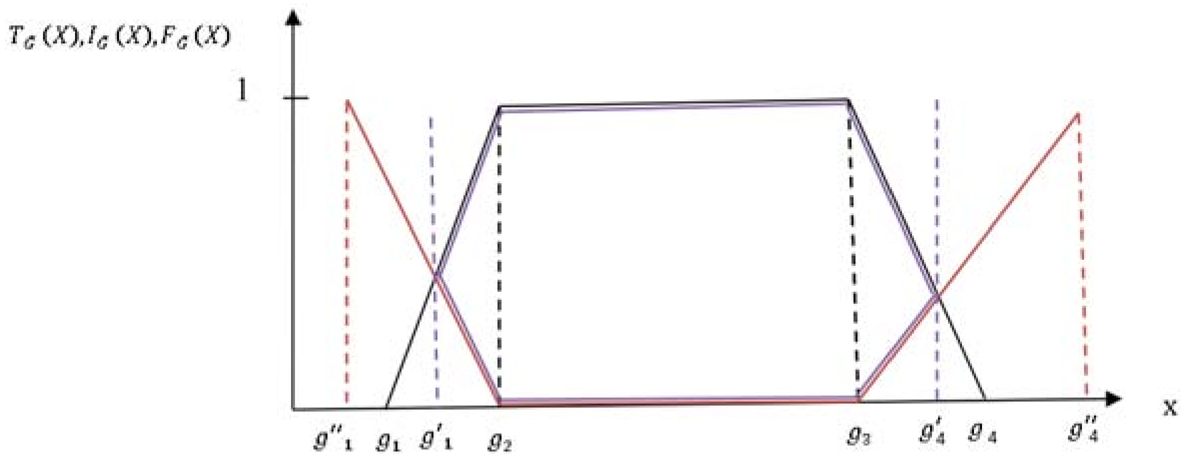

Fig. 1: Presents T, I, F membership functions of the trapezoidal neutrosophic number

We use Trapezoidal neutrosophic number because calculations with trapezoidal membership are easy and fewer complexes.

Definition 3(3). *a ranking function of neutrosophic numbers is a function* $N(R)\to R$, *where* $N(R)$ *is a set of neutrosophic numbers de?ned onset of real numbers, which transform each neutrosophic number into the real line.*

Let $\tilde{C}=\langle(c_1,c_2,c_3,c_4);\propto_{\tilde{C}},\theta_{\tilde{C}},\beta_{\tilde{C}}\rangle$ *and* $\tilde{D}=\langle(d_1,d_2,d_3,d_4);\propto_{\tilde{D}},\theta_{\tilde{D}},\beta_{\tilde{D}}\rangle$ *are two trapezoidal neutrosophic numbers, then*

1. *If* $R(\tilde{C})>R(\tilde{D})$ *then* $\tilde{C}>\tilde{D}$,
2. *If* $R(\tilde{C})<R(\tilde{D})$ *then* $\tilde{C}<\tilde{D}$,
3. *If* $R(\tilde{C})=R(\tilde{D})$ *then* $\tilde{C}=\tilde{D}$.

3 Problem Formulation and Solution Concept

The TLNLPP with neutrosophic parameters in the objective functions may be formulated as follows:

[First Level]

$$\max_{X_1} F_1 \approx \sum_{j=1}^{n}\tilde{c}_{1j}x_j\,, \tag{4}$$

Where x_2, x_3 solves

[Second Level]

$$\max_{X_2} F_2 \approx \sum_{j=1}^{n} \tilde{c}_{2j} x_j \ , \tag{5}$$

Where x_3 solves
[Third Level]

$$\max_{X_3} F_3 \approx \sum_{j=1}^{n} \tilde{c}_{3j} x_j \ , \tag{6}$$

Subject to

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i \ ; \tag{7}$$

$i = 1,2,\ldots,m$, $j = 1,2,\ldots,n$, $x_j \geq 0$, $\tilde{c}_{1j}$,$\tilde{c}_{2j}$, $\tilde{c}_{3j}$ are a trapezoidal neutrosophic number. Therefore$F_i : R^m \rightarrow R$,$(i = 1,2,3)$is the first level, the second level, and the third level objective functions, sequentially. In addition,the First Level Decision Maker (FLDM) has x_1 indicating choice of first level decision, the Second Level Decision Maker (SLDM) and the Third Level Decision Maker (TLDM) have x_2 and x_3 indicating the choice of second level decision and the choice of third level decision, sequentially.

