Journal of Statistics Applications & Probability
An International Journal

http://dx.doi.org/10.18576/jsap/100207

A Generalized Ratio-Type Estimator of Finite Population Variance using Quartiles and their Functions

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Received: 28 Aug. 2019, Revised: 21 Mar. 2020, Accepted: 23 Dec. 2020
Published online: 1 Jul. 2021

Abstract: In this paper, some ratio-type estimators for finite population variance have been proposed using known values of the parameters of an auxiliary variable such as quartiles and their functions under simple random sampling. The suggested estimator has been compared with the usual unbiased estimator of population variance under large sample approximation. An empirical study has been also conducted to judge the merits of the proposed estimator over other existing ratio estimators for the population variance.

Keywords: Study variable, Auxiliary variable, Quartiles, Bias, Mean squared error, Simple random sampling.

1 Introduction

In sampling survey, the use of the suitable auxiliary variable always increases the efficiency of an estimator of the character under study. The efficiency of the estimator can be increased at both stages selection as well as estimation. For estimating the population parameters such as population mean $\bar{Y}$ and population variance S_y^2 of the study variable y, several authors have used information on different parameters such as population mean $\bar{X}$, coefficient of variation C_x , standard deviation σ_x , coefficient of skewness $\beta_1(x)$ and coefficient of kurtosis $\beta_2(x)$ of the auxiliary variable x. However, the problem of estimating the population variance S_y^2 using auxiliary information on a supplementary variable has attracted the attention of survey statisticians. Estimating the finite population variance has played a prominent role in various fields such as industry, agriculture, medical and biological sciences where we come across the populations which are likely to be skewed. Many more situations can be brightening in practice where the estimation of population variance of the study variable y assumes importance. For these reasons various authors like [1,2,6,7,8,9,13,14,15,16,17] have paid their attention towards the estimation of population variance.

According to the property of quartiles and their functions, their values are unaffected by the extreme values or the presence of outliers in the population values. For this reason, [4,10,11] and [12] have considered the problem of estimating the population variance of the study variable y using information on quartiles, inter-quartile range, semi-quartile range and semi-quartile average of an auxiliary variable x.

The usual unbiased estimator s_y^2 , the estimators of the population variance due to [1,2,4,10,11] and [12] are presented in the Table 1 along with their biases and mean squared errors (MSEs).

Table 1: The Existing estimators of population variance S_y^2

Estimator(.)	Bias B(.)	Mean Squared Error MSE(.)
$t_0 = S_y^2$	-	$\gamma S_y^4(\lambda_{40}-1))$
$t_R = s_y^2\left(\frac{S_x^2}{s_x^2}\right)$ [1] Estimator	$\gamma S_y^2(\lambda_{04}-1)(1-c)$	$\gamma S_y^4[(\lambda_{40}-1)+(\lambda_{04}-1)(1-2c)]$
$t_1 = s_y^2\left(\frac{S_x^2-C_x}{s_x^2-C_x}\right)$ [2] Estimator	$\gamma S_y^2(\lambda_{04}-1)\theta_1(\theta_1-c)$	$\gamma S_y^4[(\lambda_{40}-1)+\theta_1(\lambda_{04}-1)(\theta_1-2c)]$

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Table 2: The Existing estimators of population variance S_y^2 Conti...

