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Steady State Performance of A Cold Stand by System With Conditional Server Replacement

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Abstract: This paper probabilistically investigates the steady state performance of a two unit cold standby system. The system consists of two identical units and a server who is meant for bringing the system back in to operation as early as possible after failure. The server failure during working is possible. The server is treatable/ diagnosable but if its treatment time exceeds a specified limit it gets replaced. The failure, repair, replacement and treatment times are assumed to be statistically independent. The semi-Markov process and regenerative point technique are used to derive expressions for steady state performance measures. The simulation results are also given for mean time to system failure, availability and profit.

Keywords: Stochastic Model, Standby System, Server Replacement, Steady State Performance

1 Introduction

The cold standby system models, consisting of two identical units and a server, are widely discussed by researchers in the literature. Gopalan and Naidu [1], analyzed a two-unit repairable system considering two type of failures along with the provision of inspection. Mahmoud et al [2] debated on the optimum preventive maintenance for a standby system permitting the patience-time for repair. Agnihotri and Statsangi [3] introduced the concept of inter-change time in describing a two-unit redundant system. Kumar et al [4] carried out probabilistic analysis of a two-unit cold standby system with the provision of directives as needed. Chander and Bhardwaj [5] introduced a 2-out-of-3 cold stand by priority system and derived the reliability and economic indices. Malik [6] highlighted the issue of maximum repair and operation times for a computer system and obtained reliability measures. Bhardwaj et al [7,8] emphasized on the failure of standby unit in a redundant system and evaluated the system performance by taking general probability distributions for repair and replacement times.

A well-known fact in relation to the repairable cold standby systems is their characterization through the server together with the standbys. Indeed, the server plays a significant role in bringing the system back into operation, at its failure. The servers high availability can ensure better system functioning. But in practice, the failure of server while doing job is a common event and such situations undesirably affect the system performance. Some studies have debated on this issue of server failure. Such as Cao and Wu [9] debated on the reliability of a two-unit cold standby system with a replaceable repair facility; Bhardwaj and Singh [10,11] proposed an inspection-repair-replacement model of a stochastic standby system with server failure.

If the server fails during operation it must be given proper treatment or diagnosis timely so as to ensure system rectification as well as its functioning. But the maximum treatment time limit i.e. the allowable time for server to return, should be indicated so as to avoid the situation where the cost involve in treatment of server due to elapsed longer period of time exceeds its replacement cost. Otherwise the system will run in huge loss. Though the concept of maximum operation and repair time has been discussed by some researchers [12] but the issue of maximum treatment time for server has not attracted much attention in previous studies. Keeping the practical significance in view, in this paper a two unit cold standby system model is developed. The system consists of two identical units and a server. The server gets

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treatment at its failure during operation and it takes time for the server to recover. If the treatment time of the server crosses a threshold limit, termed as maximum treatment time, it is replaced by an equally efficient one to ensure proper system functioning without further delay. This replacement is termed as conditional because it is initiated only when the treatment time reaches its maximum limit.The semi-Markov process and regenerative point technique are used to obtain expressions for several performance measures, in steady state with general life time distributions, such as mean time to system failure, availability, busy period, profit etc. The failure, repair, treatment and replacement times are assumed statistically independent. The simulation results are also given for exponential distribution.

2 Notations

$A_i(t)$: P$\{$the system is in up- state at instant t $|$ the system entered regenerative state S_i at $t = 0.\}$
$B_i(t)$: P$\{$the Server is busy in repair at instant t $|$ the system entered regenerative state S_i at $t = 0.\}$
$D_i(t)$: Expected number of repair of unit $(0,t]$ given that the system entered regenerative state S_i at $t = 0.\}$
$T_i(t)$: Expected number of servers treatment in $(0,t)$ given that the system entered regenerative state S_i at $t = 0$
$M_i(t)$: P$\{$ system is initially up in $S_i \in E$ is up at t without visiting other $S_j \in E$ $\}$
$z(t)/Z(t)$: pdf/ cdf of failure time of the unit.
$u(t)/U(t)$: pdf / cdf of failure time of the server.
$g(t)/G(t)$: pdf / cdf of repair time of the failed unit.
$h(t)/H(t)$: pdf / cdf of the treatment time of the server.
$s(t)/S(t)$: pdf / cdf of replacement time of the failed server.
ω : Maximum treatment time of the failed server.
$q_{i,j}(t)/Q_{i,j}(t)$: pdf / cdf of direct transition time from a regenerative state i to a regenerative state j without visiting any other regenerative state.
$q_{i,j.k}(t)/Q_{i,j.k}(t)$: pdf / cdf of first passage time from a regenerative state i to a regenerative state j or to a failed state j visiting state k once in (0,t].
$q_{i,j.k,r}(t)/Q_{i,j.k,r}(t)$: pdf / cdf of first passage time from regenerative state i to a regenerative state j or to a failed state j visiting state k, r once in (0,t].
$q_{i,j.k,r,s}(t)/Q_{i,j.k,r,s}(t)$: pdf / cdf of first passage time from regenerative state i to a regenerative state j or to a failed state j visiting state k, r and s once in (0,t].
$W_i(t)$: Probability that the server is busy in the state S_i up to time t without making any transition to any other regenerative state or returning to the same state via one or more non-regenerative states.
$m_{i,j}$: Contribution to mean sojourn time (μ_i) in state S_i when system transit directly to state j.
$(s)/(c)$: Stieltjes convolution / Laplace convolution.
$\sim$: Laplace Stieltjes Transform (LST)
$*$:Laplace Transform (LT).