Definition 4.*For any* $(x_1 \in G_1 = \{x_1 \mid (x_1,\ldots,x_m) \in G\})$ *given by FLDM and* $(x_2 \in G_2 = \{x_2 \mid (x_1,\ldots,x_m) \in G\})$ *given by SLDM, if the decision-making variable* $(x_3 \in G_3 = \{x_3 \mid (x_1,\ldots,x_m) \in G\})$ *is the Pareto optimal solution of the TLDM, then* $(x_1,\ldots,x_m)$ *is a feasible solution of TLNLPP.*

Definition 5.*If* $x^* \in R^m$ *is a feasible solution of the TLNLPP; no other feasible solution* $x \in G$ *exists, such that*$F_1(x^*) \leq F_1(x)$ *; so* x^**is the Pareto optimal solution of the TLNLPP.*

The basic idea in treating TLNLPP is to use the ranking function to transform each trapezoidal number into its equal crisp number.

If a TLNLPP is in maximization state, then the ranking function for this trapezoidal neutrosophic number can be stated as the following [3]:

$$R(\tilde{g}) = \left(\frac{g^l + g^u + 2(g^{m1} + g^{m2})}{2} \right) + (T_{\tilde{g}} - I_{\tilde{g}} - F_{\tilde{g}}) \tag{8}$$

If TLNLPP is in minimization state, then the ranking function for the trapezoidal neutrosophic number can be stated as the following [3]:

$$R(\tilde{g}) = \left(\frac{g^l + g^u - 3(g^{m1} + g^{m2})}{2} \right) + (T_{\tilde{g}} - I_{\tilde{g}} - F_{\tilde{g}}) \tag{9}$$

Where $(\tilde{g} = g^l, g^{m1}, g^{m2}, g^u \ ; \ T_{\tilde{g}}, I_{\tilde{g}}, F_{\tilde{g}})$ be a trapezoidal neutrosophic number, where g^l, g^{m1}, g^{m2}, g^u , are lower bound, ?rst, second median value and upper bound for trapezoidal neutrosophic number, respectively. Also $T_{\tilde{g}} - I_{\tilde{g}} - F_{\tilde{g}}$ are the truth, indeterminacy and falsity degree of the trapezoidal number.

4 Deterministic Three-Level Linear Programming Problem

Now, the TLNLPP can be simplified into the following deterministic Three-Level Linear Programming Problem (TLLPP) after the implementation of the maximization ranking function in Eq. (8):
[First Level]

$$\max_{X_1} F_1 = \sum_{j=1}^{n} c_{1j} x_j \ , \tag{10}$$

Where x_2, x_3 solves
[Second Level]

$$\max_{X_2} F_2 = \sum_{j=1}^{n} c_{2j} x_j \ , \tag{11}$$

Where x_3 solves
[Third Level]

$$\max_{X_3} F_3 = \sum_{j=1}^{n} c_{3j} x_j \ , \tag{12}$$

Subject to

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i \ ; \tag{13}$$

5 An Interactive Model for the Three level Linear Programming Problem

To solve the TLLPP through the adoption of the three-planner Stackelberg game [4], he FLDM provides the SLDM with the acceptable and reasonable solutions in rank order, And then the SLDM takes the FLDM's acceptable solutions to find the solutions and gradually obtain the FLDM's favored solution. The TLDM receives the solutions that are acceptable to the FLDM and the SLDM , and it progressively obtains the favored SLDM solution. Finally, according to the following satisfaction test functions the FLDM and the SLDM determine the favored solution of TLLPP:

First, the FLDM determines, by means of the following FLDM satisfaction testing function, whether the proposed solution x_1^F, x_2^S, x_3^S is a preferred and acceptable solution for him or can be modified:

$$\frac{\left\| F_1(x_1^F, x_2^F, x_3^F) - F_1\left(x_1^F, x_2^S, x_3^S\right) \right\|_2}{\left\| F_1\left(x_1^F, x_2^S, x_3^S\right) \right\|_2} < \delta^F \tag{14}$$

So x_1^F, x_2^S, x_3^S is a favored solution to the FLDM, where δ^F is a fairly small positive constant specified by the FLDM.

