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Estimation of Finite Population Mean using Two Auxiliary Variables in the Presence of Non-Response and Measurement Errors

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Abstract: The present paper focuses on the problem of estimating the mean of a finite population utilizing the information on two auxiliary variables in the simultaneous presence of non-response and measurement errors. In this paper, we have proposed some improved estimators of the population mean adopting the fundamental idea of two-phase (double) sampling scheme. The properties of the proposed estimators have been discussed in detail. An empirical study has also been carried out to strengthen the theoretical results.

Keywords: Population mean, Two-phase sampling, Auxiliary information, Measurement error, Non-response error

1 Introduction

Sample survey is frequently used technique or procedure to realize the population characteristics. The statistician's interest does not lie in providing information about individual elements of the target population, but in measuring some quantities for aggregate of elements, the whole population or domains, generally termed as parameters of interest. A sample estimate differs from the population value by varying magnitudes in different samples. This difference between the sample estimate and the population value is called the sampling error of the estimate. Moreover, the accuracy of a result is affected not only by sampling errors arising from chance variation in the selection of the sample but also by lack of precision in reporting observations or faulty canvassing of a designated random sample or faulty methods of estimation. These errors are usually grouped under the heading 'non-sampling errors'. In some situations the non-sampling errors may be large and deserve greater attention than the sampling error. The measurement error and non-response error are important non-sampling errors in a sample survey.

The problem of measurement error has been discussed by several authors including [1,2,3,4,5,6,7,8,9]. The problem of non-response was first discussed by [10]. Latter, [11] suggested a model to reduce the bias due to incomplete sample without successive call backs. Moreover, [12,13,14,15,16,17] and others have discussed the problem of non-response in the several situations.

It is usual practice that the authors generally consider only one kind of non-sampling errors at a time in estimating the population parameters. In most of the situations, it is realized that both measurement error and non-response error are of equal importance at the same time. In this connection, [18], [19] and [20] have suggested some estimators of the population mean in the presence of measurement error and non-response error. In the present paper, we have proposed some improved estimators of population mean utilizing the information on two auxiliary variables in the simultaneous presence of measurement error and non-response error. To propose the estimators, we have adopted the idea of double sampling scheme. An empirical study has also been done to support the theoretical results.

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2 Sampling Strategy

Let $P(U_1,U_2,...,U_N)$ be a finite population consisting of N units. Let X_0 and X_1 be the study and auxiliary variables with respective population means $\overline{X}_0$ and $\overline{X}_1$. Let X_2 be the additional auxiliary variable with population mean $\overline{X}_2$. The aim of the present research is to estimate the population mean $\overline{X}_0$ using the information on the auxiliary variables X_1 and X_2 under the condition that the population mean $\overline{X}_1$ is unknown and the population mean $\overline{X}_2$ is assumed to be known. It is further assumed that the measurement error occurs on all the variables X_0, X_1 and X_2 while non-response occurs on study variable only. Adopting the idea of two phase sampling scheme, we select a first phase sample of $n^{'}$ units from the entire population of N units by simple random sampling without replacement (SRSWOR) scheme and collect the information on auxiliary variables X_1 and X_2 to estimate the parameter $\overline{X}_1$. Now, we select a smaller second phase sample of size $n$from the first phase sample of $n^{'}$ units by SRSWOR scheme and collect the information on X_0. It is noted that out of n units, only n_1 units respond and n_2 units do not respond on X_0. Applying the Hansen and Hurwitz (1946) technique of sub-sampling of non-respondents, we select $h_2(h_2 = n_2/k; k > 1)$units from n_2 non-responding units by SRSWOR scheme and collect the information from all h_2 units.