Estimator(.)	Bias B(.)	Mean Squared Error MSE(.)
$t_2 = s_y^2\left(\frac{S_x^2-\beta_2(x)}{s_x^2-\beta_2(x)}\right)$ [2] Estimator	$\gamma S_y^2(\lambda_{04}-1)\theta_2(\theta_2-c)$	$\gamma S_y^4[(\lambda_{40}-1)+\theta_2(\lambda_{04}-1)(\theta_2-2c)]$
$t_3 = s_y^2\left(\frac{\beta_2(x)S_x^2-C_x}{\beta_2(x)s_x^2-C_x}\right)$ [2] Estimator	$\gamma S_y^2(\lambda_{04}-1)\theta_3(\theta_3-c)$	$\gamma S_y^4[(\lambda_{40}-1)+\theta_3(\lambda_{04}-1)(\theta_3-2c)]$
$t_4 = s_y^2\left(\frac{C_xS_x^2-\beta_2(x)}{C_xs_x^2-\beta_2(x)}\right)$ [2] Estimator	$\gamma S_y^2(\lambda_{04}-1)\theta_4(\theta_4-c)$	$\gamma S_y^4[(\lambda_{40}-1)+\theta_4(\lambda_{04}-1)(\theta_4-2c)]$
$t_5 = s_y^2\left(\frac{S_x^2+Q_2}{s_x^2+Q_2}\right)$ [10] Estimator	$\gamma S_y^2(\lambda_{04}-1)\theta_5(\theta_5-c)$	$\gamma S_y^4[(\lambda_{40}-1)+\theta_5(\lambda_{04}-1)(\theta_5-2c)]$
$t_6 = s_y^2\left(\frac{S_x^2+Q_1}{s_x^2+Q_1}\right)$ [11] Estimator	$\gamma S_y^2(\lambda_{04}-1)\theta_6(\theta_6-c)$	$\gamma S_y^4[(\lambda_{40}-1)+\theta_6(\lambda_{04}-1)(\theta_6-2c)]$
$t_7 = s_y^2\left(\frac{S_x^2+Q_3}{s_x^2+Q_3}\right)$ [11] Estimator	$\gamma S_y^2(\lambda_{04}-1)\theta_7(\theta_7-c)$	$\gamma S_y^4[(\lambda_{40}-1)+\theta_7(\lambda_{04}-1)(\theta_7-2c)]$
$t_8 = s_y^2\left(\frac{S_x^2+Q_r}{s_x^2+Q_r}\right)$ [11] Estimator	$\gamma S_y^2(\lambda_{04}-1)\theta_8(\theta_8-c)$	$\gamma S_y^4[(\lambda_{40}-1)+\theta_8(\lambda_{04}-1)(\theta_8-2c)]$
$t_9 = s_y^2\left(\frac{S_x^2+Q_d}{s_x^2+Q_d}\right)$ [11] Estimator	$\gamma S_y^2(\lambda_{04}-1)\theta_9(\theta_9-c)$	$\gamma S_y^4[(\lambda_{40}-1)+\theta_9(\lambda_{04}-1)(\theta_9-2c)]$
$t_{10} = s_y^2\left(\frac{S_x^2+Q_a}{s_x^2+Q_a}\right)$ [11] Estimator	$\gamma S_y^2(\lambda_{04}-1)\theta_{10}(\theta_{10}-c)$	$\gamma S_y^4[(\lambda_{40}-1)+\theta_{10}(\lambda_{04}-1)(\theta_{10}-2c)]$
$t_{11} = s_y^2\left(\frac{C_xS_x^2+Q_2}{C_xs_x^2+Q_2}\right)$ [12] Estimator	$\gamma S_y^2(\lambda_{04}-1)\theta_{11}(\theta_{11}-c)$	$\gamma S_y^4[(\lambda_{40}-1)+\theta_{11}(\lambda_{04}-1)(\theta_{11}-2c)]$
$t_{12} = s_y^2\left(\frac{\rho S_x^2+Q_3}{\rho s_x^2+Q_3}\right)$ [4] Estimator	$\gamma S_y^2(\lambda_{04}-1)\theta_{12}(\theta_{12}-c)$	$\gamma S_y^4[(\lambda_{40}-1)+\theta_{12}(\lambda_{04}-1)(\theta_{12}-2c)]$