3 The Model Development

3.1 The State Transition Diagram

The following figure shows all the possible transitions between various states along with the regenerative points for different states.

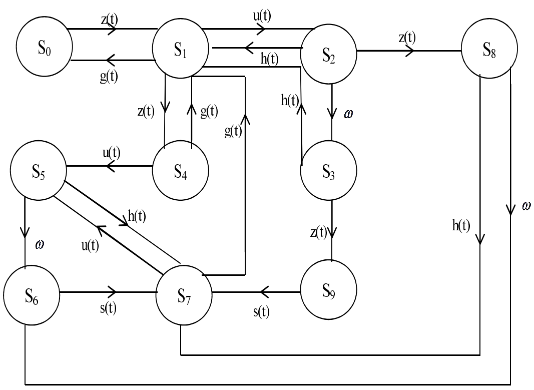

Fig. 1: System's State Transition

3.2 States Description

The system model comprises of regenerative and non-regenerative states. The states $S_i; i = 0,1,2$ are regenerative whereas $S_i; i = 3,4,\ldots,8,9$ are non-regenerative states. The detailed description of all possible states is as follows:
The regenerative states (E) :
S_0 : System up. One unit is operating and another in cold standby mode.
S_1 : System up. One unit is operating and failed unit under repair.
S_2: System up. One unit operating, another waiting for repair, failed server under treatment.
Non-regenerative states $(\bar{E})$
S_3 : System up. One unit is operating, other waiting for repair continuously, server under treatment.
S_4: System down. One unit is under repair, another unit waiting for repair.
S_5: System down. One unit is waiting for repair, waiting for repair continuously, server under treatment.
S_6: System down. Both units are waiting for repair/ continuously, server under treatment.
S_7: System down. One unit is under repair, continuously waiting for repair.
S_8: System down. One unit waiting for repair, waiting for repair continuously, server continuously under treatment.
S_9: System down. One unit waiting for repair, waiting for repair continuously, server continuously under replacement.

3.3 State Transition Probabilities

Simple probabilistic considerations yields the following expressions for the non- zero elements

$$p_{i,j} = Q_{i,j}(\infty) = \int_0^\infty q_{i,j}(t)dt \qquad (1)$$

$p_{0,1} = \int_0^\infty z(t)dt, \quad p_{1,0} = \int_0^\infty g(t)\bar{Z}(t)\bar{U}(t)dt, \quad p_{1,2} = \int_0^\infty u(t)\bar{G}(t)\bar{Z}(t)dt,$

$p_{1,4} = \int_0^\infty z(t)\bar{G}(t)\bar{U}(t)dt, \quad p_{1,1.4} = p_{1,4}(c)p_{4,1},$

$p_{1,1.4,5,6(7,5)^n} = p_{1,4}(c)p_{4,5}(c)p_{5,6}(c)p_{6,7}(c)p_{7,5}(c)p_{(5,7)^n}(c)p_{7,1},$

$p_{1,1.4,(5,7)^n} = p_{1,4}(c)p_{4,5}(c)p_{(5,7)^n}(c)p_{7,1}, \quad p_{2,1} = \int_0^\infty h(t)\bar{Z}(t)e^{-\omega t}dt,$