Let $(x^*_{0i}, x^*_{1i}, x^*_{2i})$ and $(X^*_{0i}, X^*_{1i}, X^*_{2i})$ be the observed values and the true values on the i^{th} unit $(i = 1,2,...N)$ for the variables (X_0, X_1, X_2) respectively. Let the measurement errors associated with the variables X_0, X_1 and X_2 be

$$u^*_i = x^*_{0i} - X^*_{0i} \tag{1}$$

$$v^*_i = x^*_{1i} - X^*_{1i} \tag{2}$$

$$w^*_i = x^*_{2i} - X^*_{2i} \tag{3}$$

It is assumed that the measurement errors u^*_i, v^*_i and w^*_i are random in nature and uncorrelated with means zero and variances S^2_U, S^2_V and S^2_W respectively. Let $S^2_{U(2)}$, $S^2_{V(2)}$ and $S^2_{W(2)}$ be the variances of the measurement errors u^*_i, v^*_i and w^*_i respectively for the non-responding units of the population. Let S^2_0 , S^2_1 and S^2_2 be the variances of the variables X_0, X_1 and X_2 respectively. Let $S^2_{0(2)}$, $S^2_{1(2)}$ and $S^2_{2(2)}$ be the respective variances of the variables X_0, X_1 and X_1 for the non-responding units of the population.

Thus, the Hansen and Hurwitz (1946) estimator of the population mean $\overline{X}_0$ is given as (without using auxiliary information)

$$T_{0HH} = \frac{n_1\overline{x}_{01} + n_2\overline{x}_{0h2}}{n} \tag{4}$$

where $\overline{x}_{01}$ and $\overline{x}_{0h2}$ are the means based on n_1 and h_2 units respectively for the study variable X_0.

The variance of the estimator T_{0HH} in the simultaneous presence of non-response and measurement errors is given as

$$V^*(T_{0HH}) = \left[\frac{1}{n} - \frac{1}{N}\right]\left(S^2_0 + S^2_U\right) + \frac{k-1}{n}W_2\left(S^2_{0(2)} + S^2_{U(2)}\right) \tag{5}$$

where W_2 is the non-response rate in the population.

The ratio and product-type estimators of the population mean $\overline{X}_0$ are respectively given by (using the information on auxiliary variable X_1)

$$T_{0R} = T_{0HH}\frac{\overline{X}_1}{T_{1HH}} \tag{6}$$

$$T_{0P} = T_{0HH}\frac{T_{1HH}}{\overline{X}_1} \tag{7}$$

where $T_{1HH} = \frac{n_1\overline{x}_{11} + n_2\overline{x}_{1h2}}{n}$, $\overline{x}_{11}$ and $\overline{x}_{1h2}$ are the means based on n_1 and h_2 units respectively for the auxiliary variable X_1.

The mean square errors (MSE) of the estimators T_{0R} and T_{0P} up to the first order of approximation in the simultaneous presence of non-response and measurement errors are respectively given as

$$MSE^*(T_{0R}) = \left[\frac{1}{n} - \frac{1}{N}\right]\left(S^2_0 + R^2S^2_1 - 2R\rho_{01}S_0S_1 + S^2_U + R^2S^2_V\right)$$

$$+\frac{k-1}{n}W_2\left(S^2_{0(2)} + R^2S^2_{1(2)} - 2R\rho_{01(2)}S_{0(2)}S_{1(2)} + S^2_{U(2)} + R^2S^2_{V(2)}\right) \tag{8}$$

$$MSE^{*}(T_{0P}) = \left[\frac{1}{n} - \frac{1}{N}\right]\left(S_0^2 + R^2 S_1^2 + 2R\rho_{01} S_0 S_1 + S_U^2 + R^2 S_V^2\right)$$

$$+\frac{k-1}{n} W_2 \left(S_{0(2)}^2 + R^2 S_{1(2)}^2 + 2R\rho_{01(2)} S_{0(2)} S_{1(2)} + S_{U(2)}^2 + R^2 S_{V(2)}^2\right) \tag{9}$$

where ρ_{01} and $\rho_{01(2)}$ are the correlation coefficients between X_0 and X_1 for the entire population and non-response group respectively. $R = \overline{X}_0 / \overline{X}_1$.

