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Attribute Control Charts for the New Weibull Pareto Distribution Under Truncated Life Tests

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Abstract: The present paper addresses the attribute control chart developed for the new Weibull pareto distribution under truncated life tests. Performance of the new Weibull pareto distribution control chart constructed on truncated life test is evaluated by average run length **(ARL)**, which compares the performance of the new Weibull pareto distribution with inverse gaussian.

Keywords: attribute control chart, new Weibull pareto distribution, average run length (ARL), truncated life tests.

1 Introduction

The control charts are a necessary part of quality development and a fundamental tool for monitoring the production process. The classical control charts consist of three lines: Central line (CL), the upper control limit (UCL) and lower control limit (LCL). Sometimes, the specification limits are used for the monitoring reason. These stipulation limits are set by the consumer or producer or by both. In control limits, we may understand whether a particular procedure is steady or probable over time, if plotted points lies within the control limits, the process is said to be stable. If plotted points fall outside the limits set by the system variation over time, the process is said to be unstable. Instability occurs because of some particular causes; these causes must be covered and eliminated to create more stable processes. Utilizing of control charts in business Minimizes the nonconforming item.

Control charts comprises two types: Attribute control charts and variables control charts. Attribute control charts are often used to monitor the attribute quality characteristics, while measurable quality characteristics are monitored by the variables control charts. Attribute control charts have only two values, such as good (bad), conforming (non-conforming), or acceptable (not acceptable). Both control charts have been widely used in the industry to minimize defective items. Attribute control charts involves types, e.g. C Control Chart, np Control Chart, P Control Chart (fraction defective), q control chart, and u Control Chart (number defective). The present study only addresses np chart is used when the variable of interest satisfies the binomial experiment conditions. A comprehensive review of the attribute control chart is found in the literature.[1],[2] , [3], which designed an attribute control chart to monitor distributed mean lifetime of the product based on single sampling (SS) under some distributions.

life tests are truncated as pursues. With n items placed on test, the analysis will end at$min(T_{q0}, T_0)$, where T_{q0} ,n is a random variable equal to the time at which the $q0^th$ failure occurs and T_0 is a truncation time, beyond which the experiment will not be run. Both $q0$ and T_0 are assigned before experimentation starts. If the experiment terminates at T_{q0},n (i.e. if$q0$ failures occur before time T_0), the action in terms of hypothesis testing is the rejection of some specified null-hypothesis. If the experiment terminates at time T_0 (i.e. if the $q0^th$ failure does not occur before time T_0), the action in terms of hypothesis testing is the acceptance of some specified null-hypothesis.

The control limits for in control process are given, as follows:

$$UCL = np_0 + k\sqrt{np_0(1-np_0)}$$

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$$LCL = max\left[0, np_0 + k\sqrt{np_0(1-np_0)}\right]$$

where p_0 is the probability that an item fails before the experiment time t_0 when the process is in control and k is the distance between the central line to the control limits. Efficiency of the control chart is measured using the ARL. The ARL for in control process is given as follows:

$$ARL_0 = \frac{1}{1-p_{in}^0}$$

[4] designed an attribute control chart under a truncated life test. By Burr X and XII, Inverse Gaussian and exponential lifetime truncated distributions, a Shewhart-type attribute control chart is built to display the data. Performance of the attributed control chart constructed on a truncated life test is evaluated by average run length (ARL), which compares the performance of all distributions. it was indicated that inverse Gaussian is the best distribution .

Inverse Gaussian (IG) Distribution is beneficial for modeling data that are long-tailed. The probability density function and the distribution of IG are given respectively as:

$$f(t;\mu,\lambda) = \sqrt{\lambda/2\pi t^3}\,exp(-\lambda(t-\mu)^2/2t\mu^2), \quad 0 < t < \infty,$$

$$F(t;\mu,\lambda) = \Phi\left(\sqrt{\lambda/t}\,(t/\mu-1)\right) + exp^{2\lambda/\mu}\Phi\left(-\sqrt{\lambda/t}\,(t/\mu+1)\right).$$