$p_{2,3} = \int_0^\infty \omega e^{-\omega t}\bar{H}(t)\bar{Z}(t)dt, \quad p_{2,8} = \int_0^\infty z(t)\bar{H}(t)e^{-\omega t}dt, \quad p_{2,1.3} = p_{2,3}(c)p_{3,1},$

$p_{2,1.8,(7,5)^n} = p_{2,8}(c)p_{8,7}(c)p_{7,5}(c)p_{(5,7)^n}(c)p_{7,1},$

$p_{2,1.8,6,(7,5)^n} = p_{2,8}(c)p_{8,6}(c)p_{6,7}(c)p_{7,5}(c)p_{(5,7)^n}(c)p_{7,1},$

$p_{2,1.3,9,(7,5)^n} = p_{2,3}(c)p_{3,9}(c)p_{9,7}(c)p_{7,5}(c)p_{(5,7)^n}(c)p_{7,1},$

$p_{3,1} = \int_0^\infty s(t)\bar{Z}(t)dt, \quad p_{3,9} = \int_0^\infty z(t)\bar{S}(t)dt, \quad p_{4,1} = \int_0^\infty g(t)\bar{U}(t)dt,$

$p_{4,5}=\int_0^\infty u(t)\bar{G}(t)dt, \quad p_{5,6}=\int_0^\infty \omega e^{-\omega t}\bar{H}(t)dt, \quad p_{5,7}=\int_0^\infty h(t)e^{-\omega t}dt,$
$p_{6,7}=\int_0^\infty s(t)dt, \quad p_{7,1}=\int_0^\infty g(t)\bar{U}(t)dt, \quad p_{7,5}=\int_0^\infty u(t)\bar{G}(t)dt,$
$p_{8,6}=\int_0^\infty \omega e^{-\omega t}\bar{H}(t)dt, \quad p_{8,7}=\int_0^\infty h(t)e^{-\omega t}dt, \quad p_{9,7}=\int_0^\infty s(t)dt,$
For these Transition Probabilities, it can be verified that
$p_{0,1}=p_{1,0}+p_{1,2}+p_{1,4}=p_{1,0}+p_{1,2}+p_{1,1.4}+p_{1,1.4,5,6,(7,5)^n}+p_{1,1.4,(5,7)^n}=$
$p_{2,1}+p_{2,3}+p_{2,8}=p_{2,1}+p_{2,1.3}+p_{2,1.8,(7,5)^n}+p_{2,1.8,6,(7,5)^n}+p_{2,1.3,9,(7,5)^n}=$
$p_{3,1}+p_{3,9}=p_{4,1}+p_{4,5}=p_{5,6}+p_{5,7}=p_{6,7}=p_{7,1}+p_{7,5}=p_{8,6}+p_{8,7}=p_{9,7}=1$

3.4 Mean Sojourn Times

Let T be the time to system failure then the Mean sojourn time μ_i in state S_i are given by:

$$\mu_i=E(t)=\int_0^\infty P(T>t)dt \tag{2}$$

$\mu_0=\int_0^\infty \bar{Z}(t)dt, \quad \mu_1=\int_0^\infty \bar{Z}(t)\bar{U}(t)\bar{G}(t)dt, \quad \mu_2=\int_0^\infty \omega e^{-\omega t}\bar{H}(t)\bar{Z}(t)dt$
The unconditional mean time taken by the system to transit from any state S_i when time is counted from epoch at entrance into state S_j is stated as:
$m_{i,j}=\int t dQ_{i,j}(t)=-q^{*\prime}_{i,j}(0)$
i.e $m_{0,1}=\mu_0, \quad m_{1,0}+m_{1,2}+m_{1,4}=\mu_1, m_{1,0}+m_{1,2}+m_{1,1.4}+m_{1,1.4,5,6,(7,5)^n}+m_{1,1.4,(5,7)^n}=\mu_1', \quad m_{2,1}+m_{2,3}+m_{2,8}=\mu_2, m_{2,1}+m_{2,1.3}+m_{2,1.8,(7,5)^n}+m_{2,1.8,6,(7,5)^n}+m_{2,1.3,9,(7,5)^n}=\mu_2', \quad m_{3,1}+m_{3,9}=\mu_3, \quad m_{4,1}+m_{4,5}=\mu_4, m_{5,6}+m_{5,7}=\mu_5, \quad m_{6,7}=\mu_6, \quad m_{7,1}+m_{7,5}=\mu_7, m_{8,6}+m_{8,7}=\mu_8, \quad m_{9,7}=\mu_9$