Where $\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} y_i$; (population mean of y), $\bar{X} = \frac{1}{N}\sum_{i=1}^{N} x_i$; (population mean of x), $S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(y_i-\bar{Y})^2$; (population variance of Y), $S_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}(x_i-\bar{X})^2$; (population variance of x), $S_{xy} = \frac{1}{N-1}\sum_{i=1}^{N}(x_i-\bar{X})(y_i-\bar{Y})$; (covariance between x and Y), $\rho = \frac{S_{xy}}{S_xS_y}$;(correlation coefficient between y and x), $C_x = \frac{S_x}{\bar{X}}$; (coefficient of variation of x), $C_y = \frac{S_y}{\bar{Y}}$; (coefficient of variation of y), $s_y^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i-\bar{y})^2$; (sample variance of y), $s_x^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2$; (sample variance of x), $Q_i(i=1,2,3)$; indicates the quartile, $Q_r = (Q_3-Q_1)$; inter-quartile range, $Q_d = \frac{(Q_3-Q_1)}{2}$; semi-quartile range, $Q_a = \frac{(Q_3+Q_1)}{2}$; semi-quartile average $\theta_1 = \frac{S_x^2}{S_x^2-C_x}$ $\theta_2 = \frac{S_x^2}{S_x^2-\beta_2(x)}$, $\theta_3 = \frac{\beta_2(x)S_x^2}{\beta_2(x)S_x^2-C_x}$, $\theta_4 = \frac{C_xS_x^2}{C_xS_x^2-\beta_2(x)}$, $\theta_5 = \frac{S_x^2}{S_x^2+Q_2}$, $\theta_6 = \frac{S_x^2}{S_x^2+Q_1}$, $\theta_7 = \frac{S_x^2}{S_x^2+Q_3}$, $\theta_8 = \frac{S_x^2}{S_x^2+Q_r}$, $\theta_9 = \frac{S_x^2}{S_x^2+Q_d}$, $\theta_{10} = \frac{S_x^2}{S_x^2+Q_a}$, $\theta_{11} = \frac{C_xS_x^2}{C_xS_x^2+Q_2}$, $\theta_{12} = \frac{\rho S_x^2}{\rho S_x^2+Q_3}$, $\gamma = 1/n$, $C = (\lambda_{04}-1)^{-1}(\lambda_{22}-1)$, $\lambda_{rs} = \mu_{rs}(\mu_{02}^{s/2}\mu_{20}^{r/2})^{-1}$, $\mu_{rs} = \frac{1}{N}\sum_{i=1}^{N}(y_i-\bar{Y})^r(x_i-\bar{X})^s$; (r,s being non-negative integers)
It is noticeable that the estimators $t_k(k=5,6,\cdots 11)$ due to [4,10,11] and [12] have used the quartiles $(Q_i : i=1,2,3)$ and their functions such as inter-quartile range Q_r , semi-quartile range Q_d and semi-quartile average Q_a and in additive form to sample and population variances s_x^2 and S_x^2 respectively of the auxiliary variable x. Moreover the unit of the quartiles and their function as given above is of original variable x, while the unit of S_x^2 and s_x^2 is in the square of the unit of the original variable x. This empowers authors to develop some alternative estimators for the population variance and study their properties.

The present paper aims to estimate the unknown population variance of study variable y by improving the estimators suggested by [4,10,11] and [12] using same information on an auxiliary variable x. The remaining part of the paper is organized as follows: In Section 2, the generalized family of the estimators for the population variance has been suggested along with the expressions of asymptotic biases and mean squared errors. In addition, some new members of the proposed family of the estimators have been generated with their respective properties. Section 3 is dedicated to efficiency comparison of the suggested estimator with respect to the usual unbiased estimator of the population variance and other stated estimators. Section 4 is devoted to an empirical study of the proposed ratio-type estimators for the real data sets. Conclusion is presented in Section 6.

2 The proposed family of estimators

Using the known values of quartiles $(Q_i; i=1,2,3)$ of an auxiliary variable x, a generalized family of the estimators has been proposed to estimate the population variance S_y^2 of the study variable y as follows:

$$t^* = s_y^2\left[\alpha+(1-\alpha)\left(\frac{\delta S_x^2+L^2}{\delta s_x^2+L^2}\right)\right] \tag{1}$$

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where $(\delta S_x^2+L^2)>0,(\delta s_x^2+L^2)>0$ and (δ, L) are either real constants or function of known parameters of an auxiliary variable x with $0\leq\alpha\leq 1$.
To obtain the biases and mean squared errors (MSEs) of the estimators t^* , we write $s_y^2=S_y^2(1+e_0)$, $s_x^2=S_x^2(1+e_1)$ such that $E(e_0)=E(e_1)=0$ and to the first degree of approximation ignoring finite population correction (f.p.c.) term, we have

$$\begin{aligned} E(e_0^2) &= \frac{(\lambda_{40}-1)}{n} \\ E(e_1^2) &= \frac{(\lambda_{04}-1)}{n} \\ E(e_0e_1) &= \frac{(\lambda_{22}-1)}{n} \end{aligned}$$

Now expressing (1) in terms of e's, we have

$$t^*=S_y^2(1+e_0)\left[\alpha+(1-\alpha)\left(\frac{\delta S_x^2+L^2}{\delta S_x^2(1+e_1)+L^2}\right)\right]$$

$$=S_y^2(1+e_0)\left[\alpha+(1-\alpha)\left(1+\theta^* e_1\right)^{-1}\right] \tag{2}$$

where $\theta^*=\frac{\delta S_x^2}{\delta S_x^2+L^2}$
We assume that $|\theta^* e_1|<1$,so $(1+\theta^* e_1)^{-1}$ is expandable in terms of power series. Now, we have

$$t^*=S_y^2(1+e_0)[\alpha+(1-\alpha)(1-\theta^* e_1+\theta^{*2}e_1^2-\cdots]$$

$$=S_y^2[(1+e_0-\theta^* e_1+\theta^{*2}e_1^2-\theta^* e_1e_0+\cdots)+\alpha(\theta^* e_1-\theta^{*2}e_1^2+\theta^* e_1e_0-\cdots)]$$