Second, the SLDM determines, by means of the following SLDM satisfaction testing function, whether the proposed solution x_1^F, x_2^S, x_3^T is a preferred and acceptable solution for him or can be modified:

$$\frac{\left\| F_2(x_1^F, x_2^S, x_3^S)) - F_2\left(x_1^F, x_2^S, x_3^T\right) \right\|_2}{\left\| F_2\left(x_1^F, x_2^S, x_3^T\right) \right\|_2} < \delta^S \tag{15}$$

So, x_1^F, x_2^S, x_3^T is a favored solution to the SLDM, where δ^F is a fairly small positive constant specified by the SLDM. As a result, x_1^F, x_2^S, x_3^T is a favored solution to the TLLPP.

6 An Algorithm for Solving TLNLPP

An algorithm for solving TLNLPP with neutrosophic parameters in the objective functions is outlined in the following sequence of steps:

Step 1:
DMs enter their TLNLPP with neutrosophic parameters in the objective functions.
Step 2:
If TLNLPP is in maximization state, then every neutrosophic parameters in the objective functions is converted into its equivalent crisp value by means of Eq. (8). Else using Eq. (9).
Step 3:
The TLNLPP is simplified into the equivalent deterministic TLLPP.
Step 4:
The FLDM finds the individual optimal solution of his problem x_1^F, x_2^F, x_3^F.
Step 5:
The FLDM evaluate δ^F value.
Step 6:
The SLDM defines his problem in point of view of the FLDM by setting x_1^F to the SLDM constraints.
Step 7:
Formulate the SLDM Problem.
Step 8:
The SLDM finds the optimal solution of his problemx_1^F, x_2^s, x_3^s.

Step 9:
If $\frac{\left\| F_1(x_1^F, x_2^F, x_3^F) - F_1\left(x_1^F, x_2^S, x_3^S\right) \right\|_2}{\left\| F_1\left(x_1^F, x_2^S, x_3^S\right) \right\|_2} < \delta^F$ then go to step 10 otherwise go to step 5.
Step 10:
The SLDM evaluates δ^F value
Step 11:
The TLDM defines his problem in point of view of the FLDM and SLDM by setting x_1^F and x_2^S to the TLDM constraints.
Step 12:
Formulate the TLDM problem.
Step 13:
The TLDM Find the optimal solution of his problemx_1^F, x_2^s, x_3^T.
Step 14:
If $\frac{\left\| F_2(x_1^F, x_2^S, x_3^S) - F_2\left(x_1^F, x_2^S, x_3^T\right) \right\|_2}{\left\| F_2\left(x_1^F, x_2^S, x_3^T\right) \right\|_2} < \delta^S$ then go to step 15 otherwise go to step 10.
Step 15:
So, (x_1^F, x_2^S, x_3^T) is the compromised solution to the TLNLPP. Then go to step 16.
Step 16:
Stop.

7 Numerical example

[First Level]

$$\max_{X_1} F_1 \approx (12,13,15,17)X_1 + (6,8,9,12)X_2 - (1,2,3,4)X_3$$

Where x_2, x_3 solves
[Second Level]

$$\max_{X_2} F_2 \approx (5,7,8,10)X_1 + (11,12,13,15)X_2 + (7,8,10,12)X_3$$

Where x_3 solves
[Third Level]

$$\max_{X_3} F_3 \approx (5,6,8,10)X_1 + (8,9,11,13)X_2 + (9,10,12,14)X_3$$

Subject to

$$3x_1 + 2x_2 + x_3 \leq 90$$
$$4x_1 + x_3 \leq 50$$
$$2x_1 + 5x_2 \leq 60$$
$$x_1, x_2, x_3 \geq 0$$

The following solves a TLNLPP with trapezoidal neutrosophic numbers in the objective functions. In the trapezoidal neutrosophic numbers, the sequence of the element is as follows: lower, first median, second median and finally the upper bound. confirmation degree $(T_{\tilde{g}}, I_{\tilde{g}}, F_{\tilde{g}})$ of each value of a trapezoidal neutrosophic number by DMS is (0.9, 0.1, 0.1) for the FLDM and (0.8, 0.6, 0.4) for the SLDM and (0.75, 0.5, 0.25) for the TLDM.

At first, every trapezoidal neutrosophic number is transformed into its equivalent crisp value by means of Eq. (8).