4 Stochastic Analysis

4.1 Reliability Measure

Let $\phi_i(t)$ be the c.d.f of the first passage time from regenerative state S_i to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for $\phi_i(t)$:

$$\phi_i(t)=\sum_j\{Q_{i,j}(t)+Q_{i,j,k}(t)+Q_{i,j,k,l}(t)+\cdots+Q_{i,j,k,l,m...}(t)\}(c)\phi_j(t)+\sum_f Q_{i,f}(t); i=0,1,2 \tag{3}$$

Where S_j is an un-failed regenerative state to which the given regenerative state S_i can transit and S_k is failed state to which the state S_i can transit directly.
Taking Laplace Stieltjes transform of equation (3)and solving for $\tilde{\phi}_0(s)$, we get MTSF as follow

$$MTSF=\lim_{s\to 0}R^*(s)=\lim_{s\to 0}[\frac{\{1-\tilde{\phi}_0(s)\}}{s}]=\frac{\mu_0[1-p_{1,2}(p_{2,1}+p_{2,1.3})]+\mu_1+\mu_1' p_{1,2}}{1-p_{1,0}-p_{1,2}(p_{2,1}+p_{2,1.3})} \tag{4}$$

The reliability R(t) is given by

$$R(t)=L^{-1}\{R^*(s)\}=L^{-1}[\frac{\{1-\tilde{\phi}_0(s)\}}{s}] \tag{5}$$

4.2 Economic Measures

Let the system entered the regenerative state S_i at t=0. Considering S_j as a regenerative state to which the given regenerative state S_i transits. The recursive relations for various measures contributing for profit in (0, t] are given as follows:
Availability:

$A_0(t) = M_0(t) + q_{0,1}(t)(c)A_1(t)$
$A_1(t) = M_1(t) + q_{1,0}(t)(c)A_0(t) + [q_{1,1.4}(t) + q_{1,1.4,5,6,(7,5)^n}(t) + q_{1,1.4,(5,7)^n}(t)](c)A_1(t) + q_{1,2}(t)(c)A_2(t)$
$A_2(t) = M_2(t) + [q_{2,1}(t) + q_{2,1.3}(t) + q_{2,1.8,(7,5)^n}(t) + q_{2,1.8,6,(7,5)^n}(t) + q_{2,1.3,9,(7,5)^n}(t)](c)A_1(t)$
In compact form we can expressed these equations as follows.

$$A_i(t) = M_i(t) + \sum_j \{q_{i,j}(t) + \delta_{i,j,k,l\ldots}\{q_{i,j,k}(t) + q_{i,j,k,l}(t) + \ldots\}\}(c)A_j(t); i = 0,1,2 \tag{6}$$

Similarly the compact form recursive relations for remaining measures are as follows
Busy Period due to Repair:

$$B_i^r(t) = W_i^r(t) + \sum_j \{q_{i,j}(t) + \delta_{i,j,k,l\ldots}\{q_{i,j,k}(t) + q_{i,j,k,l}(t) + \ldots\}\}(c)B_j^r(t); i = 0,1,2 \tag{7}$$

$$D_i(t) = \sum_j \{Q_{i,j}(t) + \delta_{i,j,k,l\ldots}\{Q_{i,j,k}(t) + Q_{i,j,k,l}(t) + \ldots\}\}(s)\{\delta_j + D_j(t)\}; i = 0,1,2 \tag{8}$$

$$T_i(t) = \sum_j \{Q_{i,j}(t) + \delta_{i,j,k,l\ldots}\{Q_{i,j,k}(t) + Q_{i,j,k,l}(t) + \ldots\}\}(s)\{\delta_j + T_j(t)\}; i = 0,1,2 \tag{9}$$

$$R_i^s(t) = \sum_j \{Q_{i,j}(t) + \delta_{i,j,k,l\ldots}\{Q_{i,j,k}(t) + Q_{i,j,k,l}(t) + \ldots\}\}(s)\{\delta_j + R_j^s(t)\}; i = 0,1,2 \tag{10}$$