Where λ is a scale parameter , μ is a mean and Φ is the standard normal distribution. Then, the probability of disappointment can also be written as:

$$P = F(t_0) = \Phi\left\{\sqrt{\frac{\lambda}{a(\mu/\mu_0)}}(\frac{a}{\mu/\mu_0}-1)\right\} + exp^{2\lambda}\Phi\left\{\sqrt{\frac{\lambda}{a(\mu/\mu_0)}}(\frac{a}{\mu/\mu_0}+1)\right\}$$

When the process is under control and the general as well as specified means are equal, probability of failure is:

$$p_0 = \Phi\left\{\sqrt{\frac{\lambda}{a}}(a-1)\right\} + exp^{2\lambda}\Phi\left\{\sqrt{\frac{\lambda}{a}}(a+1)\right\}.$$

However, In case of control lack probability will be:

$$p_1 = \left\{\sqrt{\frac{\lambda}{a(\mu/\mu_0)}}(\frac{a}{\mu/\mu_0}-1)\right\} + exp^{2\lambda}\Phi\left\{\sqrt{\frac{\lambda}{a(\mu/\mu_0)}}(\frac{a}{\mu/\mu_0}+1)\right\} = P = F(t_0),$$

where a is a given constant.

More details can be seen in [5],[6],[7],[8], [9],[10] and [11].

In this paper, we propose an attribute control chart based on the time truncated life test. The rest of the paper is organized as follows: Section Two presents, an overview of the new Weibull-Pareto distribution. Attribute Control Charts for the new Weibull Pareto Distribution under Truncated Life Tests and the performance of evaluation using extensive simulation studies considering in control ARL and out of control ARL are presented in Section Three. Comparisons between the new Weibull Pareto and Inverse Gaussian control charts time using the ARL criterion are handled in Section Four.

2 The New Weibull-Pareto Distribution

[12] defined and explored the new Weibull-Pareto. The probability density function of the NWP random variable is defined as follows:

$$f(t,\alpha,\theta,\beta) = \frac{\beta\alpha}{\theta}\left(\frac{1}{\theta}\right)^{\beta-1} e^{-\alpha\left(\frac{t}{\theta}\right)^{\beta}},$$

and its corresponding cumulative distribution function is given by

$$F(t,\alpha,\theta,\beta) = 1 - e^{-\alpha\left(\frac{t}{\theta}\right)^{\beta}}, \quad t > 0, \alpha, \beta, \theta > 0. \tag{1}$$

Where α and θ are shape parameters and β is the scale parameter. The percentiles of the new Weibull-pareto distribution are written as,

$$t_q = \theta\left\{\frac{1}{\alpha}ln\left(\frac{1}{1-q}\right)\right\}^{\frac{1}{\beta}},$$

$$t_q = \theta\gamma^{\frac{1}{\beta}}, \tag{2}$$

where

$$\gamma = \frac{1}{\alpha}ln\left(\frac{1}{1-q}\right)$$

3 Attribute Control Charts for the New Weibull Pareto Distribution Under Truncated Life Tests.

We propose the following np control chart under time truncated life test:

Step 1 Take a sample of size n at each subgroup from the production process and put it on a time truncated life test. Count the number of failures within n items (D).

Step 2 Declare the process as out-of-control if $H > UCL$ or $H < LCL$. Declare the process as in control if $LCL \leq H \leq UCL$.

When the procedure is in device, the random variable H (number of failures) is following a binomial distribution with parameters n and a probability of success which is typically unidentified. Therefore, the control limits of our np chart will be:

$$UCL = np_0 + k\sqrt{np_0(1-p_0)}$$

$$LCL = max\left[0, np_0 + k\sqrt{np_0(1-p_0)}\right]$$

where k is the distance between the central line and the control limits. This would be predictable from model (Sample) when the procedure is in control. Thus, the following control limits will be practically used:

$$UCL = \bar{H} + k\sqrt{\bar{H}(1-\bar{H}/n)},$$

$$LCL = max\left[0, \bar{H} + k\sqrt{\bar{H}(1-\bar{H}/n)}\right]$$

,

where $\bar{H}$ is the average number of failures over the subgroups.