Neglecting terms of e's that have power greater than the two, we have

$$t^*\cong S_y^2[(1+e_0-\theta^* e_1+\theta^{*2}e_1^2-\theta^* e_1e_0)+\alpha(\theta^* e_1-\theta^{*2}e_1^2+\theta^* e_1e_0)]$$

$$(t^*-S_y^2)\cong S_y^2[(e_0-\theta^* e_1+\theta^{*2}e_1^2-\theta^* e_1e_0)+\alpha(\theta^* e_1-\theta^{*2}e_1^2+\theta^* e_1e_0)] \tag{3}$$

Taking expectation of both sides of (3), we get the biases of t^* to the first degree of approximation as

$$B(t^*)=\frac{S_y^2}{n}[\theta(1-\alpha)\beta_2^*(x)(\theta-C)] \tag{4}$$

Squaring both sides of (3) and neglecting terms of e's having power greater than two, we have

$$(t^*-S_y^2)^2\cong S_y^4[e_0-\theta^* e_1+\alpha\theta^* e_1]^2 \tag{5}$$

Taking expectation of both sides of (5), we get the MSEs of t^* to the first degree of approximation as

$$MSE(t^*)=\frac{S_y^4}{n}[\beta_2^*(y)+\theta^*(1-\alpha)\beta_2^*(x)\{\theta^*(1-\alpha)-2C\}] \tag{6}$$

The MSE of t^* given by (6) is minimized for

$$\alpha_{opt.}=\left(1-\frac{C}{\theta}\right)$$

and the minimum MSE of t^* is given by

$$\min MSE(t^*)=\frac{S_y^4}{n}[\beta_2^*(y)-C^2\beta_2^*(x)] \tag{7}$$

Several new estimators of the population variance can be generated for different choices of the scalars (α,δ,L) from the proposed family of estimators t^* given in (1) which are presented in the Table-2.

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Table 3: Some new estimators generated from t^* for different combinations of (α,δ,L)

Estimators (.)	α	δ	**L**
$t_1^* = s_y^2\left(\frac{\rho S_x^2+Q_1^2}{\rho s_x^2+Q_1^2}\right)$	0	ρ	Q_1
$t_2^* = s_y^2\left(\frac{C_x S_x^2+Q_1^2}{C_x s_x^2+Q_1^2}\right)$	0	C_x	Q_1
$t_3^* = s_y^2\left(\frac{\beta_2(x) S_x^2+Q_2^2}{\beta_2(x) s_x^2+Q_2^2}\right)$	0	$\beta_2(x)$	Q_2
$t_4^* = s_y^2\left(\frac{\beta_2(x) S_x^2+Q_r^2}{\beta_2(x) s_x^2+Q_r^2}\right)$	0	$\beta_2(x)$	Q_r
$t_5^* = s_y^2\left(\frac{\rho S_x^2+Q_d^2}{\rho s_x^2+Q_d^2}\right)$	0	ρ	Q_d
$t_6^* = s_y^2\left(\frac{C_x S_x^2+Q_d^2}{C_x s_x^2+Q_d^2}\right)$	0	C_x	Q_d
$t_7^* = s_y^2\left(\frac{\beta_2(x) S_x^2+Q_a^2}{\beta_2(x) s_x^2+Q_a^2}\right)$	0	$\beta_2(x)$	Q_a

Similarly one can identify many other estimators from the proposed family of ratio-type estimators t^* for different combinations of (α,δ,L) . To the first degree of approximation the bias and MSE of the estimators $t_i^*(i=1,2,\cdots,7)$ are respectively given by

$$B(t_i^*) = \gamma S_y^2(\lambda_{04}-1)\theta_i^*(\theta_i^*-c); \qquad (i=1,2,\cdots,7)$$

$$MSE(t_i^*) = \gamma S_y^4[(\lambda_{40}-1)+\theta_i^*(\lambda_{04}-1)(\theta_i^*-2c]; \qquad (i=1,2,\cdots,7)$$