Here $\delta_j = \begin{cases} 1, \text{if there is a repair/treatment from } S_i \text{ to } S_j \\ 0, \text{Otherwise} \end{cases}$
and $\delta_{i,j,k,l\ldots} = \begin{cases} 1, \text{if there is a transition from } S_i \text{ to } S_j \text{ via } S_{k,l\ldots} \\ 0, \text{Otherwise} \end{cases}$
Using LT/ LST, of equations (9-10) and solving we get the results in steady state as below:

$$A_0(\infty) = \lim_{s\to 0} sA_0^*(s) = \frac{\mu_0 p_{1,0} + \mu_1 + \mu_2 p_{1,2}}{\mu_0 p_{1,0} + \mu_1' + \mu_2' p_{1,2}} \tag{11}$$

$$B_0^r = \lim_{s\to 0} sB_0^{r\sim}(s) = \frac{W_1{}^{r*}(0)}{\mu_0 p_{1,0} + \mu_1' + \mu_2' p_{1,2}} \tag{12}$$

$$D_0 = \lim_{s\to 0} sD_0^{\sim}(s) = \frac{1 - p_{1,2}(p_{2,1} + p_{1,1.3})}{\mu_0 p_{1,0} + \mu_1' + \mu_2' p_{1,2}} \tag{13}$$

$$T_0 = \lim_{s\to 0} sT_0^{\sim}(s) = \frac{p_{1,2} p_{2,1}}{\mu_0 p_{1,0} + \mu_1' + \mu_2' p_{1,2}} \tag{14}$$

$$R_0^s = \lim_{s\to 0} sR_0^{s\sim}(s) = \frac{p_{1,2} p_{2,1}}{\mu_0 p_{1,0} + \mu_1' + \mu_2' p_{1,2}} \tag{15}$$

Further, using the values of above performance measures, the profit incurred to the system model in steady state is given as below.

$$p_0 = (K_0 A_0) - (C_1 B_0{}^r + C_2 D_0 + C_3 T_0 + C_4 R_0{}^s) \tag{16}$$

K_0=Revenue per unit up time of the system.
C_1=Cost per unit time for which server is busy due to repair.
C_2=Cost per unit time for repair of the unit.
C_3=Cost per unit time for server treatment.
C_4=Cost per unit time for replacement of the server.

5 Illustration

5.1 Numerical Example (Exponential Distribution)

In the following the values of different performance measures are obtained assuming all the random variables as exponentially distributed with following density functions:
$z(t) = \lambda e^{-\lambda t}, \quad g(t) = \alpha e^{-\alpha t}, \quad u(t) = \gamma e^{-\gamma t}, \quad s(t) = \Omega e^{-\Omega t}, \quad h(t) = \beta e^{-\beta t}$
Further we choose the following values for different parameters:
Failure rate of server (γ) = 0.02 per unit time,
Failure rate of unit (λ) = 0.008 per unit time,
Repair rate of unit (α) = 0.3 per unit time,
Treatment rate of server (β) = 0.07 per unit time,
Maximum repair time (ω) = 0.08 unit time.
Replacement time of the server (Ω) = 2.0
$K_0 = 20000, \quad C_1 = 100, \quad C_2 = 700, \quad C_3 = 600, \quad C_4 = 200$
MTSF =4388.807 unit time,
Availability = 0.998837
Busy period of server for repair = 0.026264
Expected number of repairs = 0.007992
Expected number of replacements of server =0.000261
Expected number of treatments =0.000229129
System profit =19968.33 Unit

6 Tables

In this section we have constructed tables which are as follows:

Table 1: The behaviour of mean time to system failure

MTSF	$(\lambda = 0.008,$	$\gamma = 0.02,$	$\alpha = 0.3,$	$\Omega = 2.0,$	$\omega = 0.08)$
Treatment Rate (β)	0.01	0.02	0.03	0.04	0.05
$\lambda = 0.008$	4114.956	4178.083	4232.230	4279.182	4320.280
$\lambda = 0.009$	3281.182	3330.218	3372.338	3408.906	3440.949
$\lambda = 0.01$	2681.862	2720.917	2754.511	2783.712	2809.326
$\gamma = 0.02$	4114.956	4178.083	4232.230	4279.182	4320.280
$\gamma = 0.04$	3537.500	3629.818	3710.769	3782.319	3846.008
$\gamma = 0.06$	3109.793	3215.389	3309.544	3394.003	3470.177
$\alpha = 0.3$	4114.956	4178.083	4232.230	4279.182	4320.28
$\alpha = 0.4$	5403.922	5488.287	5560.611	5623.296	5678.148
$\alpha = 0.5$	6692.887	6798.490	6888.991	6967.411	7036.015
$\Omega = 2.0$	4114.956	4178.083	4232.230	4279.182	4320.28
$\Omega = 2.2$	4117.131	4180.059	4234.040	4280.853	4321.833
$\Omega = 2.4$	4118.947	4181.708	4235.551	4282.247	4323.128
$\omega = 0.08$	4114.956	4178.083	4232.230	4279.182	4320.28
$\omega = 0.09$	4175.950	4230.140	4277.145	4318.301	4354.635
$\omega = 0.1$	4228.053	4275.111	4316.325	4352.718	4385.088