Under time truncated life test, to find the probability of failure new Weibull-pareto distribution, p_0 is in convenient form. Let us consider the experiment time is t_0 as multiple of termination ratio δ_q and specified percentile lifet_{q0}. i.e. $t_0 = \delta_q t_{q0}$. After simplification, the probability of failure under new Weibull-pareto distribution is written as,

$$P = F(t_0 = \delta_q t_{q0}) = 1 - e^{-\alpha\left(\frac{\delta_q t_{q0}}{\theta}\right)^{\beta}}, \tag{3}$$

using (2) and applying it to (3), the probability of failure under new Weibull-pareto distribution can be written as:

$$P = 1 - exp\left[-\alpha\gamma\left(\frac{\delta_q}{t_q/t_{q0}}\right)^{\beta}\right]. \tag{4}$$

When the process is in-control, the percentile ratio of the p-value (t_q/t_{q0}) equals 1. Therefore, Eq (4) will be reduced to:

$$p_0 = 1 - exp\left[-\alpha\gamma(\delta_q)^\beta\right]. \quad (5)$$

Now, we consider that the percentile ratio $(t_q/t_{q0} = f = 1.0, 0.9, 0.8, 0.7, \dots)$. Then, the probability in equ (4) gets

$$p_1 = 1 - exp\left[-\alpha\gamma\left(\frac{\delta_q}{f}\right)^\beta\right]. \quad (6)$$

The probability of declaring as in control for the proposed control chart is given, as follows:

$$P_{in}^0 = P(LCL \le H \le UCL|p_0) = \sum_{h=\lfloor LCL\rfloor+1}^{\lfloor UCL\rfloor} \binom{n}{h} p_0^h (1-p_0)^{n-h}, \quad (7)$$

using (5) and applying it to (3), equ (7) can be written as:

$$P_{in}^0 = P(LCL \le H \le UCL|p_0) = \sum_{h=\lfloor LCL\rfloor+1}^{\lfloor UCL\rfloor} \binom{n}{h} \left(1 - exp\left[-\alpha\gamma(\delta_q)^\beta\right]\right)^h \left(exp\left[-\alpha\gamma(\delta_q)^\beta\right]\right)^{n-h}, \quad (8)$$

where $\lfloor . \rfloor$ denotes the largest integer which is less than or equal to the argument.
Efficiency of the control chart is measured using the ARL. The ARL for in control process is given, as follows:

$$ARL_0 = \frac{1}{1 - P_{in}^0}, \quad (9)$$

using (8) and applying it to (9), equ (9) can be written as:

$$ARL_0 = \frac{1}{1 - \left[\sum_{h=\lfloor LCL\rfloor+1}^{\lfloor UCL\rfloor} \binom{n}{h} \left(1 - exp\left[-\alpha\gamma(\delta_q)^\beta\right]\right)^h \left(exp\left[-\alpha\gamma(\delta_q)^\beta\right]\right)^{n-h}\right]}. \quad (10)$$

Similarly, the ARL, when the process is out of control is:

$$ARL_1 = \frac{1}{1 - P_{in}^1},$$

$$ARL_1 = \frac{1}{1 - \left[\sum_{h=\lfloor LCL\rfloor+1}^{\lfloor UCL\rfloor} \binom{n}{h} \left(1 - exp\left[-\alpha\gamma\left(\frac{\delta_q}{f}\right)^\beta\right]\right)^h \left(exp\left[-\alpha\gamma\left(\frac{\delta_q}{f}\right)^\beta\right]\right)^{n-h}\right]}. \quad (11)$$

We used the following algorithm to complete the tables for the proposed control chart.

1. Specify the values of ARL, say r_0, shape parameter is $\beta_0$and scale parameter is α_0.
2. Determine the values of control chart parameters and sample size n for which the ARL from Equation (10) is close to r_0 .
3. Use the values of control chart parameters obtained in step 2 to find ARL_1 according to shift constant f using Equation (11).