The ratio and exponential-type estimators of the population mean $\overline{X}_0$ under two-phase sampling scheme are respectively given as (using the information on auxiliary variable X_1)

$$T_{0R}^{'} = T_{0HH} \frac{T_1^{'}}{T_{1HH}} \tag{10}$$

$$T_{0Exp}^{'} = T_{0HH} \exp\left(\frac{T_1^{'} - T_{1HH}}{T_1^{'} + T_{1HH}}\right) \tag{11}$$

The expressions for the MSE of the estimators $T_{0R}^{'}$ and $T_{0Exp}^{'}$ up to the first order of approximation in the simultaneous presence of non-response and measurement errors are respectively given as

$$MSE\left(T_1^{**}\right) = \left(\frac{1}{n} - \frac{1}{N}\right)\left(S_0^2 + S_U^2\right) + \left(\frac{1}{n} - \frac{1}{n'}\right)\left[R^2\left(S_1^2 + S_V^2\right) - 2R\rho_{01} S_0 S_1\right] +$$

$$\frac{(k-1)}{n} W_2 \left[S_{0(2)}^2 + S_{U(2)}^2 + R^2\left(S_{1(2)}^2 + S_{V(2)}^2\right) - 2R\rho_{01(2)} S_{0(2)} S_{1(2)}\right] \tag{12}$$

$$MSE\left(T_{0Exp}^{'}\right) = \left(\frac{1}{n} - \frac{1}{N}\right)\left(S_0^2 + S_U^2\right) + \left(\frac{1}{n} - \frac{1}{n'}\right)\left[\frac{R^2}{4}\left(S_1^2 + S_V^2\right) - R\rho_{01} S_0 S_1\right] +$$

$$\frac{(k-1)}{n} W_2 \left[S_{0(2)}^2 + S_{U(2)}^2 + \frac{R^2}{4}\left(S_{1(2)}^2 + S_{V(2)}^2\right) - R\rho_{01(2)} S_{0(2)} S_{1(2)}\right] \tag{13}$$

3 Proposed Estimators

We now propose some improved estimators of finite population mean $\overline{X}_0$ utilizing the information on two auxiliary variables X_1 and X_2 in the simultaneous presence of non-response and measurement errors as

$$T_1^{**} = T_{0HH} \frac{T_{1R}^{'}}{T_{1HH}} \tag{14}$$

$$T_2^{**} = T_{0HH} \frac{T_{1Exp}^{'}}{T_{1HH}} \tag{15}$$

where $T_{1R}^{'} = T_1^{'} \frac{\overline{X}_2}{T_2^{'}}$, $T_{1Exp}^{'} = T_1^{'} \exp\left(\frac{\overline{X}_2 - T_2^{'}}{\overline{X}_2 + T_2^{'}}\right)$, $T_1^{'} = \frac{1}{n'} \sum_i^{n'} x_{1i}^{*}$ and $T_2^{'} = \frac{1}{n'} \sum_i^{n'} x_{2i}^{*}$.

To discuss the properties of the proposed estimators, we further assume

$$w_0^{*} = \frac{1}{\sqrt{n}} \sum_i^{n} \left(x_{0i}^{*} - \overline{X}_0\right) \tag{16}$$

$$w_1^{*} = \frac{1}{\sqrt{n}} \sum_i^{n} \left(x_{1i}^{*} - \overline{X}_1\right) \tag{17}$$

$$w_1^{*'} = \frac{1}{\sqrt{n'}} \sum_i^{n'} \left(x_{1i}^{*} - \overline{X}_1\right) \tag{18}$$

$$w_2^{*'} = \frac{1}{\sqrt{n'}} \sum_i^{n'} \left(x_{2i}^* - \overline{X}_2\right) \tag{19}$$