Secondly, the previous TLNLPP with trapezoidal neutrosophic numbers in the objective functions is transformed into equivalent crisp TLLPP as follows:
[First Level]

$$\max_{X_1} F_1 = 43.2\, X_1 + 26.7\, X_2 - 8.2\, X_3$$

Where x_2, x_3 solves
[Second Level]

$$\max_{X_2} F_2 = 22.3\, X_1 + 37.8\, X_2 + 27.3\, X_3$$

Where x_3 solves
[Third Level]

$$\max_{X_3} F_3 = 21.5\, X_1 + 30.5\, X_2 + 33.5\, X_3$$

Subject to

$$3x_1 + 2x_2 + x_3 \leq 90$$

$$4x_1 + x_3 \leq 50$$

$$2x_1 + 5x_2 + \leq 60$$

$$x_1, x_2, x_3 \geq 0$$

7.1 The FLDM Problem

[First Level]

$$\max_{X_1} F_1 = 43.2\, X_1 + 26.7\, X_2 - 8.2\, X_3$$

Subject to

$$3x_1 + 2x_2 + x_3 \leq 90$$

$$4x_1 + x_3 \leq 50$$

$$2x_1 + 5x_2 + \leq 60$$

$$x_1, x_2, x_3 \geq 0$$

The FLDM solution is (x_1^F, x_2^F, x_3^F) = (12, 7, 0) and $F_1 = 726.9$ and $\delta^F = 0.1$ is specified by FLDM.

7.2 The SLDM Problem

The SLDM reformulates this problem from the FLDM's perspective by setting x_1^F to the SLDM constraints.

[Second Level]

$$\max_{X_2} F_2 = 22.3\, X_1 + 37.8\, X_2 + 27.3\, X_3$$

Subject to

$$3x_1 + 2x_2 + x_3 \leq 90$$

$$4x_1 + x_3 \leq 50$$

$$2x_1 + 5x_2 + \leq 60$$

$$x_1 = 12$$

$$x_1, x_2, x_3 \geq 0$$

So, the SLDM solution is (x_1^F, x_2^S, x_3^S) = (12, 7, 2) and $F_2 = 594.36$ and $\delta^S = 0.1$ is specified by SLDM. The test function in Eq. (13) is used by the FLDM to determine whether or not the solution is acceptable.

$$\frac{\| F_1(12,7,0) - F_1(12,7,2)\, \|_2}{\| F_1(12,7,2) \|_2} = .05 \ < .1$$

So, (x_1^F, x_2^S, x_3^S) = (12, 7, 2) is acceptable solution to the FLDM.

7.3 The TLDM Problem

The TLDM reformulates this problem from the FLDM's and SLDM's perspective by setting x_1^F and x_2^S to the TLDM constraints.

[Third Level]

$$\max_{X_3} F_3 = 21.5\, X_1 + 30.5\, X_2 + 33.5\, X_3$$

Subject to

$$3x_1 + 2x_2 + x_3 \leq 90$$
$$4x_1 + x_3 \leq 50$$
$$2x_1 + 5x_2 + \leq 60$$
$$x_1 = 12$$
$$x_2 = 7$$
$$x_1, x_2, x_3 \geq 0$$

The TLDM solution is (x_1^F, x_2^S, x_3^T) = (12, 7, 2) and $F_3 = 538.5$.

The test function in Eq. (13) is used by the SLDM to determine whether or not the solution is acceptable.

$$\frac{\|F_2(12,7,2) - F_2(12,7,2)\,)\|_2}{\|F_2\,(12,7,2)\,\|_2} = 0.0 < 0.1$$

So, (x_1^F, x_2^S, x_3^T) = (12, 7, 2) is acceptable solution to the SLDM.

Finally, (x_1^F, x_2^S, x_3^T) = (12, 7, 2) is the compromised solution to the TLLPP.

Where $F_1 = 688.9$,$F_2 = 586.8$, $F_3 = 538.5$.

8 Conclusions

This paper presented a solution algorithm for solving TLNLPP with neutrosophic parameters in the objective functions. The neutrosophic nature of the problem is transformed into its equivalent crisp model in the first stage of the solution algorithm to reduce the complexity of the problem. In the second stage, an interactive algorithm is used to reach a compromised solution for the TLLPP. Finally, a numerical example is demonstrated to show the accuracy of the suggested solution algorithm.

Though, a number of points are open to future debate that should be examined and investigated in neutrosophic multi-level linear optimization such as:

1. Multi-level large scale linear decision-making problems with neutrosophic parameters in both objective functions and constraints.
2. Multi-level linear multi-objective decision-making problems with neutrosophic parameters in both objective functions and constraints and with integrality conditions.

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