We defiend the control chart parameters and ARL_1 for various values of β_0, α_0, r_0 and n as shown in Tables 1 – 4. The value of δ_q is chosen as 0.8 for all tables. They demonstrate the rapid decrease occurs in ARLs as the shift constant reduces. Note that for the shift $f = 1$, the ARLs are very close to the specified ARLs values. In this section, we use different shape ethics and scale parameters of the Burr X and XII, Inverse Gaussian , General Exponential, Pareto, New Weibull Pareto distribution and various values of target (290, 350, 400) for the in control average run length are show in table 5.1 to 5.12 according to the shift parameter from 1.0 to 0.1 for all distribution. The R program is used to generate the table. The same table can be created for any other values of shape parameter and sample size.

Table 1: ARL_s for the New Weibull Pareto distribution under time truncated life test with $q = 0.5, \alpha = 1, \beta = 1.966$ and $\delta = 0.801, n = 50$

r_0	290	350	400
K	2.95	3.053	3.67
f	**ARL1**	**ARL1**	**ARL1**
1.0	273.2	345.15	411.20
0.9	89.60	98.12	235.17
0.8	8.17	11.66	56.92
0.7	1.02	1.07	1.02
0.6	1.00	1.00	1.00
0.5	1.00	1.00	1.00
0.4	1.00	1.00	1.00
0.3	1.00	1.00	1.00
0.2	1.00	1.00	1.00
0.1	1.00	1.00	1.00

Table 2: ARL_s for the New Weibull Pareto distribution under time truncated life test with $q = 0.5, \alpha = 1, \beta = 1.966$ and $\delta = 0.801, n = 80$

r_0	290	350	400
K	2.89	3.13	3.56
f	**ARL1**	**ARL1**	**ARL1**
1.0	271.5	331.8	385.7
0.9	2.08	3.7	3.7
0.8	1.08	1.07	56.92
0.7	1.02	1.07	1.01
0.6	1.00	1.00	1.00
0.5	1.00	1.00	1.00
0.4	1.00	1.00	1.00
0.3	1.00	1.00	1.00
0.2	1.00	1.00	1.00
0.1	1.00	1.00	1.00

Table 3: Optimal parameters under Inverse Gaussian time truncated distribution with $\lambda = 2, n = 100$

r_0	290	350	400
K	3.075	2.984	2.89
f	**ARL1**	**ARL1**	**ARL1**
1.0	290.87	341.56	387.65
0.9	6.3	148.57	3.06
0.8	1.80	10.16	1.06
0.7	1.01	1.23	1.00
0.6	1.00	1.00	1.00
0.5	1.00	1.00	1.00
0.4	1.00	1.00	1.00
0.3	1.00	1.00	1.00
0.2	1.00	1.00	1.00
0.1	1.00	1.00	1.00

4 Comparison

The ARL criterion is used to compare between the new Weibull Pareto distribution control chart and Inverse Gaussian time control chart. It is noticeable when the smaller ARL value means a better performance of the graph for detecting the shift (f). A computer program written in Python language 3.7.4 is used to investigate the performance of the control charts under the new Weibull Pareto distribution and Inverse Gaussian distribution. Simulation of 1000 runs samples sizes $n = 100$ and shift (f) various from 1.0 to 0.1. The out of control ARL results are compared to the under control $ARL0 = 290$. The result of the simulation is displayed in Table 5.

Performance of the new Weibull Pareto distribution in the out of control state is better compared with that of the Inverse Gaussian distribution because it is readily known that the result of the distribution is much more operative in detecting the out of control state. For various estimations of the move in f as well as various estimations of the shape and

Table 4: Optimal parameters under Inverse Gaussian time truncated distribution with $\lambda = 3, n = 200$

r_0	290	350	400
K	3.0174	3.055	2.954
f	**ARL1**	**ARL1**	**ARL1**
1.0	276.28	343.80	2.68
0.9	6.28	3.63	3.06
0.8	1.69	1.08	1.03
0.7	1.00	1.00	1.00
0.6	1.00	1.00	1.00
0.5	1.00	1.00	1.00
0.4	1.00	1.00	1.00
0.3	1.00	1.00	1.00
0.2	1.00	1.00	1.00
0.1	1.00	1.00	1.00

scales parameters, the performance of the new Weibull Pareto distribution is given by an estimate of the probability of detecting the shift. Performance of the new Weibull Pareto distribution control chart is rather than better. The probability of detecting shifts in a shift parameter of $f = 0.1$ is extremely bad. It gets better with larger sample size and more significant shift value. Tables exhibit that the values of the percentile ratio reduced from 1.0 to 0.1. Average run length also reduced, but all other values remaind constant.