Table 2: The behaviour of system availability

Availability	($\lambda = 0.008$,	$\gamma = 0.02$,	$\alpha = 0.3$,	$\Omega = 2.0$,	$\omega = 0.08$)
Treatment Rate (β)	0.01	0.02	0.03	0.04	0.05
$\lambda = 0.008$	0.998401	0.998517	0.998608	0.998682	0.998743
$\lambda = 0.009$	0.998024	0.998165	0.998276	0.998366	0.99844
$\lambda = 0.01$	0.99761	0.997778	0.997911	0.998019	0.998108
$\gamma = 0.02$	0.998401	0.998517	0.998608	0.998682	0.998743
$\gamma = 0.04$	0.997662	0.997855	0.998011	0.998139	0.998246
$\gamma = 0.06$	0.997057	0.997298	0.997497	0.997665	0.997806
$\alpha = 0.3$	0.998401	0.998517	0.998608	0.998682	0.998743
$\alpha = 0.4$	0.998955	0.999042	0.999110	0.999165	0.99921
$\alpha = 0.5$	0.999241	0.99931	0.999365	0.999408	0.999443
$\Omega = 2.0$	0.998401	0.998517	0.998608	0.998682	0.998743
$\Omega = 2.2$	0.998423	0.998536	0.998626	0.998698	0.998758
$\Omega = 2.4$	0.998441	0.998552	0.998641	0.998712	0.998771

Table 3: The behaviour of system profit

Profit	($\lambda = 0.008$,	$\gamma = 0.02$,	$\alpha = 0.3$,	$\Omega = 2.0$,	$\omega = 0.08$)
Treatment Rate (β)	0.01	0.02	0.03	0.04	0.05
$\lambda = 0.008$	19959.69	19961.98	19963.80	19965.26	19966.47
$\lambda = 0.009$	19951.12	19953.91	19956.12	19957.91	19959.38
$\lambda = 0.01$	19941.81	19945.15	19947.79	19949.92	19951.68
$\gamma = 0.02$	19959.69	19961.98	19963.80	19965.26	19966.47
$\gamma = 0.04$	19944.81	19948.62	19951.71	19954.25	19956.37
$\gamma = 0.06$	19932.59	19937.35	19941.30	19944.61	19947.41
$\alpha = 0.3$	19959.69	19961.98	19963.80	19965.26	19966.47
$\alpha = 0.4$	19971.44	19973.17	19974.52	19975.61	19976.50
$\alpha = 0.5$	19977.57	19978.94	19980.02	19980.88	19981.58
$\Omega = 2.0$	19959.69	19961.98	19963.80	19965.26	19966.47
$\Omega = 2.2$	19960.13	19962.37	19964.15	19965.59	19966.77
$\Omega = 2.4$	19960.49	19962.70	19964.45	19965.86	19967.02

Table 4: Maximum treatment time threshold

Effect of ω on economic measures (threshold limits) with $\lambda = 0.008,\ \gamma = 0.02,\ \alpha = 0.3,\ \Omega = 0.1,\ \beta = 0.02$

(ω)	0.01	0.02	0.03	0.04	0.05
Availability	19959.69	19961.98	19963.80	19965.26	19966.47
Profit	19951.12	19953.91	19956.12	19957.91	19959.38

7 Concluding Remarks

In the current study a new stochastic model for a two unit cold standby system, with the possibility of server failure subjected to maximum treatment time, is developed. The semi-Markov process and regenerative point technique are used derive expressions for various indices of system performance. The numerical simulation of the results is given for a particular case of exponential distributions. The simulation results indicate that the maximum treatment time limit affects both the reliability as well profit of the system. All the performance indices shows rising trends with higher limit. The system performance is better with higher values of the treatment time threshold it may be because of the replacement cost involved. So the analysis infers that a system with specified value of server maximum treatment time threshold is more reliable and profitable to use. Further, a threshold limit is also provided, under the given setup. The study may find its applicability for systems in diverse areas such as communication systems, transportation systems, automobile industry, defense manufacturing, remote sensing etc. it may be a guiding document for reliability practitioners in deciding time limits for server rectification.

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Conflict of Interest

The authors declare that they have no conflict of interest.

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