where $\theta_1^* = \left(\frac{\rho S_x^2}{\rho S_x^2+Q_1^2}\right)$, $\theta_2^* = \left(\frac{C_x S_x^2}{C_x S_x^2+Q_1^2}\right)$, , $\theta_3^* = \left(\frac{\beta_2(x) S_x^2}{\beta_2(x) S_x^2+Q_2^2}\right)$, $\theta_4^* = \left(\frac{\beta_2(x) S_x^2}{\beta_2(x) S_x^2+Q_r^2}\right)$, $\theta_5^* = \left(\frac{\rho S_x^2}{\rho S_x^2+Q_d^2}\right)$, $\theta_6^* = \left(\frac{C_x S_x^2}{C_x S_x^2+Q_d^2}\right)$, $\theta_7^* = \left(\frac{\beta_2(x) S_x^2}{\beta_2(x) S_x^2+Q_a^2}\right)$,

3 Efficiency Comparison

We have derived the conditions under which the proposed family of estimators t^* are more efficient than the usual unbiased estimator s_y^2 , [1,2,4,10,11] and [12] $t_k(k=1,2,\cdots,12)$ estimator. Table 1 and (6) shows that

i

$$MSE(t^*) < MSE(s_y^2) \qquad \text{If} c > \frac{\theta^*(1-\alpha)}{2} \tag{8}$$

ii $MSE(t^*) < MSE(t_R)$ If

$$min\left[\left(\frac{1}{1-\alpha}\right),\left(\frac{2c-1}{1-\alpha}\right)\right] < \theta^* < max\left[\left(\frac{1}{1-\alpha}\right),\left(\frac{2c-1}{1-\alpha}\right)\right] \tag{9}$$

iii $MSE(t^*) < MSE(t_K); k=1,2,\cdots,12$ If

$$min\left[\left(\frac{\theta_k}{1-\alpha}\right),\left(\frac{2c-\theta_k}{1-\alpha}\right)\right] < \theta^* < max\left[\left(\frac{\theta_k}{1-\alpha}\right),\left(\frac{2c-\theta_k}{1-\alpha}\right)\right]; k=1,2,\cdots,12 \tag{10}$$

4 Empirical Study

To see the performance of the proposed estimator t^* of the population variance over the usual unbiased estimator s_y^2, [1,2, 4,10,11] and [12] estimator $t_k(k=1,2,\cdots,12)$, the description of the considered population data sets are as follows:

Population-I[5]

Y: Output for 80 factories in a region
X: Fixed capital
N = 80, n = 20, $\bar{Y}$ =51.8264, $\bar{X}$=11.2646, ρ=0.9413, S_y =18.3549, C_y =0.3542, S_x = 8.4563, C_x=0.7507, λ_{04}= 2.8664, λ_{40}= 2.2667, λ_{22}=2.2209, Q_1 = 5.1500, Q_2 =10.300, Q_3=16.975, Q_r= 11.825, Q_d = 5.9125, Q_a=11.0625

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Population-II[3]

N =70, n =25, $\bar{Y}$ =96.7000, $\bar{X}$=175.2671, ρ =0.7293, S_y=60.7140, C_y =0.6254, S_x=140.85, C_x=0.8037, λ_{04} =7.0952, λ_{40}=4.7596, λ_{22} =4.6038, Q_1=80.1500, Q_2 =160.30, Q_3=225.025, Q_r = 144.8750, Q_d=72.4375, Q_a =152.5875
Furthermore, for the purpose of efficiency comparison of the estimators $t_i^*(i=1,2,\cdots,7)$, the relative efficiencies of the estimator $t_k(k=1,2,\cdots,12)$ and $t_i^*(i=1,2,\cdots,7)$ w.r.t. s_y^2 have been computed using the formula:

$$PRE(t_k,s_y^2)=\frac{MSE(s_y^2)}{MSE(t_k)}X100 \qquad \text{for}(k=1,2,\cdots,12)$$

$$PRE(t_i^*,s_y^2)=\frac{MSE(s_y^2)}{MSE(t_i^*)}X100 \qquad \text{for}(i=1,2,\cdots,7)$$

Findings are presented in Table-3

Table 4: PRE of the estimators $t_k(k=1,2,\cdots,12)$, $t_i^*(i=1,2,\cdots,7)$ w.r.t. s_y^2