$$w_U^* = \frac{1}{\sqrt{n}} \sum_i^{n} u_i^* \tag{20}$$

$$w_V^* = \frac{1}{\sqrt{n}} \sum_i^{n} v_i^* \tag{21}$$

$$w_V^{*'} = \frac{1}{\sqrt{n'}} \sum_i^{n'} v_i^* \tag{22}$$

$$w_W^{*'} = \frac{1}{\sqrt{n'}} \sum_i^{n'} w_i^* \tag{23}$$

From equations (16) and (20), we have

$$w_0^* + w_U^* = \frac{1}{\sqrt{n}} \sum_i^{n} \left(x_{0i}^* - \overline{X}_0\right) + \frac{1}{\sqrt{n}} \sum_i^{n} u_i^*$$

From the above, we get

$$T_{0HH} = \overline{X}_0 + \frac{1}{\sqrt{n}} \left(w_0^* + w_U^*\right)$$

From equations (17) and (21), we have

$$T_{1HH} = \overline{X}_1 + \frac{1}{\sqrt{n}} \left(w_1^* + w_V^*\right)$$

Similarly, from equations (18), (22), (19) and (23), we get

$$T_1' = \overline{X}_1 + \frac{1}{\sqrt{n'}} \left(w_1^{*''} + w_V^{*'}\right)$$

$$T_2' = \overline{X}_2 + \frac{1}{\sqrt{n'}} \left(w_2^{*''} + w_W^{*'}\right)$$

Moreover, we have

$$E\left(W_0^2\right) = \left[\frac{1}{n} - \frac{1}{N}\right] \left(S_0^2 + S_U^2\right) + \frac{k-1}{n} W_2 \left(S_{0(2)}^2 + S_{U(2)}^2\right) \tag{24}$$

$$E\left(W_1^2\right) = \left[\frac{1}{n} - \frac{1}{N}\right] \left(S_1^2 + S_V^2\right) + \frac{k-1}{n} W_2 \left(S_{1(2)}^2 + S_{V(2)}^2\right) \tag{25}$$

$$E\left(W_1'^2\right) = \left[\frac{1}{n'} - \frac{1}{N}\right] \left(S_1^2 + S_V^2\right) \tag{26}$$

$$E\left(W_2'^2\right) = \left[\frac{1}{n'} - \frac{1}{N}\right] \left(S_2^2 + S_W^2\right) \tag{27}$$

$$E\left(W_0 W_1\right) = \left[\frac{1}{n} - \frac{1}{N}\right] \rho_{01} S_0 S_1 + \frac{k-1}{n} W_2 \rho_{01(2)} S_{0(2)} S_{1(2)} \tag{28}$$

$$E\left(W_0 W_1'\right) = \left[\frac{1}{n'} - \frac{1}{N}\right] \rho_{01} S_0 S_1 \tag{29}$$

$$E\left(W_0 W_2'\right) = \left[\frac{1}{n'} - \frac{1}{N}\right] \rho_{02} S_0 S_2 \tag{30}$$

$$E\left(W_1W_1^{'}\right)=\left[\frac{1}{n^{'}}-\frac{1}{N}\right]\left(S_1^2+S_V^2\right) \tag{31}$$

$$E\left(W_1W_2^{'}\right)=\left[\frac{1}{n^{'}}-\frac{1}{N}\right]\rho_{12}S_1S_2 \tag{32}$$

$$E\left(W_1^{'}W_2^{'}\right)=\left[\frac{1}{n^{'}}-\frac{1}{N}\right]\rho_{12}S_1S_2 \tag{33}$$

where $W_0=\frac{1}{\sqrt{n}}\left(w_0^*+w_U^*\right)$, $W_1=\frac{1}{\sqrt{n}}\left(w_1^*+w_V^*\right)$, $W_1^{'}=\frac{1}{\sqrt{n^{'}}}\left(w_1^{*''}+w_V^{*'}\right)$ and $W_2^{'}=\frac{1}{\sqrt{n^{'}}}\left(w_2^{*''}+w_W^{*'}\right)$. ρ_{02} and ρ_{12} are the correlation coefficients between X_0 and X_2 and X_1 and X_2 respectively for the entire population.