Table 5: Performance of the control charts under Inverse Gaussian and new Weibull Pareto distributions $r_0 = 290$.

	Inverse Gaussian $a = 0.5$		New Weibull Pareto $q = 0.5$	
f	$K = 3.05$ $d = 2$	$K = 3.01$ $d = 3$	$K = 3.21$ $\beta = 1.96$ $\delta = 0.801$	$K = 2.9$ $\beta = 1.7$ $\delta = 0.7$
1.0	290.98	276.38	301.7	297.6
0.9	6.23	6.29	8.09	8.25
0.8	1.80	1.69	1.71	1.62
0.7	1.00	1.00	1.00	1.00
0.6	1.00	1.00	1.00	1.00
0.5	1.00	1.00	1.00	1.00
0.4	1.00	1.00	1.00	1.00
0.3	1.00	1.00	1.00	1.00
0.2	1.00	1.00	1.00	1.00
0.1	1.00	1.00	1.00	1.00

5 Conclusion

the present study handles presenting a new np control chart if the lifetime of the product follows new Weibull pareto distribution. The chart constants for the modern employed broad tables are presented. Simulated data a clarified methodology. We note decline in ARLs values. The proposed control chart can be adopted for some different distributions as a future research.

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References

[1] M. Aslam , N. Khan, and C. H. Jun, A control chart for time truncated life tests using Pareto distribution of second kind, *Journal of statistical computation and simulation*, **86**(11), 2113-2122 (2016).
[2] M., Azam, and C. H. Jun, , A HEWMA-CUSUM control chart for the Weibull distribution, *Communications in Statistics-Theory and Methods*, **47**(24), 5973-5985(2018).
[3] S. Balamurali, S. Jeyadurga, and M. Usha , Designing of Bayesian multiple deferred state sampling plan based on gamma Poisson distribution, *American Journal of Mathematical and Management Sciences*, **35**(1), 77-90 (2016).
[4] A. Shafqat, J. Hussain , A. D. Al-Nasser and M. Aslam , Attribute control chart for some popular distributions, *Communications in Statistics-Theory and Methods*, **47**(8), 1978-1988 (2018).
[5] S. Larpkiattaworn . *A neural network approach for multi-attribute process control with comparison of two current techniques and guidelines for practical use*. M.A. thesis, University of Pittsburgh, USA, (2004).
[6] Z. Wu, H. Luo and X. Zhang , Optimal np control chart with curtailment, *European Journal of Operational Research*,**174** (3), 1723-1741 (2006).
[7] K. R. Skinner,G. C. Rungerv and D. C. Montgomery , Process monitoring for multiple count data using a deleted-Y statistic, *Quality Technology and Quantitative Management*, **3**(3), 247-261 (2006).
[8] K. Derya and H. Canan , Control charts for skewed distributions: Weibull, gamma, and lognormal, *Metodoloski zvezki*, **9**(2), 95 (2012).
[9] M. Aslam, and C. H. Jun, Attribute control charts for the Weibull distribution under truncated life tests, *Quality Engineering*,**27** (3), 283-288 (2015).
[10] G.Rao , A Control Chart for Time Truncated Life Tests Using Exponentiated Half Logistic Distribution, *Appl. Math. Inf. Sci*. **12**(1), 125-131(2018).
[11] N. Khan, M. Aslam, M. Khan and C. H. Jun , A Variable Control Chart under the Truncated Life Test for a Weibull Distribution, *Technologies*, **6**(2), 55 (2018).
[12] S. Nasiru and A. Luguterah , The new weibull-pareto distribution, *Pakistan Journal of Statistics and Operation Research*, **11**(1), 103-114 (2015).

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