Estimators (.)	**Population I**	**Population II**
$t_R=s_y^2\left(\frac{S_x^2}{s_x^2}\right)$	183.2345	142.0218
$t_1=s_y^2\left(\frac{S_x^2-C_x}{s_x^2-C_x}\right)$	179.6210813	142.01093
$t_2=s_y^2\left(\frac{S_x^2-\beta_2(x)}{s_x^2-\beta_2(x)}\right)$	169.2398	141.9261
$t_3=s_y^2\left(\frac{\beta_2(x)S_x^2-C_x}{\beta_2(x)s_x^2-C_x}\right)$	181.9786	142.0202
$t_4=s_y^2\left(\frac{C_xS_x^2-\beta_2(x)}{C_xs_x^2-\beta_2(x)}\right)$	164.4934	141.9028
$t_5=s_y^2\left(\frac{S_x^2+Q_2}{s_x^2+Q_2}\right)$	226.8671	144.1754
$t_6=s_y^2\left(\frac{S_x^2+Q_1}{s_x^2+Q_1}\right)$	206.6417	143.1002
$t_7=s_y^2\left(\frac{S_x^2+Q_3}{s_x^2+Q_3}\right)$	247.2471	145.0414
$t_8=s_y^2\left(\frac{S_x^2+Q_r}{s_x^2+Q_r}\right)$	232.1285	143.9687
$t_9=s_y^2\left(\frac{S_x^2+Q_d}{s_x^2+Q_d}\right)$	209.8595	142.9965
$t_{10}=s_y^2\left(\frac{S_x^2+Q_a}{s_x^2+Q_a}\right)$	229.5416	144.0721
$t_{11}=s_y^2\left(\frac{C_xS_x^2+Q_2}{C_xs_x^2+Q_2}\right)$	238.1734	144.6995
$t_{12}=s_y^2\left(\frac{\rho S_x^2+Q_3}{\rho s_x^2+Q_3}\right)$	249.8426	146.1556
$t_i^*=s_y^2\left[\alpha+(1-\alpha)\left(\frac{\delta S_x^2+L^2}{\delta s_x^2+L^2}\right)\right]$	**270.6324**	**230.8138**
$t_1^*=s_y^2\left(\frac{\rho S_x^2+Q_1^2}{\rho s_x^2+Q_1^2}\right)$	266.3895	222.2802
$t_2^*=s_y^2\left(\frac{C_xS_x^2+Q_1^2}{C_xs_x^2+Q_1^2}\right)$	270.3844	218.7164
$t_3^*=s_y^2\left(\frac{\beta_2(x)S_x^2+Q_2^2}{\beta_2(x)s_x^2+Q_2^2}\right)$	270.6075	185.8162
$t_4^*=s_y^2\left(\frac{\beta_2(x)S_x^2+Q_r^2}{\beta_2(x)s_x^2+Q_r^2}\right)$	266.8421	178.7482
$t_5^*=s_y^2\left(\frac{\rho S_x^2+Q_d^2}{\rho s_x^2+Q_d^2}\right)$	270.6148	214.4873
$t_6^*=s_y^2\left(\frac{C_xS_x^2+Q_d^2}{C_xs_x^2+Q_d^2}\right)$	268.1147	210.3682
$t_7^*=s_y^2\left(\frac{\beta_2(x)S_x^2+Q_a^2}{\beta_2(x)s_x^2+Q_a^2}\right)$	269.7894	182.2565

Table 3 indicates that the performance of the proposed estimator and the new estimators generated from the proposed family of the estimators $t_i^*(i=1,2,\cdots,7)$ are better than the usual unbiased estimator s_y^2, [1], [2], [4], [10], [11] and [12] $t_k(k=1,2,\cdots,12)$ estimator.

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5 Conclusion

In this paper, the family of ratio-type estimator for finite population variance S_y^2 using known values of the parameters of an auxiliary variable such as quartiles and their functions has been proposed under simple random sampling. The bias and mean squared error of the proposed family of estimators have been obtained under large sample approximation. Furthermore, the condition has been derived under which the proposed estimator $t_i^*(i=1,2,\cdots,7)$ performs better than the usual unbiased estimator s_y^2 , [1,2,4,10,11] and [12] $t_k(k=1,2,\cdots,12)$ estimator. The performance of the suggested family of estimators has been assessed. It was detected that it is more efficient than the other considered estimators for known natural population data sets under certain conditions.

Acknowledgment

The author is grateful to the editor and the anonymous referees for the constructive comments and suggestions which greatly improved the quality of the paper.

Conflicts of interest

The authors declare no conflicts of interest regarding the publication of this paper.

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