Now, we can express equation (14) in terms of W_0, W_1, $W_1^{'}$ and $W_2^{'}$ as

$$T_1^{**}=\frac{\left(\overline{X}_0+W_0\right)}{\left(\overline{X}_1+W_1\right)}\frac{\left(\overline{X}_1+W_1^{'}\right)\overline{X}_2}{\left(\overline{X}_2+W_2^{'}\right)} \tag{34}$$

From equation (34), we get

$$T_1^{**}=\left(\overline{X}_0+W_0\right)\left(1+\frac{W_1^{'}}{\overline{X}_1}\right)\left(1-\frac{W_1}{\overline{X}_1}+\frac{W_1^2}{\overline{X}_1^2}-...\right)\left(1-\frac{W_2^{'}}{\overline{X}_2}+\frac{W_2^{'2}}{\overline{X}_2^2}-...\right)$$

Simplifying the expression and neglecting the terms involving powers of W_0, W_1, $W_1^{'}$ and $W_2^{'}$ greater than two, we get

$$T_1^{**}-\overline{X}_0=W_0+\frac{R}{\overline{X}_2}\left(W_1W_2^{'}-W_1^{'}W_2^{'}\right)-\frac{R}{\overline{X}_1}\left(W_1^{'}W_1-W_1^2\right)+R\left(W_1^{'}-W_1\right)-\frac{1}{\overline{X}_2}W_0W_2^{'}-R^{'}W_2^{'}+$$

$$\frac{R^{'}}{\overline{X}_2}W_2^{'2}+\frac{1}{\overline{X}_1}\left(W_0W_1^{'}-W_0W_1\right) \tag{35}$$

where $R^{'}=\frac{\overline{X}_0}{\overline{X}_2}$.

Taking expectation on both the sides of equation (35), we get the bias of T_1^{**} up to the first order of approximation as

$$Bias\left(T_1^{**}\right)=\frac{1}{\overline{X}_1}\left(\frac{1}{n}-\frac{1}{n^{'}}\right)\left[R\left(S_1^2+S_V^2\right)-\rho_{01}S_0S_1\right]+\frac{1}{\overline{X}_2}\left(\frac{1}{n^{'}}-\frac{1}{N}\right)\left[R^{'}\left(S_2^2+S_W^2\right)-\rho_{02}S_0S_2\right]+$$

$$\frac{1}{\overline{X}_1}\frac{(k-1)}{n}W_2\left[R\left(S_{1(2)}^2+S_{V(2)}^2\right)-\rho_{01(2)}S_{0(2)}S_{1(2)}\right] \tag{36}$$

Squaring both the sides of equation (35) and then taking expectation on neglecting the terms involing powers of W_0, W_1, $W_1^{'}$ and $W_2^{'}$ higher than two, we get the MSE of T_1^{**} up to the first order of approximation as

$$MSE\left(T_1^{**}\right)=\left(\frac{1}{n}-\frac{1}{N}\right)\left(S_0^2+S_U^2\right)+\left(\frac{1}{n}-\frac{1}{n^{'}}\right)\left[R^2\left(S_1^2+S_V^2\right)-2R\rho_{01}S_0S_1\right]+$$

$$\left(\frac{1}{n^{'}}-\frac{1}{N}\right)\left[R^{'2}\left(S_2^2+S_W^2\right)-2R^{'}\rho_{02}S_0S_2\right]+$$

$$\frac{(k-1)}{n}W_2\left[S_{0(2)}^2+S_{U(2)}^2+R^2\left(S_{1(2)}^2+S_{V(2)}^2\right)-2R\rho_{01(2)}S_{0(2)}S_{1(2)}\right] \tag{37}$$

Now, we express equation (15) in terms of W_0, W_1, $W_1^{'}$ and $W_2^{'}$ as

$$T_2^{**}=\frac{\left(\overline{X}_0+W_0\right)}{\left(\overline{X}_1+W_1\right)}\left(\overline{X}_1+W_1^{'}\right)\exp\left[\frac{\overline{X}_2-\left(\overline{X}_2+W_2^{'}\right)}{\overline{X}_2+\left(\overline{X}_2+W_2^{'}\right)}\right] \tag{38}$$

From the above equation (38), we get

$$T_2^{**} = (\overline{X}_0 + W_0)\left(1+\frac{W_1^{'}}{\overline{X}_1}\right)\left(1-\frac{W_1}{\overline{X}_1}+\frac{W_1^2}{\overline{X}_1^2}-...\right)\exp\left[-\frac{W_2^{'}}{2\overline{X}_2}\left(1+\frac{W_2^{'}}{2\overline{X}_2}\right)^{-1}\right]$$

Expanding the above expression on neglecting the terms having powers of W_0, W_1, $W_1^{'}$ and $W_2^{'}$ greater than two, we get

$$T_2^{**} - \overline{X}_0 = W_0 + R\left(W_1^{'} - W_1\right) + \frac{R}{\overline{X}_1}\left(W_1^2 - W_1^{'}W_1\right) + \frac{R}{2\overline{X}_2}\left(W_1W_2^{'} - W_1^{'}W_2^{'}\right) + \frac{1}{\overline{X}_1}\left(W_0W_1^{'} - W_0W_1\right) - \frac{R^{'}}{2}W_2^{'} - \frac{1}{2\overline{X}_2}W_0W_2^{'} + \frac{3R^{'}}{8\overline{X}_2}W_2^{'2} \tag{39}$$

Taking expectation on both sides of equation (39), we get the bias of T_2^{**} up to the first order of approximation as

$$Bias\,(T_2^{**}) = \frac{1}{\overline{X}_1}\left(\frac{1}{n}-\frac{1}{n^{'}}\right)\left[R\left(S_1^2+S_V^2\right)-\rho_{01}S_0S_1\right] + \frac{1}{\overline{X}_2}\left(\frac{1}{n^{'}}-\frac{1}{N}\right)\left[\frac{3}{8}R^{'}\left(S_2^2+S_W^2\right)-\frac{1}{2}\rho_{02}S_0S_2\right] + \frac{1}{\overline{X}_1}\frac{(k-1)}{n}W_2\left[R\left(S_{1(2)}^2+S_{V(2)}^2\right)-\rho_{01(2)}S_{0(2)}S_{1(2)}\right] \tag{40}$$

Squaring both sides of equation (39) and then taking expectation on ignoring the terms having powers of W_0, W_1, $W_1^{'}$ and $W_2^{'}$ greater than two, we get the MSE of T_2^{**} up to the first order of approximation as

$$MSE\,(T_2^{**}) = \left(\frac{1}{n}-\frac{1}{N}\right)\left(S_0^2+S_U^2\right) + \left(\frac{1}{n}-\frac{1}{n^{'}}\right)\left[R^2\left(S_1^2+S_V^2\right)-2R\rho_{01}S_0S_1\right] + \frac{1}{\overline{X}_2}\left(\frac{1}{n^{'}}-\frac{1}{N}\right)\left[\frac{1}{4\overline{X}_2}\left(S_2^2+S_W^2\right)-2R^{'}\rho_{02}S_0S_2\right] + \frac{(k-1)}{n}W_2\left[S_{0(2)}^2+S_{U(2)}^2+R^2\left(S_{1(2)}^2+S_{V(2)}^2\right)-2R\rho_{01(2)}S_{0(2)}S_{1(2)}\right] \tag{41}$$

4 Empirical Study

To verify the performance of the proposed estimators, it is imperative to carry out an empirical study. Due to non-availability of real data set, we have generated some numerical data from the normal distribution using *R* software. First, we have generated values of the auxiliary variable $\overline{X}_1$ from the normal population $N(5,10)$. We have taken the values of study variable $\overline{X}_0$ and auxiliary variable $\overline{X}_2$ by considering the transformations $X_0 = X_1 + N(0,1)$ and $X_2 = X_1 + N(0,3)$. We have N = 5000, $n^{'}$ = 500, n = 200 ,$\overline{X}_0$ = 5.208249, $\overline{X}_1$ = 5.227365, $\overline{X}_2$ = 5.179607, R= 0.9963431, $R^{'}$ = 1.00553, ρ_{01} = 0.9950057, $\rho_{01(2)}$ = 0.7462543, $\rho_{01(2)}$ = 0.9536 S_U = 3.053276, S_V = 3.011117, S_W = 3.051285, $S_{U(2)}$ = 2.644215, $S_{V(2)}$ = 2.607704, $S_{W(2)}$ = 2.642491,S_0 = 10.16078, S_1 = 10.09635, S_2 = 10.48225, $S_{0(2)}$ = 8.799495, $S_{1(2)}$ = 8.743697 and $S_{2(2)}$ = 9.077898.

Table 1 depicts the MSE/Variance of T_{0HH}, $T_{0R}^{'}$, $T_{0Exp}^{'}$, T_1^{**} and $T_2^{**}$for the different choices of non-response rate W_2 and inverse sampling rate k.

Table 1: MSE/Variance of T_{0HH}, T'_{0R}, T'_{0Exp}, T_1^{**} and T_2^{**}

W_2	k	MSE/Variance				
		T_{0HH}	T'_{0R}	T'_{0Exp}	T_1^{**}	T_2^{**}
0.1	2.0	0.582519	0.287005	0.341787	0.136259	0.253708
	2.5	0.603624	0.300169	0.353756	0.149422	0.266871
	3.0	0.62473	0.313332	0.365725	0.162586	0.280035
	3.5	0.645836	0.326495	0.377695	0.175749	0.293198
0.2	2.0	0.62473	0.313332	0.365725	0.162586	0.280035
	2.5	0.666942	0.339659	0.389664	0.188913	0.306362
	3.0	0.709153	0.365986	0.413603	0.215239	0.332689
	3.5	0.751365	0.392313	0.437541	0.241566	0.359015
0.3	2.0	0.666942	0.339659	0.389664	0.188913	0.306362
	2.5	0.730259	0.379149	0.425572	0.228403	0.345852
	3.0	0.793576	0.418639	0.46148	0.267893	0.385342
	3.5	0.856893	0.45813	0.497388	0.307383	0.424833
0.4	2.0	0.709153	0.3659856	0.413602	0.215239	0.3326885
	2.5	0.793576	0.4186394	0.461479	0.267893	0.3853422
	3.0	0.877999	0.4712931	0.509357	0.320546	0.4379959
	3.5	0.962422	0.5239468	0.557234	0.373200	0.4906496

5 Concluding Remarks

We have proposed some improved ratio and exponential-type estimators of finite population mean using the information on two auxiliary variables in the simultaneous presence of non-response and measurement errors. The estimators have been proposed under the situation in which population mean of the auxiliary variable X_1 is unknown but population mean of the auxiliary variable X_2 is assumed to be known. Hence, we have adopted the idea of two-phase sampling scheme to propose the estimators. The expressions for the biases and mean square errors of the proposed estimators have been derived up to the first order of approximation. An empirical study has been carried out to strengthen the theoretical results. The MSE/Variance comparison of the proposed estimators T_1^{**} and T_2^{**} with the existing estimators T_{0HH}, T'_{0R} and T'_{0Exp} has been presented in Table 1. Table 1 shows that the proposed estimators perform better than the other existing estimators. It is seen that the MSE/ Variance of the estimators increases with increase in W_2 and k. Such kind of result is instinctively anticipated.

Conflict of interest There is no conflict of interest in this work.

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